Vectors and tensors

We have seen that the coordinates transform according to

 $x''^{\beta} = \Lambda_{x}^{\beta} x''; \quad \eta_{x\beta} \Lambda_{y}^{\alpha} \Lambda_{s}^{\beta} = \eta_{ys}$

In general any quantity that transforms as

 $\sqrt{\beta} = \sqrt{\beta} \sqrt{\alpha}$

is called a contravariant vector.

In contrast there are also covariant vectors that transform as

 $U_{B} = \Lambda_{R}^{\alpha} U_{\alpha}$ where

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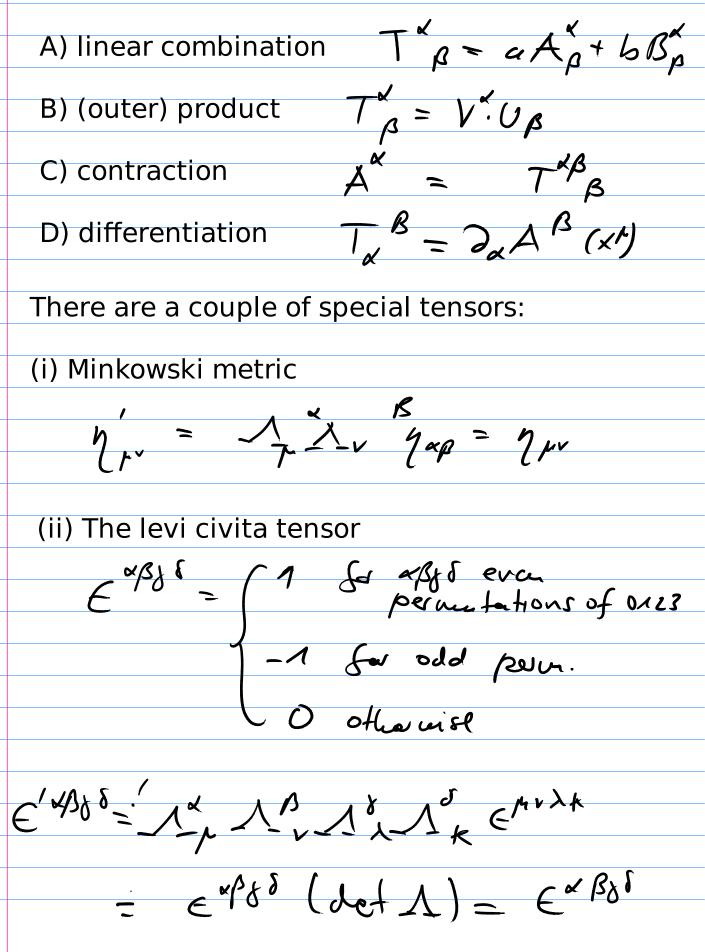
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Notice that these Lorentz trafos are inverse to each other in the sense

 $\frac{\Lambda^{x}}{\beta} = \frac{\Lambda^{x}}{\beta} = \frac{\Lambda^{y}}{\beta} =$ It follows that the contraction of contravariant and covariant vectors are scalars (invariant). $U_{\alpha}^{\prime} V^{\prime \alpha} = \Lambda_{\alpha}^{\mu} \Lambda_{\alpha}^{\prime} U_{\mu} V^{\nu} = U_{\mu} V^{\mu}$ Contravariant vectors can be transformed into covariant vectors using the metric = Maplip NUB AR US My Agus $\Delta_{\alpha}(\gamma_{v_{\lambda}}(x)) = \angle$ Another example for a scalar is the line element y dxtdx V 25

The differential Oxt is contraviariant. $x'' = \Lambda''_{\nu} \times V'$ The derivative operator is covariant: $\frac{d}{dx'} = \frac{dx'}{dx'} \frac{d}{dx'} = \Lambda_{\mu} \frac{d}{dx'}$ $u = \partial \mu = \Delta \mu \vee \partial \nu$ We can use this to contruct covariant vector fields out of scalar fields. q(xt); Drq(xt) yn quqqq Scalas cov. vectal Scalar xrdry scala There are also quantities with several indices. These are tensors. The indices can be either covariant or contravariant. TUBI = ALASTNU

