Announcements
What this lecture aims at, and what it does not ...

http://www-h1.desy.de/~jung/qcdColliderPhysics_2005
All what you need to know is the QCD Langrangian:

\[ \mathcal{L}_{\text{classical}} + \mathcal{L}_{\text{gauge fixing}} + \mathcal{L}_{\text{ghost}} \]

\[ \mathcal{L}_{\text{classical}} = -\frac{1}{4} F_{\alpha \beta}^A F_{A}^{\alpha \beta} + \sum_{\text{flavors}} \bar{q}_a (i \not{D} - m)_{ab} q_b \]

\[ F_{\alpha \beta}^A = \left[ \partial_\alpha A_\beta^A - \partial_\beta A_\alpha^A - g f^{ABC} A_\alpha^B A_\beta^C \right] \]

It is the third 'non-Abelian' term which distinguishes QCD from QED.
Literature

- Applications of pQCD  
  R.D. Field  
  Addison-Wesley 1989

- Basics of perturbative QCD  
  Yu. Dokshitzer, V. Khoze, A. Mueller, S. Troyan  
  Edition Frontiers 1991

- Collider Physics  
  V.D. Barger & R.J.N. Phillips  
  Addison-Wesley 1987

- Deep Inelastic Scattering.  
  R. Devenish & A. Cooper-Sarkar  
  Oxford 2

- Handbook of pQCD  
  G. Sterman et al

- High-Energy Particle Diffraction  
  V. Barone & E. Predazzi  
  Springer 2002

- Quarks and Leptons,  
  F. Halzen & A.D. Martin  
  J.Wiley 1984

- QCD and collider physics  
  R.K. Ellis & W.J. Stirling & B.R. Webber  
  Cambridge 1996

- QCD at HERA  
  M. Kuhlen  
  Springer 1999

- The Lund Model  
  Bo Andersson  
  Cambridge 1998

- The Partonic Structure of the Photon  
  M. Erdmann  
  Springer 1997

- The Structure of the Proton  
  R.G. Roberts  
  Cambridge 1990
Outline of the lectures

- 9.11. Intro to Deep Inelastic Scattering: kinematics, cross sections, QPM
- 16.11. Factorization and DGLAP
- 23.11. QCD improved parton model: $F_2$ in NLO, fact. Schemes
- 30.11. DGLAP parton showers, leading log resummations, MCs
- 7.12. Jets/Final states: LO and NLO
- 14.12. small x: BFKL/CCFM
- 21.12. Exercises

NEW YEAR

- 11.1. Heavy quarks
- 18.1. Hadronization/fragmentation
- 25.1. Diffraction
- 1.2. High density systems: multiple interactions,
- 8.2. High density systems: saturation, BK and GLR
- 15.2. Structure of the photon
- 22.2. Exercises
Homework Exercises
- Will be given after each lecture
- Hand them back, any time you want
  - for lectures this year, before 21. Dec. 2005
  - for lectures next year, before 22. Feb. 2006
- Will be checked by Zhenyu Ye (Frank) (DESY/HERMES)

Copy of transparencies as lecture notes ...

Please fill in list of participants ...
Requests to you ...

- If things go wrong .. lecture is too easy... too trivial ... too complicated, too chaotic or too boring ...
- PLEASE complain immediately!
- PLEASE ask questions any time!
The structure of matter

How can the structure of matter be probed?

➔ analogy to optics:
➔ resolve with \( d > \lambda \)

Size of charge radius of proton = 0.8 fm = 0.8 \(10^{-15}\) m

Constituents of protons/neutrons: \( d \ll r_{proton} \)

Wavelength of particle at 1eV: \( \sim 10^{-6}\) m

Use natural units: \( \hbar = c = 1 \)

Conversion:
\[ \hbar c \approx 200 \text{ MeV fm} \]
\[ (\hbar c)^2 = 0.389 \text{ GeV}^2 \text{ mbarn} \]
Four-vector kinematics

- Fundamental laws ... same form in all Lorentz frames
- Speed of light ... same in all Lorentz frames
- Special relativity: \( (E, p) \equiv (p^0, p^1, p^2, p^3) = p^\mu \equiv p \)

Basic Lorentz invariant:
\[ E^2 - p^2 \]

Free particle (on-mass shell):
\[ E^2 - p^2 = m^2 \]

Product of 4-vectors:
\[ A.B \overset{\text{def}}{=} A^0 B^0 - A B \]

Example: collision of 2 particles, one with mass \( M \)
- Head on collision, opposite momenta:
  \[ s = (p_1 + p_2)^2 = 4E_{cm}^2 \]
  \[ E_{lab} = \frac{s - M_2^2}{2M_2} \]

