Spin-orbit tracking simulations and spin resonance strengths for deuterons in COSY

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The purpose of science is not to lead us to everlasting wisdom, but to place a limit on everlasting error.

– Bertolt Brecht

Claims of unexpected behaviour of proton and deuteron spins at the COSY ring

Unexpected enhancements and reductions of rf spin resonance strengths
M. A. Leonova et al.
Phys. Rev. ST Accel. Beams 9, 051001 (2006)

Reply to Comment on “Spin manipulation of 1.94 GeV/c polarized protons stored in the COSY cooler synchrotron ”
V. S. Morozov et al.
Phys. Rev. ST Accel. Beams 8, 099002 (2005)

Unexpected reduction of rf spin resonance strength for stored deuteron beams
A. D. Krisch et al.
Phys. Rev. ST Accel. Beams 10, 071001 (2007)

Beam request 180.2 for PAC-36, December 2008:
Still the suggestion that the measured values of the resonance strengths of deuteron beams in COSY might mean that the deuterons in COSY are exhibiting some unspecified macroscopic quantum behaviour.

November 2006: Advice

Report for PAC-32 on Proposal 170 by the SPIN@COSY Collaboration
D. P. Barber
November 2006!

Notation

- $\hat{x}$ is horizontal (radial in a flat ring).
- $\hat{y}$ is vertical
- $\hat{s} = \hat{x} \times \hat{y}$ (longitudinal in a flat ring).
- $\vec{S}$ is the rest-frame, single-particle spin expectation value.
  The "spin" ! – classical equation of motion.
  For spin-1/2 normalise to unit length.
- s is the distance around the ring of circumference C.
- $\theta = 2\pi s/C$: use a generalised angle instead of a distance.
- Angular frequency of circulation: $\omega_{\mathrm{c}} = d\theta/dt = 2\pi c\beta/C$.
- $y' = dy/ds$.
- $\rho(s)$ is local bending radius.

NMR and ESR – a reminder

A spin in sample in a (say) vertical magnetic "holding" field $\vec{B}_{\rm H}$, precesses around the field at angular frequency $|\Omega_{\rm H}| = \frac{eg}{2m} B_{\rm H}$

$$\frac{d\vec{S}}{dt} = \frac{eg}{2m}\,\vec{S} \times \vec{B}_{\rm H} \equiv \vec{\Omega}_{\rm H} \times \vec{S} \qquad \vec{S} \cdot \vec{B}_{\rm H} = \text{constant}$$

Add a horizontal rf magnetic field (from a microwave generator) and view that as two counter-rotating constant fields.

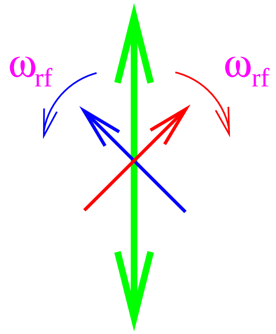
$$\vec{B}_{\rm rf} = 2\,b_{\rm rf}\,\cos(\omega_{\rm rf}\,t)\,\hat{s} = b_{\rm rf}\,\{\cos(\omega_{\rm rf}\,t)\,\hat{s} + \sin(\omega_{\rm rf}\,t)\,\hat{x}\} + b_{\rm rf}\,\{\cos(\omega_{\rm rf}\,t)\,\hat{s} - \sin(\omega_{\rm rf}\,t)\,\hat{x}\}$$

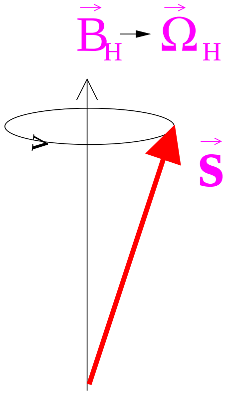
- Choose the component rotating in the same sense as the spin in $\vec{B}_{\rm H}$.
- Transform into the coordinate system where **that** component is constant $= b_{\rm rf}$.
- In that frame the angular frequency of precession around the vertical is $\Omega_{\rm H} - \omega_{\rm rf}$ so that the effective vertical field is $\frac{\Omega_{\rm H} - \omega_{\rm rf}}{\Omega_{\rm H}} \vec{B}_{\rm H}$.
- At **resonance** $|\Omega_{\rm H} - \omega_{\rm rf}| = 0$, the total magnetic field in that frame is horizontal $= b_{\rm rf}$ and an initially vertical spin tumbles from up to down and back, precessing around the horizontal.
- Close to this resonance the other component is normally rotating at high frequency "backwards" in this frame and its effect usually averages away.

The rotating wave approximation for spin resonance

in a stationary sample in a vertical magnetic field

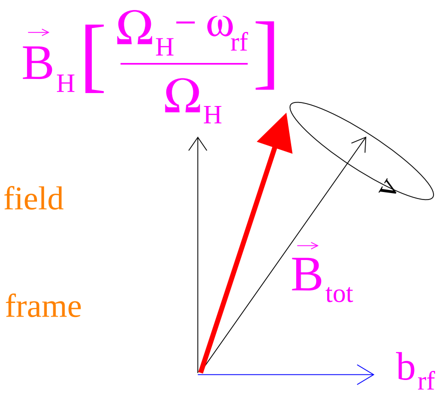
Counter rotating
horizontal
magnetic field
vectors





Apply the horizontal RF field

and transform to the new frame



In the lab. frame without the RF field

In the ‘‘blue’’ rotating frame

−−ignoring the other counter−rotating component

cont.....