New tensors can be constructed from old ones via

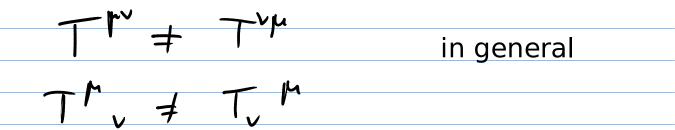


For the proof notice that the RHS is totally antisymmetric and that for $\sqrt{3}\times\delta$ =0123 the expression is the determinant.
antisymmetric and that for $\sqrt{3} \times \delta = 0123$
 the expression is the determinant.
 (iii) zero tensors

comments:

Since η_{μ} and η^{μ} are tensors, they can be used to lower and raise indices.

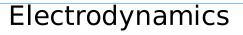
Notice that the order of the indices is important

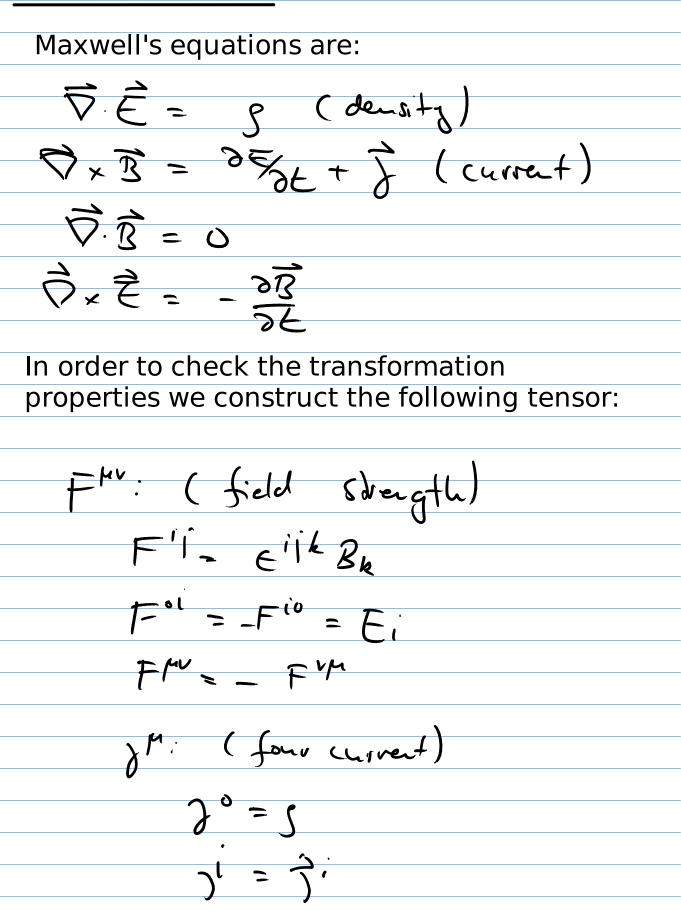


Remember that Λ_{ρ} and Λ_{ρ} are inverse of each other each other

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The Maxwell equations then read $\frac{d}{dx} \propto F^{\alpha\beta} = -\gamma^{\beta}$ $\begin{array}{c} \epsilon^{\mu}\beta\delta & \frac{d}{dx}\beta & F_{\beta} = 0 \\
 \epsilon_{x}^{\mu}\delta\delta & \frac{\partial}{\partial\rho}F^{\lambda} = 0 \\
\end{array}$ $F^{\mu\nu} = \begin{pmatrix} 0 & \overline{E} \\ 0 & B_3 - B_2 \\ \overline{E} & 0 & B_3 - B_3 \\ \overline{E} & 0 & B_3$ F'r = NBNa For $\Lambda: (^{1}R)$ z'- RÊ $\vec{R}' = \vec{R}\vec{B} - i erecise$