Supersymmetric GUTs

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In unbroken SU(5) Grand Unified Theories (GUTs) we expect

$$g_{GUT} = g_s = g = \sqrt{5/3}\, g' \tag{1}$$

and

$$\sin^2\theta_W = \frac{g'^2}{g^2 + g'^2} = \frac{3/5\, g_{GUT}^2}{g_{GUT}^2 + 3/5\, g_{GUT}^2} = \frac{3}{8}\,. \tag{2}$$

However, these parameters are not observed in low-energy experiments. Furthermore, we do not observe interactions through the 12 additional gauge bosons of SU(5). Thus the GUT symmetry must be broken at a high scale M_{GUT}. In that case, the gauge couplings of the three Standard Model groups evolve independently below the GUT scale due to the renormalization of divergent radiative corrections to the gauge vertices.

1 Evolution of Gauge Couplings

The evolution of the gauge couplings g_i ($g_3 = g_s$, $g_2 = g$, $g_1 = \sqrt{5/3}\, g'$) is described by the beta function:

$$\beta_i \equiv \frac{dg_i}{d\ln(\mu/\mu_0)} = \mu\,\frac{dg_i}{d\mu} = \frac{1}{16\pi^2}\, b_i g_i^3 + \mathcal{O}(g_i^5), \tag{3}$$

where the constants b_i are determined from the one-loop corrections to the gauge vertices. The beta function can also be rewritten in terms of $\alpha_i = g_i^2/4\pi$

$$\mu\,\frac{d\alpha_i}{d\mu} = \mu\,\frac{d(g_i^2/4\pi)}{d\mu} = \mu\,\frac{g_i}{2\pi}\frac{dg_i}{d\mu} = \frac{1}{2\pi}\, b_i\alpha_i^2 + \mathcal{O}(\alpha_i^3), \tag{4}$$

The first order beta function for the inverse gauge couplings becomes very simple,

$$\mu\,\frac{d\alpha_i^{-1}}{d\mu} = -\frac{1}{\alpha_i^2}\cdot\mu\,\frac{d\alpha_i}{d\mu} = -\frac{1}{2\pi}\, b_i\,, \tag{5}$$

and can easily be integrated using separation of variables:

$$\int_M^\mu d\alpha_i^{-1}(\mu') = -\frac{1}{2\pi}\, b_i \int_M^\mu \frac{d\mu'}{\mu'} \tag{6}$$

leading to to the evolution equation

$$\alpha_i^{-1}(\mu) = \alpha_i^{-1}(M) - \frac{1}{2\pi}\, b_i \ln(\mu/M). \tag{7}$$

Under the assumption that there are no new particles (except for SM singlets) between the electroweak (or SUSY mass) scale and the GUT scale, M_{GUT} and the unified gauge coupling α_{GUT} can be determined from the intersection of two gauge couplings according to equation (1):

$$\alpha_{GUT}^{-1} = \alpha_i^{-1}(m_Z) - \frac{b_i}{2\pi} \ln(M_{GUT}/m_Z) = \alpha_j^{-1}(m_Z) - \frac{b_j}{2\pi} \ln(M_{GUT}/m_Z) \tag{8}$$

$$M_{GUT} = m_Z \exp\left(\frac{2\pi}{b_i - b_j} \left(\alpha_i^{-1}(m_Z) - \alpha_j^{-1}(m_Z)\right)\right) \tag{9}$$

In order to solve these equations we need the measured input parameters at the electroweak scale:

$$\begin{aligned} \alpha_{em}^{-1}(m_Z) &\simeq 128\,, \\ \sin^2\theta_W(m_Z) &\simeq 0.231\,, \\ \alpha_s(m_Z) &\simeq 0.118\,, \\ m_Z &\simeq 91.2\,\text{GeV}. \end{aligned} \tag{10}$$

From these we can determine α_1 and α_2 at the electroweak scale:

$$\begin{aligned} \alpha_1^{-1}(m_Z) &= \frac{3}{5}\frac{\cos^2\theta_W(m_Z)}{\alpha_{em}(m_Z)} = 59.1\,, \\ \alpha_2^{-1}(m_Z) &= \frac{\sin^2\theta_W(m_Z)}{\alpha_{em}(m_Z)} = 29.6\,. \end{aligned} \tag{11}$$

In addition we need to know the constants b_i for $SU(3)_c$, $SU(2)_L$ and $U(1)_Y$. These depend on the particle content of the theory.

