Spin Dynamics and Polarization
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Today: Spin introduction .... 'mathematics'

- History: Stern-Gerlach experiment and its interpretation
- Group theory
- Quantum numbers
- Dirac equation

Tomorrow: Spin in experiments ..... 'phenomenology'

- Some applications (spread in the physics examples)
- Physics at RHIC: spin crisis of the proton
- Physics at HERA: protons crisis, right-currents and all that
- Physics at the ILC: physics, sources and depolarization
Begin of last century

Status of begin of last century: Bohr model

- electron only at quantized orbits: 'space quantization'
- many physicists had strong doubts, that 'spatial orientation of atoms is something physically true' (Debye, Pauli, etc.)

Hydrogen excitation spectra: complex splitting patterns in a magnetic field not understandable (Zeeman effect)

\[ J^2 \Psi = j(j+1) \Psi \quad J=L+S \]
\[ J_z \Psi = m \Psi \]
Spin history 1

Proof of 'spatial quantization': Stern-Gerlach experiment (1922):

- beam of silver atoms in inhomogeneous magnetic field of 0.1 T and 10 T/cm
- classical theory predicts two levels: splitting of silver beam was only 0.2 mm
- misalignments of collimating slits by more than 0.01mm spoiled experiment!

( precise alignment needed to discover new physics .... 'same problem today' ! )
On the way to the spin

Stern-Gerlach continued:

- historical 'postcard' in 1922

- still doubts from Einstein, Ehrenfest etc.: how could exact splitting occur when atoms entered field with random orientation?

Complete explanation (together with Zeeman effect) only with 'spin':

- 1924 Pauli: two-valued quantum degree of freedom
- 1925 Pauli: exclusion principle
- 1925 Kronig, Goudsmit and Uhlenbeck: electron self-rotation, has 'spin'
- 1927 Pauli: theory of spin based on quantum mechanics, but non-relativistic
Spin history, cont.

1928 Dirac: published Dirac equation

- description of relativistic electron spin via 'spinor'
- connection between 'relativity' and 'quantum effect'!

1940 Pauli: spin-statistic-theorem

- particle classification in fermions = anti-symmetric and bosons = symmetric
  states whose ensembles obey different statistics, Fermi-Dirac or Bose-Einstein

- breakthrough in description of particle phenomena!

Summary:

- spin has no classical description (only quasi-classical,......, see Des lecture)
- description as 'rotating particle' wrong, but helps understanding
- behaves like a kind of 'angular momentum'
- relativistic description via 'spinors'=vector
Group theory and Dirac equation

Questions:

- a) what are the kinematic properties of a particle? Only mass and spin?
- b) how to describe the states of a relativistic particles with spin?
- c) mathematical principle from which mass and spin follow deductively?

Solution:

- notion of mass is related with special relativity: Lorentz transformations
- notion of spin is related with rotation ('angular momentum'): rotations

Group needed which embraces Lorentz transformations and rotations and allows a definition of mass and describes the spin ...
Group theory, introduction

Definition of a group \( G \): 

1. product \( ab \) belongs to \( G \) if \( a \) and \( b \) belong to \( G \) [ \( a \in G, b \in G \Rightarrow ab \in G \) ]
2. associativity: \( (bc) = (ab) c \) for all \( a,b,c \)
3. unit element: \( e \in G \), such that \( a e = e a = a \) for all \( a \)
4. every \( a \in G \) has inverse \( a^{-1} \in G \), such that \( (a a^{-1}) = (a^{-1} a) = e \)

Order of a continuous group: number of parameters

\( \Rightarrow \) e.g. for rotations: 2 for fixing direction of axis, 1 angle for rotation around axis

\( \Rightarrow \) present rotation as matrix with 3 parameters

Infinitesimal operations:

\( \Rightarrow A = \exp(\alpha) = I + \alpha + \alpha^2/2! + \ldots \) i.e. \( \alpha = \lim_{n \to \infty} n (A^{1/n} - I) \)

\( \Rightarrow \alpha \) has to fulfill the Lie algebra of the group \( G \)

\( \Rightarrow g_n = \) basis matrices of algebra: \( \alpha = c_1 g_1 + c_2 g_2 + \ldots + c_n g_n \) ‘infinitesimal generators’
SU(2): group of unitary 2 x 2 matrices with \( U U^+ = 1 \) and \( \det U = 1 \) ( 'S' )

- 8 elements - (4 unitary +1 'special' ) conditions = 3 parameters

- generators: \( U = \exp(ih) \) with \( h^T = h^+ \) i.e. Lie algebra of SU(2) are hermitian matrices with the 3 Pauli matrices as basis: \( h = c_1 \sigma_1 + c_2 \sigma_2 + c_3 \sigma_3 \)

\[
\begin{array}{ccc}
\sigma_1 &=& \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \\
\sigma_2 &=& \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \\
\sigma_3 &=& \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}
\end{array}
\]

