Stern-Gerlach Forces and Spin Splitters

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Abstract. I comment on suggestions that Stern-Gerlach forces be used to separate ensembles of antiprotons in opposite spin states as a way of providing beams or ensembles of polarised antiprotons.

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INTRODUCTION

"Why, sometimes, I've thought of as many as six impossible things before breakfast"
The Queen in *"Through the Looking Glass"*, Lewis Carroll, born at Daresbury, 1832.

Although Stern-Gerlach (SG) [1] forces are extremely small in comparison with the Lorentz forces acting on charged particles in storage rings, it was suggested about twenty-two years ago at the workshop at Bodega Bay that SG forces could be used to separate ensembles of antiprotons with opposite spin states into sub-ensembles with non-zero polarisation [2]. In the meantime two basic approaches have been under consideration: the use of transverse SG forces (the TSG effect) and the use of longitudinal SG forces (the LSG effect). For the TSG effect the fields of special strong quadrupoles or skew quadrupoles cause the "spin-up" and "spin-down" (or "spin-left" and "spin-right") ensembles to oscillate coherently in antiphase with gradually increasing amplitudes so that they eventually separate and can be identified. For the LSG effect specially designed cavity resonators would provide oscillating magnetic fields in a TE mode and this would cause the "spin forward" and "spin-backward" sub-ensembles to separate. Such systems are now called "spin splitters".

In this article I comment on the schemes against the background of practical accelerator physics in which many additional effects must be considered which could overwhelm the influence of SG forces. My comments will be based on classical physics: if the schemes have no chance of working on the basis of classical physics, the inclusion of the uncertainties associated with quantum mechanics will not make the schemes more viable. Nevertheless, I must remind the reader that the original observation of the SG effect involved electrically neutral objects, namely silver atoms [1], and that since that time many people have questioned whether the combination of the Lorentz forces and the Uncertainty Principle would make the effect unobservable for electrically charged

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particles [3, 4, 5, 6]. On the other hand the so-called “continuous SG effect” has been used for measurements in magnetic traps of the magnetic moment of the free electron [7, 8, 9] and of electrons bound in atomic ions [10]. Moreover, spin splitting of electron systems by SG forces might be occuring in special materials [11, 12]. Some discussion of quantum effects in storage rings can be found in [17, 18].

I will not describe the schemes for the TSG and LSG effects in detail. Instead, I will direct the reader to the original literature and confine myself to rather general comments. Since there is a large number of very similar papers and reports on the use of the TSG and LSG effects, by the same group of authors, I have cited those that appear to be the most relevant for the narrative. I apologise to those whose papers are not mentioned.

In the following the word “spin” will refer to the expectation value of the spin operator in the rest frame of a particle. I assume that the reader is familiar with the basic physics of magnetic moments, spins and nucleon g-factors and the relations between them.

THE TRANSVERSE STERN-GERLACH EFFECT

Schemes employing the TSG effect are based on two key components [2, 13, 14, 15, 16, 17, 18]. The first is spin-orbit resonance whereby TSG kicks to the particle should be close to resonance with the orbital motion so that although the kicks are very small, their effect might add up coherently to produce significant shifts to the trajectories.

The second component is a set of quadrupoles or skew quadrupoles with gradients large enough to generate the required relatively large TSG forces. We assign the unit vectors $\hat{x}$, $\hat{y}$ and $\hat{s}$ to the radial, vertical and longitudinal directions respectively. Then the transverse fields in a quadrupole are $B_x = gy$ and $B_y = gx$ where $g = \partial B_y/\partial x$ in obvious notation. In a skew quadrupole the fields are $B_x = \tilde{g}x$ and $B_y = -\tilde{g}y$ where $\tilde{g} = \partial B_x/\partial x$ [19]. Moreover, after transforming naively to the laboratory frame the TSG force is $\vec{F}_{\mathrm{TSG}} = \vec{\nabla}(\vec{\mu}\cdot\vec{B})$ where $\vec{\mu}$ is the magnetic moment in the rest frame and $|\vec{\mu}| \approx 1.4\times 10^{-26}$ Joule/Tesla $\approx 8.8\times 10^{-14}$ MeV/Tesla for antiprotons. The TSG force then causes deflections:

$$\delta x' = \frac{L_q}{\beta c p}(\mu_x \tilde{g} + \mu_y g) \qquad \delta y' = \frac{L_q}{\beta c p}(\mu_x g - \mu_y \tilde{g})\,, \tag{1}$$

where L_q is the length of the magnet and p is the momentum. These forces are independent of the transverse position of the particle in the magnet but dependent on the orientation of the spin.