1.1 Standard Model

In the Standard Model the constants in the one-loop beta functions are given by

$$b_i = \frac{2}{3}\, T(R_i)d(R_j)d(R_k) + \frac{1}{3}\, T(S_i)d(S_j)d(S_k) - \frac{11}{3}\, C_2(G_i). \tag{12}$$

The fermion multiplets transform according to the representation R_i with respect to G_i and the bosons according to S_i. $T(R)$ is the normalization of the generators T^a in the representation R:

$$\mathrm{Tr}\left[T^a T^b\right] = T(R)\delta^{ab}. \tag{13}$$

The generators of the fundamental representation of SU(N) are usually normalized to $T(R) = 1/2$. $d(R)$ is the dimension of the representation R and $C_2(R)$ is the quadratic Casimir operator for the representation R:

$$T^a_{ik} T^a_{kj} = C_2(R)\delta_{ij}. \tag{14}$$

It is easy to see that $T(R)$ and $C_2(R)$ are related by the identity

$$C_2(R)d(R) = T(R)d(G), \tag{15}$$

where $d(G)$ is the dimension of the adjoint representation (i.e. the number of generators of the group). For the adjoint representation of SU(N) we have

$$T(G) = C_2(G) = N. \tag{16}$$

For a representation of U(1) one has $C_2(G) = 0$ and $T(R) = Y^2$, where Y is the appropriately normalized hypercharge.

For SU(3) we only have contributions from the SU(3) quark triplets

$$b_3 = \frac{2}{3}\cdot\frac{1}{2}\left(2\cdot 1 + 1\cdot 1 + 1\cdot 1\right) N_G - \frac{11}{3}\cdot 3 = \frac{4}{3}\,N_G - 11. \tag{17}$$

In the SU(2) case we have contributions from one SU(2) quark doublet, one SU(2) lepton doublet and SU(2) Higgs doublets

$$\begin{aligned} b_2 &= \frac{2}{3}\cdot\frac{1}{2}\left(3\cdot 1 + 1\cdot 1\right) N_G + \frac{1}{3}\cdot\frac{1}{2}\left(1\cdot 1\right) N_H - \frac{11}{3}\cdot 2 \\ &= \frac{4}{3}\,N_G + \frac{1}{6}\,N_H - \frac{22}{3} \end{aligned} \tag{18}$$

and in the U(1) case we have contributions from all particles according to their hypercharge (with a normalization factor of 3/5)

$$\begin{aligned} b_1 = \frac{2}{3}\cdot\frac{3}{5}\Bigg[&\left(\frac{1}{6}\right)^2\cdot 2\cdot 3 + \left(-\frac{2}{3}\right)^2\cdot 1\cdot 3 + \left(\frac{1}{3}\right)^2\cdot 1\cdot 3 + \left(-\frac{1}{2}\right)^2\cdot 2\cdot 1 \\ &+ 1^2\cdot 1\cdot 1\Bigg] N_G + \frac{1}{3}\cdot\frac{3}{5}\left[\left(-\frac{1}{2}\right)^2\cdot 2\cdot 1\right] N_H \\ = \frac{4}{3}\,N_G + \frac{1}{10}\,N_H\,. \end{aligned} \tag{19}$$