Rest frame (fixed target):

Literature: Halzen & Martin, chapter 3.2
Probing the Structure of Matter

- virtual photons (in analogy to optics)
- BUT also:
  - Z/W exchange
  - Jet production
  - Heavy quarks
- Example Deep Inelastic Scattering:
  - Kinematics:

\[ s = (e + p)^2 \]
\[ q = e - e' \]
\[ Q^2 = -q^2 \]
\[ y = \frac{q \cdot p}{e \cdot p} \]
\[ W^2 = (q + p)^2 \]
\[ x = \frac{Q^2}{2p \cdot q} \]

- in p-rest frame: (watch out: different def. of \( \nu \) on the market)

\[ p = (M, 0) \]
\[ \nu = \frac{p \cdot q}{M} = \frac{M q_0}{M} = E - E' \]

- using

\[ W^2 = (q + p)^2 = M^2 + 2q \cdot p - Q^2 \]
\[ \nu = \frac{Q^2 + W^2 - M^2}{2M} \]
**Inelastic Scattering: the early days**

- Elastic peak at
  \[ \nu = \frac{Q^2}{2M} \]

  or at
  \[ W \simeq 1 \text{ GeV} \]

- at larger W nucleon resonances appear

- at even larger W observe continuum excitation. In analogy to scattering on nuclei, this can be due due to quasi elastic scattering on nucleus constituents

- Scattering at large W and large momentum transfer \( Q^2 \) is called DIS

- Idea of Quark Parton Model

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**Figure 8.6** Excitation curve of inelastic \( ep \) scattering, obtained at the DESY electron accelerator (Bartel et al. 1968). \( E \) and \( E' \) are the energies of the incident and the scattered electron, and \( W \) is the mass of the recoiling hadronic state. The peaks due to the pion-nucleon resonances of masses 1.24, 1.51, and 1.89 GeV are clearly visible.

From Perkins, p270

Experiment at DESY in 1968
Deep Inelastic Scattering

- General form of DIS can be obtained from leptonic and hadronic tensor:

\[ L_{\mu\nu}^e = \frac{1}{2} \text{Tr}((k' + m)\gamma^\mu(k + m)\gamma^\nu) \]

\[ W^{\mu\nu} = -W_1 g^{\mu\nu} + \frac{W_2}{M^2} p^{\mu} p^{\nu} + \frac{W_4}{M^2} q^{\mu} q^{\nu} + \frac{W_5}{M^2} (p^{\mu} q^{\nu} + q^{\mu} p^{\nu}) \]

- And

\[ d\sigma \sim L_{\mu\nu}^e W^{\mu\nu} \]

- The hadronic tensor \( W^{\mu\nu} \) serves to parameterize our total ignorance of the form of the propagator at the proton side.

- Current conservation results in only two of the \( W \)'s are independent, which are called structure functions...

- Differential cross section as function of the structure functions \( F_1 \) and \( F_2 \):

\[ \frac{d\sigma}{dxdy} = \frac{4\pi\alpha^2 s}{Q^4} \left[ xy^2 F_1 + (1 - y) F_2 \right] \quad \text{with} \quad F_1 = MW_1 \quad F_2 = \nu W_2 \]
The early days of QPM

R.P. Feynman

Photon-Hadron Interactions (1972) p 134

... Another way to look at this is to take a dynamic view of the parts in the rest system and assume finite energy of interaction among the parts so as time goes on they change their momenta, are created or annihilated, etc... in finite times. But moving at large momentum $P$ these times are dilated by the relativistic transformation so as $P$ rises these things change more and more slowly, until ultimately they appear not to be interacting at all.

Infinite momentum frame

Quark parton model assumptions
Partons with finite $x$ and fixed transverse size are distributed on the Lorentz contracted disc and the number of partons per unit of long. Phase space $dx/x$ is rather small.

Partons with very small $x$ ($x \sim 1\text{GeV}/P$), so called wee partons are not confined to the Lorentz contracted disc, but acc. to uncertainty principle:

$$\Delta z \sim \frac{1}{(xP)}$$
Space time picture: free partons?

