Proton structure from DIS
and impact of the LHC data

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Atom: Electrons + Nucleus

**nucleons:** protons, neutrons
mass $M_N \sim 1$ GeV
partons (quarks & gluons)

valence quarks \( (u, d) \)

\[ M_u \approx 0.003 \, M_N, \quad M_d \approx 0.006 \, M_N \]

Where does the mass of the nucleon come from?
Feynman’s Parton Model

- The nucleon is made up of point-like constituents (partons)
- Partons behave **incoherently**
- Probability $f(x)$ for a parton $f$ to carry the fraction $x$ of the nucleon momentum is an intrinsic property of the nucleon, i.e. **process independent**

Learn about the nucleon structure via lepton-nucleon scattering
Feynman’s Parton Model

- The nucleon is made up of point-like constituents (partons)
- Partons behave incoherently
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Learn about the nucleon structure via lepton-nucleon scattering

Electron-proton scattering in parton picture

Electron scatters off a charged constituent (parton) of the proton

Identify the charged partons with quarks
Kinematics of $ep$ Scattering in Parton Model

$\gamma$ exchange

\[ x = -\frac{q^2}{2p \cdot q} \quad \text{Bjorken scaling} \]
Kinematics of $ep$ Scattering in Parton Model

Infinite proton momentum frame:

partons do not interact, move parallel to the proton, massless, no transverse momentum
parton $i$ carries fraction $x_i$ of $P_p$

\[ x = \frac{-q^2}{2p \cdot q} \quad \text{Bjorken scaling} \]
Kinematics of $ep$ Scattering in Parton Model

$\gamma$ exchange

\[ x = -\frac{q^2}{2p \cdot q} \quad \text{Bjorken scaling} \]
\[ Q^2 = -q^2 \quad \text{photon virtuality} \]
Kinematics of $ep$ Scattering in Parton Model

 Kinematics:
\[ x = -\frac{q^2}{2p \cdot q} \quad \text{Bjorken scaling} \]
\[ Q^2 = -q^2 \quad \text{photon virtuality} \]

4-momentum transfer $Q^2$ defines distance scale $r$ at which proton is probed

\[ r \approx \frac{\hbar c}{Q} = 0.2\,[\text{fm}] / Q[\text{GeV}] \]
Kinematics of $ep$ Scattering in Parton Model

\[ x = -\frac{q^2}{2p \cdot q} \]  
Bjorken scaling

\[ Q^2 = -q^2 \]  
photon virtuality

4-momentum transfer $Q^2$ defines distance scale $r$ at which proton is probed

HERA collider: $r_{\text{min}} \approx R_p/1000$
**Hadron - Electron Ring Accelerator**

**World-only $ep$ collider**

$E_p = 460-920$ GeV

$E_e = 27.6$ GeV

**protons**

**electrons**

**Unique tool to study proton structure**

HERA: 6.3km circumference accelerator of electrons and protons.

End of running 30/6/07
HERA Collider Experiments

Collider experiments H1 & ZEUS
\[ \sqrt{s_{\text{max}}} = 318 \text{ GeV} \]

Integrated luminosity
\[ \sim 0.5 \text{ fb}^{-1}/\text{experiment} \]
Deep Inelastic Scattering

Scatter both electron/positrons.

Neutral Current: $\gamma, Z^0$ exchange

Charged Current: $W^{\pm}$ exchange
**ep Scattering at HERA**

**Deep Inelastic Scattering**

\[ e^\pm, k \rightarrow e^\pm, \bar{\nu}^{\prime} \]

\[ q = k - k' \]

\[ \gamma/Z^0, W^{\pm} \]

\[ P_q = x P_p \]

- Scatter both electron/positrons
- Neutral Current: \( \gamma, Z^0 \) exchange
- Charged Current: \( W^{\pm} \) exchange

\( \gamma, Z : \) Neutral Current  \( ep \rightarrow e X \)

Isolated energetic scattered \( e^\pm \)
Deep Inelastic Scattering

\[ e^\pm, k \rightarrow e^\pm, \nu, k', q = k - k' \]
\[ \gamma/Z^0, W^\pm \rightarrow P_q = x P_p \]

Scatter both electron/positrons

Neutral Current: \( \gamma, Z^0 \) exchange

Charged Current: \( W^\pm \) exchange

\( \gamma, Z : \) Neutral Current \( ep \rightarrow e X \)

Isolated energetic scattered \( e^\pm \)

\( W^\pm : \) Charged Current \( ep \rightarrow \nu X \)

Large missing transverse momentum
**DIS Cross Section and Proton Structure**

E.g. for Neutral Current: \( e^\pm p \rightarrow e^\pm X \)

\[
\frac{d^2 \sigma^{e^\pm p}}{dxdQ^2} \propto \frac{2\pi \alpha^2}{xQ^4} \left[ (1 + (1 - y)^2) (\cancel{F_2} - y^2 F_L \mp x F_3) \right]
\]

