Communications in Astrophysics, Vol. 18, p. 69

A New Theory to Unify Synchrotron Radiation and Inverse Compton Scattering

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A new approach to synchrotron radiation theory was published recently in the Astrophysical Journal by R. Lieu and W. I. Axford. The method is elegant in that it uses the concept of Inverse Compton scattering to explain with unprecedented simplicity all the classical and quantum electrodynamic properties of synchrotron radiation, thereby unifying two fundamental processes in physics and astrophysics. Their work provides new insights into the meaning of Schwinger's "first order quantum correction" of the synchrotron emissivity and the origin of spin-flip radiation asymmetry. We illustrate possible applications for radiation characteristics in non-uniform magnetic fields.

Key Words: *radiation mechanism, synchrotron radiation, Inverse Compton scatter*

INTRODUCTION

Synchrotron radiation and Inverse Compton scattering are two fundamental processes in physics and astrophysics. Specific mechanisms which couple them together offer powerful insights for interpreting astrophysical data. An example is the "Synchrotron-Self-Compton" process. Within the classical limit ($\hbar \equiv 0$) three striking similarities between them have long been noticed:

(a) At non-relativistic speeds both cyclotron radiation and Thomson scattering have a dipolar angular distribution.

Comments Astrophys.
1995, Vol. 18, No. 2, pp. 69–79
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(b) At relativistic speeds synchrotron radiation and Inverse Compton scattering produce broadband spectra with a characteristic frequency $\sim \gamma^2\omega$. For the former ω is the cyclotron frequency, and for the latter it is the frequency of the incoming electromagnetic radiation.

(c) At all speeds υ the energy loss rate of a randomly moving charge equals $4\sigma_{Th}\gamma^2\upsilon^2 U/3c$, where σ_{Th} is the Thomson cross section; for synchrotron radiation U is the energy density of the (uniform) magnetic field, and for Inverse Compton scattering U is the energy density of the "gas" of randomly propagating photons.

These apparent similarities remind us of relativistic Coulomb bremsstrahlung, in which an electron of speed $\upsilon \rightarrow c$ passing close to an atomic nucleus emits radiation. The problem has been treated quite satisfactorily as Inverse Compton scattering of equivalent photons (Fermi 1924, von Weizsäcker 1934, Williams 1935; hereafter FWW). In particular, the spectral cutoff at the particle energy $\hbar\omega = \gamma mc^2$ can be explained as a purely kinematic effect, viz., that of Compton recoil. Despite the simplicity of the FWW procedure it was not obvious how it could be applied to unify synchrotron radiation and the Compton effect. In a synchrotron the charge undergoes continuous acceleration, so that its own reference frame is not inertial. Most of us, like Baylis, Schmid and Lüscher (1967), had to settle for a vague feeling that we might have "understood" the connection. Even if you take a simpler approach by looking for a wave that mimics the static field (see, e.g., the model given on p. 419 of Harwit 1973) you cannot get much further.

The situation was dramatically changed by Richard Lieu and Ian Axford (1993). The authors have successfully treated synchrotron radiation as an Inverse Compton effect by deriving all the standard results of classical radiation using the Thomson cross section. The technique used is a generalized version of FWW which coherently sums the Inverse Compton scattered amplitudes from infinitesimal segments of the particle trajectory. The most remarkable aspect of Lieu and Axford's work lies in its ability to go beyond the classical limit, to obtain full quantum electrodynamic (QED) corrections of the radiation properties in the limit of high γB, by applying the appropriate Compton cross section and taking into account particle recoil. The results can be compared with those of QED derivations for the specific case of a circular orbit in a uniform field. For spinless particles the agreement is exact, and for spin-1/2 particles the agreement is highly accurate. Since the Compton cross section which they used for either particle species can also be calculated with a classical electromagnetic field, this leads one to conclude that genuine QED effects (i.e., second quantization) do not play an important role in synchrotron radiation. The quantum corrections are due to (i) a kinematic effect associated with recoil (as alluded to by Schwinger 1954 and by Baier and Katkov 1968), and (ii) an additional spin-related term in the Compton cross section from scattering off the electron magnetic moment. The recoil correction ensures that the emitted spectrum cuts off at the kinetic energy of the electron. There is thus a resemblance to the FWW approach to bremsstrahlung, except that the treatment here is even more neat, as the spectral cut-off is secured without the need for a "minimum impact parameter."