Name	Scalars ϕ^i	Fermions χ_L^i	$(\mathrm{SU}(3)_c,\, \mathrm{SU}(2)_L)_Y$
Sleptons, leptons	$\tilde{L}^i = \begin{pmatrix} \tilde{\nu}_L^i \\ \tilde{e}_L^{-\,i} \end{pmatrix}$	$L^i = \begin{pmatrix} \nu_L^i \\ e_L^{-\,i} \end{pmatrix}$	$(\mathbf{1},\, \mathbf{2})_{-\frac{1}{2}}$
	$\tilde{E}^{*\,i} = \tilde{e}_R^{-\,*\,i}$	$E^{c\,i} = e_R^{-\,c\,i}$	$(\mathbf{1},\, \mathbf{1})_{+1}$
Squarks, quarks	$\tilde{Q}_h^i = \begin{pmatrix} \tilde{u}_{L,h}^i \\ \tilde{d}_{L,h}^i \end{pmatrix}$	$Q_h^i = \begin{pmatrix} u_{L,h}^i \\ d_{L,h}^i \end{pmatrix}$	$(\mathbf{3},\, \mathbf{2})_{+\frac{1}{6}}$
	$\tilde{U}_h^{*\,i} = \tilde{u}_{R,h}^{*\,i}$	$U_h^{c\,i} = u_{R,h}^{c\,i}$	$(\bar{\mathbf{3}},\, \mathbf{1})_{-\frac{2}{3}}$
	$\tilde{D}_h^{*\,i} = \tilde{d}_{R,h}^{*\,i}$	$D_h^{c\,i} = d_{R,h}^{c\,i}$	$(\bar{\mathbf{3}},\, \mathbf{1})_{+\frac{1}{3}}$
Higgs, higgsinos	$H_d = \begin{pmatrix} H_d^0 \\ H_d^- \end{pmatrix}$	$\tilde{H}_d = \begin{pmatrix} \tilde{H}_d^0 \\ \tilde{H}_d^- \end{pmatrix}$	$(\mathbf{1},\, \mathbf{2})_{-\frac{1}{2}}$
	$H_u = \begin{pmatrix} H_u^+ \\ H_u^0 \end{pmatrix}$	$\tilde{H}_u = \begin{pmatrix} \tilde{H}_u^+ \\ \tilde{H}_u^0 \end{pmatrix}$	$(\mathbf{1},\, \mathbf{2})_{+\frac{1}{2}}$

Table 1: Chiral supermultiplets of the MSSM.

We can summarize these results and insert the number of fermion generations $N_G = 3$ and Higgs doublets $N_H = 1$:

$$\begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} = N_G \begin{pmatrix} \frac{4}{3} \\ \frac{4}{3} \\ \frac{4}{3} \end{pmatrix} + N_H \begin{pmatrix} \frac{1}{10} \\ \frac{1}{6} \\ 0 \end{pmatrix} - \begin{pmatrix} 0 \\ \frac{22}{3} \\ 11 \end{pmatrix} = \begin{pmatrix} \frac{41}{10} \\ -\frac{19}{6} \\ -7 \end{pmatrix}. \tag{20}$$

Using these parameters the gauge couplings do not unify at the high scale. The three intersection points range from $M_{GUT} \simeq 10^{13}\,\mathrm{GeV}$ to $M_{GUT} \simeq 10^{17}\,\mathrm{GeV}$ and correspond to a unified coupling in the range $\alpha_{GUT}^{-1} \simeq 40$ to $\alpha_{GUT}^{-1} \simeq 47$.

Thus new physics beyond the Standard Model is needed to make GUTs viable. One possibility is to introduce additional particles in the SM. For instance six Higgs doublets at $\mathcal{O}(m_Z)$ lead to a unification of gauge couplings at $M_{GUT} \simeq 4\times 10^{13}\,\mathrm{GeV}$. Another possibility is to change the particle content due to the introduction of supersymmetry.