\( y \): transferred photon energy fraction

**Quark-Parton Model:** \( F_2 \propto x \sum_f q_f + \bar{q}_f \)

**Parton Distribution Functions (PDFs):**

probability to find a parton \( q \) in a proton carrying fraction \( x \) of it’s momentum

**Bjorken scaling:** if partons do not interact, \( q = q(x); F_2 = F_2(x) \)
Quarks do interact via gluon exchange. Probability via splitting functions:

Interpretation of PDFs: number of partons in the proton, carrying momentum between $xP$ and $(x+dx)P$, as resolved at $Q^2$. $F_2(x) \rightarrow F_2(x, Q^2)$, $q(x) \rightarrow q(x, Q^2)$
Quantum Chromodynamics Picture

Quarks do interact via gluon exchange. Probability via splitting functions:

\[
\begin{align*}
\frac{\partial q(x, Q^2)}{\partial \ln Q^2} &\propto \int_x^1 \frac{dz}{z} \left[ q(z, Q^2)P_{qq} \left( \frac{x}{z} \right) + g(z, Q^2)P_{gq} \left( \frac{x}{z} \right) \right] \\
\frac{\partial g(x, Q^2)}{\partial \ln Q^2} &\propto \int_x^1 \frac{dz}{z} \left[ q(z, Q^2)P_{gq} \left( \frac{x}{z} \right) + g(z, Q^2)P_{gg} \left( \frac{x}{z} \right) \right]
\end{align*}
\]

Interpretation of PDFs: number of partons in the proton, carrying momentum between \(xP\) and \((x+dx)P\), as resolved at \(Q^2\). \(F_2(x)\rightarrow F_2(x, Q^2), \ q(x)\rightarrow q(x, Q^2)\)

Additional dependence on \(Q^2\) quantitatively described in perturbative QCD via Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) Evolution Equations

Quark and gluon distributions coupled in DGLAP equations
Scaling Violations at Highest Precision

JHEP 01 (2010) 109: combined H1 and ZEUS data from HERA I, $L \sim 115$ pb$^{-1}$

$$\sigma_r = F_2(x, Q^2) - \frac{y^2}{1 + (1 - y)^2} F_L(x, Q^2)$$

H1 and ZEUS data averaged:

- global fit of 1402 measurements
- 110 sources of systematic errors
- account for systematic correlations (cross calibration of experiments)
- total uncertainty: 1-2% for $Q^2 < 500$ GeV$^2$
- covered kinematics:
  $$10^{-7} < x < 0.65$$
  $$0.05 < Q^2 < 30000$$ GeV$^2$
Scaling Violations at Highest Precision

JHEP 01 (2010) 109: combined H1 and ZEUS data from HERA I, $\mathcal{L} \sim 115$ pb$^{-1}$

$$\sigma_r = F_2(x, Q^2) - \frac{y^2}{1 + (1 - y)^2} F_L(x, Q^2)$$

small $x$: $F_2$ rises with $Q^2$, gluon splits into quark pair

large $x$: $F_2$ falls with $Q^2$, quarks radiate gluons
QCD fit to HERA DIS Cross Sections

Neutral Current

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<tr>
<th>$x$</th>
<th>$Q^2$ GeV$^2$</th>
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Charged Current

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Result of the QCD analysis of combined HERA data: HERAPDF
Structure function factorization: for an exchange-Boson $V (\gamma, Z, W^{\pm})$

$$F_2^V(x, Q^2) = \sum_{i=q,\bar{q},g} \int_0^1 dz \times C_{2,i}^V(\frac{x}{z}, Q^2, \mu_F, \mu_R, \alpha_S) \times f_i(z, \mu_F, \mu_R)$$

$x$-dependence of PDFs is not calculable in perturbative QCD:

- parameterize at a starting scale $Q^2_0$:
  $$f(x) = Ax^B(1-x)^C(1+Dx+Ex^2)$$
- evolve these PDFs using DGLAP equations to $Q^2 > Q^2_0$
- construct structure functions from PDFs and coefficient functions:
  predictions for every data point in $(x, Q^2)$ – plane
- $\chi^2$- fit to the experimental data

determined using measured cross sections

calculable in pQCD

number of active flavors defines factorization scheme

PDF
HERA parton density functions

**HERAPDF1.5:** most precise DIS data, not yet available for other PDF groups

DGLAP fit at NLO and NNLO

Scales: $\mu_r = \mu_f = Q^2$

**Experimentally very precise**

**NLO Parameterization at starting scale:**

\[ xg(x) = A_g x^{B_g} (1 - x)^{C_g} \]

\[ xu_v(x) = A_{u_v} x^{B_{u_v}} (1 - x)^{C_{u_v}} (1 + E_{u_v} x^2) \]

\[ xd_v(x) = A_{d_v} x^{B_{d_v}} (1 - x)^{C_{d_v}} \]

\[ x\overline{U}(x) = A_{\overline{U}} x^{B_{\overline{U}}} (1 - x)^{C_{\overline{U}}} \]

\[ x\overline{D}(x) = A_{\overline{D}} x^{B_{\overline{D}}} (1 - x)^{C_{\overline{D}}} \]