The primary example of such a scheme employs a Siberian Snake [20, 21]. This is a device which rotates a spin by 180 degrees around an axis in the machine plane independently of the energy of the particle. One could, for example, use a strong solenoid to rotate spins around the longitudinal axis. [2] Then, the vector $\hat{n}_0$, the 1-turn periodic solution to the T-BMT equation on the closed orbit, lies in the horizontal plane [22, 23]. Moreover, the closed-orbit spin tune ν_{spin} is 1/2. Thus a vertical spin returns to its

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[2] Of course, with a solenoid, the usually small energy spread in the beam would lead to a small spread around 180 degrees.

original direction only on every second turn. See the diagrams in [23, p.71]. Of course, only spins parallel to $\hat{n}_0$ and on the closed orbit show 1-turn periodic orientations. Spin-orbit resonance in this case implies that the fractional part of a transverse orbital tune [22] should be close to 1/2. (In a variation of this scheme, the orbital tune should be close an integer). See eq. (17) in [22]. The TSG force provides an inhomogeneous term in otherwise homogeneous equations of orbital motion [22] which adds a forced motion to the basic transverse motion. If the relevant transverse tune is 1/2, the amplitude of the forced solution increases essentially linearly with time. If the fractional part of the transverse tune is only close to 1/2, the amplitude of the forced solution varies at a rate which decreases with the distance from resonance.

A specific layout is shown in figure 1 [13, 16]. The snake consists of a solenoid and this is surrounded by pairs of strong skew quadrupoles which ensure that the horizontal and vertical transverse motions are uncoupled when viewed from a place in the arc. Then the transverse tunes can be identified with pure horizontal and vertical motion in the arcs. It happens that this decoupling is achieved for opposite polarities of the skew quadrupoles before and after the solenoid. Then a spin pointing vertically upward before the solenoid points downward after the solenoid and the TSG forces from the skew quadrupoles before and after the solenoid act vertically and reinforce each other.

One can always treat an unpolarised ensemble as if it contains equal numbers of spin-up and spin-down particles. The dynamics just explained would tend to separate the particles with spin-up from those with spin-down so that the two sub-ensembles would oscillate coherently in antiphase and eventually become separated by an amount comparable to the vertical beam size. Then, with a suitable detector and electronics to keep track of the vertical oscillations, it might be possible to associate events from scattering in an internal target or polarimeter with one or the other of the sub-ensembles. However, as is well known, particle beams are not stable when a transverse tune is close to 1/2. Thus extra, special electric-magnetic devices have been proposed [18] to shift the spin tune and enable spin-orbit resonance to be maintained at a vertical tune away from 1/2. It is estimated that at a momentum of a few hundred MeV/c and with superconducting skew quadrupoles providing gradients of the order of 100 Tesla/metre, the sub-ensembles would have separated at the rate of a few millimetres per hour in LEAR [13, 16]. Schemes which would generate transverse separation of the sub-ensembles are called “transverse spin splitters”.

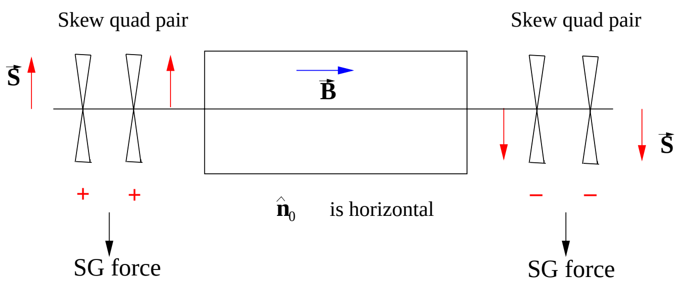

FIGURE 1. The spin orientations and the SG forces in a side view of a spin splitter based on a solenoid snake with a longitudinal magnetic field $\vec{B}$.

While such an estimate whets the appetite, it ignores some aspects of beam dynamics. It also ignores an important aspect of the spin-orbit coupling inherent in the T-BMT equation [22] as I now explain.

