Using the equation of state Ansatz, one obtains for energy-momentum conservation

$$\frac{\partial}{\partial t}\left(e a^3\right) = -3 p a^3 \frac{\dot{a}}{a}$$

$$(\Rightarrow) \quad \frac{\partial}{\partial a}\left(e a^3\right) = -3 p a^2 \approx -3 c e a^2$$

$$\frac{1}{1+c} \frac{de}{e} \simeq -3 \frac{da}{a}$$

$$e \approx a^{-3(1+c)}$$

$\underline{c = 0:}$ matter domination

$$e \propto a^{-3} \; ; \; \left(e a^3\right) = E_{total} = const.$$

$$e \propto 1/V$$

The energy density is made from the mass of the particles in the plasma.
The energy density goes down with the volume (~dilution).

$\underline{c = \frac{1}{3}:}$ radiation domination

$$e \propto a^{-4}$$

There is still dilution due to the volume increase, but also the wavelength of the radiation (think photons) is stretched.

$\underline{c = -1}$: Cosmological constant (CC)

$$\rho = \text{const} \ , \quad E_{total} \propto a^3 \propto V$$

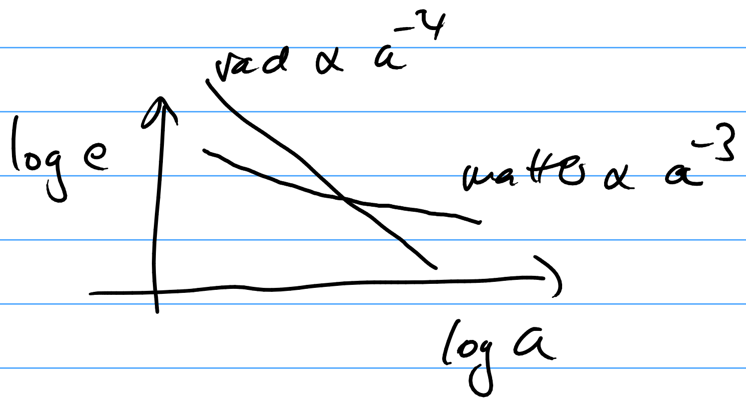
$c = -\frac{1}{3}$: curvature

$$\rho \propto a^{-2}$$

Now imagine that there are different components in teh Universe, e.g. there is a dominant contribution of radiation and a subdominant contribution by matter.

Since the Universe is expanding, the radiation is more diluted than matter

$$RD \propto a^{-4} \quad ; \quad MD \propto a^{-3}$$



In general: radiation->matter->curvature->CC

In our Universe, curvature seems to be small. We had radiation domination in the beginning, then matter domination and entered CC domination some time ago.

Friedmann equation:

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho \propto a^{-3(1+c)}$$

$$\frac{da}{a^{-\frac{3(1+c)}{2}+1}} = dt$$

$$a(t) \propto t^{\frac{2}{3(1+c)}}$$

RD $\left(c = \frac{1}{3}\right)$ MD $(c=0)$ CC $(c=-1)$ curvature $(c=-\frac{1}{3})$

$a \propto t^{1/2}$ $a \propto t^{2/3}$ $a \propto e^{Ht}$, $H = \frac{\dot{a}}{a} = \text{const}$ $a \propto t$

Some comments:

- A static Universe requires a negative CC. This was proposed by Einstein ('big blunder')

- Olbers paradox: night sky would be bright if the Universe was homogenous+static+infinite

- if the Universe was dominated by radiation or matter in the beginning, the age of the Universe is finite ->
  there was a very hot and dense state in the beginning -> big bang

- redshift:

$ds^2 = 0$ for light $\quad$ $= 0$ for a radial ray

$$dt^2 = a^2(t) \frac{1}{1-kr^2} \left(dr^2 + \underbrace{r^2 d\Omega^2}_{=0 \text{ for a radial ray}}\right)$$

$$\hookrightarrow \int_{t_i}^{t_e} \frac{dt}{a} = \int_{r_i}^{r_e} \frac{dr}{\sqrt{1-kr^2}} = f(r_e, r_i)$$

Consider the time it takes to travel from fixed r_i to r_e for subsequent light signals that are δt_i apart

$$\int_{t_i}^{t_e} \frac{dt}{a(t)} = \int_{t_i+\delta t_i}^{t_e+\delta t_e} \frac{dt}{a(t)} = \text{const.}$$

$$\hookrightarrow \frac{\delta t_e}{a(t_e)} \approx \frac{\delta t_i}{a(t_i)}$$

So if there is a periodic signal, the observed signal differs in frequency from the emitted one by a factor

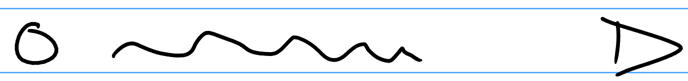
$$\frac{a(t_e)}{a(t_1)}$$

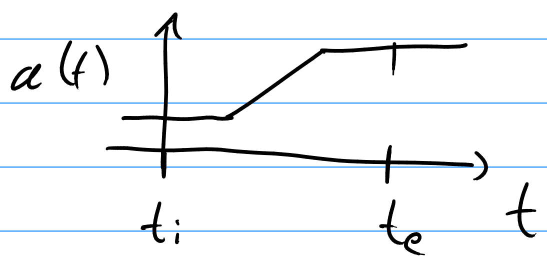
Stars that are far away are "redshifted"

There is a relationship between the distance of a star and its redshift and in leading order this relationship determines the Hubble parameter today

$$H = \frac{\dot{a}}{a}$$

There is often a misconception about this: This redshift is not equivalent to the Doppler-Effect:





no Doppler, but redshift

thermodynamics

$$dU = -p\,dV + T\,dS$$

$$\to U(V,S) \qquad p = \frac{dU}{dV}\Big|_S \qquad T = \frac{\partial U}{\partial S}\Big|_V$$

free energy:

$$F = U - TS \to F(V,T)$$

$$dF = -p\,dV - S\,dT$$

$$p = -\frac{\partial F}{\partial V}\Big|_T \qquad S = -\frac{\partial F}{\partial T}\Big|_V$$

$$F \propto V \to F = -pV$$

$$S = -\frac{\partial F}{\partial T}\Big|_V = V\frac{\partial p}{\partial T}\Big|_V = Vs$$

S entropy, s entropy density

$$Ve = U = F + TS = V(-p + Ts)$$

$$\boxed{e = T\frac{\partial p(T)}{\partial T} - p}$$

$$F = U - \frac{\partial U}{\partial S}S$$

$$\frac{\partial e}{\partial s}s = (p+e)$$

Compare this to energy-momentum conservation:

$$a \frac{\partial e}{\partial a} = -3(e+p) = -3 \frac{\partial e}{\partial s} s$$

$$(s a^3) = \text{const} = S$$

All these relations can be explicitly shown using the expression for the energy momentum tensor, e.g.

$$p = \int \frac{d^3p}{2E} f\left(\frac{E}{T}\right) p^2$$

$$T \frac{\partial p}{\partial T} = T \int \frac{d^3p}{2E} f'(E/T)\, p^2 \cdot \left(-\frac{E}{T^2}\right)$$

$$f' \frac{\partial E}{\partial p T} = \partial_p f(E/T)$$

+ partial integration

$$\hookrightarrow \frac{\partial p}{\partial T} T = e + p$$