1.2 Minimal Supersymmetric Standard Model

In the supersymmetric case there is a scalar partner for each fermion that transforms according to the same representations with respect to the gauge groups. Thus the contributions of fermions and bosons to b_i can be combined in one term. The fermionic partners of the gauge bosons, the gauginos, also transform according to the adjoint representation and therefore alter the

Name	Gauge bosons	Gauginos	$(\mathrm{SU}(3)_c,\, \mathrm{SU}(2)_L)_Y$
B boson, bino	$A_\mu^{(1)} = B_\mu$	$\lambda^{(1)} = \tilde{B}$	$(\mathbf{1},\, \mathbf{1})_0$
W bosons, winos	$A_\mu^{(2)\,a} = W_\mu^a$	$\lambda^{(2)\,a} = \tilde{W}^a$	$(\mathbf{1},\, \mathbf{3})_0$
gluons, gluinos	$A_\mu^{(3)\,a} = G_\mu^a$	$\lambda^{(3)\,a} = \tilde{g}^a$	$(\mathbf{8},\, \mathbf{1})_0$

Table 2: Gauge supermultiplets of the MSSM.

coefficient of the $C_2(G_i)$ term. One obtains:

$$b_i = T(R_i) d(R_j) d(R_k) - 3\, C_2(G_i). \tag{21}$$

Inserting the particle content of the MSSM one has

$$b_3 = \frac{1}{2}\,(2 \cdot 1 + 1 \cdot 1 + 1 \cdot 1)\, N_G - 3 \cdot 3 = 2\, N_G - 9, \tag{22}$$

$$\begin{aligned} b_2 &= \frac{1}{2}\,(3 \cdot 1 + 1 \cdot 1)\, N_G + \frac{1}{2}\,(1 \cdot 1 + 1 \cdot 1)\, N_{2H} - 3 \cdot 2 \\ &= 2\, N_G + N_{2H} - 6 \end{aligned} \tag{23}$$

and

$$\begin{aligned} b_1 = \frac{3}{5}\Bigg[&\left(\frac{1}{6}\right)^2 \cdot 2 \cdot 3 + \left(-\frac{2}{3}\right)^2 \cdot 1 \cdot 3 + \left(\frac{1}{3}\right)^2 \cdot 1 \cdot 3 + \left(-\frac{1}{2}\right)^2 \cdot 2 \cdot 1 \\ &+ 1^2 \cdot 1 \cdot 1 \Bigg] N_G + \frac{3}{5}\left[\left(-\frac{1}{2}\right)^2 \cdot 2 \cdot 1 + \left(\frac{1}{2}\right)^2 \cdot 2 \cdot 1 \right] N_{2H} \\ = 2\, N_G + \frac{3}{5}\, N_{2H}\, . \end{aligned} \tag{24}$$

Inserting the number of fermion generations $N_G = 3$ and the number of pairs of Higgs doublets $N_{2H} = 1$ the results can be summarized as:

$$\begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} = N_G \begin{pmatrix} 2 \\ 2 \\ 2 \end{pmatrix} + N_{2H} \begin{pmatrix} \frac{3}{5} \\ 1 \\ 0 \end{pmatrix} - \begin{pmatrix} 0 \\ 6 \\ 9 \end{pmatrix} = \begin{pmatrix} \frac{33}{5} \\ 1 \\ -3 \end{pmatrix} \tag{25}$$

With these altered coefficients the one-loop calculation results in a unification of gauge couplings at $M_{GUT} \simeq 2 \times 10^{16}$ GeV, corresponding to a unified gauge coupling $\alpha_{GUT}^{-1} \simeq 24$.

In fact, using the precision input parameters from LEP, one has to consider the two-loop beta function for the evolution of the gauge couplings including also one-loop threshold effects due to particle masses (at the electroweak scale and also at the GUT scale).