In SI units, the vector $\vec{\Omega}$ in the T-BMT equation takes the form

$$\vec{\Omega} = -\frac{e}{m}\left[\left(\frac{g}{2}-1+\frac{1}{\gamma}\right)\vec{B}_{\perp}+\frac{g}{2\gamma}\vec{B}_{//}-\frac{1}{c}\left(a+\frac{1}{1+\gamma}\right)(\vec{\beta}\times\vec{E})\right]. \tag{2}$$

where the vectors $\vec{B}_{\perp}$ and $\vec{B}_{//}$ are the magnetic fields perpendicular and parallel to the trajectories, $\vec{E}$ is the electric field and the other symbols have their usual meanings. The quantity $a = (g-2)/2$ is the gyromagnetic anomaly. For protons $a \approx 1.7928$. As explained in [24], the T-BMT equation can be deduced from the Hamiltonian [25]

$$h^{dk} = h^{dk}_{orb} + \frac{\hbar}{2}\vec{S}\cdot\vec{\Omega}\,, \tag{3}$$

where:

$$h^{dk}_{orb} = c\sqrt{(\vec{p}-e\vec{A})^2+m^2c^2}+e\Phi\,, \tag{4}$$

in the usual notation [26] and $\vec{S}$ is the expectation value for the spin operator in the rest frame. This Hamiltonian consists of a purely orbital part of zeroth order in $\hbar$ and a spin part of first order in $\hbar$. The equations of orbital motion emerge from Hamilton's equations in the usual way [27] and we see that the term $\frac{\hbar}{2}\vec{S}\cdot\vec{\Omega}$ is a Stern-Gerlach energy which gives rise to a TSG force in inhomogeneous fields. Note however, that whereas the TSG force in eq. (1) is proportional to $g/2$ where g is the nucleon g-factor, that obtained from eq. (3) is proportional to $g/2-1+1/\gamma$. The piece $-1+1/\gamma$ is due to Thomas precession and for spin-1/2 fermions it arises naturally from a Foldy-Wouthuysen reduction of the Dirac equation [25]. It is insignificant at very low energy but its presence in eq. (3) indicates that care is needed in deducing the TSG force at high energy. In fact, the exact form of the TSG force depends on which coordinates are used to describe the position of the particle. See [28] for a careful discussion of the differences between TSG forces based on canonical coordinates and on covariant coordinates. Since TSG forces are so small, great care is needed.

If the accelerating/bunching cavities are switched off, this Hamiltonian is explicitly time independent so that the total energy is conserved. Then, as pointed out in the semiclassical discussion in [29, 30, 31], an increase in the energy associated with orbital motion, due to an increase in the amplitude for vertical motion (see eq. (4) in [22]), would require the spin to reverse direction at some point in order to conserve energy. Then the orbit amplitude would decrease again. The possibility of spin reversal is already clear by analogy with coupled orbital motion where energy can be exchanged between orbital modes. In particular, spin splitters operate near spin-orbit resonance so that the spins are unstable and very sensitive to the details of the orbital motion. So vertical spins can be very strongly disturbed by the magnetic fields on vertical betatron trajectories in the quadrupole fields. Equivalently, the angle between the so-called invariant spin field and $\hat{n}_0$ will be large [32, 33, 34]. Then the motion of the spins will be erratic and it cannot be assumed that they will stay nicely lined up with the directions assumed in the basic concept. The potential for the tumbling of the spins is illustrated for the case when spins are almost parallel to $\hat{n}_0$ in [35]. There, the empty 6×2 matrix in $\hat{M}$ in eq. (13) in [22] is replaced with elements representing the influence of TSG forces on the orbital motion. As explained in [35] we have an immediate analogy with the exchange of energy between transverse orbital modes [22, eq. (4)] that can take place near to orbital resonance in a ring with skew quadrupoles. In a spin splitter, the exchange of energy would be between the spin motion and the orbital motion. The discussion in [29, 30, 31] does not address a specific layout but points out that the long term effect of the TSG forces is to produce a small orbital tune shift without significant separation of the sub-ensembles.