A GENERALIZED WEIZSÄCKER–WILLIAMS METHOD FOR CURVED TRAJECTORIES

A key concept in this paper concerns the nature of the equivalent photons appropriate for a Compton scattering model of synchrotron radiation. Since the classical synchrotron loss rate can also be expressed as the product of the Thomson cross section and the Poynting energy flux as seen in the instantaneous rest frame of the moving charge, this strongly suggests that the equivalent photons propagate in the direction of the Poynting flux, which is anti-parallel to the velocity **v** of the charge. In an instantaneous rest frame the charge experiences transverse (**E**, **B**) fields which are equal in magnitude to an accuracy of $1/\gamma^2$. The fields resemble that of an electromagnetic wave, except that they are static. The necessary dynamics would become apparent once there is a way of linking the different instantaneous frames along the orbit, without necessarily appealing to a non-inertial system.

Before proceeding to describe how Lieu and Axford employed the technique of coherent addition to overcome this difficulty, it is instructive to consider the usual procedure of deriving the radiation field of an accelerated charge. There are two standard results: the first is classical cyclotron and synchrotron radiation, where the acceleration is due to motion in a uniform magnetic field, and the second is Thomson scattering, where the acceleration is due to interaction with a monochromatic plane wave. (It is noteworthy that this is the order in which the results are presented in Jackson 1975). The radiation is, in general, associated with acceleration of the particle perpendicular to its direction of motion. The emissivity in the far field limit is determined by a Fourier time integral, which is effectively an integration along the entire particle trajectory. In the case of Thomson scattering the phase of the scattered wave is completely related to that of the incident wave. Thus the cross section, given by the ratio of the energy fluxes, carries a polarization dot product which "locks" the phases of the two waves. Obviously the same statement applies to Compton scattering. In this sense the term "scattering," implying a random process, is a misnomer.

Given that Thomson and Compton scattering are inherently phase-coherent processes, it is possible to connect them with synchrotron radiation by the following procedure. Consider first the radiation emitted when a charge travels transversely across an elementary plane slab of magnetic field. In the frame of the charge the interaction is assumed to be Compton scattering with an equivalent electromagnetic wave in the form of a pulse, the duration of which is determined by the (infinitesimal) laboratory width of the slab. Figure 1 illustrates the interaction as viewed in the different frames. The scattered pulse has a well-defined frequency, angular and phase distribution, and the trajectory of the electron is slightly deflected by a known amount. The properties of the emitted radiation when the particle is made to interact with a continuous sequence of such slabs along its trajectory is obtained by adding the elementary contributions constructively *after* they have been transformed to the laboratory frame (since there is not a unique rest frame). *Thus, within the context of this approach, the Fourier time integral has a completely different meaning, viz., that it represents a coherent addition of infinitesimal Inverse Compton radiation amplitudes from all parts of the particle trajectory.* This new way of thinking opens additional avenues for generalizing synchrotron radiation theory to include the important regime of non-uniform fields and (consequently) non-circular orbits. *Since each elementary slab is treated independently, there is no reason why the field should be the same from slab to slab provided that the variation*