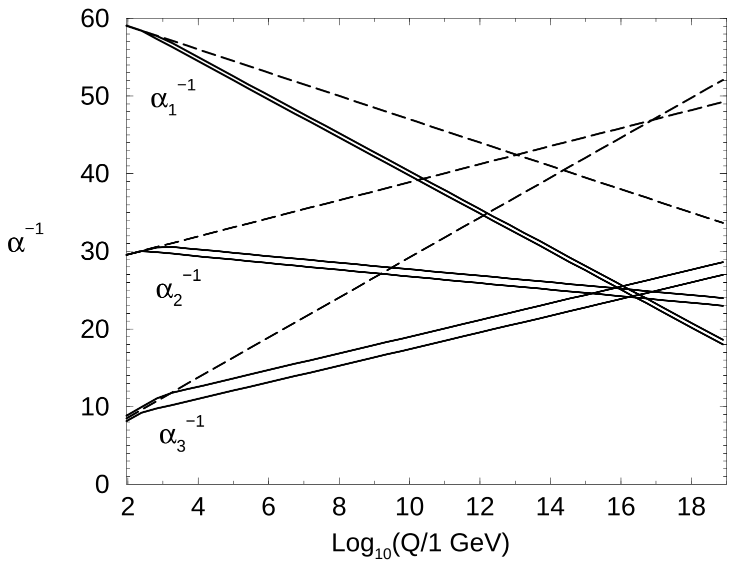

Figure 1: Renormalization group evolution of the inverse gauge couplings in the Standard Model (dashed) and the MSSM (solid) including two-loop effects. The sparticle mass thresholds are varied between 250 GeV and 1 TeV. Figure taken from [4].

2 Higgs Sector of Minimal SU(5) SUSY GUT

In order to break the SU(5) GUT group down to $SU(3)_c \times SU(2)_L \times U(1)_Y$ one introduces a new Higgs multiplet Σ in the adjoint representation (**24**) of SU(5). To maintain the electroweak symmetry breaking to $SU(3)_c \times U(1)_{em}$ and to generate masses for the fermions two additional Higgs multiplets H (**5**) and $\bar{H}$ ($\bar{\mathbf{5}}$) that contain the usual SU(2)-doublet Higgs multiplets as well as color-triplet partners of these have to be included.

The superpotential of the minimal SUSY SU(5) model is given by

$$\begin{aligned} W = & \frac{1}{2} fV \operatorname{Tr} \Sigma^2 + \frac{1}{3} f \operatorname{Tr} \Sigma^3 + \lambda \bar{H}_\alpha \left(\Sigma^\alpha_\beta + 3\, V \delta^\alpha_\beta \right) H^\beta \\ & + \sqrt{2} Y_d^{ij} \psi_i^{\alpha\beta} \phi_{j\alpha} \bar{H}_\beta + \frac{1}{4} Y_u^{ij} \varepsilon_{\alpha\beta\gamma\delta\epsilon} \psi_i^{\alpha\beta} \psi_j^{\gamma\delta} H^\epsilon, \end{aligned} \tag{26}$$

where $i, j = 1, 2, 3$ are generation indices and Greek indices are SU(5) indices. The chiral superfields ψ (**10**) and ϕ ($\bar{\mathbf{5}}$) are left-handed matter supermultiplets. The adjoint Higgs multiplet is given by

$$\Sigma = \Sigma^a T^a = \begin{pmatrix} \Sigma_{(8,1)} & \Sigma_{(3,2)} \\ \Sigma_{(\bar{3},2)} & \Sigma_{(1,3)} \end{pmatrix} + \frac{1}{2\sqrt{15}} \begin{pmatrix} 2 \cdot \mathbf{1}_{3\times3} & 0 \\ 0 & -3 \cdot \mathbf{1}_{2\times2} \end{pmatrix} \Sigma_{(1,1)} \,. \tag{27}$$

We neglect the Yukawa part for this discussion and rewrite the superpotential as

$$W = \frac{1}{2}\, fV\Sigma^a\Sigma^b\, \mathrm{Tr}\, T^aT^b + \frac{1}{3}\, f\Sigma^a\Sigma^b\Sigma^c\, \mathrm{Tr}\, T^aT^bT^c. \tag{28}$$