The reports [29, 30, 31] also suggest ingenious ways to overcome the tumbling of the spins, namely by applying an external radio frequency magnetic field in resonance with the spin motion. The reader should consult [29, 30, 31] for details. Note that this solution requires that the beams be bunched, so that "accelerating" cavities would be running. In the presence of the external radio frequency field and the cavities, the Hamiltonian becomes explicitly time dependent so that there is then no need for the total energy to be conserved. Of course, even if the Hamiltonian were time independent, imperfections in the optics of the ring, the inclusion of sextupoles and terms beyond quadratic in positions and momenta in the approximations to the square root in h_{orb}^{dk}, might cause energy exchange between horizontal and vertical orbital oscillations so that the propensity for spin flip might be weakened.

Other approaches suggested in [29, 30, 31] include using high frequency skew quadrupole fields and coherent excitation of betatron motion, using the fact that the coherent and incoherent motions have different tunes. The use of a beam whose vertical phase space density peaks on an annulus is also suggested.

Further to these matters, one should be aware that at low energy and except in beams of very low intensity, the TSG forces are orders of magnitude smaller than the forces on an antiproton resulting from the electric and magnetic fields from the rest of the beam. Thus, and just for orientation, in a beam with 10^{10} antiprotons per metre and with a radius of 1 millimetre, at low energy, the outward force on an antiproton at the surface of the beam is of the order of 10^{10} times larger than the TSG force in a skew quadrupole with a gradient of 100 Tesla/metre! However, one could at least expect that intrabeam forces would tend to push the sub-ensembles apart. Note also that such forces lead to a shift and spread in the betatron tune. The particles are also subject to wake fields and other disturbances which are much larger than TSG forces and whose effects are often so strong that powerful feedback systems are needed to stabilise the beam. The Lorentz force due to the magnetic field at a distance of 1 millimetre from the centre of a skew quadrupole is of the order of 10^{12} larger than the TSG force, and this ratio is independent of the gradient.

Thus, although the naive treatments of transverse orbital motion provided by proponents of transverse spin splitters suggest that an antiproton beam could be separated into two sub-ensembles with opposite orientations of their spins, it is far from clear whether that would be possible in practice. Clearly, there are many subtleties to be considered. These matters can only be clarified with the aid of very detailed spin-orbit tracking simulations going far beyond those discussed in [22]. In particular, in the absence of wake fields etc, and SG forces, the orbital motion should be strictly symplectic in order to ensure that when the SG forces are included, any increase in orbital amplitudes can be clearly attributed to the TSG force [36, 37, 38]. Then, when spin motion is included, all spin transformations must be described by orthogonal transport matrices. Finally, the effect of inter-particle forces and wake fields should be included in a fully consistent way so that the SG forces are not mimicked or masked by errors in the simulation. Since the very strong quadrupoles or skew quadrupoles would produce very high natural chromaticity, a carefully designed sextupole scheme would be needed. The rate of separation by the TSG effect is sensitive to the orbital tunes and takes place over a long period of time. So it is important that the tunes remain stable and that simulations reproduce shifts and spreads in orbital tunes. For hints about the effects of space-charge see [39].

Software for such precise simulations does not exist and even if the underlying mathematics of the algorithms could be formulated and implemented in a code in a sufficiently consistent way, which is doubtful, simulations for the large number of turns associated with the time scales involved, would require the use of extremely powerful parallel processors. Simulations are also desirable as a tool for optimising the design of a spin splitter and for diagnosing sources of trouble. Needless to say, no simulations of the required sophistication have ever been carried out and it is unlikely that such simulations will be available in the future. It would be very risky to invest in hardware before such simulations had been made.

The reader will note that in [22] some approximations can be made in the description of spin motion. That suffices to estimate the polarisation rate and to estimate the depolarisation rate due to orbital motion. SG forces are irrelevant in [22]. But in a spin splitter the SG forces depend crucially on the orientations of the spins. Then approximations cannot be permitted. However, the approximations of [22] are useful for estimating the depolarisation that might result from running at a spin-orbit resonance in the presence of noise injected into the particle trajectories by intrabeam scattering.