'spin matrices'

SU(3): group of unitary 3 x 3 matrices

- 18 elements - (9 unitarity +1 'special' ) conditions = 8 parameters

- 8 hermitian matrices as generators
Rotation group

R₃ = 'group of 3-dim rotations': is subgroup of SU(3) but with real matrices

→ 3 only three parameters, i.e. 3 infinitesimal generators

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J₁ = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & -1 & 0 \end{pmatrix},
J₂ = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix},
J₃ = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}

→ any hermitian 3 x 3 matrix with Tr=0 can be expressed via J₁, J₂, J₃

Total angular momentum defined as

J² = J₁² + J₂² + J₃²

→ J² has the eigenvalues: j(j+1)

→ third component J₃ has (2j +1) eigenvalues m ∈ [ -j, -j+1, ..., +j ]

→ basis vectors denoted | j m >
Relation between SU(2) and rotations

Connection between SU(2) and $R_3$: generators fulfill same 'commutation' relations

$$[J_1, J_2] = J_1 J_2 - J_2 J_1 = i J_3 \quad [J_2, J_3] = i J_1 \quad [J_3, J_1] = i J_2$$

same with $\sigma_i$, $i=1,2,3$

with every unitary matrix $A$ of SU(2) there is an element of $R_3$ associated

$$X = x \sigma = \begin{pmatrix} x_3 & x_1 - i x_2 \\ x_1 + i x_2 & -x_3 \end{pmatrix}$$

transformation $X' = A X A^+$ is pure rotation!

rotation of any 3 dim vector $x$ can be expressed via unitary 2 x 2 matrices $A$

Representations of the rotation group

for $j = \text{spin } 1/2 : J_i = \frac{1}{2} \sigma_i$ \quad Pauli- matrices = 'spin' matrices
Lorentz group

Quantities characterizing an experiment, when referred to two frames S and S', are related by the Lorentz transformation, i.e. ‘invariance under a change in description, not under a change in the experimental setup’.

Homogeneous Lorentz group: $x'_\mu = \Lambda_{\mu\nu} x_\nu$

- with under Lorentz transformations conserved quantity $x^2 = g_{\mu\nu} x_\mu x_\nu$ and

- metric $g = ( g_{\mu\nu} ) =
\begin{pmatrix}
  +1 & 0 & 0 & 0 \\
  0 & -1 & 0 & 0 \\
  0 & 0 & -1 & 0 \\
  0 & 0 & 0 & -1 \\
\end{pmatrix}$

- $x^2 = x_0^2 - \vec{x}^2$
Examples of the Lorentz group

Lorentz transformation in x-direction with velocity $v$:

$$\Lambda_1(v) = \begin{pmatrix}
\gamma & \gamma v & 0 & 0 \\
\gamma v & \gamma & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 \\
\end{pmatrix}, \text{with } \gamma = (1 - v^2)^{-1/2} \text{ (c=1 .... as usual ;-))}
$$

Rotation around the z-direction with angle $\theta$:

$$\Lambda_3(\theta) = \begin{pmatrix}
1 & 0 & 0 & 0 \\
0 & \cos \theta & -\sin \theta & 0 \\
0 & \sin \theta & \cos \theta & 0 \\
0 & 0 & 0 & 1 \\
\end{pmatrix}
$$

Clearly that's a group, all 4 x 4 matrices conserve $x^2$ with $\Lambda^T g \Lambda = g$

⇒ 16 real elements - 10 conditions (symmetric matrix) = 6 parameters

⇒ 6 infinitesimal generators: $i K_1, i K_2, i K_3 \rightarrow \Lambda_1(v), \Lambda_2(v), \Lambda_3(v)$ and $i J_1, i J_2, i J_3 \rightarrow \Lambda_1(\theta), \Lambda_2(\theta), \Lambda_3(\theta)$
Lorentz group, cont.

Done: generators for the homogeneous Lorentz group, have to fulfill the following commutation relations:

\[
\begin{align*}
[ J_1, J_2 ] &= i J_3 \\
[ J_1, K_1 ] &= 0 \\
[ J_1, K_2 ] &= i K_3 \\
[ K_1, K_2 ] &= -i J_3
\end{align*}
\]

- boost unchanged by rotation
- 'rotation group'
- K transform like 'vector'
- 'Thomas precession'

What's still needed?