- By varying $\omega_{\rm rf}$ from far below resonance to far above at a rate $<< |\Omega_{\rm H}|$ the total field $\vec{B}_{\rm tot}$ tilts from (say) up to down and the spin, which continues to precess around the field, flips from up to down.
  At very high scan rate, the spins get left behind $\Longrightarrow$ no flip. See Froissart-Stora formula later.
- Can do it in QM: just a two-level system, absorbing/emitting photons
  – in fact have used the **rotating wave approximation** familiar in radiation emission and absorption calculations in atomic and nuclear physics, e.g., in sources of polarised protons or deuterons!
  Also: general two-level, **non-spin**, systems are often handled using Pauli matrices.
  Connection to Froissart-Stora formula.
- A huge literature on the theory and approximations. Nothing mysterious!
- Can also get flip by varying the main field $\vec{B}_{\rm H}$!
- $b_{\rm rf} \Longrightarrow$ **resonance strength**!

Spin motion of moving particles – the T-BMT equation

$$\frac{d\vec{S}}{dt} = \vec{\Omega} \times \vec{S}$$

Use $a = \frac{g-2}{2} \quad (= G) : G \neq g$!!

$$\vec{\Omega} = -\frac{e}{m\gamma}\left[(a\gamma+1)\,\vec{B} - \frac{a\gamma^2\beta^2}{1+\gamma}(\hat{\beta}\cdot\vec{B})\hat{\beta} - \frac{\beta\gamma}{c}\left(a+\frac{1}{1+\gamma}\right)(\hat{\beta}\times\vec{E})\right].$$

$$\vec{\Omega} = -\frac{e}{m\gamma}\left[(a\gamma+1)\,\vec{B}_\perp + \frac{g}{2}\vec{B}_{//} - \frac{\beta\gamma}{c}\left(a+\frac{1}{1+\gamma}\right)(\hat{\beta}\times\vec{E})\right].$$

Several equivalent forms. Very easy to get it wrong!

Now calculate w.r.t. the design orbit which itself rotates (precesses) with $\vec{\Omega}_{\mathrm{CO}} = -\frac{e}{m\gamma}\vec{B}_{\mathrm{guide}}$.

$$\vec{\Omega} = -\frac{e}{m\gamma}\left[(a\gamma+1)\,\vec{B} - \vec{B}_{\mathrm{guide}} - \frac{a\gamma^2\beta^2}{1+\gamma}(\hat{\beta}\cdot\vec{B})\hat{\beta} - \frac{\beta\gamma}{c}\left(a+\frac{1}{1+\gamma}\right)(\hat{\beta}\times\vec{E})\right].$$

$\Longrightarrow$ In a flat ring the number of spin precessions per turn around the ring on the design orbit is

$\nu_0 \equiv a\gamma = G\gamma$!

ν_0 is called the **spin tune on the design orbit**,

– the generalisation of $|\Omega_{\mathrm{H}}| = \frac{eg}{2m}B_{\mathrm{H}}$.

In transverse fields: $\delta\phi_{\mathrm{spin}} = (a\gamma+1)\,\delta\phi_{\mathrm{traj}}$

Storage rings – flipping with a local rf radial field

Generate a local rf radial magnetic field $\vec{B}_{\mathrm{rfd}}$ with a dipole fed from an AC source.

With field length $L << C$ the particle is usually not in the rf field!

Assume that the speed βc (and the momentum $p = m\gamma\beta c$) is fixed. Define $Q_{\mathrm{rf}} = \omega_{\mathrm{rf}}/\omega_{\mathrm{c}}$.

Approximate the field $\vec{B}_{\mathrm{rfd}}$ seen by a particle with a modulated Dirac comb:

$$\vec{B}_{\mathrm{rfd}}(\theta) = R\cos(Q_{\mathrm{rf}}\theta + \chi) \sum_{k=-\infty}^{k=+\infty} \delta(\theta - 2\pi k)\,\hat{x} = R\frac{1}{2\pi}\sum_{n=-\infty}^{n=+\infty} \cos\{(n + Q_{\mathrm{rf}})\theta + \chi\}\,\hat{x}\ ;\quad R = \frac{2\pi}{C} B_{\mathrm{rfd}}^{\mathrm{max}} L$$

A tricky singular expression replacing the local kicks but a description in which the particle is effectively continually immersed in a superposition of smoothly oscillating fields.

$$\frac{d\vec{S}}{dt} = \vec{\Omega} \times \vec{S} \Longrightarrow \frac{d\vec{S}}{d\theta} = \vec{\mho} \times \vec{S}\,, \quad \vec{\mho} = \frac{1}{\omega_{\mathrm{c}}}\vec{\Omega}$$

The $\vec{\mho}$ for the rf dipole is:

$$\vec{\mho}_{\mathrm{rfd}}(\theta) = -\frac{e}{m\gamma\omega_{\mathrm{c}}}\left[(a\gamma + 1)\,\vec{B}_{\mathrm{rfd}}(\theta)\right]$$

$$\vec{\mho}_{\mathrm{rfd}}(\theta) = (a\gamma + 1)\,\Theta_y \frac{1}{2\pi}\sum_{n=-\infty}^{n=+\infty} \cos\{(n + Q_{\mathrm{rf}})\theta + \chi\}\,\hat{x}$$

where Θ_y is the maximum trajectory kick angle from the rf dipole.