**Model assumptions:**

\[ Q_0^2 = 1.9 \text{ GeV}^2, \quad \alpha_s(M_Z) = 0.1176, \quad m_c = 1.4 \text{ GeV}, \quad m_b = 4.75 \text{ GeV}, \quad f_s(Q_0^2) = 0.31 \]

Gluons and sea quarks: dominant partons at low $x$
HERAPDF1.5: most precise DIS data, not yet available for other PDF groups

HERAPDF1.5 NLO and NNLO: recommended by HERA and PDF4LHC
available in LHAPDF: allows for studies of parametrization and model input
Modern Understanding of Nucleon Structure

• Nucleon is made up of quarks exchanging gluons, gluons split in quark pairs
• Gluon and sea distributions dominate. Mass of baryonic matter: QCD energy
• Parton distribution functions: probability density finding a parton in a proton carrying its momentum fraction $x$ at scale $Q^2$

PDFs are process independent:
use this knowledge to describe nucleon in any other process
PDFs from HERA to Tevatron and LHC

PDFs:
- intrinsic property of nucleon i.e. process independent

From HERA to kinematics of Tevatron, LHC:
- evolution in $Q^2$ via DGLAP
Prediction based on HERAPDF agrees well with Tevatron measurements of jets or W and Z production.
Jet production measurement: sensitive to gluon distribution, $\alpha_S(M_Z)$

$W$ muon charge asymmetry:
敏感于 $u$ 和 $d$ 之间的差异

$$A_W = \frac{W^+ - W^-}{W^+ + W^-} \approx \frac{u_v - d_v}{u_v + d_v + 2u_{sea}}$$

$\mu$ 保有到 $W^+ + W^-$

CMS preliminary $234$ pb$^{-1}$ at $\sqrt{s} = 7$ TeV

Top pair production:
敏感于高 $x$ 处的胶子，$\alpha_S(M_Z)$

$QCD \otimes$ HERAPDF 描述了结果在不确定性范围内
Role of PDFs Precision in pp collisions

Structure: $f_i(x, Q^2) = q_i(x, Q^2), g(x, Q^2), f_i$ - beam parameters, **process independent**

Hard 2-parton interaction calculable in pQCD
Role of PDFs Precision in pp collisions

Structure: $f_i(x,Q^2) = q_i(x,Q^2), g(x,Q^2)$,
$f_i$ - beam parameters, process independent

Hard 2-parton interaction calculable in pQCD

Factorization: $PDF \otimes$ hard sub-process ME

\[
\sigma(s) = \sum_{i,j} \int_{\tau_0}^{1} \frac{d\tau}{\tau} \cdot \frac{dL_{ij}(\mu_F^2)}{d\tau} \cdot \hat{s} \cdot \hat{\sigma}_{ij}
\]

\[
\tau \cdot \frac{dL_{ij}}{d\tau} \propto \int_{0}^{1} dx_1 dx_2 (x_1 f_i(x_1, \mu_F^2) \cdot x_2 f_j(x_2, \mu_F^2)) + (1 \leftrightarrow 2)\delta(\tau - x_1 x_2)
\]
Role of PDFs Precision in pp collisions

\[ \sigma(s) = \sum_{i,j} \int_{\tau_0}^{1} \frac{d\tau}{\tau} \cdot \frac{dL_{ij}(\mu_F^2)}{d\tau} \cdot \hat{s} \cdot \hat{\sigma}_{ij} \]

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Precision of PDFs essential!

Structure: \( f_i(x, Q^2) = q_i(x, Q^2), \ g(x, Q^2) \)
\( f_i \) - beam parameters, \textit{process independent}

Hard 2-parton interaction calculable in pQCD

Factorization: PDF \( \otimes \) hard sub-process ME

proton structure | hard interaction

\[ s = 4E_1E_2 \]

\[ \hat{s} = \tau s = x_1 x_2 s \]
Example of PDF and PDF Errors

HERAPDF1.5NNLO (DGLAP, massive HQ in VFNS, 13 parameter fit)

Parameterization uncertainty:
shapes of the parton distributions at the starting scale $Q_0$

Variation of model assumptions:
fraction of strange quarks in the sea, heavy quark masses, minimum $Q^2$ of the included data

The variation of $\alpha_S (M_Z)$ has to be considered in addition

Experimental uncertainty:
errors of the data in the fit, correlations of errors between the different data sets

HERAPDF Structure Function Working Group March 2011

H1 and ZEUS HERA I+II PDF Fit

$Q^2 = 10 \text{ GeV}^2$

- HERAPDF1.5 NNLO (prel.)
- exp. uncert.
- model uncert.
- parametrization uncert.
PDFs obtained from data of fixed target, HERA, Tevatron

