Polarisation preservation in electron(positron) storage rings

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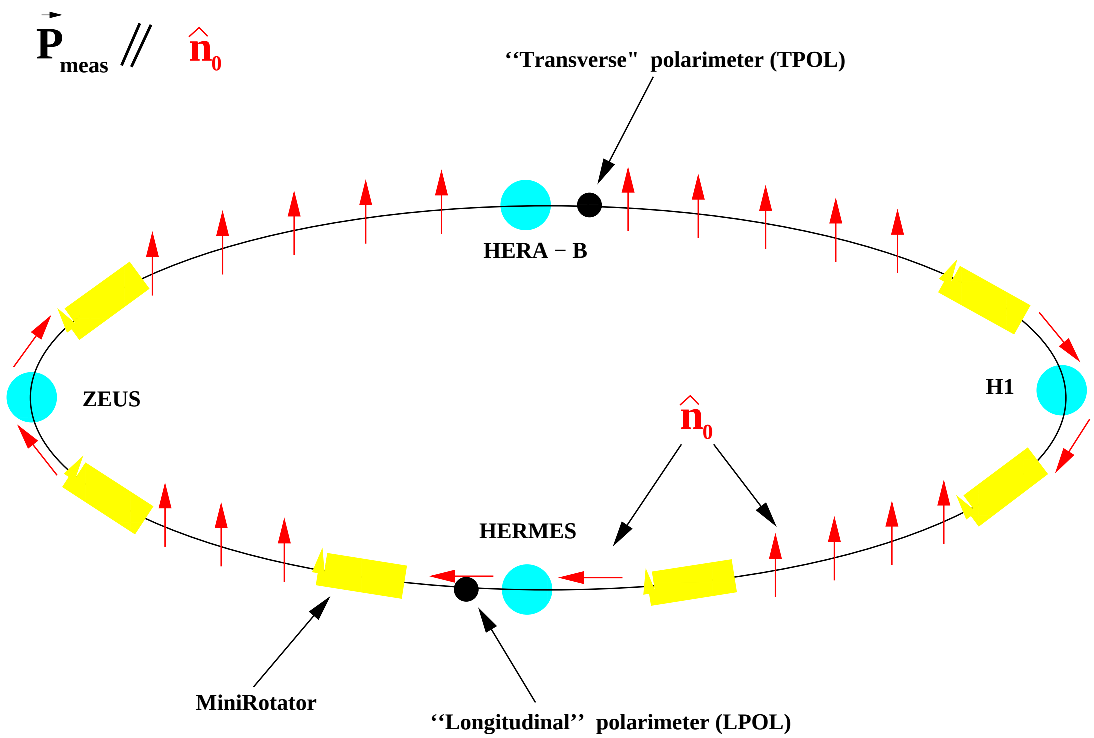
HERA electron/positron ring 2001 ––