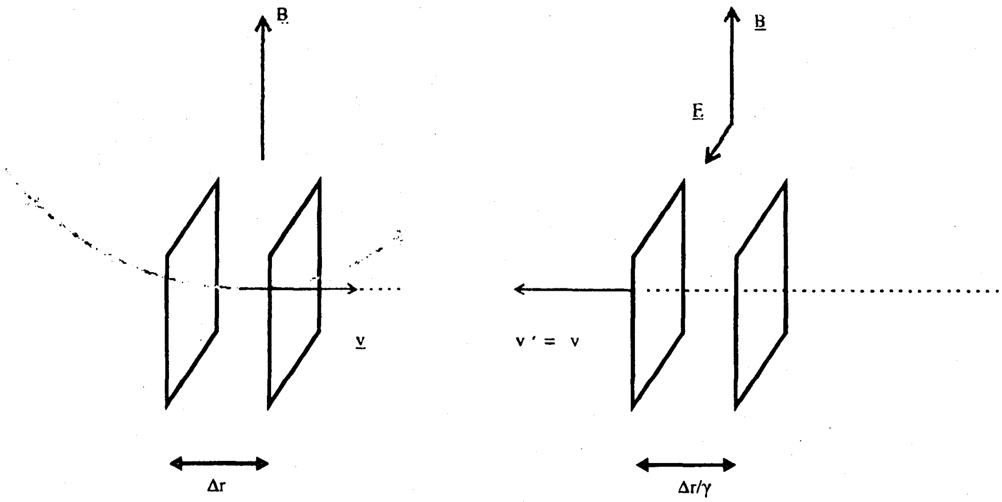

FIGURE 1 A schematic illustration of how synchrotron radiation from an elementary segment of the charge particle orbit may be considered as Inverse Compton scattering of equivalent photons. The segment is bounded by two planes perpendicular to the instantaneous velocity **v** of the charge. Such an imaginary system has a magnetic field **B** inside, which when viewed in the particle frame appears like a "slab" of (**E**$'$, B$'$) crossed field of equal magnitude (since $v \to c$), rushing past the particle like an electromagnetic pulse undergoing Compton interaction with it. The resultant scattered radiation amplitude is determined by E$'$, B$'$ and the slab velocity **v**$'$, which in turn are determined by the laboratory field and orbit. This elementary amplitude can be transformed to the laboratory frame, and coherently added with amplitudes from other segments of the orbit to produce the total synchrotron radiation which we observe.

is continuous. Thus we can describe everything about synchrotron radiation as coherent Inverse Compton scattering.

CLASSICAL AND QUANTUM SYNCHROTRON RADIATION AS AN INVERSE COMPTON EFFECT

For the problem of classical radiation from a charge moving at arbitrary velocity and acceleration, Lieu and Axford followed the above prescription using the Thomson cross section, to derive exact formulae for the differential emission rate, viz., equation (14.67) of Jackson (1975). This means even the *cyclotron* limit of $\gamma \sim 1$ is also properly accounted for. It is clear that in such a limit only the rest frame electric field matters, because the large difference between B' and E' has, strictly speaking, destroyed the symmetry necessary for an equivalent photon method. For the specific case of synchrotron radiation (i.e., $\gamma >> 1$) the result reduces to the standard expression for $d^2I/d\omega d\Omega$, the energy per unit solid angle and per unit radian frequency during one sweep of the synchrotron beam, as given in equation (14.83) of Jackson (1975). Recently, the authors have also produced a more elegant and compact version of the entire classical derivation (Lieu and Axford 1994). In this version the incoming equivalent photons have only one polarization mode, which is along the orbital plane and perpendicular to their propagation direction $-\mathbf{v}/\upsilon$, where (as before) υ is the velocity of the moving charge.

In this way the framework for unifying synchrotron radiation and Compton scattering is established within the classical domain. Now the interesting question is whether the technique can also calculate synchrotron radiation properties in that quantum limit where the particle orbit remains classical but the photon energy at the peak of the synchrotron spectrum approaches the electron energy: $\hbar\omega \to \gamma mc^2$ (and in this sense the assumption of $\hbar \equiv 0$ is no longer valid). Defining a parameter $B_c = m^2c^3/e\hbar \simeq 4.4 \times 10^{13}$ G (the Schwinger critical field), this limit is reached whenever

$$\gamma >> 1 \quad \text{and} \quad \gamma B \geq B_c \tag{1}$$

and occupies a substantial fraction of synchrotron parameter space. Before Lieu and Axford it was treated exactly by QED only for the case of circular orbits (see, e.g., Sokolov and Ternov 1986).

