Resume from last lecture
Approaches to beyond NLO
  Parton showers
  unintegrated pdfs
Hadronization
Diffraction

http://www-h1.desy.de/~jung/qcdCollider_physics_2005
NLO for $F_2$: $O(\alpha_s)$

NLO for dijets: $O(\alpha_s^2)$

NLO for 3-jets: $O(\alpha_s^3)$

NOTE: NLO for dijets is NOT NNLO for $F_2$
Reduced Scale Dependence in NLO

- dependence of the specific choice of the scale for renormalization and factorization shows sensitivity to higher order contributions, which are not included.
- scale is unphysical parameter
- physical observables must be independent of scale
- in NLO scale dependence significantly reduced compared to lowest order

Catani, Seymour hep-ph/9609521
**k\_t\text{-factorization and collinear NLO}**

- off-shell matrix elements (\(k_t\) – factorization) includes part of NLO corrections:

  \[ \text{LO} \quad \text{NLO} \quad \text{NLO} \]

  \[
  \begin{align*}
  k_t &= 0 \\
  k_t &= 0 \\
  k_t &= 0 \\
  k \neq 0
  \end{align*}
  \]

  \[ k_t \text{ – factorization} \]

- even soft \(k_t\) region is properly treated (not the case in part.level NLO calc)
- in addition contributions to all orders are included
“Perfect” agreement of NLO(FMNR) calc with CASCADE on quark and hadron level for x<0.01
The need for unintegrated PDFs

- using integrated pdfs ignores proper kinematics
- large NLO corr comes from wrong kinematics in LO

Collins, Zu, JHEP 03, 059 (2005)

Collinear factorization is wrong if details of final state are investigated
Need for fully unintegrated PDFs
Need for double uPDFs

\[ k^2 = -\frac{k_t^2}{1-x} \]

\[ k^2 = -\frac{k_t^2}{1-x} \left(1 + x \frac{m_{\text{rem}}^2}{k_t^2}\right) \]
Explicit parton evolution: parton showers

- use LO matrix elements
  - for light quarks, cutoffs are needed
- apply initial and final state parton showers (PS)
  - matching of cutoff in ME with parton showers
- obtain cross sections fully differential in any observable

BUT:
- only in LO (attempts to include NLO: Collins et al, MC@NLO, etc)
DGLAP evolution again and again...

- differential form:
  \[ t \frac{\partial}{\partial t} f(x, t) = \int \frac{dz}{z} \frac{\alpha_s}{2\pi} P_+(z) f \left( \frac{x}{z}, t \right) \]

- differential form using \[ f / \Delta_s \] with
  \[ \Delta_s(t) = \exp \left( - \int_{max} d\bar{z} \int_{t_0}^t \frac{\alpha_s}{2\pi} \frac{dt'}{t'} \tilde{P}_2(z) \right) \text{ with } \tilde{P}_2 \sim \frac{1}{1 - z} \]

\[ t \frac{\partial}{\partial t} \frac{f(x, t)}{\Delta_s(t)} = \int \frac{dz}{z} \frac{\alpha_s}{2\pi} \frac{\tilde{P}(z)}{\Delta_s(t)} f \left( \frac{x}{z}, t \right) \]

- integral form

\[ f(x, t) = f(x, t_0) \Delta_s(t) + \int \frac{dz}{z} \int \frac{dt'}{t'} \cdot \frac{\Delta_s(t)}{\Delta_s(t')} \tilde{P}(z) f \left( \frac{x}{z}, t' \right) \]

no – branching probability from \[ t_0 \] to \[ t \]
**DGLAP for parton showers**

\[
f(x, t) = f(x, t_0) \Delta_s(t) + \int \frac{dz}{z} \int \frac{dt'}{t'} \cdot \frac{\Delta_s(t)}{\Delta_s(t')} \tilde{P}(z) f \left( \frac{x}{z}, t' \right)
\]

