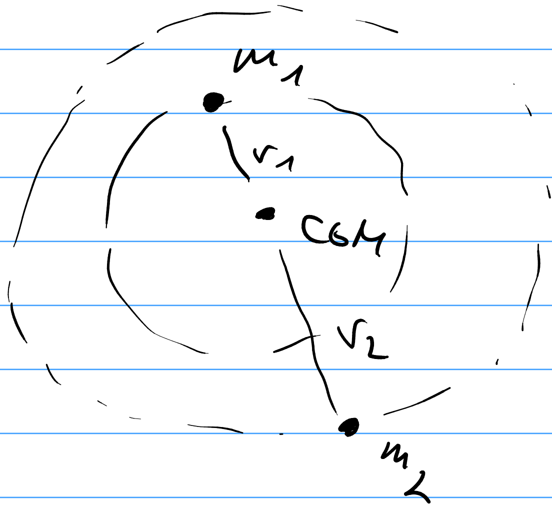
CoM: Center of mass

step 2:



forces:

$$r_1 m_1 \omega^2 = r_2 m_2 \omega^2 = \frac{m_1 m_2}{R^2} G$$

$$\boxed{\frac{r_1}{r_2} = \frac{m_2}{m_1}}$$

$$1 + \frac{r_1}{r_2} = 1 + \frac{m_2}{m_1} \Rightarrow \frac{r_2 + r_1}{r_2} = \frac{m_1 + m_2}{m_1}$$

$$\frac{r_2}{R} = \frac{m_1}{M}$$

$M = m_1 + m_2$

$R = r_1 + r_2$

quadrupole:

$$D_{ij} = \sum_{(n)} \vec{x}_i^{(n)} x_j^{(n)} T_{(n)}^{00}$$

$$T_{(n)}^{00} = m_{(n)}$$

$$\vec{x}_1 = r_1 \begin{pmatrix} \cos \omega t \\ \sin \omega t \\ 0 \end{pmatrix}$$

$$x_2 = r_2 \begin{pmatrix} -\cos \omega t \\ -\sin \omega t \end{pmatrix}$$

$$D \propto (m_1 r_1^2 + m_2 r_2^2) \begin{pmatrix} \\ \\ \end{pmatrix}$$

same as in lecture!

$$D \propto \left(m_1 \frac{m_2^2}{M^2} R^2 + m_2 m_1^2 / M^2 \, R^2 \right)$$

$$\propto \underbrace{\frac{m_1 m_2}{M}}_{\mu} R^2$$

reduced mass

step 3:

$$E_{kin} = \frac{1}{2} m_1 \underbrace{\omega^2 r_1^2}_{v^2} + \frac{1}{2} m_2 \omega^2 r_2^2$$

$$= \frac{1}{2} \omega^2 \mu R^2$$

$$E_{pot} = - G \frac{m_1 m_2}{R}$$

$$\cancel{m_1} r_1 \omega^2 = \frac{\cancel{m_1} m_2}{R^2} G$$

$$\boxed{\frac{R}{M} \omega^2 = \frac{G}{R^2}} \qquad \boxed{R^3 \omega^2 = GM}$$

$$E_{pot} = - \frac{R^2}{M} \omega^2 \cdot m_1 m_2 = - \mu \omega^2 R^2$$

$$= -2 E_{kin}$$

$$\boxed{E_{tot} \propto \mu \omega^2 R^2}$$

$$P = \dot{E}_{tot}$$

$$P_{GW} \propto \frac{2G}{5} \omega^6 \left[D_{ij} D_{ij}^* - \frac{1}{3} |D_{ii}|^2 \right]$$

$$P_{GW} \propto G\omega^6 \mu^2 R^4$$

$$E = \mu\omega^2 R^2 \propto \mu\omega^2 \left(\frac{GM}{\omega^2}\right)^{2/3}$$

$$\propto \mu\ \omega^{2/3} (GM)^{2/3}$$

$$\dot{E} = \mu\omega^{-1/3}\dot{\omega}\ (GM)^{2/3}$$

$$\stackrel{!}{=} G\omega^6\mu^2 \left(\frac{GM}{\omega^2}\right)^{4/3}$$

$$= G^{7/3}\omega^{10/3}\mu^2 M^{4/3}$$

$$\dot{\omega}\omega^{-11/3} = G^{5/3} \underbrace{M^{2/3}\mu}_{\mathcal{M}^{5/3}}$$

$$\boxed{\mathcal{M} = M^{2/5}\mu^{3/5}}$$

$$\omega^{-8/3} = G^{5/3} t\, \mathcal{M}^{5/3}$$

$$\boxed{\omega \propto G^{-5/8}\, t^{-3/8}\, M^{-5/8}}$$