- mass can only enter if energy and momentum are involved ....
- energy \( \hat{E} = i \partial / \partial t \) and momentum \( \hat{p}_x = -i \partial / \partial x \) (\( p_y, p_z \) analogous)
- \( \exp( i a \hat{p} ) f(x) = \exp( i a \partial / \partial x ) f(x) = f(x) + a \partial / \partial x f(x) = f(x+a) \) 'Taylor series'
- momentum = 'generator' of translations
- inclusion of translations leads to the 'inhomogeneous Lorentz group'
Inhomogeneous Lorentz and Poincare group

Inhomogeneous Lorentz group: \( x'_\mu = \Lambda_{\mu\nu} x_\nu + a_\mu \) denoted by \( \{a, \Lambda\} \)

→ four more generators needed \( iP_0, iP_1, iP_2, iP_3 \)

Additional commutation relations:

\[
[ P_i, P_0, j ] = 0 \quad [ J_1, P_2 ] = i P_3 \quad [ J_1, P_{0,1} ] = 0 \quad [ K_1, P_0 ] = i P_1 \quad [ K_1, P_1 ] = i P_0
\]

→ 'Abelian' \( \text{P transforms like vector}'

→ 'Rotation unchanged by translations'

→ 'energy changed by boost'

→ 'momentum changed under boost'

Inhomogeneous Lorentz group with \( \Lambda_{00} \geq 1 \) is 'Poincare group'

→ Relativistic invariance := invariance under Poincare group

→ generators: \( P_0, P_i, J_i, K_i \) with \( i = 1, 2, 3 \)

→ \( P^2 = P_0^2 - P_i^2 \) commutes with all generators

→ \( P^2 = M^2 \) conserved quantum number → that's the squared 'mass'
Casimir operators of Poincare group

Casimir operators commute with all generators

→ 'conserved quantities'

One Casimir operator of the Poincare group already known:

→ $P^2 = P_\mu P^{\mu} = M^2$

The other one is the 'spin-operator' or 'Bargmann-Wigner-operator':

→ $W^2 = W_\mu W^{\mu}$ with $W_\mu = 1/2 \epsilon_{\mu\nu\rho\sigma} M^{\nu\rho} P^{\sigma}$

Matrix $M =
\begin{bmatrix}
0 & K_1 & K_2 & K_3 \\
-K_1 & 0 & J_3 & -J_2 \\
-K_2 & -J_3 & 0 & J_1 \\
-K_3 & J_2 & -J_1 & 0 \\
\end{bmatrix}$

→ $W_0 = \vec{P} \vec{J}$ and $W = P_0 \vec{J} - \vec{P} \times \vec{K}$
**Status**

What has been managed so far?

- describe boosts, rotations and translation within one group: Poincare group

What is still missing?

- relativistic description of spin

we know: spin \( \frac{1}{2} \) described via Pauli matrices (SU(2) group)

What’s was relation between \( R_3 \) and SU(2)?

- any 3 dim vector can be expanded via hermitian 2 x 2 SU(2) matrices: \( X = x \sigma \)

- now do a 3 dim rotation: \( x' = R x \)

- 'rotated' 3 dim vector again expanded via Pauli matrices: \( X' = x' \sigma \)

- transformation matrix \( X' \) obtained via unitary transformation: \( X' = A X A^+ \)

- 3 dim metric is conserved under this unitary transformation: \( x'^2 = x^2 \)
**Relation: Lorentz Gruppe vs. SL(2C)**

- **Goal**: describe Lorentz transformations via 2 x 2 matrices possible?
  - would enable to embed **spin** within Lorentz transformations

- **First step**, take 4 dim Minkowski space, i.e. describe any 4 dim vectors:
  - any 4-vector can be expanded: \( X = x_0 \hat{1} - \vec{x} \vec{\sigma} = x_\mu \sigma_\mu \) with \( \sigma_\mu = (\sigma_0, \sigma_i) \)
  - apply **Lorentz transformation**: \( x'_\mu = \Lambda_{\mu\nu} x_\nu \)
  - still valid: \( X' = A X A^+ \), with \( A = 2 \times 2 \), but \( A \) no longer unitary! **reasonable?**
  - **yes**, since relativistic metric conserved: \( x'^2 = x^2 \)

- **The group corresponding to SU(2) (R³ vs SU(2))**, but applied for **Lorentz transformations in Minkowski space**: \( SL(2C) \)
  - 2 x 2 matrices \( A \) with \( \det A = 1 \) (keeps metric invariant), but **not unitary!**
  - \( A^+ \neq A^{-1} \) (not unitary) which indicates that \( X \neq X^+ \) (not hermitian)
Last steps to the Dirac equation

Lorentz group has 6 generators: $J_1, J_2, J_3, K_1, K_2, K_3$

Generators of SL(2C), have to be 2 x 2 matrices: $J_i = \sigma_i / 2$ and $K_i = i \sigma_i / 2$

→ fulfills all commutation relations we had for the Lorentz transformations

Keypoint: same commutation relations also fulfilled by 2\textsuperscript{nd} choice of generators, namely $J_i = \sigma_i / 2$ and $K_i = -i \sigma_i / 2$ !