THE LONGITUDINAL STERN-GERLACH EFFECT

Equation (1) shows that the rate of separation of sub-ensembles due to the TSG effect decreases with energy so that even if the method could work it would not be useful at (say) hundreds of GeV. This has lead to proposals to use the longitudinal SG forces acting on longitudinal spins when a non-zero $G_{//} \equiv \partial B_{//}/ds \neq 0$ is produced using resonators running in a TE mode [40]. The change in energy due to the LSG force is proportional to the energy and the change in the relative energy is

$$\frac{\Delta E}{E} = \mu \frac{G_{//} L_C}{\beta^2 mc^2} \; . \tag{5}$$

Spin-orbit resonance is not needed. Then, for high energy antiprotons with a typical fractional energy spread of 10^{-4} and with $G_{//} \approx 75$ Tesla/metre with 10 metres of useful cavity length L_C, antiprotons with longitudinal spins would accumulate a fractional energy change equal to the fractional energy spread in about 8 hours in a ring like the HERA proton ring [33, 34] in the absence of synchrotron oscillations. The vector $\hat{n}_0$ would be vertical in the arcs of the ring and made longitudinal at the cavities using pairs of spin rotators as in figure 2. Such a system is called a “longitudinal spin

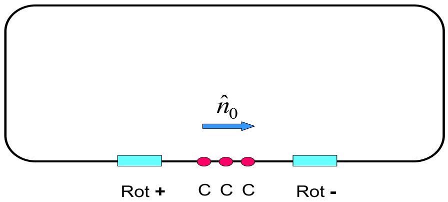

FIGURE 2. A schematic layout for the rotators (Rot±) and some TE cavities (C) for exploiting the longitudinal Stern-Gerlach effect.

splitter". However, since the particles execute synchrotron oscillations, the TE cavities would have to "be modulated with the same frequency of the phase oscillation" i.e., at the frequency of synchrotron motion [41, 42] to prevent the energy gains from being dissipated. But then as shown in [41, 42], the particle distribution would exhibit so-called filementation in the longitudinal phase space. Since it cannot be assumed that the synchrotron frequency is fixed, the TE cavities would need an appropriate tuning range. In any case, these calculations employ incomplete descriptions of the electromagnetic fields of the TE cavities.

However, one should not be so cavalier. In particular, it has been shown in [43, 44, 45] that the rate of gain in energy or longitudinal momentum can be expressed in terms of certain total differentials with respect to s. One then finds that for a particle passing from one field-free region to another, the net energy gain contains no terms beyond zeroth order in γ. The calculations in [43, 44] work with the spin-orbit Hamiltonian of eq. (3). This includes the effective modification of $g/2$ by Thomas precession. In [44] it is pointed out that one must consider two kinds of kinetic momenta: one which includes a spin term, and one that does not. The momentum including the spin term can exhibit a high rate of change, due to spin precession, so that care is needed if it is used for calculating the LSG force [46]. The calculation in [45] associates a "Thomas precession potential" with the Thomas precession.

The proponents of the use of the LSG effect now seem to agree that the energy gain is indeed insignificant when the effects of the edge fields of the cavities are included to ensure that Maxwell's equations are satisfied [47]. So, in order to still achieve a useful energy gain, the authors of [47] now propose a more complicated arrangement involving pairs of contiguous rectangular cavities running in the TE_{011} mode with overlapping, cancelling, edge fields. The arrangement is designed so that the oscillating vertical magnetic field B_y in [47] is non-zero with non-zero $\partial B_y/\partial s$ and so that B_y peaks between the two cavities. The longitudinal magnetic field vanishes on the axis. See figure 3 in [47]. Then vertical spins experience a LSG force proportional to $\partial B_y/\partial s$ and γ^2 [42]. However, the overall energy gain vanishes. To overcome this, each pair of cavities is embedded in a superconducting dipole magnet with a radial magnetic field $B_x\hat{x}$. The spins are longitudinal on entering the first cavity, as in the first scheme (figure 2), and the radial field rotates them by 360 degrees so that they are longitudinal at the exit of the second. It is then claimed that the overall energy gain is substantial and proportional to γ^2. It is also stated that terms due to Thomas precession are neglected. Apart from the obvious technical problems of embedding cavity resonators in superconducting dipoles and, at the same time, exciting them in the required way, this claim is incompatible with the basic results of [43, 44, 45]. Note that [43] and [44] predate [47] by a decade. In any case, an energy gain proportional to γ^2 implies a fractional energy gain proportional to the particle energy and, of course, at sufficiently high energy the fractional gain could exceed unity. This appears to be unphysical. However, an examination of [47] shows that more care is needed. In particular, as explained in [48], while a spin is precessing around $\hat{x}$ in the radial dipole field, it will also precess around the oscillating field $B_y\hat{y}$ and acquire a small radial component. Then, at the exit, there is an additional LSG force proportional to the non-zero $\partial B_x/\partial s$ at the end of the imposed radial field and to γ^2, and this eliminates the energy gain proportional to γ^2 of [47]. So, the apparent incompatibility between the predictions in [43, 44, 45] and in [47] is explained. In fact, in [48] it is shown that cancellation of the terms proportional to γ^2 would already occur if just the first cavity of the pair, and a spin rotation of just 180 degrees, were used. It is then explained how the cancellation occurs for the two cavities and a rotation of 360 degrees. See [28] and [49, 50, 51, 52] for more on Lagrangians and equations of spin motion.