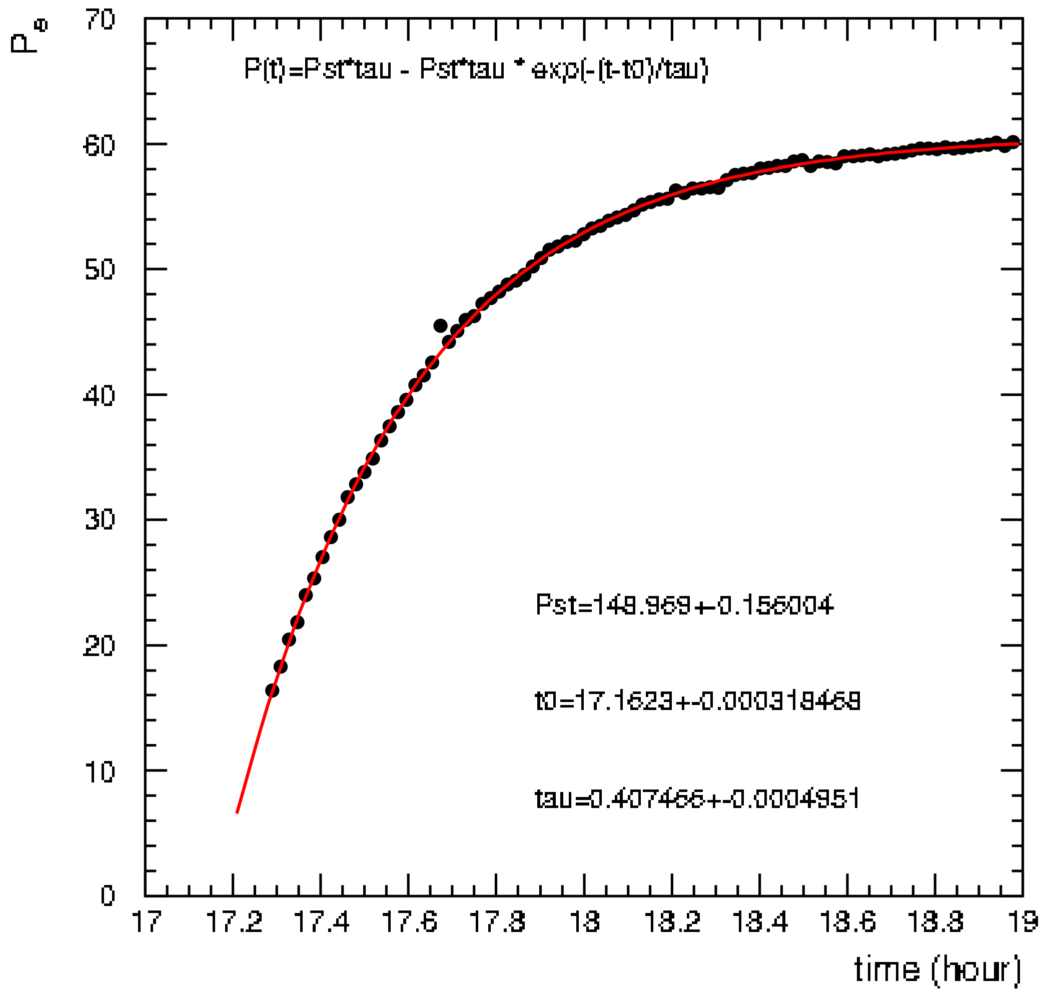


Polarisation vertical in the arcs – to drive the **Sokolov-Ternov** effect

The first and only $e^{\pm}$ ring to supply longitudinal polarisation at high energy

— via the **Sokolov-Ternov** effect – also at 3 IP's simultaneously!

June 2007, the Fabry-Perot-Compton polarimeter of the POL2000 Project: Calibrating polarimeters



3 pairs of rotators (so max. **Sokolov-Ternov** polarisation = 83 %), solenoids on, no beam-beam

The T-BMT equation

$$\frac{d\vec{S}}{ds} = \vec{\Omega}(\gamma, \vec{v}, \vec{B}, \vec{E}) \times \vec{S}$$

$\vec{S}$: unit length single-particle spin expectation value (for fermions).

s: distance around the ring.

Periodic solution $\hat{n}_0$ on closed orbit.

$\hat{n}_0$: **direction** of measured equilibrium radiative polarization.

Closed orbit **spin tune** ν_0: number of precessions per turn around $\hat{n}_0$ for a spin on the closed orbit.

In a flat simple ring, $\nu_0 = a\gamma$.

$a = (g-2)/2$ where g is the electron g factor.

At 3.3 GeV $a\gamma \approx 7.5$

The **value** of the polarization is the same at all azimuths — time scales.

The Baier-Katkov-Strakhovenko equilibrium radiative polarisation

$$P_{ST} \to P_{\rm bks}$$

$$\vec{P}_{\rm bks} \;=\; -\frac{8}{5\sqrt{3}}\,\hat{n}_0\,\frac{\oint ds \frac{\hat{n}_0(s)\cdot\hat{b}(s)}{|\rho(s)|^3}}{\oint ds \frac{\left[1-\frac{2}{9}(\hat{n}_0(s)\cdot\hat{s})^2\right]}{|\rho(s)|^3}}$$

=====> Need $\hat{n}_0$ vertical in the arcs to drive the **Sokolov-Ternov** effect.

But need longitudinal polarisation at the IP's !!

=====> **spin rotators!**

Spin motions: protons vs. electrons

- Protons: largely deterministic — unless IBS etc.

- Electrons/positrons:
  If a photon causes a spin flip, what are the other $\approx 10^{10}$ photons doing? ===>

  – Stochastic/damped orbital motion due to synchrotron radiation
  – with inhomogeneous fields
  – and spin–orbit coupling via T–BMT

  ===> spin diffusion i.e. **depolarisation!!!**

Self polarisation: Balance of poln. and depoln. ===>

$$P_{\infty} \approx P_{\rm bks} \, \frac{1}{1 \; + \; (\frac{\tau_{\rm dep}}{\tau_{\rm bks}})^{-1}}$$

In any case:

$\tau_{\rm dep}^{-1} \; \propto \; \gamma^{2N} \; \tau_{\rm bks}^{-1}$ (actually a polynomial in γ^{2N})

===> Trouble at high energy!

Spin–orbit resonances

$$\nu_{\rm spin}(\approx \nu_0) \; = \; k + k_{_I}\nu_{_I} + k_{_{II}}\nu_{_{II}} + k_{_{III}}\nu_{_{III}}$$

$\nu_{\rm spin}$: amplitude dependent spin tune $\approx$ closed orbit spin tune = precessions /turn on CO

- Orbit "drives spins" ===> Resonant enhancement of spin diffusion.
- Resonance order: $|k_{_I}| + |k_{_{II}}| + |k_{_{III}}|$
- First order: $|k_{_I}| + |k_{_{II}}| + |k_{_{III}}| = 1$ e.g. SLIM like formalisms.
- Strongest beyond first order:
  synchrotron sidebands of first order parent betatron or synchrotron resonances

$$\nu_{\rm spin} \; = \; k + k_i\nu_i + k_{_{III}}\nu_{_{III}}, \quad i = I, II \text{ or } III \;\; (e.g. \quad x,\; y,\; s)$$

Rotators: need the T-BMT equation.

$$\frac{d\hat{n}_0}{ds} = \vec{\Omega} \times \hat{n}_0$$

- Dipole rotators: in transverse fields:

$$\delta\theta_{spin} = a\gamma \cdot \delta\theta_{orbit}$$

$a = (g-2)/2$ where g is the electron g factor.