→ two different 2 x 2 representations of the Lorentz group !

Two different 'spin states': $|\sigma, p, 1\rangle$ and $|\sigma, p, 2\rangle$

→ these two spin states are not independent, but related !
And now we are there ...

Lorentz transformation on the two spin states:

\[ |\sigma, p, 1> = D (L(p)) |\sigma', p' > \]
\[ |\sigma, p, 2> = D (L(p)) |\sigma', p' > \]
\[ \text{i.e. } |\sigma, p, 1> = D (L(p)) D (L(p)) |\sigma, p, 2> \]

With explicit Lorentz transformation:

\[ |\sigma, p, 1 > = \frac{(p_0 - p \sigma)}{m} |\sigma, p, 2 > \]
\[ |\sigma, p, 2 > = \frac{(p_0 + p \sigma)}{m} |\sigma, p, 1 > \]

Last step: take 4-dim denotation (with 4 x 4 $\gamma$-matrices)

\[ (\gamma_\mu p_\mu - m)_{\alpha'\alpha} u_\alpha (p) = 0 \text{ with the wave functions } u_1 = |1/2 p 1>, u_2 = |1/2 p 1>, u_3 = |1/2 p 2>, u_4 = |1/2 p 2> \]

Lorentz invariance indicates two spin states (of particle and antiparticle):

4-spinor wave function with mass and spin naturally from relativistic invariance!
**Little group for \( m \neq 0 \) and \( m=0 \)**

Definition: transformations that do not change \( p \) is called 'little group'

- transformation within Poincare group that do not change \( p \)

**Case 1: particles with \( m \neq 0 \)**

- \( P = (m, 0, 0, 0) \) and \( P^2 = m^2 \)
- Spin-operator \( W = m (0, J_1, J_2, J_3) \)

\[
W_3 \mid m, p, s_3 > = m s_3 \mid m, p, s_3 >
\]

\[
W^2 \mid m, p, s_3 > = - m^2 s (s+1) \mid m, p, s_3 >
\]

- \( W_3 \) and \( W^2 \) do not change \( p \), form elements of 'little group'

**Case 2: particles with \( m=0 \)**

- **Problem:** \( m=0 \) \( \rightarrow \) \( P^2 = 0 \) and \( W^2 = 0 \) and \( W_\mu P^\mu = 0 \)

- But all conditions are fulfilled if \( W_\mu = \lambda \ P_\mu \) with \( \lambda = \text{helicity} = \frac{s \ p}{|p|} \)
Conclusion, part 1

Stern-Gerlach and Zeeman effect led to introduction of 'spin property'

Kinematic properties of particles via mass and spin quantum numbers

Total angular momentum \( J = L + S \)

- angular momenta are generators of rotations

- eigenvalues of \( J^2 = j(j+1) \), \( J_z = m \in [-j, ..., +j] \)

Fundamental principle: relativistic invariance

- rotation, boosts and translations contained within Poincare group

- relativistic description of spin leads to two different representations

- Dirac equation expresses relativistic spin via 4-dim spinor wave function

Group theory and commutation relations: \( SU(3) \rightarrow R^3 \rightarrow SU(2) \rightarrow SL(2C) \)

'Spin' defined for particles with \( m \neq 0 \) in their rest frame

- if \( m=0 \) 'no' boost into rest frame possible, but defined via 'helicity'
Outline, part 2

Part two: Spin in experiments ..... 'phenomenology'

- Some applications (addition of angular momenta, spin description, SUSY transformations, spread over the following physics topics)

- Physics at RHIC: spin crisis of the proton

- Physics at HERA: proton spin crisis and new physics searches

- Physics at the ILC: new physics searches, polarized sources and possible depolarization effects in beam interactions
Addition of angular momenta

Short summary of known facts:

- angular momenta a axial-vectors
- orbital angular momenta \( L \) are integers: \( l = 0,1,2,... \)
- spin \( S \) is half-integer for fermions, integer for bosons
- total angular momentum: \( \mathbf{J} = \mathbf{L} + \mathbf{S} \)
- eigenvalues: \( J^2, L^2, S^2 = j(j+1), \) etc. and of \( J_z, L_z, S_z = m \) (\( \hbar = 1, \) as usual ;-

How to add angular momenta? Quantum numbers and eigenstates?

1. take two states: \( | j_1, m_1 >, \) \( | j_2, m_2 > \)

- \( J_{1,z} | j_1, m_1 > = m_1 | j_1, m_1 > \) and \( J_{1}^2 | j_1, m_1 > = j_1(j_1+1) | j_1, m_1 > \)
- \( J_{2,z} | j_2, m_2 > = m_2 | j_2, m_2 > \) and \( J_{2}^2 | j_2, m_2 > = j_2(j_2+1) | j_2, m_2 > \)
Angular momenta, cont.