- Compare lifetime of proton fluctuation $\tau$ with time of interaction $\tau_0$
- In CM frame $P$ splits into $q_1 = xP$ and $q_2 = (1-x)P$, with $k_t$

\[
\tau = \frac{1}{\Delta E} \\
q_1 = (E_1, \vec{k}_t, xP) \\
q_2 = (E_2, -\vec{k}_t, (1-x)P) \\
E_1 \sim xP + \frac{1}{2} \frac{k_t^2}{xP} \\
E_2 \sim (1-x)P + \frac{1}{2} \frac{k_t^2}{(1-x)P} \\
\Delta E = E_1 + E_2 - E_0 = \frac{k_t^2}{2x(1-x)P} \sim \frac{k_t^2}{2xP} \\
\gamma^* \text{ four-vector } q = (E_\gamma, q_t, 0) \\
\text{with } x = \frac{Q^2}{2q_p} \text{ obtain } E_\gamma = \frac{Q^2}{2xP} \\
\tau_0 = \frac{1}{E_\gamma} \\
\frac{\tau}{\tau_0} \sim \frac{k_t^2}{Q^2}
\]

- Lifetime of proton fluctuation long compared to interaction time
Space time picture in $p$-rest frame

- How is the picture of proton viewed from a different frame?
- How does the photon see the proton?
Light-cone variables:

\[ V = (V^0, V^1, V^2, V^3) = (V^0, V_t, V^3) \]
\[ V^+ = \frac{1}{\sqrt{2}} (V^0 + V^3) \]
\[ V^- = \frac{1}{\sqrt{2}} (V^0 - V^3) \]
\[ V = (V^+, V^-, V_t) \]
\[ V.W = V^+W^- + V^-W^+ - V_t W_t \]
\[ V^2 = 2V^+V^- - V_t^2 \]

Boosts:

\[ V'^0 = \frac{V^0 + \nu V^3}{\sqrt{1 - \nu^2}} \]
\[ V'^3 = \frac{\nu V^0 + V^3}{\sqrt{1 - \nu^2}} \]
\[ V'^+ = V^+ e^\psi \]
\[ V'^- = V^- e^{-\psi} \]
\[ \psi = \frac{1}{2} \ln \frac{1+\nu}{1-\nu} \]
Color Dipole Picture: formation time

- In target rest frame:
  \[ q = (q^+, q^-, 0) = (q^+, \frac{-Q^2}{2q^+}, 0) \]
  Photon splits into \( q\bar{q} \) with momenta
  \[ \kappa = (zq^+, \frac{\kappa^2}{2zq^+}, \kappa) \]
  \[ \kappa' = ((1 - z)q^+, \frac{\kappa^2}{2(1-z)q^+}, -\kappa) \]
  \[ (\kappa + \kappa') = (q^+, \frac{\kappa^2}{z(1-z)q^+}, 0) \]
  \[ M_{q\bar{q}}^2 = \frac{\kappa^2}{z(1-z)} \]
  \[ \kappa^0 = \frac{1}{\sqrt{2}} \left( q^+ + \frac{\kappa^2}{2q^+ z(1-z)} \right) \]
  \[ q^0 = \frac{1}{\sqrt{2}} \left( q^+ - \frac{Q^2}{2q^+} \right) \]
- Formation time is
  \[ \Delta E = E_{pair} - E_\gamma = \frac{Q^2}{4\sqrt{2}q^+} \left( 1 + \frac{M_{q\bar{q}}^2}{Q^2} \right) \sim mx \]
  \[ \tau_f \sim \frac{1}{\Delta E} \sim \frac{1}{mx} \]
- Transverse size of the dipole is frozen during the interaction with the proton!
Inelastic Scattering: \( x\)-section, phase space, ME

- Cross section definition:
  \[
d\sigma = \frac{1}{F} d\text{Lips} \left| ME \right|^2
\]

- With initial flux
  \[
  F = 4 \sqrt{(p_1 p_2)^2 - m_1^2 m_2^2}
\]

- And Lorentz invariant phase space

\[
d\text{Lips} = (2\pi)^4 \delta^4(-p_1 - p_2 + \sum_i p_i) \sum_i \frac{d^4 p_i}{(2\pi)^3} \delta(p_i^2 - m_i^2)
\]

\[
d\text{Lips} = (2\pi)^4 \delta^4(-p_1 - p_2 + \sum_i p_i) \sum_i \frac{d^3 p_i}{(2\pi)^3 2E_i}
\]

\[
d\text{Lips} = (2\pi)^4 \delta^4(-p_1 - p_2 + \sum_i p_i) \sum_i \frac{1}{(2\pi)^3} \frac{dp_i^+}{p_i^+} d^2 p_t_i
\]
Cross section: example