In any case, beams of particles can excite high order extra modes in cavities. What effect would higher order modes in the cavity fields have on the spins? Bunched beams can also exhibit longitudinal instabilities which require longitudinal feedback systems for their control. It would be interesting to know whether the impedance from the special cavities would enhance such effects and to know how precisely the beam would have to be aligned to prevent the transverse electric fields from causing unwanted motion. Hands-on experience with real high energy beams suggests that it is unrealistic to expect the beams to be sufficiently stable over long periods of time.

Of course, even if a configuration can be found which suggests that LSG forces can be exploited, nothing can be decided without sophisticated simulations of spin-orbit motion of the kind described above for the TSG effect. For an example of extensive simulations of high energy proton motion which includes these kinds of effects see [53]. For an impression of the effects of non-linear orbit motion see [54]. These simulations should include the fact that at high energy, $\hat{n}_0$ does not represent the natural spin quantisation axis. Instead one should think in terms of the invariant spin field $\hat{n}$ [32, 33, 34]. This specifies the direction of the equilibrium polarisation at each point in phase space. In particular, unless special measures are taken, e.g., with the help of Siberian Snakes, at very high energy and with closed orbit distortion, the spread of the equilibrium polarisation directions across a bunch can be many tens of degrees. In fact, the invariant spin field could even be isotropic at the LHC [33]!

CONCLUSIONS

If, in the future, members of the particle physics community, who are usually not experts on beam dynamics or the subtleties of the relativistic Stern-Gerlach effect, are again confronted by claims that a scheme has been found whereby Stern-Gerlach forces can be used to obtain polarised antiprotons, they should challenge the proponents to:

- show that they are familiar with the relevant literature beyond that of their immediate circle and to compare their results with those of the other literature and, in particular, explain why they make no mention of the arguments in [43, 44, 45] when discussing the LSG effect,
- show that they have a full understanding of the basic equations of particle and spin motion, their meaning and their limitations,
- show that they have an understanding of all of the processes in storage rings that influence particle motion and stability,
- produce results of numerical simulations which are based on fully symplectic/orthogonal simulations i.e., with full spin-orbit coupling (in both directions) and which include such things as direct and mirror space charge forces, Landau damping, instabilities and feedback etc,etc,etc.
- show by simulations or otherwise, that if the simulations mentioned above suggest that partial separation of the sub-ensembles is possible, the polarisation that can be ascribed to scattering events observed in a detector, can be known with sufficient precision to enable a publishable analysis of the data to be carried out.

Of course, even if such detailed simulations for the proposed scheme were to show that the SG effect could be effectively utilised, this would provide no definite proof that the scheme would work in practice because it is impossible, even with numerical simulations to take all effects into account. Then an experimental test would be needed. However, one can be sure that if a simulation were to predict that a scheme would not work, it would not work in practice.

In any case, until these matters have been addressed in a serious manner, efforts to obtain polarised antiprotons should be directed elsewhere. The proponents of the use of Stern-Gerlach forces are to be admired for their ingenuity and creativity but if they believe that polarised antiprotons can be obtained by utilising Stern-Gerlach forces, they should at least prove it at the level of classical simulations which are as complete as computer technology and our understanding of accelerator physics allow. If, after so many years, they cannot prove it at this level, and soon, it will be time to allow these schemes to slip gracefully from the realm of the “undead” [55] into oblivion.

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