2. Algebra for angular momenta now applied on the sum: \( \mathbf{J} = \mathbf{J}_1 + \mathbf{J}_2 \)

\( \rightarrow \) \( J^2 \ | j m \rangle = j (j+1) \ | j m \rangle \) and \( J_z \ | j m \rangle = m \ | j m \rangle \)

\( \rightarrow \) with \(-m \leq j \leq +m\) 'magnetic quantum number'

3. For a given \( j_1 \) and \( j_2 \) the total sum \( j \) can have the following values:

\( \rightarrow j = j_1 + j_2, (j_1 + j_2 -1), ..., |j_1 - j_2| := j_1 \times j_2 \)

4. How to construct the corresponding eigenstates?

\( \rightarrow \ | j m \rangle = \sum_{m_2} c(m, m_2) \ | j_1 m_1 \rangle \ | j_2 m_2 \rangle \) with \( m_1 + m_2 = m \)

\( \rightarrow c(m, m_2) = '\text{Clebsch-Gordon-coefficient}' \)

\( \rightarrow \text{denotation: } \ | j m \rangle = \ | j_1 j_2 m m \rangle \) and \( \ | j_1 m_1 \rangle \ | j_2 m_2 \rangle = \ | j_1 j_2 m_1 m_2 \rangle \)

\( \rightarrow \text{obviously: } \ | j m \rangle \text{ also eigenstate of } J_1^2 \text{ and } J_2^2, \text{ but not of } J_{1,z} \text{ and } J_{2,z} ! \)
Examples

Let's sum orbital angular momentum with spin 1/2: $J = L + S$

- $j = l \times \frac{1}{2}' = l + \frac{1}{2}, l - \frac{1}{2}$ for $l \neq 0$
- $j = \frac{1}{2}$ for $l = 0$

- Number of states:
  
  - $J_z | j m > = \sum_{m_s} c(m, m_s) (L_z + S_z) | l m > | \frac{1}{2} m_s >$
  
  - $= \sum_{m_s} c(m, m_s) (m_l + m_s) | l m > | \frac{1}{2} m_s >$
  
  - $= (m_l + m_s) | j m > = m | j m >$

- number of states conserved ✓
  
  sounds promising

Spin of the pions: (1) $\pi^+ \rightarrow \mu^+ + \nu_{\mu}$, (2) $\pi^0 \rightarrow 2 \gamma$ in rest frame

- (1) could have $s = \frac{1}{2} \times \frac{1}{2}' = 1, 0$ and (2) could have $s = \frac{1}{2} \times 1' = 2, 0$

- since angular distributions of $\mu$ 's and $\gamma$ 's isotropic: only $s=0$ possible!
Spin physics at RHIC

RHIC polarizes protons (see Desmonds lecture):

- polarization = fix orientation of spin, i.e. fix magnetic spin quantum number

\[ P = \frac{\# N_{\text{up}} - \# N_{\text{down}}}{\# N_{\text{up}} + \# N_{\text{down}}} \]

At RHIC at about \( P \sim 60\% \)

Why polarized protons wanted?

- Still one serious problem: proton spin \( \frac{1}{2} \) not explainable
- 'naive': proton = 3 quarks, \( \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{2} \times \frac{1}{2} \) and \( 0 \times \frac{1}{2} = 3/2 \) and \( \frac{1}{2} \)
- Spin = \( \frac{1}{2} \) should work, if two quarks 'up' and the third has 'down' orientation!
- In 1980': 'naïve' picture is wrong! .... spin crises ....
- experiments showed that 'quarks' provide only minor part of the spin!

Goal at RHIC: unravel the structure of the nucleons and explain the origin of the proton spin
Polarized protons at RHIC

What's the proton picture today?

Sea quarks

Valence quarks

Gluons

Valence and sea quarks and gluons have spin and orbital momentum!

Complicated case to sum up all spin and orbital momenta

- Parton distribution function of the quarks and gluons describe the structure

Idea: with polarized protons one gets information about the spin dependent distribution functions

- Gives mainly information about the gluon distribution and polarization since the polarized protons directly interact with the gluons

- From W production also information about quark polarization
Spin Physics at HERA

HERA = e p collider: use polarized electrons and positrons

- reached 1995 (HERMES) at about 70% e\(^+\) polarization (see Desmonds lecture)
- polarization degree depends on number of spin rotators
- in 2003: 6 rotators = 2 at each experiment (HERMES, ZEUS, H1) worked → 50%!

Why are the rotators needed?