At 27.5 GeV $a\gamma = 62.5$

===>Suitable for high energy

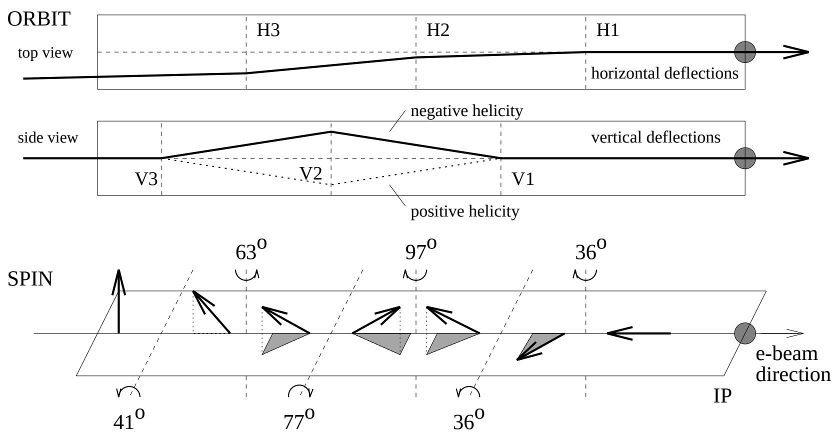
- Solenoid rotators:

$$\delta\theta_{spin} \propto \frac{B_{\mathrm{long}}}{E}$$

===>Only suitable for low energy.

. Also nontrivial spin–orbit coupling.

HERA MiniRotator: Buon + Steffen

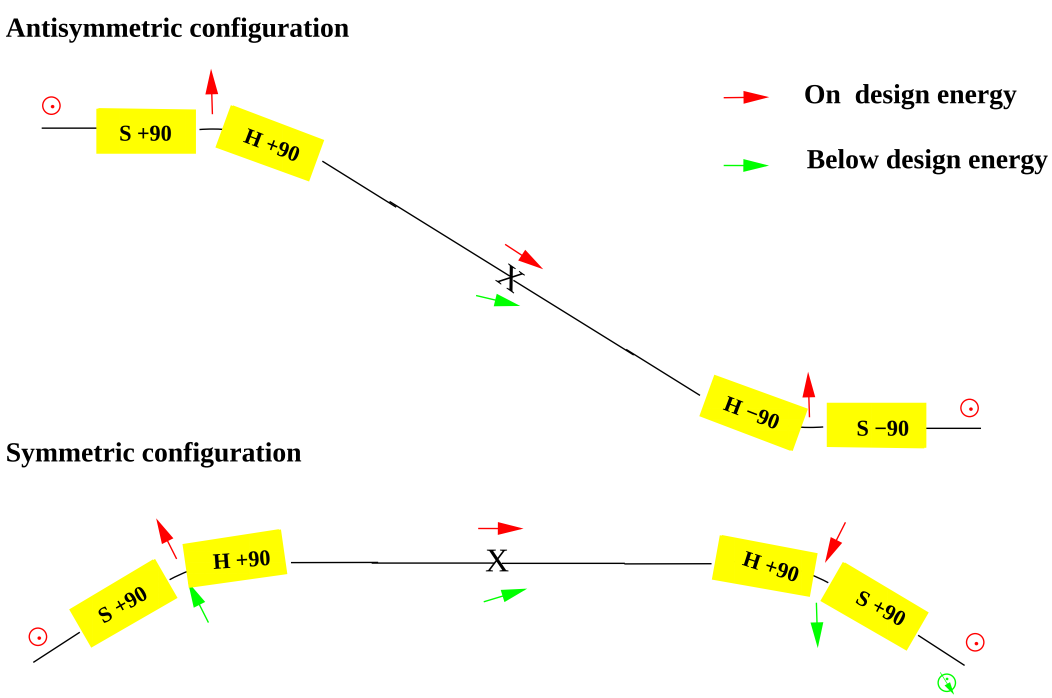


56 m ("short") → no quads.

27 – 39 GeV, both helicities, variable geometry

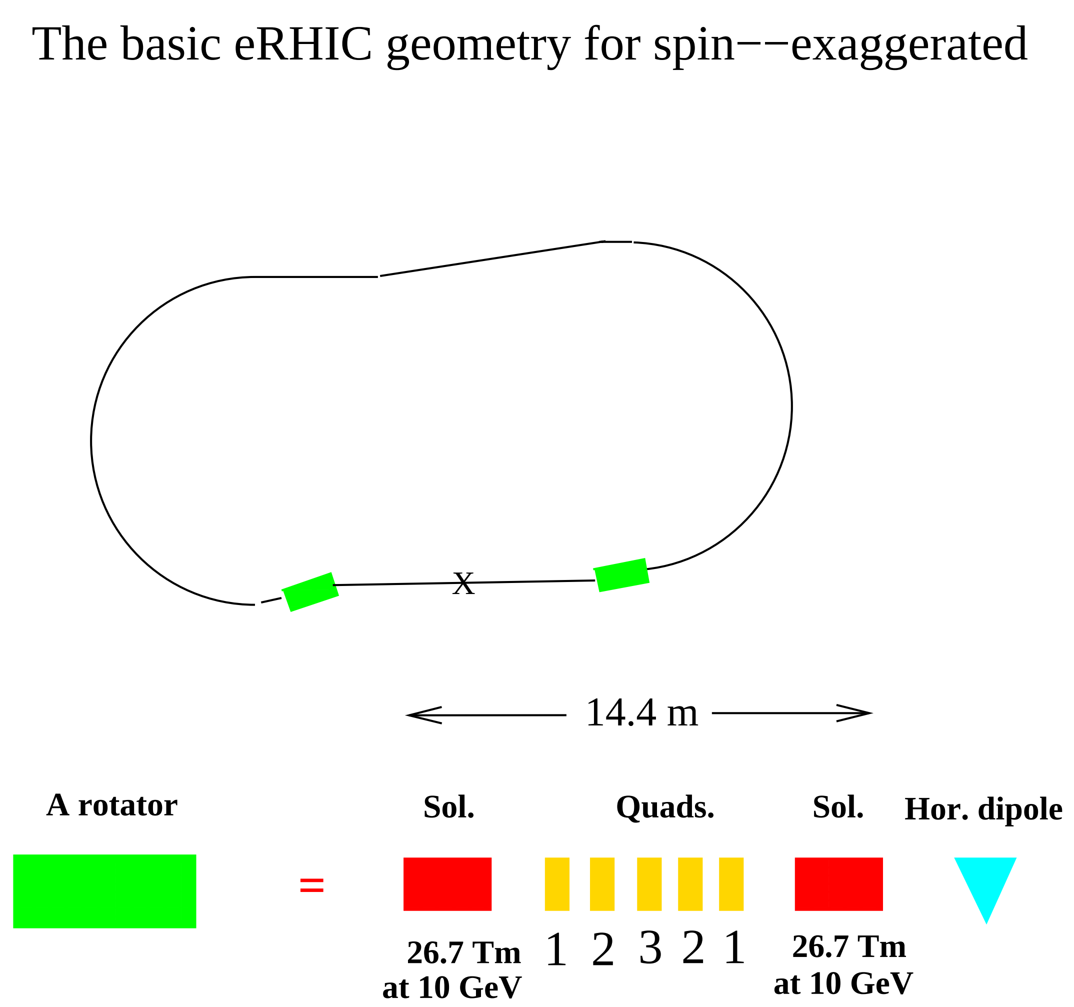
Fixed ring geometry but variable rotator geometry and fields. NO QUADRUPOLES