**HERA measurements:**
- covers most of the \((x,Q^2)\) plane,
- best constraints at low, medium \(x\)

**backbone of all available PDFs**

6 phenomenology (PDF) groups
- ABKM, CTEQ, GJR,
- HERAPDF, NNPDF, MSTW

**HERA coverage in \(x\)**
Introduction

Total cross sections versus $\sqrt{s}$ at the Tevatron ($\sqrt{s} = 1.96$ TeV) for $M_H = 180$ GeV

**NNLO gg $\rightarrow H$ at the Tevatron ($\sqrt{s} = 1.96$ TeV) for $M_H = 180$ GeV**

**NNLO (approx.) $t\bar{t}$ cross sections at the LHC ($\sqrt{s} = 7$ TeV)**

**Differences between the PDF groups:**

- data used in the fit and estimation of uncertainties
- choice of $\alpha_S$ and running of strong coupling
- different treatment of heavy quarks

**Summary**

Dominant uncertainty on HERAPDF: parameterisation, model

G. Watt

http://projects.hepforge.org/mstwpdf/pdf4lhc/
DIS:
- *ep (HERA) data*: quarks and gluon at small $x$ ($F_L$), jets (moderate $x$), flavour separation (in CC), heavy quark structure functions

- **fixed target data**: higher $x$

- **neutrino DIS**: flavor decomposition, $x>0.01$

**Drell-Yan ($\gamma, Z\rightarrow l^+l^-$):**
quark-antiquark annihilation ; high-$x$ sea quarks, deuterium target: $u/d$ asymmetry

**High $p_T$ - jets (at colliders)**: high-$x$ gluon

**W/Z production**: different quark contributions
# PDF group landscape

<table>
<thead>
<tr>
<th>PDF Order</th>
<th>MSTW08</th>
<th>CTEQ6/CT10/CT12 (prel.)</th>
<th>NNPDF2.0/2.1</th>
<th>HERAPDF1.0/1.5</th>
<th>ABKM09/ABM11</th>
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HERAPDF sets are based only on combined H1 and ZEUS data proper correlations of the systematic uncertainties: use $\Delta \chi^2 = 1$ criterion for proper statistical uncertainties; no need for nuclear corrections

HERAPDF tools allow for usage of different Heavy Flavour schemes
- close collaboration with theory groups MSTW, CTEQ, ABKM
- test and tune different pheomenological approaches

PDF fit is performed in line with the analysis of experimental data
- most precise inclusive DIS cross sections in the broad kinematic range
- semi-inclusive data provide further constraints
- test assumptions on the PDF parametrization and model parameters

examples:
- choice of $\alpha_s$ and running of strong coupling
- different treatment of heavy quarks
PDF fits using HERA jet data: fixed $\alpha_s$

Inclusive DIS data:
combined HERAI+HERAII

Jet data:
H1 high $Q^2$, *EPJ C65* (2010)

ZEUS incl. jets *PLB547* (2002)
incl.+2jets *NP B765* (2007)

PDF Fit:
- $\alpha_s(M_Z)$ fixed

Inclusion of jet data into the PDF fit using fixed $\alpha_s$ does not have large impact
PDF fits with free $\alpha_s(M_Z)$

H1 and ZEUS HERA I+II PDF Fit

Q$^2 = 10$ GeV$^2$

no jet data

HERAPDF1.5f (prel.)
free $\alpha_s(M_Z)$

- exp. uncert.
- model uncert.
- parametrization uncert.

$Q^2 = 10$ GeV$^2$

H1 and ZEUS HERA I+II PDF Fit

March 2011
PDF fits with free $\alpha_s (M_Z)$

Inclusion of jet data into the PDF fit **decouples** the gluon and $\alpha_s (M_Z)$
\( \alpha_s (M_z) \) from PDF fits including HERA jet data

Scan of the \( \alpha_s (M_z) \) in the PDF fit

PDF and \( \alpha_s (M_z) \) determined in the common fit:
\[
\alpha_s (M_z) = 0.1202 \pm 0.0013_{\text{exp}} \pm 0.0007_{\text{model/param}} \pm 0.0012_{\text{had}} + 0.0045_{\text{scale}}
\]

From including the Jet data in the PDF fit: determine gluon and \( \alpha_s (M_z) \)
Charm data in the PDF fit

Charm production probes gluon directly. **Do charm data influence the gluon?**

Heavy quark treatment in PDFs is quite some issue. Different schemes exist, (treatment of mass terms in perturbative calculation) assume different $m_c$

PDFs and quality of PDF fit using charm data is sensitive to the value of $m_c$
Charm data in the PDF fit

Study the sensitivity of the PDF fit to the value of $m_c$

⇒ pin down the value of $m_c$ for different heavy quark schemes

Different HQ schemes prefer different optimal $m_c$ ($\star$)

parameter of a specific heavy flavour scheme in PDF fit
Charm data in the PDF fit