The resonance strength of the rf dipole

As with NMR/ESR decompose the rf perturbation into counter-rotating vectors.

Then select the harmonic close to resonance with the spin precession in the vertical dipole fields!

More convenient to reformulate in complex form:

$$\frac{1}{2\pi}\cos\{(n+Q_{\rm rf})\theta+\chi\} = \frac{1}{4\pi}\left(e^{+i\{(n+Q_{\rm rf})\theta+\chi\}} + e^{-i\{(n+Q_{\rm rf})\theta+\chi\}}\right)$$

Write

$$\vec{\mathcal{O}}_{\rm rfd}(\theta) = \mathcal{O}_x\hat{x} + \mathcal{O}_s\hat{s}$$

and make a spectral decomposition:

$$\mathcal{O}_x - i\mathcal{O}_s = \sum_{\kappa}\epsilon_\kappa e^{-i\kappa\theta}$$

Then trivial to fish out the **resonance strengths** ϵ_κ:

$$|\epsilon_\kappa^{\rm rfd}| = \frac{(a\gamma+1)\Theta_y}{4\pi} \quad !!$$

– the same for all harmonics. Note the $1/4\pi$!!

Or evaluate:

$$\epsilon_\kappa = \lim_{N\to\infty}\frac{1}{2\pi N}\int_0^{2\pi N}(\mathcal{O}_x - i\mathcal{O}_s)e^{i\kappa\theta}d\theta \qquad \text{at resonance} \quad \nu_0 = n + Q_{\rm rf}$$

In this picture, the pairs of counter-rotating fields are represented by the pairs: $e^{+i\kappa\theta}$ and $e^{-i\kappa\theta}$.

The Froissart-Stora formula

When sweeping through $Q_{\rm rf} + n = \nu_0$ with $S_y^{\rm initial} = 1$:

$$S_y^{\rm final} = 2\exp(-\frac{\pi|\epsilon|^2}{2\alpha}) - 1$$

where $\alpha = \frac{dQ_{\rm rf}}{d\theta}$

The F-S formula also gives the final time-averaged polarisation $\vec{P}^{\rm final}$– which is vertical in a ring like COSY. For narrow sweeps, need modified version by A.W. Chao.

To obtain an unknown $|\epsilon|$ (e.g., for $\epsilon^{\rm tot}$, below), measure the dependence of $P_y^{\rm final}/P_y^{\rm initial}$ on α.

Unexpected?

The quantity $\vec{S} \cdot \hat{n}$ is an adiabatic invariant, where $\hat{n}$ is the vector of the ISF at each point in phase space.

The Froissart-Stora formula quantifies the degree of invariance of $\vec{S} \cdot \hat{n}$ when tunes or other parameters are changed.

Testing the picture and the formula for ϵ^{rfd}

Only include the vertical dipole fields and ignore the y' terms (explained later):

- Do the Fourier transform with a tracking simulation – later
- Check the dependence of S_y^{final} on α.
- Check the geometry of the Invariant Spin Field (ISF).

TRIVIAL

The ISF is particularly useful for testing the limits/validity of this, the conventional mathematical model.

The contribution from the quadrupoles etc.

Rf dipoles add an inhomogeneous term to the equations of orbital motion. Solve using standard text-book methods $\Longrightarrow$

$$y = y_{\text{orig}} + y_{\text{induced}} + y_{\text{forced}}$$

Spectral analysis:

y_{orig} and y_{induced} are free betatron oscillations containing just Q_y

y_{forced} is a **superposition** of free betatron oscillations covering all the harmonics $n + Q_{\text{rf}}$!

So y_{forced} in the fields in the rest of the ring can contribute to the resonance strength!

y_{forced} diverges as $Q_{\text{rf}} \Longrightarrow Q_y$ because of small denominators.

So the total resonance strength ϵ^{tot} can show marked dependence on Q_y for fixed Q_{rf} !!

In general need

$$\epsilon_\kappa = \lim_{N\to\infty} \frac{1}{2\pi N} \int_0^{2\pi N} (\mathcal{O}_x - i\mathcal{O}_s) e^{i\kappa\tilde{\theta}} d\theta \qquad \tilde{\theta} = \int \frac{ds}{\rho}$$

For a rf solenoid (rfs): the inhomogeneous term in the orbit equations from a rf solenoid produces no significant excitation of orbital motion except extremely close to resonance with orbital motion.

For ϵ^{rfs}: $\hat{x} \Longrightarrow \hat{s}$, $\Theta_y \Longrightarrow \Theta_s = eB^{\text{max}}_{\text{rfs}} L/p$ and $1 + a\gamma \Longrightarrow 1 + a$.