Solenoid spin rotators (from above): best at low energy

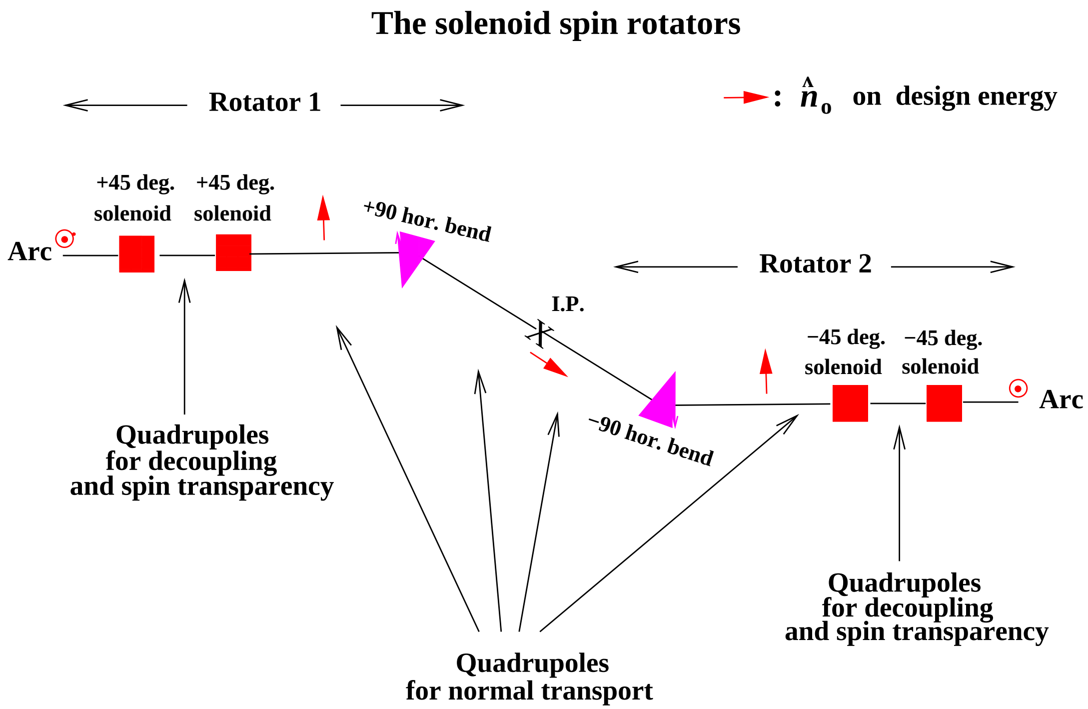


D.P. Barber et al., Part. Acc. vol. 17 1985

eRHIC: ring-ring option



eRHIC: ring-ring option



$\hat{n}_0$ exactly longitudinal at just at one energy.

BUT! for **storage**

Spin rotators are usually potentially dangerous for polarisation!!

- Spin rotators orient the spins so that they are particularly susceptible to diffusion.
- Spin rotators can be “spin-opaque”

The SLIM formalism for estimating depolarisation at first order (Chao 1981).

Skip the fancy theory: heuristics instead for today!

$$\vec{S} = \sqrt{1-\alpha^2-\beta^2}\ \hat{n}_0(s) + \alpha\hat{m}_0(s) + \beta\hat{l}_0(s) \quad \approx \quad \hat{n}_0(s) + \alpha\hat{m}_0(s) + \beta\hat{l}_0(s)$$

$\hat{m}_0$ and $\hat{l}_0$ orthogonal to $\hat{n}_0$. All obey the T-BMT eqn.

α, β: 2 **small** spin tilt angles — have subtracted out the big rotations!