- for physics the electrons beams have to be longitudinally polarized, since the electron mass is very small, remember spin definition for m=0
- definition of polarization via 'helicity':

\[ P = \frac{\# N_{\text{left}} - \# N_{\text{right}}}{\# N_{\text{left}} + \# N_{\text{right}}} \]

- 'automatic' polarization in a storage ring gives only transverse polarization

What are the physics goals?

- again proton spin crisis but also searches for new physics
Polarized $e^+ / e^-$ at Hermes experiment

HERMES: target experiment

- target also polarized with about 95% (polarized H atoms)

- electron can only interact with quarks that have opposite polarization:

  * interaction occurs via photon (Spin ± 1) and conserves helicity: if incoming $e^-$ right-handed, q has to be left-handed .... $ \vec{\hbar} (m_e + m_q) = \vec{\hbar} m = +1$
  
- incoming (polarized) electrons are deflected / scattered at target

- change target polarization, change in scattering leads to asymmetry

- such an asymmetry gives information about the spin structure of the quarks

One HERMES result as example:

- valence quarks contribution to the proton spin only about 28%!

- sea quarks probably only minor important

- largest contribution comes from the gluons!
Polarized $e^+/e^-$ at ZEUS and H1

What is the physics at the other HERA experiments?

a) nucleon structure: measure the (quark) parton distribution functions
   - longitudinal lepton polarization affects the neutral (Z-boson exchange) and charged (W-boson exchange) currents
   - with different choices for the lepton charge as well as the polarization, one gets complementary information about the parton distribution function

b) searches for physics beyond the Standard Model
   - test/check whether right-handed charged currents exist ('$W_R$')
   - search for scalar leptoquarks, R-parity violating supersymmetry
   - search for excited leptons, in particular neutrinos

Polarized leptons, $e^+$ and $e^-$ would enlarge the physics potential and improve the results
Spin physics at the ILC

What are the goals of the ILC?

- Discovery of New Physics (NP)
  - complementary to the LHC
  - large potential for direct searches and indirect searches
- Unraveling the structure of NP
  - precise determination of underlying dynamics and the parameters
  - model distinction through model-independent searches
- High precision measurements
  - test of the Standard Model (SM) with unprecedented precision
  - even smallest hints of NP could be observed

Beam polarization = decisive tool for direct and indirect searches!
Where are spin effects at the ILC?

Spin in physics processes:
- spin formalism to include all spin and polarization effects
- effects of beam polarization for the physics analysis

How to polarize the beams at the ILC?
- spin effects at the source

Theoretical aspects to match the required precision
- depolarization effects at the beams
Spin formalism

1. Polarized beams: introduce 'dreibein' (better 'vierbein' \((p, s^a, a=1,2,3)\) )

- longitudinal \(s^3 = 1/m (|p_1|, E_1 \hat{p}_1)\), transverse \(s^2 = (0, \hat{p}_1 \times \hat{p}_3), s^1 = (0, \hat{p}_1 \times s_2)\)

2. To include all spin and polarization effects also from the intermediate states: calculate amplitude squared \(|T|^2\) with complete spin correlations

\[
|T|^2 = |\Delta_{f_3}|^2|\Delta_{f_4}|^2 \sum_{\text{spins}} \left(P^{\lambda_f_3 \lambda_f_4}P^{*\lambda_{f_3}' \lambda_{f_4}'}\right) \times \left(Z^{\lambda_f_3 \lambda_f_4'}\right) \times \left(Z^{*\lambda_{f_3}' \lambda_{f_4}'}\right)
\]

production and decay processes are coupled by interference terms between various polarization states of the decaying fermions
Well-known statistical examples

A warm-up: gain effective polarization $P_{\text{eff}}$ and $A_{LR}$

For many processes ($V$, $A$ interactions) the cross section is given by:

$$
\sigma(P_{e^-} P_{e^+}) = (1 - P_{e^-} P_{e^+}) \sigma_0 \left[1 - P_{\text{eff}} A_{LR}\right] \quad \text{with} \quad P_{\text{eff}} = \frac{P_{e^-} - P_{e^+}}{1 - P_{e^-} P_{e^+}}
$$

- $P(e^+)$ = strong 'lucrative' factor  \[\rightarrow\] both beams should be polarized!
- $P(e^+)$ of about 60% sufficient with $\Delta P / P = 0.5\%$ for physics studies
**High precision measurements of the SM**

Process: \( e^+e^- \rightarrow Z \rightarrow f\bar{f} \) at the Z-pole

Measurement of the mixing angle \( \sin\theta_{\text{eff}} \) via left-right asymmetry \( A_{LR} \)

\[
A_{LR} = \frac{2(1 - 4\sin^2\Theta_{\text{eff}}^\ell)}{1 + (1 - 4\sin^2\Theta_{\text{eff}}^\ell)^2}
\]

Requested by physics: \( \Delta\sin\theta_{\text{eff}} < 1.3 \times 10^{-5} \)

Sensitive to tiny traces of new physics!