To handle this regime Lieu and Axford again enlisted FWW but introduced Compton corrections: recoil is important and a more elaborate cross section must be used. Provided $\gamma >> 1$ a Compton scattered photon of laboratory frequency ω is related to an incident photon of frequency ω/η, where η is the Compton redshift ratio which to an accuracy of $1/\gamma^2$ is given by

$$\eta = 1 - \hbar\omega/\gamma mc^2. \tag{2}$$

Two remarkable features are noteworthy about η. First, it does not depend on any angle because the incident equivalent photon always propagates in the $-\mathbf{v}/v$ direction. For relativistic particles this correlation makes the redshift completely insensitive to the propagation direction of the scattered photon. Second, when the scattered photon has energy $\hbar\omega \rightarrow \gamma mc^2$ it results from an incident photon of infinite energy, implying that a cutoff must exist at γmc^2. Thus energy conservation in the quantum limit is enforced naturally by energy-momentum balance during recoil, and *not* by some illusive QED effect.

Once the redshift is taken care of, the next modification is a transition from Thomson to Compton cross section. Note that for the latter a distinction must be made between spinless and spin-1/2 particles. The coherent addition along the orbit then applies, and Lieu and Axford arrived at formulae for synchrotron radiation in the QED limit, valid for arbitrary planar orbits in a non-uniform magnetic field (full 3-D orbits will be discussed later) and with significantly fewer algebraic manipulations. A particularly fascinating point is that, for $\hbar\omega/\gamma mc^2 << 1$, the differential *number* of emitted photons is simply given by its classical formula with the replacement $\omega \rightarrow \omega/\eta \approx \omega(1 + \hbar\omega/\gamma mc^2)$. This well-known "first order quantum correction" (Schwinger 1954) now has a new meaning. Rather than talking intuitively about "recoil", the correction factor is identified with electron recoil accompanying the scattering of equivalent photons. Thus the similarity between synchrotron and Inverse Compton processes is now appreciated at a level considerably deeper than the three superficial reasons given at the beginning of this article. For comparison of the full formulae with QED, however, it is necessary to adopt the restricted case of a circular orbit, since QED has no exact results elsewhere.

We first discuss the spinless case. Here there is complete agreement with QED for a circular orbit. Such a result has implications on the long debated issue of radiation reaction in the limit given by (1). It has often been argued that in this limit, although the orbit in the absence of radiation is classical, the substantial change of the particle energy during photon emission prevents us from using an unperturbed orbit for calculations. The agreement with QED refutes the argument once and for all, since Lieu and Axford used an unperturbed classical orbit. There is a simple explanation as well. Although a photon, once emitted, changes the particle orbit drastically, the *probability* of emission within one radiation length scale (i.e., synchrotron arc) is $<< 1$, unless γB exceeds $B_c = 4.4 \times 10^{13}$ G by several orders of magnitude. This means the average fractional energy loss within such a distance remains small, so in fact the orbit is not affected significantly by radiation.

For spin-1/2 particles, the agreement with QED is not exact, but the two theories are indistinguishable until $\gamma B/B_c \sim 10$. Even in this highly non-classical domain they exhibit a slight difference only in the spectral peak region. However, as announced by Richard Lieu at the SPIN94 conference held at Indiana University in September 1994, he has recently identified the source of this discrepancy, which has been a small blemish preventing complete unification. This achievement is not just of academic interest, but could directly lead to a better appreciation of the Sokolov–Ternov effect.