Linearised (T-BMT) equations of motion for α, β: ===> spin-orbit coupling matrix $\mathbf{G_{2\times 6}}$:

$$\begin{pmatrix} x \\ x' \\ y \\ y' \\ l \\ \delta \\ \alpha \\ \beta \end{pmatrix}(s_2) \;=\; \begin{pmatrix} \mathbf{M_{6\times 6}} & \mathbf{0_{6\times 2}} \\ \mathbf{G_{2\times 6}} & \mathbf{D_{2\times 2}} \end{pmatrix} \begin{pmatrix} x \\ x' \\ y \\ y' \\ l \\ \delta \\ \alpha \\ \beta \end{pmatrix}(s_1)$$

with $\delta = \delta E/E_0$

Unified transport formalism for trajectory and spin.

Spin–orbit covariance matrix

$$\begin{pmatrix} \sigma_x^2 & \sigma_{xx'} & . & . & . & . & | & . & . \\ \sigma_{x'x} & \sigma_{x'}^2 & . & . & . & . & | & . & . \\ . & . & . & . & . & . & | & . & . \\ . & . & . & . & . & . & | & . & . \\ . & . & . & . & . & . & | & . & . \\ . & . & . & . & . & \sigma_\delta^2 & | & . & . \\ - & - & - & - & - & - & - & - & - \\ . & . & . & . & . & . & | & \sigma_\alpha^2 & \sigma_{\alpha\beta} \\ \sigma_{\beta x}. & . & . & . & . & . & | & \sigma_{\beta\alpha} & \sigma_\beta^2 \end{pmatrix}$$

$$\Delta P = 1 - \Delta < \sqrt{1-\alpha^2-\beta^2} > \approx -\frac{1}{2}\Delta(<\alpha^2+\beta^2>) = -\frac{1}{2}\Delta(\sigma_\alpha^2+\sigma_\beta^2) \Longrightarrow \frac{dP}{dt} \approx -\frac{1}{2}\frac{d}{dt}(\sigma_\alpha^2+\sigma_\beta^2)$$

Random walk in plane orthogonal to $\hat{n}_0$.

Calculate analytically (stochastic differential equations or Fokker-Planck methods) (Brownian motion). Everything needed is in 1-turn matrices.

Or brute force with a Monte-Carlo with multi-turn tracking – in fact the best in the end.

No damping mechanism for spin – but the S-T effect works to restore the polarisation along $\hat{n}_0$.

Just need the **slope** $\frac{dP}{dt}$.

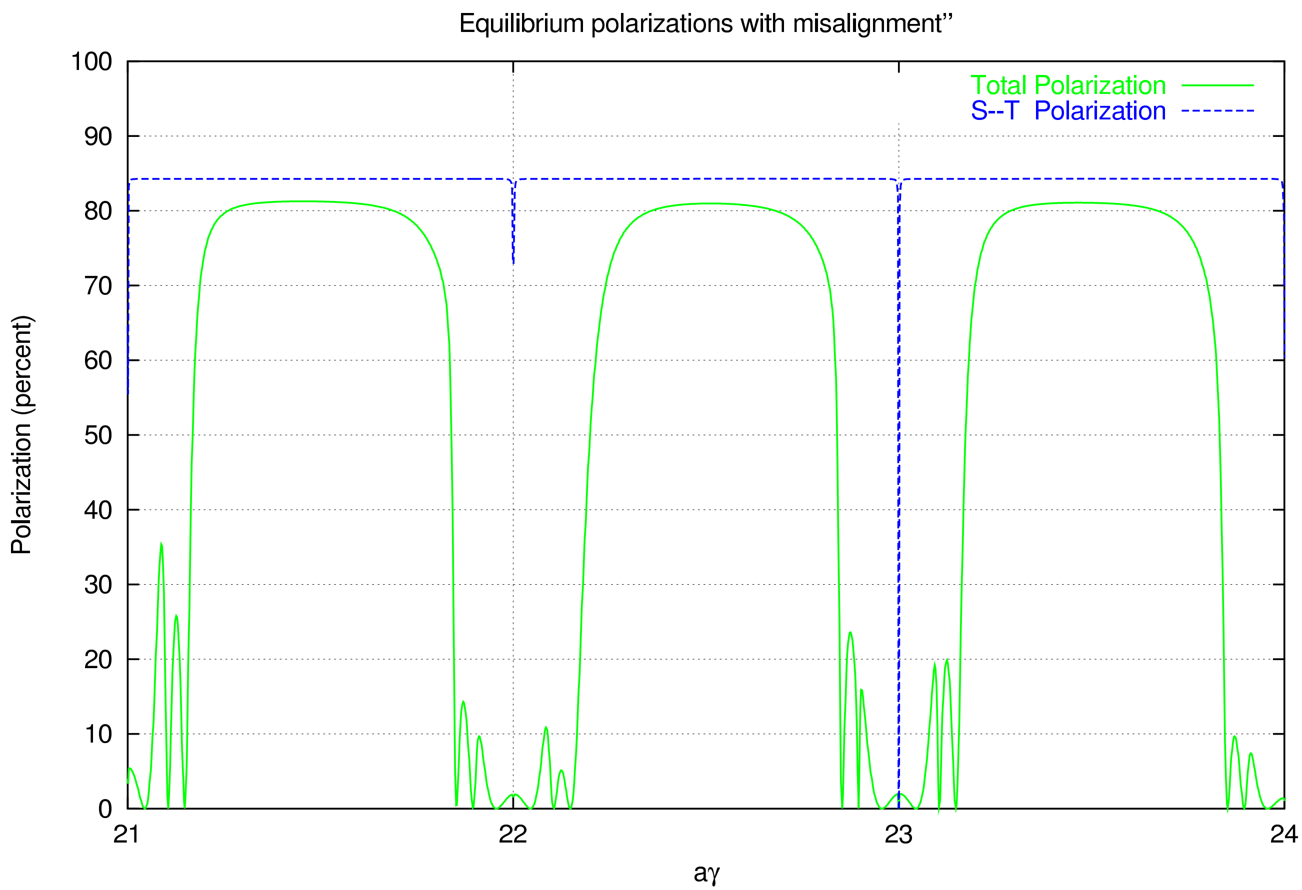
Linear spin matching

$$\hat{\mathrm{M}}_{8\times 8} = \begin{pmatrix} \mathbf{M_{6\times 6}} & \mathbf{0_{6\times 2}} \\ \mathbf{G_{2\times 6}} & \mathbf{D_{2\times 2}} \end{pmatrix}$$

To minimize depolarization:
minimize appropriate bits of $\mathbf{G_{2\times 6}}$ for appropriate stretches of ring
===> lots of independent quadrupole circuits.