With no significant excitation of orbital motion, measured ϵ^{tot} agrees with ϵ^{rfs}. **OBVIOUS!**

Components of $\vec{\mathcal{O}}$ due to y_{forced}

$$\vec{\mathcal{O}} = -\frac{e}{m\gamma\omega_c}\left[(a\gamma+1)\,\vec{B}_\perp + \frac{g}{2}\vec{B}_{//} - \frac{\beta\gamma}{c}\left(a + \frac{1}{1+\gamma}\right)(\hat{\beta}\times\vec{E})\right].$$

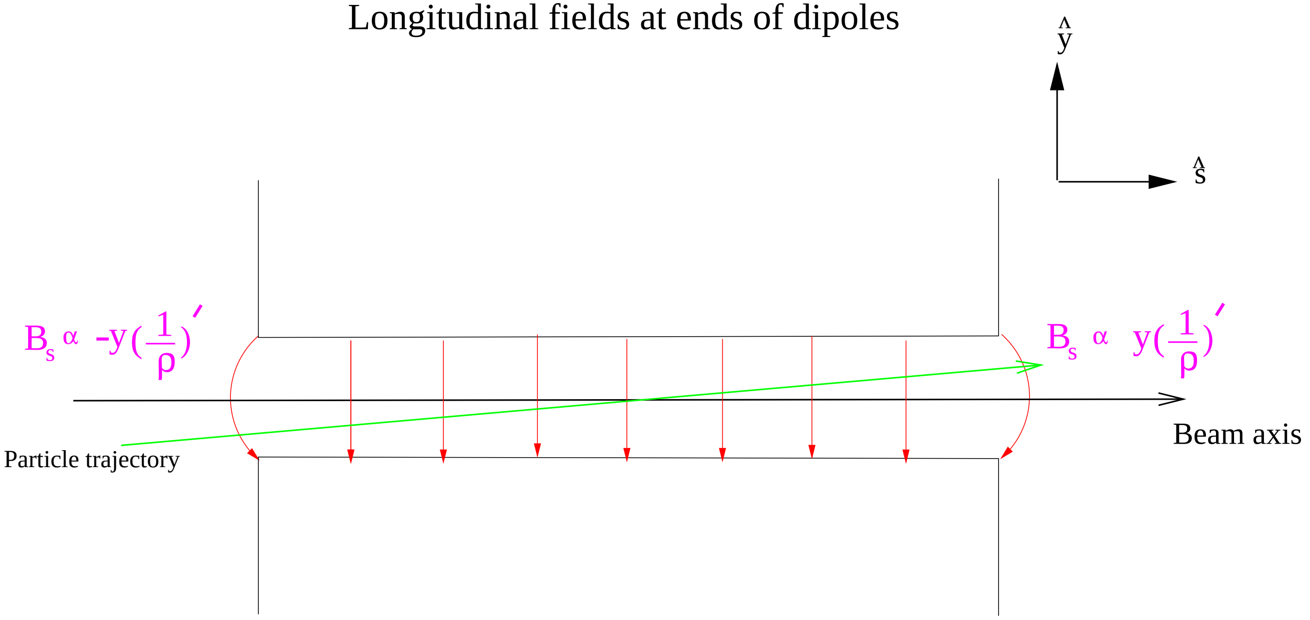
Paraxial approximation: linearise as, for example, in E. Courant + R. Ruth BNL-51270 (1980 !!!).

- $\mathcal{O}_x \propto (1+a\gamma)y''$: radial fields in quadrupoles and the rf dipole.
- $\mathcal{O}_s \propto a\gamma(1-\frac{1}{\gamma})\frac{y'}{\rho}$: vertical fields in dipoles – includes projection of $\vec{B}_{\text{dip}}$ along the trajectory.
- $\mathcal{O}_s \propto (1+a)y(\frac{1}{\rho})'$: $\text{curl}\vec{B} = 0$ induces a B_s at the ends of the dipoles. Spin rotation angle $\propto \pm(1+a)y\Delta(\frac{1}{\rho})$ after integration through the fringe treating it as having zero length.

$a_{\text{proton}} \approx 1.79285...$, $a_{\text{deuteron}} \approx -0.14298...$

To get ϵ^{tot}:
get the Fourier transform for the **whole** forced solution – which **includes** the effect of the rf dipole, or -
combine the real and imaginary parts of ϵ^{rfd} and ϵ^{ring}

Longitudinal fields at ends of dipoles



The SLIM/SLICK formalism

Linearised orbital and spin motion for first order analytical estimates of radiative depolarisation in electron storage rings, e.g., HERA, eRHIC, ELIC, ENC@FAIR, SuperB, LHeC.....

Attach an orthonormal coordinate system $\hat{n}_0(s), \hat{m}_0(s), \hat{l}_0(s)$ to the closed orbit.

$\hat{n}_0(s), \hat{m}_0(s), \hat{l}_0(s)$ obey the T-BMT equation on the closed orbit.