Study the sensitivity of the PDF fit to the value of $m_c$

⇒ pin down the value of $m_c$ for different heavy quark schemes

Different HQ schemes prefer different optimal $m_c$ (★)

parameter of a specific heavy flavour scheme in PDF fit

NB:
Study in FixedFlavourNumberScheme (S.Alekhin, S.Moch, KL) allows for determination of the $m_c (\overline{\text{MS}})$ using DIS NC, CC and charm data (DIS2012 conference)

From including the charm data in the PDF fit decouple the $g(x)$ and $m_c$ in the fit
Value of $m_c$ in PDF important for W/Z at LHC

Dominant error on predictions for W and Z cross sections due to $m_c$ in PDF different heavy flavour schemes use their preferred assumptions:

- 7% error due to different schemes

\[ \chi^2 \]

\[ \sigma_{W^+} / \text{nb} \]

\[ W^+ (\sqrt{s} = 7 \text{ TeV}) \]

HERAPDF1.0 + $F_2^{c\bar{c}}$ (prel.)

August 2010

HERA Inclusive Working Group

Dominant error on predictions for W and Z cross sections due to $m_c$ in PDF different heavy flavour schemes use their preferred assumptions:
Value of $m_c$ in PDF important for W/Z at LHC

Dominant error on predictions for W and Z cross sections due to $m_c$ in PDF different heavy flavour schemes use their preferred assumptions:

Uncertainty on $\sigma_W$ prediction due to HF treatment in PDFs reduced to 1%
Tool for PDF fits: HERAFitter

Developed/supported by HERA experts, follows HERAPDF approach:
✓ experimentalists perform QCD analysis of the PDF-sensitive data and tune/test phenomenology
✓ very close collaboration with theory groups
✓ implies possibility to use different available heavy flavour schemes

Open source code, available on HEPForge: http://projects.hepforge.org/herafitter/

HERAFitter
HERAFitter is a set of PDF fitting tools jointly developed by the H1 and ZEUS collaborations for determination of the parton density functions. The HERAFitter codes were used to obtain the HERAPDF sets.

The current distribution contains a BETA-version of the first code released within the HERAFitter package, the H1FITTER program.
Tool for PDF fits: HERAFitter

**Experimental Input**
- HERA, Tevatron, LHC, fixed target
- NC, CC DIS, jets, diffraction, heavy quarks (c,b,t)
- Drell-Yan, W production

**Theoretical Calculations/Tools**
- Heavy quark schemes: MSTW, CTEQ, ABM
- Jets, W, Z production: fastNLO, Applgrid
- Top production: NNLO (Hathor)
- QCD Evolution: DGLAP (QCDNUM)
- $k_T$ factorisation
- Alternative tools: NNPDF reweighting
- Other models: Dipole model
- + Different error treatment models
- + Tools for data combination (HERAaverager)
PDF constraints from the LHC data

W and Z-boson production at LHC sensitive to the light sea decomposition:

NNLO QCD analysis using HERAFitter

Strange density enhanced wrt. assumptions following previous measurements
Isolated $\gamma$ production at the LHC (ATLAS, CMS) in QCD analysis:

direct sensitivity to $g(x)$:
D. D’Enteria, J. Rojo NPB 860 (2012) 311

Analysis using the NNPDF reweighing tool
(Update of the existing PDF probability distribution accounting the constrains from new data)

Isolated photon production at LHC: impact on $g(x)$ for $5 \times 10^{-3} < x < 0.1$:
reduction of ~20% of gluon PDF (and gg-H cross-section) uncertainty
PDF constraints from the LHC data

CMS top-pair production cross sections compared to NNLO\textsubscript{approx} (QCD):

- Experimental error much more precise than PDF uncertainty of prediction

Data sensitive to gluon distribution at large $x$:
will help constraining model parameters in QCD analysis
Understanding of the proton structure:

- one of the fundamental questions in particle physics
- prerequisite for accurate predictions (e.g. analyses and searches at LHC)

HERA DIS data provide highest precision at low and medium x

- heavy quarks and $\alpha_s$: quite some issue in QCD analyses

HERA charm and jet data provide constraints in PDF fits

- more to learn from the LHC: W&Z, top quarks, jets, prompt photons

- QCD analysis tools available: close collaboration between experiments and theory
Back up
HERAPDF1.5 NNLO is based on HERA I + II inclusive DIS data uses more flexible parametrisation then NLO

HERAPDF1.5NNLO: eigenvectors available in LHAPDF allows for many studies of parametrization and model parameters
Heavy Quarks and PDF Fits

Factorization: 

\[ F_2^V(x, Q^2) = \sum_{i=l, q, g} \int_x^1 d\frac{z}{x} \times C_2^{V,i}(\frac{x}{z}, Q^2, \mu_F, \mu_R, \alpha_S) \times f_i(z, \mu_F, \mu_R) \]

\( i \) - number of active flavours in the proton: defines the factorization (HQ) scheme
Heavy Quarks and PDF Fits