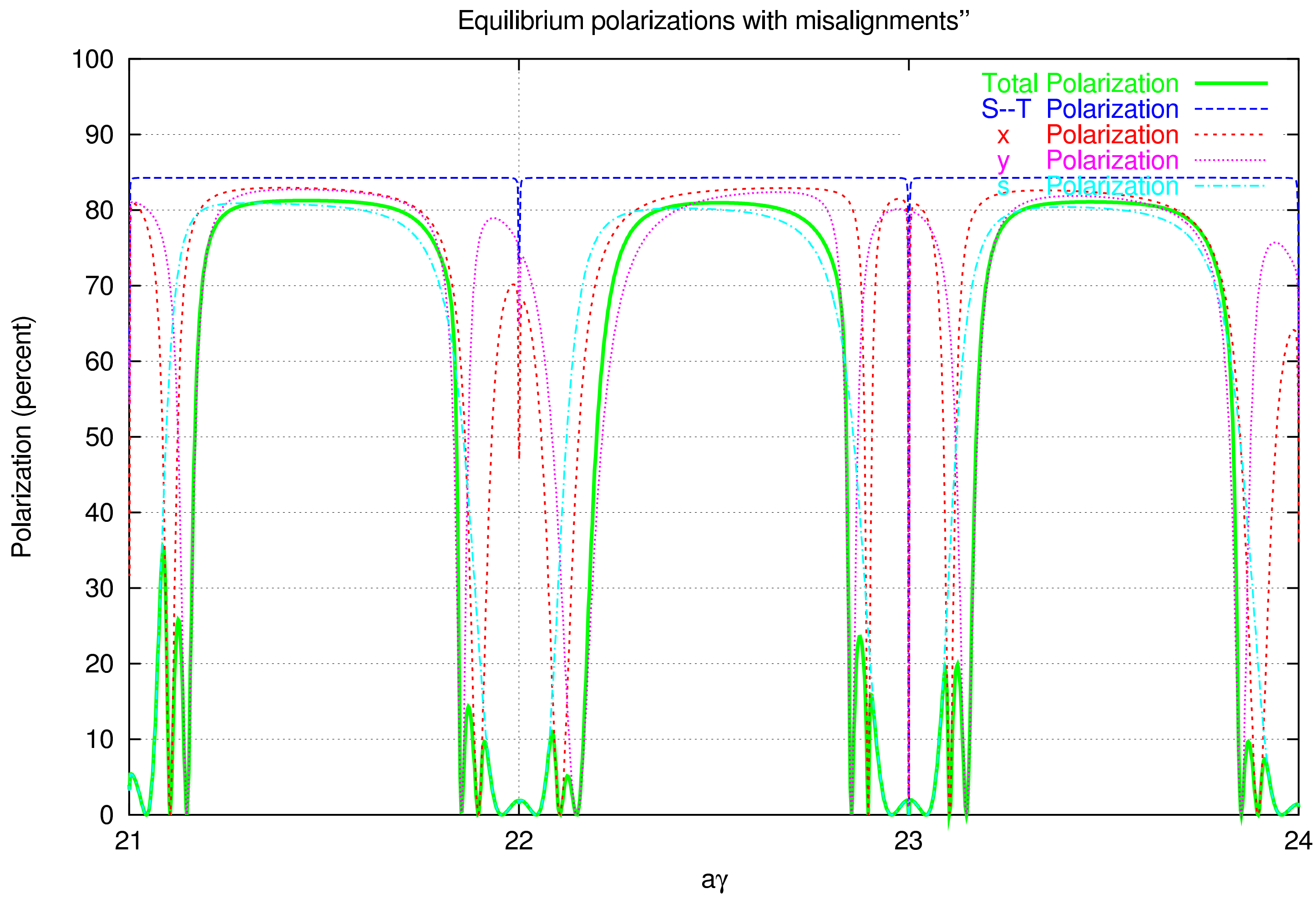
"Spin transparency"!!! – not the simple kind on the design orbit!

eRHIC with SLICKTRACK: all monitors on: just ONE example!