- solve integral equation via explicit iteration:

\[
f_0(x, t) = f(x, t_0) \Delta(t)
\]

from \( t' \) to \( t \)

w/o branching

branching at \( t' \)

\[
f_1(x, t) = f(x, t_0) \Delta(t) + \int_{t_0}^{t} \frac{dt'}{t'} \frac{\Delta(t)}{\Delta(t')} \int \frac{dz}{z} \tilde{P}(z) f(x/z, t_0) \Delta(t')
\]

from \( t_0 \) to \( t' \)

w/o branching
Parton showers for the initial state

spacelike $(Q<0)$ parton shower evolution
- starting from hadron (fwd evolution)
  
or from hard scattering (bwd evolution)
- select $q_1$ from Sudakov form factor
- select $z_1$ from splitting function
- select $q_2$ from Sudakov form factor
- select $z_2$ from splitting function
- stop evolution if $q_2 > Q_{\text{hard}}$
Parton showers to solve DGLAP evolution

- for fixed $x$ and $Q^2$ chains with different branchings contribute
- iterative procedure, spacelike parton showering

\[
    f(x, t) = \sum_{k=1}^{\infty} f_k(x_k, t_k) + f_0(x, t_0) \Delta_s(t)
\]

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Parton showers for the final state

timelike ($Q > 0$) parton shower evolution

- starting with hard scattering

- select $q_1$ from Sudakov form factor

- select $z_1$ from splitting function

- select $q_2$ from Sudakov form factor

- select $z_2$ from splitting function

- stop evolution if $q_2 < q_0$
Matching of ME - PS

- Approximation to higher orders.....
- using initial and final state radiation according to DGLAP
- ME sets maximum scale for parton showers
- check sensitivity on particular choice

\[
\frac{1}{N} \frac{dn}{d\eta(K_\phi)}(MC)
\]

- \( ptcut = 2.5 \text{ GeV} \)
- \( ptcut = 3 \text{ GeV} \)
- \( ptcut = 4 \text{ GeV} \)
- \( ptcut = 5 \text{ GeV} \)
Hadronic final state: Energy flow

- $E_t$ flow in DIS at small $x$ and forward angle ($p$-direction):
  - $O(\alpha_s)$ processes not enough
  - even DGLAP parton showers are not sufficient at small $x$

- need higher order contributions...
Hadronic final state: Energy flow

- $E_t$ flow in DIS at small $x$ and forward angle (p-direction):
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- need higher order contributions...
- $k_t$-factorization very good !!!!!
Parton Showers versus fixed NLO

- MC@NLO

Top production at LHC

- Advantage of PS at small pt and phi
- Advantage of fixed order at large pt

Frixione, Nason, Webber hep-ph/0305252

Solid: MC@NLO
Dashed: Herwig
Dotted: NLO
Fragmentation Models

- describe transition from quarks to hadrons
  - quarks fragment independently
  - gluon are split: \( g \rightarrow q\bar{q} \)
  - fragmentation depends on momentum (energy), but *not* on virtuality
  - *not* Lorentz invariant
  - with 4 parameters can describe broad features of 2-jet and 3-jet
  - for qq is similar to independent fragmentation
  - **BUT** is covariant and has no leftover
  - constraints on fragmentation function: \( q\bar{q} \) symmetric
  - transverse momentum distribution from tunneling effect
- **Cluster Fragmentation** (Webber NPB 238 (1984) 492)
  - pre-confinement of color
  - gluon split \( g \rightarrow q\bar{q} \)
**Fragmentation: simple example**

- **process** \( e^+ e^- \rightarrow q \bar{q} \)
- \[ \frac{d\sigma}{d\cos\theta d\phi} = \frac{\alpha_{em}^2}{4s} \left( 1 + \cos^2 \theta \right) \]
- **BUT** what about fragmentation/ hadronization ???
- use concept of **local parton-hadron duality**

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**linear confinement potential:** \( V(r) \sim -1/r + \kappa r \)
with \( \kappa \sim 1 \text{ GeV/fm} \)

qq connected via color flux tube of transverse size of hadrons (~1 fm)

- color tube: uniform along its length \( \rightarrow \) linearly rising potential

\[ \rightarrow \text{Lund string fragmentation} \]
in a color neutral qq-pair, a color force is created in between

color lines of the force are concentrated in a narrow tube connecting q and q, with a string tension of:

\[ \kappa \sim 1 \text{GeV}/\text{fm} \sim 0.2 \text{GeV}^2 \]

as q and q are moving apart in qq rest frame, they are de-accelerated by string tension, accelerated back etc ... (periodic oscillation)

viewed in a moving system, the string is boosted
Lund string fragmentation (cont'd)