A SIMPLE DERIVATION OF SPIN-FLIP RADIATION PROPERTIES

It was predicted by Sokolov and Ternov (1964) that electrons circulating in the vertical guide field of a storage ring should become spin polarized antiparallel to the field as a result of spin-flip synchrotron radiation. This occurs because there is a difference in the rates for flip from spin down to spin up and vice versa. The simplest version of the theory predicts an equilibrium polarization of 92.4%. This polarization effect was observed experimentally for the first time in 1970 (see Montague 1984 for a review of early experimental work) and is now observed routinely even at the highest energy electron storage rings HERA, TRISTAN and LEP (Barber 1993), although various depolarizing effects, which can increase strongly with energy, prevent the maximum value of 92.4% from being reached. At HERA it has recently been possible even to rotate the polarization into the beam direction, so as to make longitudinally polarized electrons available for high-energy physics experiments (Barber 1994).

When we refer to the "spin" of an electron, we mean the spin angular momentum in the rest frame of the particle. Thus the discussion of spin motion in terms of laboratory variables inevitably leads to a consideration of the Thomas precession of spins in electric and magnetic fields, and indeed these terms are present in the commonly used covariant formulation of spin motion (Bargmann, Michel and Telegdi 1959). Thomas precession terms appear explicitly in the effective semiclassical Hamiltonian describing non-radiative spin motion (Jackson 1976).

They also appear in the radiative part of the Hamiltonian used by Jackson to calculate the spin-flip rates. One counter-intuitive effect of these terms is that the Sokolov–Ternov polarization does not vanish when the g factor of the electron is zero as might be expected, but at a g factor of 1.198 instead.

So clearly, it would be instructive to do the entire spin-1/2 calculation consistently in the rest frame of the electron. As we have seen this is precisely where the Lieu–Axford formalism begins. And in fact it was the attempt to explain the spin-flip radiation asymmetry which led to an identification of the origin of the discrepancy in the Lieu and Axford (1993) calculation. The point is that spin-flip effects cannot be handled by applying the spin-averaged Compton cross section (Klein and Nishina 1929) from the start. Instead, a correct procedure will involve using the spin-dependent Compton cross section, which is simpler than the Jackson (1976) approach because it contains only a magnetic dipole (i.e., $\mathbf{s} \cdot \mathbf{B}$) interaction term. This is followed by coherent integration of the amplitudes along the orbit. By taking these effects into account one can then arrive at a general emissivity formula valid for spin-1/2 particles, which reduces to the QED result for the case of circular orbits. By pursuing this philosophy it will be possible to reproduce the Sokolov–Ternov result for the equilibrium polarization. That would allow the spin-flip asymmetry to be viewed from a completely new and hopefully more transparent point of view. Lieu and Axford will submit these calculations for publication in the very near future.

We also mention that by applying successive (continuous) coordinate rotations to the scattered amplitudes from the trajectory segments before coherent integration, it is possible to generalize the emissivity formula further to include the case of 3-D orbits in a non-uniform field (Lieu and Axford 1994). The reason is that a relativistic particle in any magnetic field configuration experiences in its own frame equivalent photons propagating in the direction of $-\mathbf{v}$, *irrespective* of the laboratory angle between $\mathbf{v}$ and $\mathbf{B}$. Thus the large amounts of unnecessary complexities which tend to confine QED to handling only circular orbits is altogether by passed.

APPLICATIONS OF THE NEW THEORY

Turning to the practical aspects of the new approach, a potentially important application to synchrotron radiation calculations has to do with the so-called "beamstrahlung" in high energy e^+e^- linear colliders. During the collision of the tightly focused electron and positron beams, the particles in one beam experience the very strong collective fields of the oncoming beam and they emit high energy photons as their trajectories are bent. Such energetic particles experience collective field effects reaching the limit given by equation (1), or beyond. Under these conditions, the beamstrahlung can in principle remove a large fraction of the initial energy of the beam, and poses a major problem for linear collider designs. Thus a thorough understanding of the nature of this radiation is vital. In particular, since the field gradient can be large even within a synchrotron arc length, the standard QED formula for circular orbits is not useful, and the general formula in Lieu and Axford (1993) provides a new framework for thinking about the problem. We note that the same result is also of interest to astrophysicists, since the limit (1) can be reached in radio pulsar magnetospheres (Cheng, Ho and Ruderman 1986) and possibly in jets of active galactic nuclei near a black hole (Lovelace, MacAuslon and Burns 1979).