$\hat{n}_0(s)$ is the 1-turn periodic solution – “the stable spin direction”.

$$\vec{S} \approx \hat{n}_0(s) + \alpha\hat{m}_0(s) + \beta\hat{l}_0(s)$$

α, β: 2 small spin tilt angles — have subtracted out the big rotations!

$$\hat{\mathbf{M}}_{8\times 8} = \begin{pmatrix} \mathbf{M_{6\times 6}} & \mathbf{0_{6\times 2}} \\ \mathbf{G_{2\times 6}} & \mathbf{D_{2\times 2}} \end{pmatrix}$$

acting on $\vec{u} = (x,\ x',\ y,\ y',\ l,\ \delta)$ and α, β

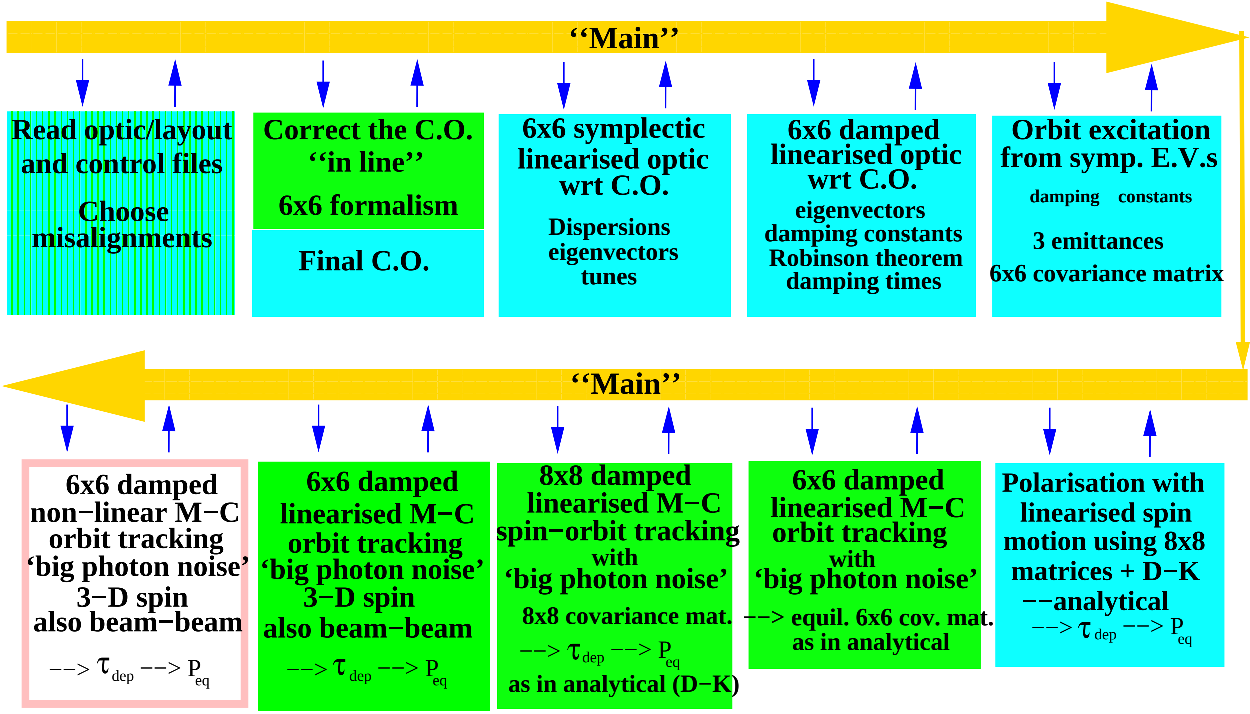
$\mathbf{G_{2\times 6}}$ represents the linearised solution of the T-BMT equation for α, β .

This is the **SLIM formalism**: originally A.W. Chao 1981 – working at DESY.

Theory and codes developed further by H. Mais and G. Ripken, D. Barber.

D. Barber: also a thick-lens version, SLICK and with Monte-Carlo extensions, SLICKTRACK.

The structure of SLICKTRACK



‘‘Main’’

Read optic/layout and control files

Choose misalignments

Correct the C.O. ‘‘in line’’

6x6 formalism

Final C.O.

6x6 symplectic linearised optic wrt C.O.

Dispersions eigenvectors tunes

6x6 damped linearised optic wrt C.O.

eigenvectors damping constants Robinson theorem damping times

Orbit excitation from symp. E.V.s

damping constants

3 emittances

6x6 covariance matrix

‘‘Main’’

6x6 damped non−linear M−C orbit tracking ‘big photon noise’ 3−D spin also beam−beam

--> τ_{dep} --> P_{eq}

6x6 damped linearised M−C orbit tracking ‘big photon noise’ 3−D spin also beam−beam

--> τ_{dep} --> P_{eq}

8x8 damped linearised M−C spin−orbit tracking with ‘big photon noise’

8x8 covariance mat.

--> τ_{dep} --> P_{eq}

as in analytical (D−K)

6x6 damped linearised M−C orbit tracking with ‘big photon noise’

--> equil. 6x6 cov. mat. as in analytical

Polarisation with linearised spin motion using 8x8 matrices + D−K −−analytical

--> τ_{dep} --> P_{eq}

🟦 = old (SLICK) 🟩 = New (done) ▥ = Old + New (done) ☐ = New (in progress) ☐ = Planned

Also: acceleration and spin flip

Using $G_{2\times 6}$ to get $\epsilon^{\mathrm{tot}}, \epsilon^{\mathrm{rfd}}$ etc

See for example:

G.H. Hoffstaetter, *"High Energy Polarised Proton Beams: a Modern View"*, Springer Tract in Modern Physics, Vol 218 (2006).

Flyer distributed at SPIN-2006 (Kyoto).