\( \Delta A_{LR} < 10^{-4} \) needed

Only via Blondel scheme possible:

`express \( A_{LR} \) via polarized \( \sigma_{LR}, \sigma_{LL}, \sigma_{RL}, \sigma_{RR} \)`

Polarized \( e^- \) and \( e^+ \) required

\( P(e^+) \) of 40% up to 60% sufficient!
General remarks about the coupling structure

Def.: left-handed = \( P(e^\pm) < 0 \) 'L' 

right-handed = \( P(e^\pm) > 0 \) 'R'

Which configurations are possible in annihilation channels?

contributions from LR, RL: SM and (?) NP (\( \gamma, Z \))

contributions only from LL, RR: NP!

Which configurations are possible in scattering channels?

depends on \( P(e^+) \)!

helicity of e- not coupled with helicity of e+!

depends on \( P(e^-) \)!
**Supersymmetry (SUSY)**

Why SUSY today and here? ➡ unique extension of Poincare group!

- $Q |\text{boson}\rangle = |\text{fermion}\rangle$ and $Q |\text{fermion}\rangle = |\text{boson}\rangle$

- SUSY operator $Q$ changes spin (behaviour under spatial rotations) by $\frac{1}{2}$

- Invariance under local coordinate change: general relativity can be included!

- SUSY transformation influences in general both space-time and internal quantum numbers!

Particle spectrum -- compared to the Standard Model:

![Particle Spectrum Diagram](image-url)
Supersymmetry (SUSY): gives fermions a bosonic partner and vice versa

- all quantum numbers except the spin identical for SM $\leftrightarrow$ SUSY partner!
- i.e. left-, right-handed electrons need scalar SUSY partners, the left and right selectrons:

\[
\begin{align*}
\tilde{e}_L^-, R^+ & \quad \text{and} \quad \tilde{e}_R^+, L^- \\

\text{Problem: scalar particles have spin zero and have no helicity} & \quad \text{.....}
\end{align*}
\]

but they should carry the L,R quantum number in their couplings

How to prove experimentally that such SUSY partners exist?

Strategy: a) study pair production with polarized beams

b) extract the pair \[ \tilde{e}_R^-, e_L^+ \] from other produced particles

c) this pair reflects the unique relation between SM and SUSY partner!
Test of SUSY quantum numbers

Association of chiral electrons to scalar partners and:

\[ e_{L,R}^- \leftrightarrow \tilde{e}_{L,R}^- \]
\[ e_{L,R}^+ \leftrightarrow \tilde{e}_{L,R}^+ \]

s-channel \hspace{1cm} t-channel

1. separation of scattering versus annihilation channel

2. test of 'chirality': only may survive at \( P(e^-) > 0 \) and \( P(e^+) > 0 \)!

Even high \( P(e^-) \) not sufficient, \( P(e^+) \) is substantial!
**Transversely polarized beams**

Use spin rotators to rotate longitudinally into transversely polarized beams

\[
\begin{align*}
\text{spin rotator} = \text{\textquoteleft magnets\textquoteright} \\
\end{align*}
\]

Only effects if polarized e- and e+ !

*effect proportional to \[ P(e^-) P(e^+) \] \]

→ sensitive to CP-violating effects

→ CP-asymmetries w.r.t. azimuthal angle

→ rather large values even for small CP-phases!

Further examples in the polarization POWER report (hep-ph/0507011)!
Beam polarization at linear colliders

Polarized beams at linear e⁻e⁺ colliders:

- synchrotron radiation due to longitudinal acceleration negligible
- beams have to be polarized at the source!

Polarized e⁻ source:

- at the SLAC Linear Collider (SLC): excellent e⁻ polarization of about 78%
- led to precision measurement of the weak mixing angle:
  \[ \sin \theta_{\text{eff}} = 0.23098 \pm 0.00026 \] (SLD)
  \[ \text{(LEP: } 0.23221 \pm 0.00029 \text{)} \]

Polarized sources at the ILC:

- expected e⁻ polarization between 80% and 90%
- e⁺ polarization as an absolute technical innovation:
  expected polarization about 60% with full intensity
How to produce (polarized) $e^+$ at the ILC?

- Conventional scheme: only unpolarized $e^+$:
  - Challenges: thermal stress in target
  - Thick rotating target needed (360 m/s)

- Undulator-based scheme: polarized $e^+$ via circularly polarized photons
  - Thin target sufficient, $e^+$ polarization depends on undulator length
  - (alternative scheme: instead of undulator use multiple laser scattering)
Prototypes of e+ sources for the ILC

Polarized schemes are absolute innovations, do prototypes exist?