Now we can substitute $\mathrm{Tr}\, T^aT^b = 1/2\, \delta^{ab}$ and

$$\begin{aligned}\mathrm{Tr}\, T^aT^bT^c &= \mathrm{Tr}\, \frac{1}{2}\, T^a \left(\left[T^b,\, T^c\right] + \left\{T^b,\, T^c\right\}\right) \\ &= \mathrm{Tr}\, \frac{1}{2}\, T^a \left(f^{bcd}T^d + d^{bcd}T^d\right) = \frac{1}{4}\left(f^{abc} + d^{abc}\right)\end{aligned} \tag{29}$$

with $f^{abc} = 2\, \mathrm{Tr}\, T^a \left[T^b,\, T^c\right]$ and $d^{abc} = 2\, \mathrm{Tr}\, T^a \left\{T^b,\, T^c\right\}$. The superpotential becomes

$$W = \frac{1}{4}\, fV \left(\Sigma^a\right)^2 + \frac{1}{12}\, fd^{abc}\Sigma^a\Sigma^b\Sigma^c. \tag{30}$$

The conditions for a SUSY conserving minimum of the superpotential are

$$\frac{\partial W}{\partial \Sigma^a} = 0, \quad \frac{\partial W}{\partial H} = 0, \quad \frac{\partial W}{\partial \bar{H}} = 0 \tag{31}$$

and they are at the same time approximate conditions for the minimum of the scalar potential. We find

$$\frac{\partial W}{\partial \Sigma^a} = \frac{1}{2}\, fV\Sigma^a + \frac{1}{4}\, fd^{abc}\Sigma^b\Sigma^c = 0 \tag{32}$$

and see that $\langle \Sigma^a \rangle = 0$ is a solution that does not break SU(5). Therefore we choose a VEV for $\Sigma_{(1,1)}$ and use

$$d^{12\,12\,12} = 4\, \mathrm{Tr}\, T^{12}T^{12}T^{12} = 4\, \frac{3 \cdot 2^3 - 2 \cdot 3^3}{2^3 \sqrt{15}^3} = \frac{1}{\sqrt{15}} \tag{33}$$

giving the condition

$$\frac{1}{2}\, f \left\langle \Sigma_{(1,1)} \right\rangle \left(V - \frac{1}{2\sqrt{15}} \left\langle \Sigma_{(1,1)} \right\rangle \right) = 0 \tag{34}$$

that is solved by $\left\langle \Sigma_{(1,1)} \right\rangle = 2\sqrt{15}\, V$, so that

$$\langle \Sigma \rangle = \left\langle \Sigma_{(1,1)} \right\rangle T^{12} = V \begin{pmatrix} 2 & & & & \\ & 2 & & & \\ & & 2 & & \\ & & & -3 & \\ & & & & -3 \end{pmatrix}. \tag{35}$$

This VEV breaks SU(5) to SU(3)× SU(2)×U(1). In a similar way one can find a VEV that breaks SU(5) to SU(4)×U(1):

$$\langle\Sigma\rangle = V \begin{pmatrix} 1 & & & & \\ & 1 & & & \\ & & 1 & & \\ & & & 1 & \\ & & & & -4 \end{pmatrix} . \tag{36}$$

These three minima are degenerate and the minimal SU(5) SUSY GUT does not explain why the breaking to SU(3)× SU(2)×U(1) should be chosen.

Now we want to check the other two conditions in equation (31):

$$\begin{aligned} \frac{\partial W}{\partial H} &= \lambda \bar{H} \left(\Sigma + 3\,V \cdot \mathbf{1}\right) = 0\,, \\ \frac{\partial W}{\partial \bar{H}} &= \lambda \left(\Sigma + 3\,V \cdot \mathbf{1}\right) H = 0\,. \end{aligned} \tag{37}$$

These conditions require that the color-triplet components do not acquire a VEV. On the other hand, a VEV for the SU(2)-doublet Higgs bosons does not destroy the picture and allows for electroweak symmetry breaking at low energy. Inserting the VEV into the superpotential we see that it is fine-tuned such that the SU(2) Higgs doublets are massless while the color-triplet Higgs bosons obtain a mass parameter

$$M_{H_c} = M_{\bar{H}_c} = 5\lambda V. \tag{38}$$

The Higgs VEV $\langle\Sigma\rangle$ generates masses for the SU(5) gauge bosons X and Y. This can be seen from the kinetic Lagrangian of the **24** Higgs bosons:

$$\mathcal{L} \supset \mathrm{Tr}(D_\mu\Sigma)^\dagger(D^\mu\Sigma) \tag{39}$$

with the covariant derivative of the adjoint Higgs matrix

$$D_\mu\Sigma = \partial_\mu\Sigma + ig_{GUT}\left[A_\mu,\, \Sigma\right]. \tag{40}$$