M. Vogt, PhD Thesis, University of Hamburg, Germany, DESY-THESIS-2000-054 (2000): http://www-library.desy.de/preparch/desy/thesis/desy-thesis-00-054.pdf

Hard copies distributed isotropically by airmail including Ann Arbor in 2000.

D.P. Barber and G. Ripken in *"Handbook of Accelerator Physics and Engineering"*, Eds: A.W. Chao and M. Tigner, World Scientific, 3rd edition (2006).

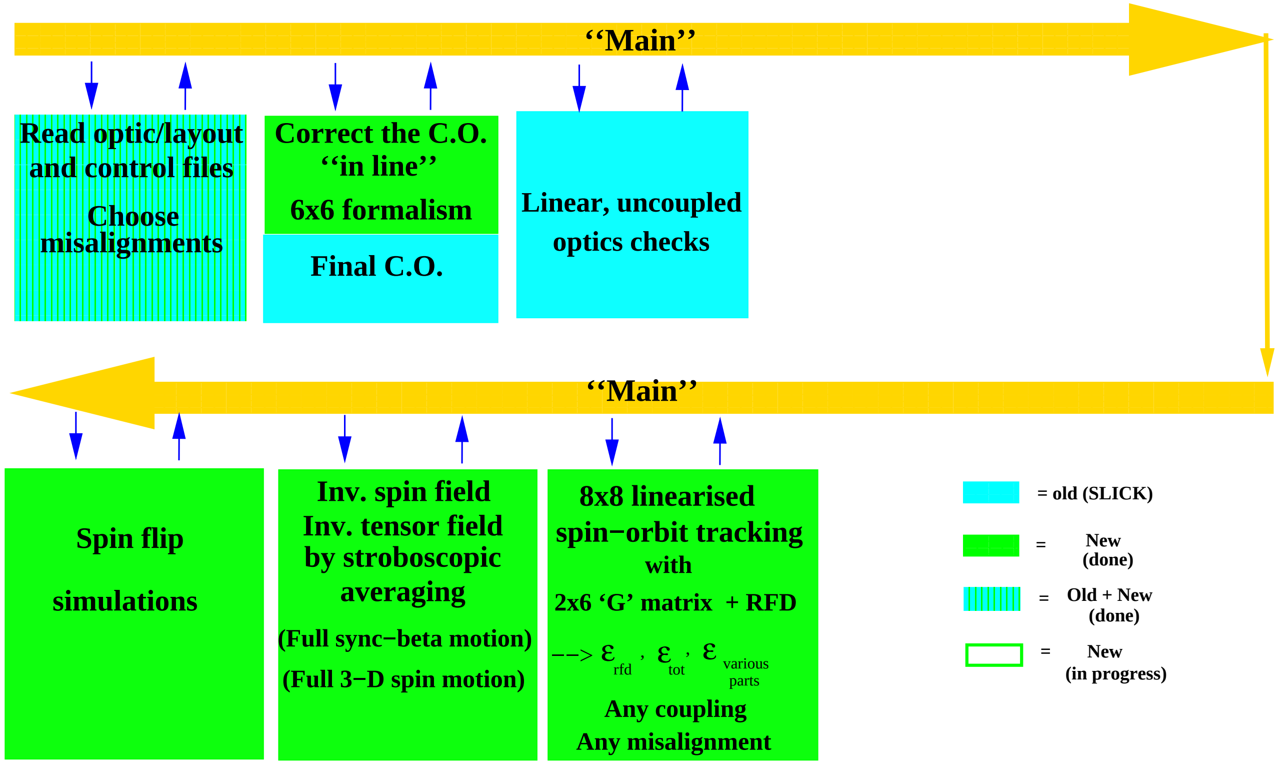
Just need the 1-turn $G_{2\times 6}$ for the homogeneous problem.

Extend to long term tracking and averaging to get the Fourier integral with the (inhomogeneous) rf dipole contribution.

Any ring geometry, any misalignment, any linear coupling.
Full 6-D orbital motion!

Getting ϵ by tracking:
including the (inhomogeneous) rf dipole with the matrix $G_{2\times6}$

The structure of EpsSLICK



“Main”

Read optic/layout and control files
Choose misalignments

Correct the C.O. “in line”
6x6 formalism
Final C.O.

Linear, uncoupled optics checks

“Main”

Spin flip
simulations

Inv. spin field
Inv. tensor field
by stroboscopic averaging
(Full sync−beta motion)
(Full 3−D spin motion)

8x8 linearised spin−orbit tracking with
2x6 ‘G’ matrix + RFD
--> ε_{rfd} , ε_{tot} , $\varepsilon_{various\ parts}$
Any coupling
Any misalignment

🟦 = old (SLICK)
🟩 = New (done)
▨ = Old + New (done)
□ = New (in progress)

Diagnostics! Diagnostics! Diagnostics!

Switch spin-orbit coupling off/on to see what does what.

For example:
Check that ϵ^{rfd} comes out correctly.
Study contributions from y'', y', y.
Getting the relative signs right.