- color force materialize a massless qq pair on a point on the string
- string separates into two independent (color neutral) strings
  analogy with electric field coupled to particles suggest:
  \[
  \frac{dP}{dxdt} = C \exp\left(\frac{-\pi m^2}{\kappa}\right)
  \]
  ... tunneling probability through potential barrier
- production of different flavor in hadronization
  \[
  P \propto \exp\left(\frac{-\pi m^2}{\kappa}\right)
  \]
  with \( m_u = m_d = 0, m_s = 0.25 \text{ GeV}, m_c = 1.2 \text{ GeV} \)
  \[ u:d:s:c = 1:1:0.37:10^{-10} \]
- typical example of Monte Carlo approach
**Fragmentation in the String Model**

- hadronization: iterative process
- string breaks in qq pairs (still respecting color flow)
- select transverse motion with $m=m_{qq}$ (and flavor)

\[ P \sim \exp \left( -\frac{\pi m_i^2}{\kappa} \right) = \exp \left( -\frac{\pi m^2}{\kappa} \right) \exp \left( -\frac{\pi p_i^2}{\kappa} \right) \]

- suppression of heavy quark production
  \[ u : d : s : c \sim 1 : 1 : 0.37 : 10^{-10} \]
  actually leave it as a free parameter
- longitudinal fragmentation
  symmetric fragmentation function (from either $q$ or $\bar{q}$)
  \[ f(z) \sim z^a(1-z)^a \exp(-b m_t^2/z) \]
  harder spectrum for heavy quarks
- start from $q$ or $\bar{q}$
- repeat until cutoff is reached
- heavy use of random numbers and importance sampling method

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Hadronization: particle masses and decays

- particle masses
  - taken from PDG, where known, otherwise from constituent masses
- particle widths
  - in hard scattering production process short lived particles ($\rho, \Delta$) have nominal mass, without mass broadening
  - in hadronization use Breit-Wigner:
    \[ P(m)dm \propto \frac{1}{(m - m_0)^2 + \Gamma^2/4} \]
- lifetimes
  - related to widths ... but for practical purpose separated
  - \( P(\tau)d\tau \sim \exp(-\tau/\tau_0) \, d\tau \)
  - calculate new vertex position \( \nu' = \nu + \tau \, p/m \)
- decays
  - taken from PDG, where known
  - assume momentum distribution given by phase space only
  - exceptions, like \( \omega, \phi \rightarrow \pi^+ \pi^- \pi^0 \) , or \( D \rightarrow K\pi, \bar{D}^* \rightarrow K\pi\pi \)
  and some semileptonic decays use matrix elements
Gluons in string fragmentation

- process $e^+e^- \rightarrow qqg$
- watch out color flow !!!
- gluons act as kicks on strings
- string effect seen in experiment

TPC (PEP) H. Aihara, ZPC 28, 31 (1985)
Color Flow in String Fragmentation

- quarks carry color
- anti-quarks carry anticolor
- gluons carry color – anticolor
- connect to color singlet systems

\[ x_n \quad k_{tn} \]
\[ x_{n-1} \quad k_{tn-1} \]
\[ x_0 \quad k_{t0} \]

\[ q_n \quad p_{tn} \]
\[ q_{n-1} \quad p_{tn-1} \]
\[ q_1 \quad p_{t1} \]
Cluster Fragmentation

- Pre-confinement of color
- Gluon split $g \rightarrow q\bar{q}$
Rapidity Gaps during Hadronization

- assume a statistical distribution of particles, uniform in rapidity:
  \[ \frac{dN}{d\eta} \sim c \]

- all correlations between particles are local in rapidity

\[ \rightarrow \] probability of rapidity gap of size \( \Delta \eta \) is:

\[ P \sim e^{-\Delta \eta} \]

\[ \rightarrow \] coming from Poisson distribution

\[ \rightarrow \] Hadronization produces exponentially suppressed rap-gap distributions
Rapidity Gap Events: measurements

desy 94-133

H1 data
----- DIS