Factorization: 
\[ F_2^V(x, Q^2) = \sum_{i=1,\bar{q},g} \int_x^1 dz \times C_{2, i}^V \left( \frac{x}{z}, Q^2, \mu_F, \mu_R, \alpha_S \right) \times f_i(z, \mu_F, \mu_R) \]

\( i \) - number of active flavours in the proton: defines the factorization (HQ) scheme

• \( i \) fixed: Fixed Flavour Number Scheme (FFNS)
  
  only light flavours in the proton: \( i = 3 \ (4) \)

  \( c \) - (\( b \)) quarks massive, produced in boson-gluon fusion

  \( Q^2 \gg m_{HQ}^2 \): can be less precise, NLO coefficients contain terms \( \sim \ln \left( \frac{Q}{m_{HQ}} \right) \)
Heavy Quarks and PDF Fits

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  - \( i \) variable: Variable Flavour Number Scheme (VFNS)
    
    - Zero Mass VFNS: all flavours massless. Breaks down at \( Q^2 \sim m_{HQ}^2 \)
    
    - Generalized Mass VFNS: different implementations provided by PDF groups

smooth matching with FFNS for \( Q^2 \rightarrow m_{HQ}^2 \) must be assured
Heavy Quarks and PDF Fits

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QCD analysis of the proton structure: treatment of heavy quarks essential
Heavy Quark Mass Definition in PDFs

Usually HQ coefficient functions use a pole mass definition

BUT: pole mass defined for free quarks
Corrections due to loop integrals receive large contributions $\sim O(\Lambda_{\text{QCD}})$

large higher order corrections
bad convergence of perturbative series

Another way of defining quark mass: via renormalization
Heavy Quark Mass Definition in PDFs

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bad convergence of perturbative series

Another way of defining quark mass: via renormalization

running coupling
running mass
Heavy Quark Mass Meaning in PDFs

Massive HQ coefficient functions are calculated at NLO using pole mass
Smith et al NPB 395,162 (1993)

Used by the global fit groups: MSTW, CTEQ, ABKM, GJR, HERAPDF

ZMVFNS: $m_{HQ}$ defines a threshold at which HQ appears as an active flavour

GMVFNS: $m_{HQ}$ is also used as a parameter at which FFNS turns into VFNS
Massive HQ coefficient functions are calculated at NLO using pole mass

Used by the global fit groups: MSTW, CTEQ, ABKM, GJR, HERAPDF

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**PDG values:** 1.66±0.18 / 4.79

PDF fits assume pole mass definition for heavy quarks

Values of $m_c$ as used by most PDF groups too low wrt. PDG
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HQ treatment in PDF fits, meaning and values of HQ masses non trivial..

**Heavy quark data can help!**
Heavy Quark Production at HERA

Heavy quarks in $e p$ scattering produced in boson-gluon fusion

Contribution to total DIS cross section:

- charm: $\sim 30\%$ at large $Q^2$
- beauty: at most $1\%$

Gluon directly involved:
cross-check of $g(x)$ from NC and CC DIS cross sections

HQ contributions to the proton structure function $F_2$: (e.g. charm)

$$
\sigma^{cc} \propto F_2^{cc}(x, Q^2) - \frac{y^2}{1 + (1 - y)^2} F_L^{cc}(x, Q^2)
$$

Direct test of HQ schemes in PDF fits
Charm Data in the PDF Fit

Charm production probes gluon directly. **Do charm data influence the gluon?**

Consider 2 values of $m_c$

$m_c = 1.65$ GeV: **better fit**

steeper gluon distribution

**PDFs and PDF fit using charm data is sensitive to the value of $m_c$**
Study the sensitivity of the PDF fit to the value of $m_c$

PDF fit to inclusive DIS

Weak dependence on $m_c$
Charm Mass as a Model Parameter in PDF

Study the sensitivity of the PDF fit to the value of $m_c$

PDF fit to inclusive DIS

\[
\chi^2 / \text{ndf}
\]

$m_c^{\text{model}} \, (\text{opt}) = 1.308 \pm 0.100 \text{ GeV}$
HERAPDF1.0
RT standard
- flexible param
- standard param

Weak dependence on $m_c$

PDF fit to inclusive DIS + charm data

\[
\chi^2 / \text{ndf}
\]

$m_c^{\text{model}} \, (\text{opt}) = 1.572 \pm 0.018 \text{ GeV}$
HERAPDF1.0 + $F_2^c$ (prel.)
RT standard
- flexible param
- standard param

Strong dependence on $m_c$
Different HQ Schemes in PDFs

Value of $m_c$: how different for various HQ schemes in PDF Fits?

Test different HQ schemes
(used by different PDF groups)
Value of $m_c$: how different for various HQ schemes in PDF Fits?

Test different HQ schemes
(used by different PDF groups)

RT: MSTW PDFs
ACOT: CTEQ PDFs
ZMVFNS: NNPDF

NB: ZMVFNS does not describe charm data even at $Q^2 >> m_c^2$
Value of $m_c$ : how different for various HQ schemes in PDF Fits?