Recapitulation

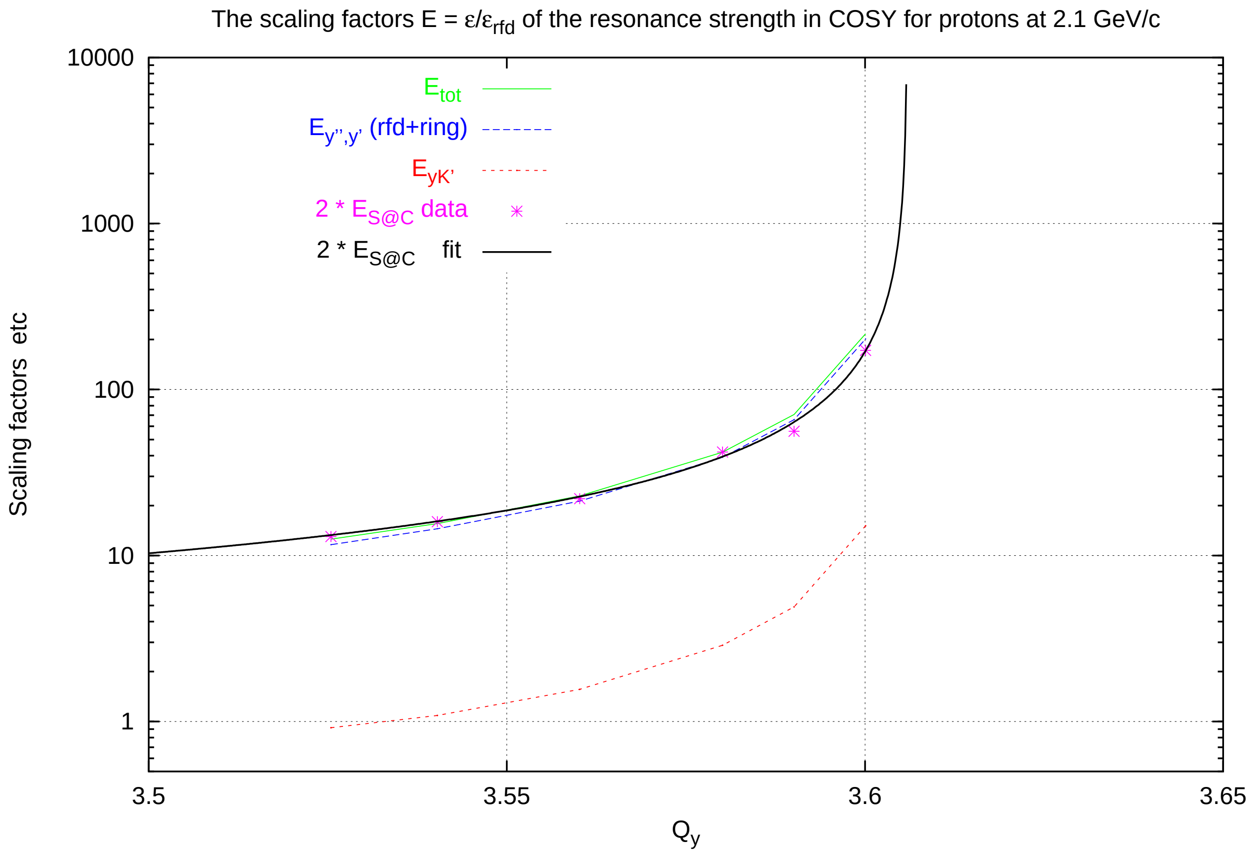
- Spins lined up with the periodic solution $\hat{n}_0(\theta)$ of the T-BMT equation on the closed orbit are perturbed by $\vec{\mathcal{O}}_{\substack{\text{ring}\\ +\text{rfd}}}(\theta)$
- Write

$$(\mathcal{O}_x - i\mathcal{O}_s)_{\substack{\text{ring}\\ +\text{rfd}}}(\tilde{\theta}) = \sum_{\kappa} \epsilon_\kappa e^{-i\kappa\tilde{\theta}}$$