CONCLUSIONS

In conclusion the belief in the equivalence of synchrotron radiation and inverse-Compton scattering long held by astrophysicists is now shown to be valid. The approach of Weizsäcker (1934) and Williams (1935) was generalized and applied successfully by Lieu and Axford to this problem. The standard method of calculating the radiation emissivity of a moving charge, viz., an integral of the orbital path, can now be associated with a radically different interpretation—this integral represents the coherent addition of Inverse Compton amplitudes. The result is an alternative way of understanding the nature of synchrotron radiation which is elegant and insightful. A glimpse of the potential of the new theory is afforded by the immediate availability of novel radiation formulae for arbitrary trajectories in the QED limit, which are relevant to laboratory beam physics (including accelerator design) and to high energy astrophysics. The new framework also enables us to consider other QED processes in magnetic fields, such as pair creation and nuclei disintegrations, in much simpler ways.

Acknowledgment

We are grateful to Dr. Pisin Chen at SLAC for his critical reading of this manuscript.

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References

Barber, D.P., 1993, *Proceedings of the 10th International Symposium on High Energy Spin Physics, Nagoya, Japan 1993,* **83**. Universal Academic Press. Barber, D.P. *et al.*, 1995, *Phys. Lett. B,* **343**, 436.
Barber, D.P., 1994, *Proceedings of the 11th International Symposium on High Energy Spin Physics, Bloomington, USA, September 1994,* AIP Conference Proceedings to be published.
Baier, V.N. & Katkov, V.M., 1968, *Sov. Phys. JETP,* **26**, 854.
Bargmann, V., Michel, L. and Telegdi, V.L., 1959, *Phys. Rev. Lett.,* **2**, 435.
Baylis, W.E., Schmid, W.M. and Lüscher, E., 1967, *Z. Astrophys.* **66**, 271.
Cheng, K.S., Ho, C. and Ruderman, M.A., 1986, *Ap. J.,* **300**, 500.
Fermi, E., 1924, *Z. Physik* **29**, 315.
Harwit, M., 1973, *Astrophysical Concepts,* Wiley.
Jackson, J.D., 1975, *Classical Electrodynamics,* Wiley, New York.
Jackson, J.D., 1976, *Rev. Mod. Phys.* **48 (3)**, 417.
Klein, O. and Nishina, Y., 1929, *Z. Physik* **52**, 853.
Lieu, R. and Axford, W.I., 1993, *Ap. J.,* **416**, 700.
Lieu, R. and Axford, W.I., 1995, *Ap. J.,* to appear in the July 1 issue.
Lovelace, R.V.E., MacAuslon, J. and Burns, M.C., 1979, *AIP Conf. Proc.,* **56**, 399.
Montague, B.W., 1984, *Physics Reports* **113 (1)**.
Schwinger, J.S., 1954, *Proc. Natl. Acad. Sci. USA,* **40**, 132.
Sokolov, A.A. and Ternov, I.M., 1964, *Sov. Phys. Doklady,* **8**, 1203.
Sokolov, A.A. and Ternov, I.M., 1986, *Radiation from Relativistic Electrons,* A.I.P., New York.
Weizsäcker, C.F., 1934, *Z. Physik* **88**, 612.
Williams, E.J., 1935, *Kgl. Danske Videnskab. Skelskab, Mat-Fyz. Medd* **13**, No. 4.