Test different HQ schemes (used by different PDF groups)

Different HQ schemes prefer different optimal $m_c$
Different HQ Schemes in PDFs

Value of $m_c$: how different for various HQ schemes in PDF Fits?

Test different HQ schemes
(used by different PDF groups)

Different HQ schemes prefer
different optimal $m_c$

Parameter of a specific
HQ scheme in PDF fits
What is the Meaning of $m_c$ in PDF Fits?

Recent theory developments: (ABKM group, DESY, arXiv:1011.5790)

HQ coefficient functions provided in $\overline{\text{MS}}$ scheme using running $m_{HQ}$

Perturbative series converge better

Consistent treatment of HQ in PDF fits

$m_c(m_c)$ determined using DIS data
Heavy Quarks in PDFs and W/Z at LHC

Prediction of $W^\pm$ cross section @ LHC: dominant uncertainty due to PDF

Prediction using $m_c=1.4$ GeV

Error band: PDF uncertainty

Experimental error

Parametrization variation

Model assumptions

including $m_c$ variation

$m_c$ variation in PDF: significant uncertainty on W@LHC in central region
Heavy Quarks in PDFs and W/Z at LHC

Vary the charm mass in the PDF. Use resulting PDFs for LHC predictions.

Larger $m_c \rightarrow$ more gluons, less charm $\rightarrow$ more light quarks $\rightarrow$ larger $\sigma_W$. 

\( W^+ (\sqrt{s} = 7 \text{ TeV}) \)
HERAPDF1.0 + $F_2^c$(prel.) 

\[
\frac{\sigma_{W^+}}{\text{nb}} \quad \text{as a function of } m_c^{\text{model}} \text{ / GeV}
\]
Heavy Quarks in PDFs and W/Z at LHC

Vary the charm mass in the PDF. Use resulting PDFs for LHC predictions

Only one HQ scheme

\[ \sigma_{W^+} (\sqrt{s} = 7 \text{ TeV}) \]

HERAPDF1.0 + \[ F_2^\sigma \text{(prel.)} \]

\[ m_c \] variation in PDF

\[ 1.4 < m_c < 1.65 \text{ GeV} \]

3% uncertainty on W prediction
Heavy Quarks in PDFs and W/Z at LHC

Vary the charm mass in the PDF. Use resulting PDFs for LHC predictions

Several HQ schemes

\[ W^+ (\sqrt{s} = 7 \text{ TeV}) \]
\[ \text{HERAPDF1.0 + } F_2^{\sigma} \text{(prel.)} \]

\[ \sigma_{W^+} / \text{nb} \]

\[ m_c \text{ variation in PDF} \]
\[ 1.4 < m_c < 1.65 \text{ GeV} \]

3% uncertainty on W prediction

Using different HQ schemes:

+ 7% uncertainty

Large uncertainty on \( \sigma_W \) prediction due to HQ treatment in PDFs
Charm at HERA and W/Z at LHC

Use the optimal $m_c$ for HQ schemes in PDFs fixed by HERA charm data

Uncertainty on $\sigma_W$ prediction due to HQ treatment in PDFs reduced to 1 %

★ Optimal $m_c$ using $F_2 + F_2^c$

ZMVFNS not considered
Combination Procedure

Minimized value:

$$\chi^2 (\bar{m}, \bar{b}) = \sum_i \left( \frac{m^i - \sum_j \gamma_j^i m^i b_j - \mu^i}{(\delta_{i,\text{stat}}^i \mu^i)^2 + (\delta_{i,\text{unc}} m^i)^2} \right) + \sum_j b_j^2$$

- $\mu^i$ measured value at point $i$
- $\delta_i$ statistical, uncorrelated systematic error
- $\gamma_j^i$ – correlated systematic error
- $b_j$ – shift of correlated systematic error sources
- $m^i$ – true value (corresponds to min $\chi^2$)

Measurements performed sometimes in slightly different range of $(x, Q^2)$ swimming to the common $(x, Q^2)$ grid via NLO QCD in massive scheme
Think of scattering of longitudinal and transverse polarized photons:

\[ y \text{ (or } Y = 1 \pm (1-y)^2 \text{) related to photon polarization} \]

Cross section:

\[ \sigma \sim \sigma_T + \frac{2(1-y)}{Y} \sigma_L \]

Kinematics:

\[ x = -\frac{q^2}{2p \cdot q} \text{ Bjorken scaling variable} \]
\[ Q^2 = -q^2 \text{ photon virtuality} \]
\[ y = \frac{p \cdot q}{p \cdot k} \text{ transferred } \gamma \text{ energy fraction} \]
**ep Scattering in Quark-Parton Picture**

Think of scattering of longitudinal and transverse polarized photons:

\[ y \quad \text{or} \quad Y_\pm = 1 \pm (1-y)^2 \] related to photon polarization

**Cross section:**

\[ \sigma \sim \sigma_T + \frac{2(1-y)}{Y_+} \sigma_L \]