- and evaluate

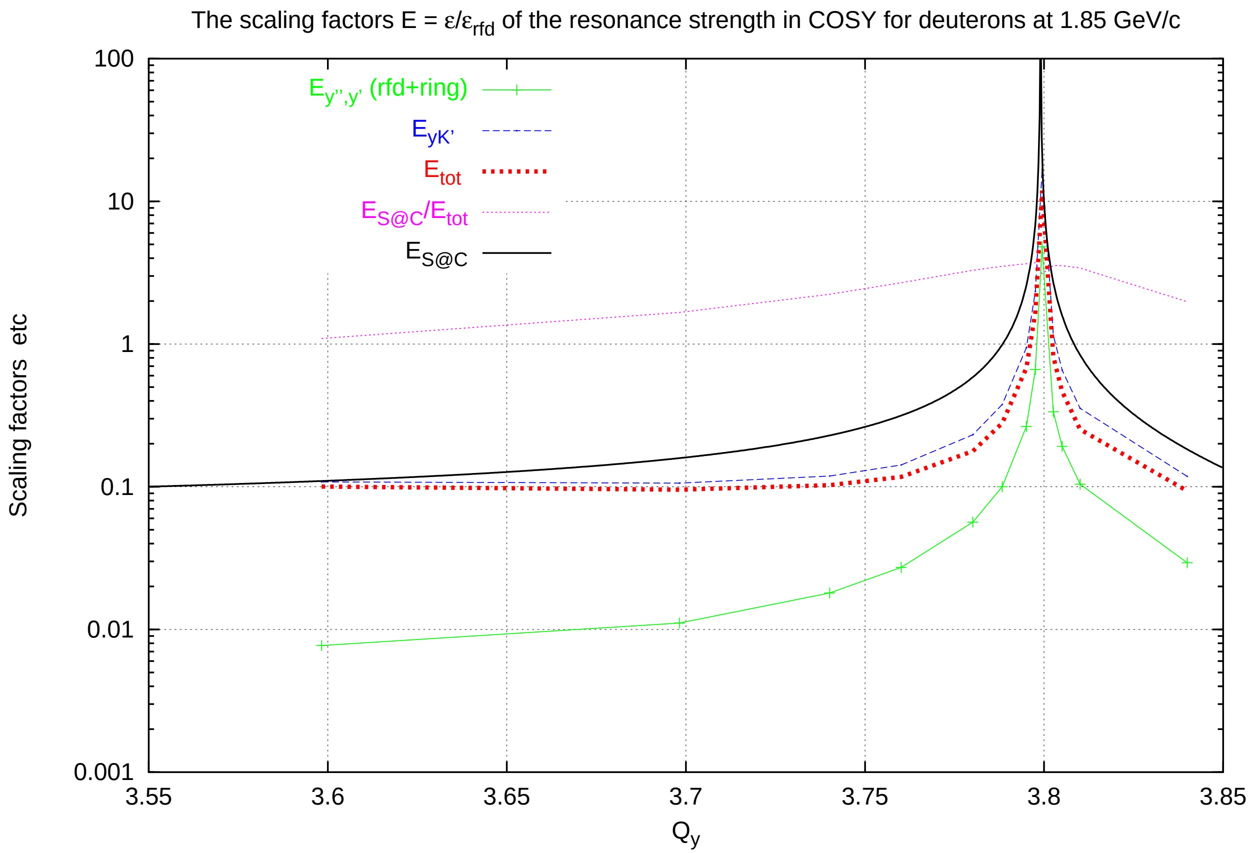
$$\epsilon_\kappa = \lim_{N\to\infty} \frac{1}{2\pi N} \int_0^{2\pi N} (\mathcal{O}_x - i\mathcal{O}_s)_{\substack{\text{ring}\\ +\text{rfd}}} e^{i\kappa\tilde{\theta}} d\theta \quad \text{for} \quad \kappa = \nu_0$$

  by spin-orbit tracking using simple spin kicks for the rf dipole and the $G_{2\times 6}$ matrix for the ring.



The values for $E_{y''}(\mathrm{rfd}+\mathrm{ring})$ confirm earlier preliminary results (2006!) from A. Lehrach.

The values for $E_{y'',y'}(\mathrm{rfd}+\mathrm{ring})$ are confirmed by M. Vogt with the code SPRINT.



A.M Kondratenko and S.R. Mane: for deuterons at small γ, $E_{y'',y'}(\mathrm{rfd}+\mathrm{ring})$ should be small.

In fact the contributions from the rf dipole and the y'', y' terms from the ring substantially cancel!

The entry and exit terms of the longitudinal end fields of the dipoles almost cancel leaving the small difference of much larger numbers which may be poorly known. But this small difference is dominant.

Note: y can be many millimeters near orbital resonance ($Q_y = 3.8$).

Conclusions

For protons: absolutely nothing unexpected!

For deuterons: absolutely nothing unexpected for the basic magnitudes.
Comparison probably limited by lack of knowledge of the
geometry of the dipole fringe fields.

That there is a large difference between the proton and
deuteron enhancements is predicted by standard theory.

The relative sizes of the $E_{y'',y'}(\mathrm{rfd} + \mathrm{ring})$ **and** $E_{yK'}$ **contributions**
are exchanged for proton $\Leftrightarrow$ **deuteron.**

Just do the calculations!!!!!

At this level it's a student's warm-up problem.

Why would anyone want to flip spins in a way guaranteed to kill the luminosity?

No need for special "natural" coordinate systems

No need to modify the T-BMT equation, e.g., $a\gamma + 1 \Longrightarrow a\gamma$ **in the rf dipole.**

No need to modify the Lorentz force equation.

No need to invoke quantum mechanical behaviour for deuterons which "might be relevant for quantum computing".

See also the already-published works:

Deuteron spin-flip resonance widths and the spin response function
S. R. Mane
Phys. Rev. ST Accel. Beams 10, 111001 (2007)

Comment on “Unexpected reduction of rf spin resonance strength for stored deuteron beams”
S. R. Mane
Phys. Rev. ST Accel. Beams 11, 069001 (2008)

Analysis of data for stored polarized beams using a spin flipper
Yu. M. Shatunov and S. R. Mane
Phys. Rev. ST Accel. Beams 11, 094002 (2008)

Calculation of spin resonance strength at COSY accelerator
A. M. Kondratenko, M. A. Kondratenko and Yu. N. Filatov
Physics of Particles and Nuclei Letters 5(6) (2008)
(Pis’ma v Zhurnal Fizike Elementarnykh Chastits i Atomnogo Yadra 6(148), 902 (2008))