Exercises

Topic 2

(1) Prove that $\omega_0 = \hat{t} \times \dot{\hat{t}}$.

Topic 3

(1a) Consider a flat ring consisting of a continuous dipole. Use the phase function $2\pi\nu_0 s/C$. Sketch the orientation and motion of the vectors $\hat{l_0}, \hat{n}_0, \hat{m}_0$ and the vectors $\hat{l}, \hat{n}_0, \hat{m}$.

(1b) Consider a flat ring consisting of alternating dipoles and drifts of the same length. Sketch the orientation and motion of the vectors $\hat{l_0}, \hat{n}_0, \hat{m}_0$ and the vectors $\hat{l}, \hat{n}_0, \hat{m}$.

(2) Explain, without lots of explicit mathematics, why it's obvious that rotation matrices are orthogonal: $R^T R = I$ or $RR^T = I$.

(3) Consider a flat ring for which the magnetic field on the design orbit is vertical. The design-orbit spin tune is ν_0. Add a very short section of radial field at one point on the ring and imagine that it doesn't change the orbit. Discuss the qualitative behaviour of $\hat{n}_0$ as this ν_0 approaches an integer.
Relate your observation to a familiar concept in spin-orbit dynamics.

(4) Prove that when the 3×3 matrix A is antisymmetric, $\exp(A)$ is an orthogonal 3×3 matrix.

(5) Prove that at least one eigenvalue of an orthogonal 3×3 matrix is 1.

(6) Consider spin motion on the design orbit of a flat ring. Write down the 3×3 spin transport matrix for one turn around the ring in terms of $\nu_0 = 2\pi a\gamma$. Then calculate the complex eigenvalues of the matrix in the form of complex exponentials and comment on the values of the exponents.

Topic 4

(1) Why is it difficult to run with polarisation in the VEPP4M ring when it is set up for producing $\tau^{\pm}$ pairs?

(2) Use an integrating factor and semi-quantitative arguments to establish that $\sqrt{(P_l^2 + P_m^2)}$ settles down to $O\left((c\tau_0)^{-1}/2\pi\nu_0\right)$.

Topic 5

(1) The spin components γ, α, β transform according to the group $SO(3)$.
Under which matrix group do $\gamma, \alpha + i\beta, \alpha - i\beta$ transform?
$SO(3)$ is a N parameter subgroup of which group? What is N?

(2) Prove that the exponential of an antihermitian matrix is unitary.

(3) What is $\exp(\lambda J_{2\times 2})$ for some parameter λ and what has this to do with calculating the matrix G.

Topic 6

(1) Compare the difficulties of maintaining the polarisation of protons and muons during acceleration to high energy.

(2) Why would it be a bad idea to adopt the 2-snake layout of RHIC for an electron storage ring?