- transverse polarized \( \gamma \) helicity \( \pm 1 \)
- longitudinally polarized \( \gamma \) helicity 0

Parton Model: scattering off a quark \( (s=\frac{1}{2}) \):

**Kinematics:**

- \( x = -\frac{q^2}{2p \cdot q} \) Bjorken scaling variable
- \( Q^2 = -q^2 \) photon virtuality
- \( y = \frac{p \cdot q}{p \cdot k} \) transferred \( \gamma \) energy fraction

Helicity conservation \( \Rightarrow \sigma_L = 0 \)
Cross Section of ep scattering expressed via proton structure functions

\[
\frac{d^2\sigma}{dx dQ^2} = \frac{2\pi\alpha^2}{xQ^4} \left[ (1 + (1 - y)^2)F_2 - y^2 F_L \pm xF_3 \right]
\]

measured

Kinematics:

\( x = -q^2 / 2p \cdot q \)  Bjorken scaling variable
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Proton Structure Functions

Cross Section of ep scattering expressed via proton structure functions

\[
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\]

measured

Quark-Parton-Model:

\[
F_L \sim \sigma_L = 0
\]

\[
F_2 = \sum_q x e_q^2 (q(x) + \bar{q}(x))
\]

Parton Distribution Functions (PDFs):

probability to find a q in a proton carrying x fraction of its momentum

Kinematics:

\[
x = -\frac{q^2}{2p \cdot q} \quad \text{Bjorken scaling variable}
\]

\[
Q^2 = -q^2 \quad \text{photon virtuality}
\]

\[
y = p \cdot q / p \cdot k \quad \text{transferred } \gamma \text{ energy fraction}
\]
Another way to access the gluon directly: $F_L$

Remind of photon-scattering: $F_2 \sim (\sigma_T + \sigma_L)$, $F_L \sim \sigma_L$

Angular momentum conservation: spin $\frac{1}{2}$ quark absorbs spin-1 photon

**QPM**

$$\begin{aligned}
&\text{quark helicity } \pm \frac{1}{2}, \quad F_L = 0 \\
&\text{off-shell quarks may absorb longitudinal photons}
\end{aligned}$$

**QCD**

```
\[
F_L = \frac{\alpha_s}{4\pi} x^2 \int_x^1 \frac{dz}{z^3} \left[ \frac{16}{3} F_2 + 8 \sum_q e_q^2 \left(1 - \frac{x}{z}\right) z g(z) \right]
\]
```

**QCD**

- **Quarks** radiating a gluon
- **Gluons** splitting into quarks
Extraction of $F_L$

**Measurements @ Same $Q^2, x$**

- $E_p = 920 \text{ GeV}$
- $E_p = 575 \text{ GeV}$
- $E_p = 460 \text{ GeV}$

**Intercept:** $F_2$

**Slope:** $F_L$

**HERA Inclusive Working Group**

**March 2010**
HERA PDF Fits at NNLO

QCD using NNLO PDF predicts different $F_L$ shape

H1 and ZEUS

First HERA PDF Fits at NNLO:

Ihapdf grids available  https://www.desy.de/h1zeus/combined_results/

NNLO has impact on $F_L$ at low $Q^2$
HQ Contribution to the Proton Structure

Can be determined experimentally: e.g. “charm structure function”:

\[ F_{2}^{cc} \propto \frac{Q^2}{m_c^2} \int dx \frac{\epsilon^2 g(x_g, Q^2)}{x} C(...). \]

- use and combine different charm tagging methods

measure cross sections of charm and beauty production in DIS:

\[ \sigma^{cc} \propto F_{2}^{cc}(x, Q^2) - \frac{y^2}{1 + (1 - y)} F_{L}^{cc}(x, Q^2). \]

- Direct test of different schemes of HQ treatment in PDF fits

- Can be included in the full QCD analysis of DIS cross sections
  additional constrain on the gluon density in the proton
  reduce parameterization uncertainty
PDFs From HERA to Tevatron and the LHC

Kinematics in pp collisions

Center-of-mass energy:
\[ s = 4 \cdot E_1 \cdot E_2 \]

2-parton interaction:
\[ \hat{s} = x_1 \cdot x_2 \cdot s \geq M \]

Energy scale \( M = Q \)

\[ x_{1,2} = \frac{M}{\sqrt{s}} \cdot \exp(\pm y) \]

rapidity
Proton collisions at the LHC

LHC: \( p-p \) collisions at \( \sqrt{s} = 7, 10, 14 \text{ TeV} \)

Goal @ LHC: Higgs and new physics

Main challenge: Background suppression

Main Background: QCD

Hard processes > 80% gluon-gluon fusion

Cross section \( \sim |g(x)|^2 \)

*Precision of the gluon density essential!*

Luminosity: e.g. \( ud \rightarrow W^+ \rightarrow l^+\nu_l \)

*Precision of light quark densities essential!*

Key issue: understanding of the proton
Predictions based on HERAPDF in agreement with TEVATRON data.