In CM system for \( 2 \rightarrow 2 \) process with \( s = (p_1 + p_2)^2 \)

\[
\frac{d\sigma}{dt} = \frac{1}{64\pi s} \frac{1}{|\mathbf{p}_i^{cm}|^2} |ME|^2
\]

neglecting masses of incoming particle

\[
\frac{d\sigma}{dt} = \frac{1}{16\pi s^2} |ME|^2
\]

using

\[
|M_E(eq \rightarrow eq)|^2 = 2(4\pi\alpha)^2 \frac{s^2 + u^2}{t^2}
\]

gives for \( eq \rightarrow eq \)

\[
\frac{d\sigma}{dt} = \frac{2\pi\alpha^2}{t^2} \frac{s^2 + t^2}{s^2}
\]
Calculating matrix elements

- Calculate matrix elements involves dirac algebra
- Use symbolic manipulation programs to do trace calculation ....
- Free software FORM program by J. Vermaseren (http://www.nikhef.nl/~form/)
  - or REDUCE, MATHEMATICA, MAPLE
- Some commands in FORM:
  - Local expression to be calculated
  - Identification is a substitution or replacement

<table>
<thead>
<tr>
<th>FORM notation</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>gi_(i)</td>
<td>identity</td>
</tr>
<tr>
<td>g_(i,m)</td>
<td>$\gamma^m$</td>
</tr>
<tr>
<td>g5_(i)</td>
<td>$\gamma^5$</td>
</tr>
<tr>
<td>g6_(i)</td>
<td>$1 + \gamma^5$</td>
</tr>
<tr>
<td>g7_(i)</td>
<td>$1 - \gamma^5$</td>
</tr>
</tbody>
</table>

- Check example in FORM manual ...
Calculating matrix elements: example

Example for $e^+ e^- \rightarrow e^+ e^-$ using FORM

Source Program

Matrix Element for $e^+ e^- \rightarrow e^+ e^-$ (ala Halzen Martin, p 123, eq (6.20))
Vectors k,kp,p,pp;
Indices nu,mu;
Symbols m1,m;
Symbols s,u,t;
Symbols y,Q2;
write statistics;
Local $M_1 = \frac{(g_{(1,kp)+m1})g_{(1,mu)}(g_{(1,k)+m1})g_{(1,nu)/2}}{t}$;
Local $M_2 = \frac{(g_{(2,pp)+m})g_{(2,mu)}(g_{(2,p)+m})g_{(2,nu)/2}}{t}$;
Local $ME = M1*M2$;
trace4,1;
trace4,2;
Id p.p=m**2;
Id pp.pp=m**2;
Id k.k=m1**2;
Id kp.kp=m1**2;
Id k.kp=-t/2 + m1**2;
Id p.pp=-t/2 + m**2;
Id pp.kp=-t/2 +m**2/2 + m1**2/2;
Id p.kp=s/2 - m**2/2 -m1**2/2;
Id pp.kp=s/2 - m**2/2 -m1**2/2;
Id m1=0;
Id m=0;
*Id u = (y-1)*;s;
Bracket t,s;
print;
.end

Program Output:

Time = 0.00 sec  Generated terms = 3
$M_1$  Terms in output = 3
Bytes used = 74

Time = 0.00 sec  Generated terms = 3
$M_2$  Terms in output = 3
Bytes used = 74

Time = 0.00 sec  Generated terms = 2
$ME$  Terms in output = 2
Bytes used = 58

$M_1 =$
+ t^-1 * ( 2*k(nu)*kp(mu) + 2*k(mu)*kp(nu) )
+ d_(nu,mu);

$M_2 =$
+ t^-1 * ( 2*p(nu)*pp(mu) + 2*p(mu)*pp(nu) )
+ d_(nu,mu);

$ME =$
+ s^2*t^-2 * ( 2 )
+ t^-2 * ( 2*u^2 ) ;
**Inelastic Scattering: QPM**

- Key factor in QPM explanation is that over a short time in which the hard scattering takes place, the quarks behave as if they are free, i.e. No interaction between them.
- In the asymptotic limit \( Q^2 \rightarrow \infty \) the theory should describe quarks as free particles.
- Equivalent demanding that effective charge in theory should vanish as smaller and smaller distances are probed.
- Until 1973 in theories the reverse was true: because of screening of charge at larger distances coupling becomes smaller (QED).
- BREAK-THROUGH by 't Hooft (1972), Gross, Wilczek & Politzer (1973) non-Abelian theory describing asymptotic behaviour

**QCD**

- As in QED there is screening at large distances by the color charge of quarks and gluons, but this is more than compensated by antiscreening (splitting) of gluons. Thus for \( Q^2 \rightarrow \infty \) the effective coupling tends to vanish!