The mass terms for the gauge bosons are then given by

$$g_{GUT}^2\,\mathrm{Tr}\left[A_\mu,\, \langle\Sigma\rangle\right]^2. \tag{41}$$

The Standard Model gauge bosons commute with $\langle\Sigma\rangle$ and therefore remain massless. The masses for the X and Y bosons become

$$\begin{aligned} & g_{GUT}^2 V^2\,\mathrm{Tr}\left[\begin{pmatrix} & & & X_1 & Y_1 \\ & & & X_2 & Y_2 \\ & & & X_3 & Y_3 \\ X_1 & X_2 & X_3 & & \\ Y_1 & Y_2 & Y_3 & & \end{pmatrix},\begin{pmatrix} 2 & & & & \\ & 2 & & & \\ & & 2 & & \\ & & & -3 & \\ & & & & -3 \end{pmatrix}\right]^2 \\ & = -50\,g_{GUT}^2 V^2\left(X_1^2 + X_2^2 + X_3^2 + Y_1^2 + Y_2^2 + Y_3^2\right) \end{aligned} \tag{42}$$

$$\Rightarrow \qquad M_X = M_Y = 5\sqrt{2}\, g_{GUT} V\,. \tag{43}$$

We will call this mass the GUT scale and from $\alpha_{GUT}^{-1} \simeq 24$ we find $g_{GUT} \simeq 0.7$ and therefore $V \simeq 4 \times 10^{15}$ GeV.

In addition one can find the masses for the **24** Higgs bosons from the first two terms in the superpotential. The results are:

$$M_{\Sigma_{(8,1)}} = M_{\Sigma_{(1,3)}} = \frac{5}{2} fV, \tag{44}$$

$$M_{\Sigma_{(3,2)}} = M_{\Sigma_{(\bar{3},2)}} = 5\sqrt{2}\, g_{GUT} V\,, \tag{45}$$

$$M_{\Sigma_{(1,1)}} = \frac{1}{2} fV. \tag{46}$$

Open Questions In the minimal SU(5) SUSY GUT model the mass of the **5** and $\bar{\mathbf{5}}$ Higgs bosons has to be fine-tuned such that the SU(2) doublet Higgs bosons have vanishing mass and the color-triplet Higgs bosons have masses of $\mathcal{O}(M_{GUT})$ in order to suppress proton decay. This is known as the doublet-triplet splitting problem.

There are several models in the literature to obtain massless Higgs doublets without explicit fine-tuning of the parameters in the model:

- In the sliding singlet model one introduces a singlet superfield Z and adds a term $\lambda \bar{H} Z H$ to the superpotential. This would alter equation (37) to

  $$\lambda\,(\Sigma + Z)\, H + MH = 0 \tag{47}$$

  and for a nonvanishing VEV of the Higgs doublet the new singlet acquires a VEV such that $-3\,V\lambda + \lambda\,\langle Z \rangle + M = 0$.

- The missing doublet model uses a superpotential without a mass term $M\bar{H}H$ and gives masses to the color-triplet Higgs bosons through mixing with particles in another representation. To achieve this one has to use larger representations of SU(5): Instead of the **24** one uses a **75** and one introduces the additional **50** and $\bar{\mathbf{50}}$. The Higgs doublet has no partner in **50** and therefore remains massless.

Apart from the problem to generate the doublet-triplet splitting it is not clear that the color-triplet Higgs bosons are heavier than the GUT scale: In a perturbative theory the dimensionless couplings have to be small. Thus we have $\lambda \lesssim 1$ and the mass of the color-triplet Higgs bosons becomes

$$M_{H_c} = M_{\bar{H}_c} = 5\lambda V \lesssim M_{GUT}. \tag{48}$$

Thus the Higgs triplet mass might be too low to suppress rapid proton decay.

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