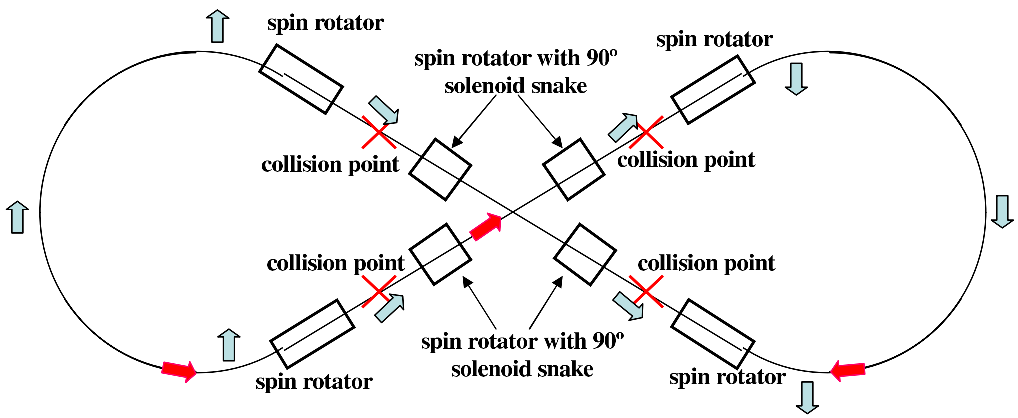
The rotator region is spin transparent for hor. and long. motion. Zholents + Litvinenko 1981.

eRHIC with SLICKTRACK: all monitors on: DIAGNOSTICS



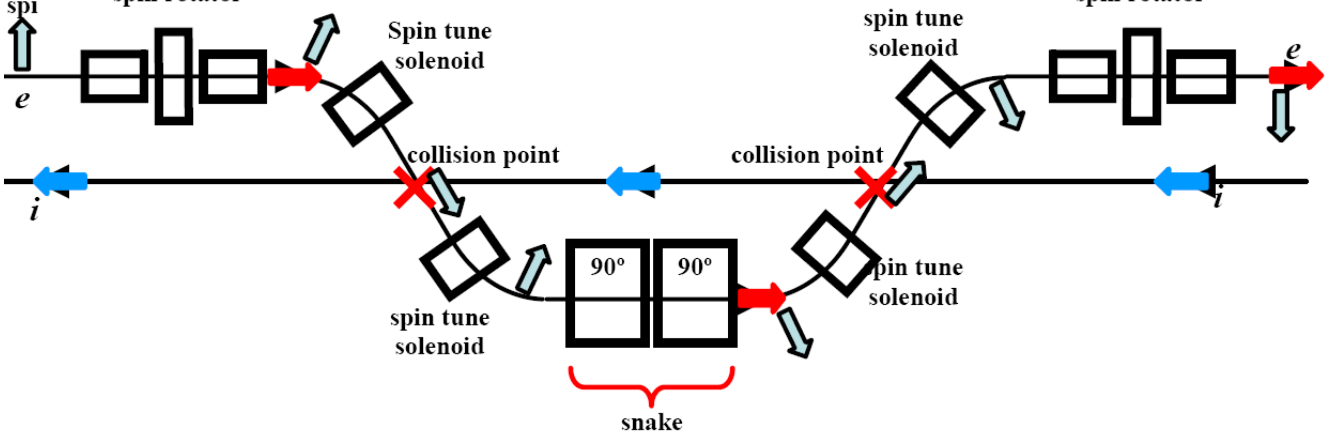
ELIC: version using stored beams

Snakes and reverse bends ensure that the Sokolov-Ternov effect does not average away.

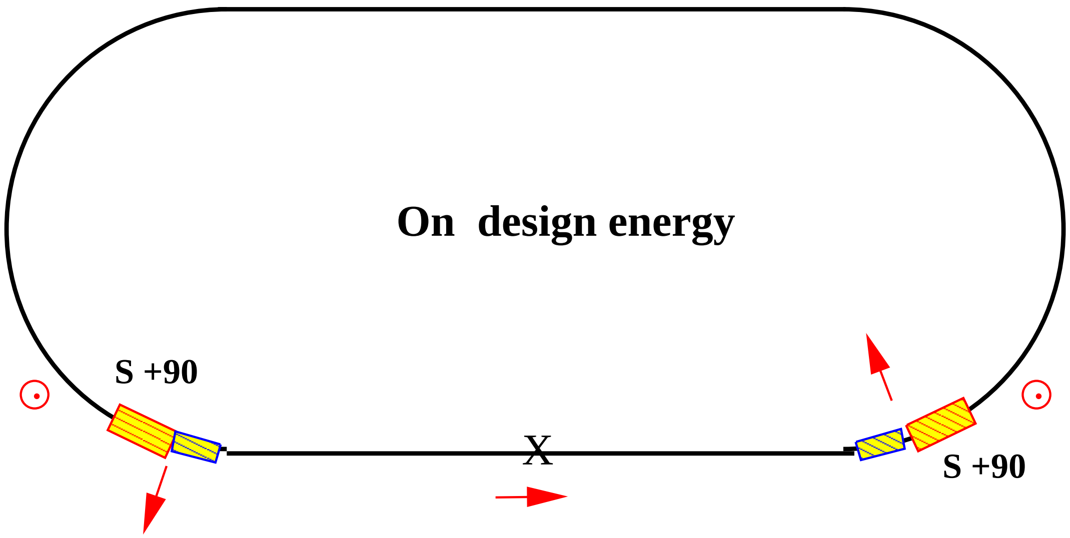


ELIC: version using stored beams

Fixed geometry but correct $\hat{n}_0$ orientation at all energies by tuning the solenoids