- proof of principle: experiment at SLAC 'E166'

use 50 GeV e- beam at SLAC in conjunction with 1m long helical undulator
→ photons on target  → analyze polarization of γ's and e+

- Institutes: SLAC, DESY, Daresbury, etc.

- physics runs in 2005: polarized e+ successfully verified!

Prototype of a helical undulator for ILC beam parameters

- currently under construction at RAL and Daresbury

- collaboration within heLiCal group (working group of Cockcroft Institute: CCLRC, DESY, Durham, Liverpool -- 'All aspects of the e+ production for the ILC')

ILC baseline design for the e+ source

- undulator-based source!
  'the polarized source'
What is the required precision?

Precision for polarization required by physics: $\Delta P/P = 0.5\% \rightarrow 0.25\%$

Possible sources of depolarization at the ILC:

- cradle-to-grave analysis of depolarization effects: task of the heLiCal group!
- probably largest depolarization during beam-beam interaction at the IP!

Depolarization effects during beam-beam interaction:

a) spin precession:

$$\text{p(e^-, S)} \rightarrow \text{B(e^+)} \rightarrow \text{p(e^+)}$$

Thomas -- Bargman-Michel-Telegdi (T-BMT) equation:

$$S = \Omega \times S$$

b) spin-flip processes due to synchrotron radiation (Sokolov-Ternov effect):

in case of colliding beams known as: beamstrahlung
Beam-beam interactions: higher-order processes

Absolutely needed for precision measurements at linear colliders:

- precise knowledge about all possible depolarization effects
- only one analytically-based simulation code exists: CAIN
- processes included so far in approximations and not with complete spin effects

Higher-order QED effects exactly with full spin correlations needed

- apply the spin density formalism, calculate these higher-order processes and provide precise theoretical predictions for the depolarization effects
- relevant for both spin precession process (T-BMT in strong fields) and synchrotron radiation processes (so far only in virtual $\gamma$ approximation)

within the UK heLiCal collaboration

Important for all linear collider designs (ILC, CLIC)
Expected size of depolarization effects

- Polarimeters measure polarization before or after the IP
  → different from the actual polarization in the physics process

- Expected size of depolarization at the ILC with 500 GeV:
  → measurement before IP: $\Delta P \approx +0.3\%$ and measurement after IP: $\Delta P \approx -0.4\%$

- Estimates for depolarization at CLIC with 3 TeV:
  → CLIC (2001): total depolarization about 25%, effective lumi-weighted about 7%

- To match the required precision: update of CAIN program absolutely needed!
  → comparison with existing Monte-Carlo-based codes Guinea-Pig / Merlin
How to flip the helicity of the e+?

- Precision requirements e.g. at the Z-pole ('GigaZ')
  - Need of e- and e+ polarization with both helicities
  - to fulfill the high precision: fast flipping needed!
  - e- flipping no problem, only change laser polarity can be done bunch-by-bunch

Helicity flip of e+?

- current proposal for flipping λ(e+): two parallel spin rotators in damping rings, fast kickers change between them can be done pulse-by-pulse (sufficient for physics requirements)
  - depolarization due to spin rotators about 3%

Alternative idea for helicity flipping of the e+

- tricky combination of different undulator sectors...... more elegant, cheaper
  - Under work
Conclusions, part 2

- Sum of angular momenta: eigenvalues + states
- Spin physics at RHIC: polarized p to unravel the proton spin crisis
- Spin physics at HERA: polarized e\(^-\)/e\(^+\) to unravel the proton spin crisis and new physics
- Spin effects at the ILC: polarized e\(^-\) and e\(^+\)
  - precise environment at the ILC allows to exploit spin effects of intermediate particles (spin correlations)
  - beam polarization at the ILC: crucial to determine the structure of new physics

- Undulator -based (polarized) positron source for the ILC baseline design!
- 'Cradle-to-grave' spin tracking needed to get all possible 'depolarization' effects under control
  - needed to match the required precision for physics!
Some literature ...

Derivation of the Dirac equation with group theory:
- R. Omnes, Introduction to Particle Physics
- Ryder, Relativistic Quantum Mechanics

'Historical' derivation of the Dirac equation:
- Bjorken/Drell, Relativistic Quantum Mechanics
- Bjorken/Drell, Relativistic Quantum Field Theory

Group theory:
- Wu-Ki Tung, Group Theory in Physics

Phenomenology:
- H. Haber, `Spin Formalism', Proc. of 21\textsuperscript{th} SLAC Summer Inst. , Stanford 1993
- TESLA TDR, HERA TDR
- Polarization at the ILC: POWER report, hep-ph/0507011