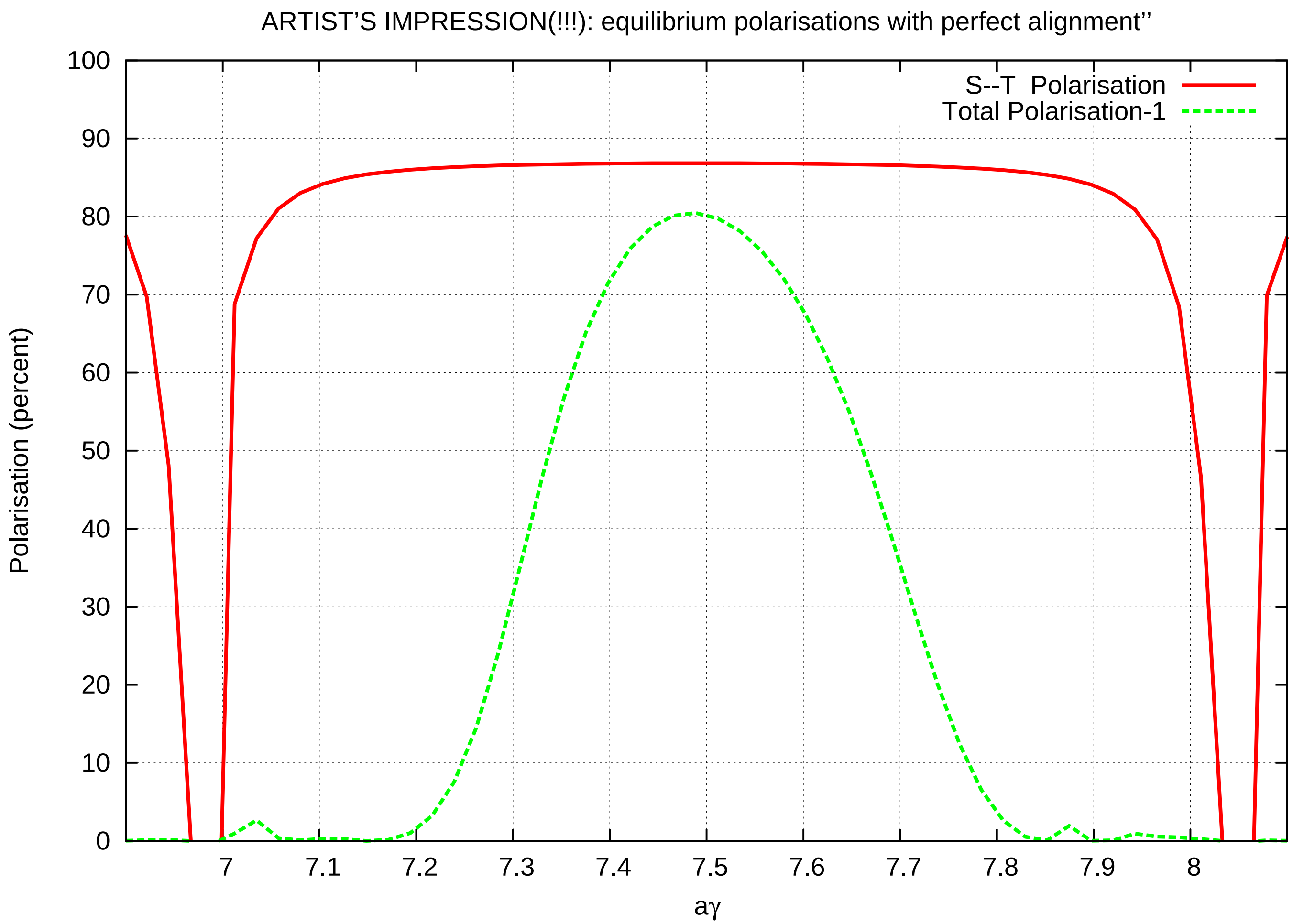
ENC Solenoid rotator layout



$\approx$ 3 GeV. So use solenoids.

Need for geometrical compatibility with the HESR: symmetric solenoids.

For the total polarisation: AN ARTIST'S IMPRESSION! of the aim



Commentary

$$\tau_{\rm tot}^{-1} = \tau_{\rm dep}^{-1} + \tau_{\rm bks}^{-1} \longrightarrow \tau_{\rm tot} \leq \tau_{\rm bks}\,, \qquad P_\infty \approx P_{\rm bks}\,\frac{\tau_{\rm tot}}{\tau_{\rm bks}}$$

- At 3.3 GeV with the current layout:
  $\tau_{\rm bks} \approx 12000$ seconds. $P_{\rm bks} \approx 86\%$ with perfect alignment.
- Polarisation can be lost during acceleration through resonances. So must inject a pre- and highly polarised beam (80 % ?) at FULL ENERGY.
- The injected polarisation relaxes (up or down) to P_∞ on a time scale $\tau_{\rm tot}$.
- Ideally, we need $\tau_{\rm dep} \gg \tau_{\rm bks}$ to ensure a high P_∞ .
  Or, at least, the polarisation should remain high during a fill.
- We aim for sufficient spin matching (==> transparency) to maximise $\tau_{\rm dep}$ at the design energy.
  Hoping to take advantage of the low energy: "only 3.3 GeV " .
- Away from the design energy (3.3 GeV) $\hat{n}_0$ tilts in the arcs so that depolarisation can turn on.
- How much polarisation survives beam-beam effects (direct and indirect) ? .
- Etc....

Recommendation

It is not trivial to avoid depolarisation.

Build in appropriate facilities for electron polarisation right at the beginning!

Otherwise it is very unlikely that polarisation can be patched in later.

Preparations for electrons are also useful for polarised positrons

Immediate plans:

Keep designing!

Keep calculating:

- Linearised formalism for first looks.
- Full 3-D spin motion with M-C tracking.
- Beam-beam effects.
- Full nonlinear orbital motion.
- Etc..

Richard Feynmann:

"The whole purpose of physics is to find a number, with decimal points, etc! Otherwise you haven't done anything."