QCD and Monte Carlo simulation VI

H. Jung (DESY, University Antwerp) hannes.jung@desy.de

http://www.desy.de/~jung/qcd_and_mc_2015/

Outline of the lectures

- 12. Oct Intro to Monte Carlo techniques and structure of matter
- 13. Oct DGLAP: solution with MCs
- 26. Oct DGLAP/BFKL/CCFM: evolution for small x
- 27. Oct DGLAP/BFKL/CCFM: evolution for small x
- 16. NovW/Z production in pp and soft gluon resummation
- 19. Nov Multiparton interactions & latest LHC results: small x, multiparton interactions, QCD in high luminosity phase: Higgs as a gluon trigger
- Exercises
- 14 & 15 Oct
- 28 & 29 Oct
- 17 & 18 Nov

Recap of last lecture

From DY to $pp \rightarrow jets$

Jet production in pp

- x-section (i.e. for light and heavy quarks ($t\overline{t}$) production)
 - $\sigma(\mathbf{p}\mathbf{p}\to\mathbf{q}\bar{\mathbf{q}}\mathbf{X}) = \int \frac{dx_1}{x_1} \frac{dx_2}{x_2} x_1 G(x_1,\bar{q}) x_2 G(x_2,\bar{q}) \times \hat{\sigma}(\hat{s},\bar{q})$

 $xG(x,\bar{q})$

• hard x-section:

• with gluon densities

$$\frac{d\sigma}{dt} = \frac{1}{64\hat{s}^2} |M_{ij}|^2$$



Lowest Order Diagrams



$2 \rightarrow 2$ processes

	Process	$ar{\sum} \mathcal{M} ^2/g^4$	$\theta^* = \pi/2$
invariant matrix elements for 2 $\rightarrow 2$ processes with	$\boxed{qq' \rightarrow qq'}$	$\frac{4}{9} \frac{\hat{s}^2 + \hat{u}^2}{\hat{t}^2}$	2.22
massless partons	$q\bar{q'} ightarrow q\bar{q'}$	$\frac{4}{9} \frac{\hat{s}^2 + \hat{u}^2}{\hat{t}^2}$	2.22
	$qq \to qq$	$\frac{4}{9} \left(\frac{\hat{s}^2 + \hat{u}^2}{\hat{t}^2} + \frac{\hat{s}^2 + \hat{t}^2}{\hat{u}^2} \right) - \frac{8}{27} \frac{\hat{s}^2}{\hat{u}\hat{t}}$	3.26
	$q\bar{q} \to q'\bar{q'}$	$\frac{4}{9} \frac{\hat{t}^2 + \hat{u}^2}{\hat{s}^2}$	0.22
	$q\bar{q} ightarrow q\bar{q}$	$\frac{4}{9} \left(\frac{\hat{s}^2 + \hat{u}^2}{\hat{t}^2} + \frac{\hat{t}^2 + \hat{u}^2}{\hat{s}^2} \right) - \frac{8}{27} \frac{\hat{u}^2}{\hat{s}\hat{t}}$	2.59
	$q\bar{q} ightarrow gg$	$\frac{32}{27}\frac{\hat{t}^2 + \hat{u}^2}{\hat{t}\hat{u}} - \frac{8}{3}\frac{\hat{t}^2 + \hat{u}^2}{\hat{s}^2}$	1.04
	$gg \to q\bar{q}$	$\frac{1}{6} \frac{\hat{t}^2 + \hat{u}^2}{\hat{t}\hat{u}} - \frac{3}{8} \frac{\hat{t}^2 + \hat{u}^2}{\hat{s}^2}$	0.15
	$gq \rightarrow gq$	$-\frac{4}{9}\frac{\hat{s}^2 + \hat{u}^2}{\hat{s}\hat{u}} + \frac{\hat{u}^2 + \hat{s}^2}{\hat{t}^2}$	6.11
Ellis, Stirling, Webber	$gg \rightarrow gg$	$\frac{9}{2} \left(3 - \frac{\hat{t}\hat{u}}{\hat{s}^2} - \frac{\hat{s}\hat{u}}{\hat{t}^2} - \frac{\hat{s}\hat{t}}{\hat{u}^2} \right)$	30.4

QCD & collider physics p249

Remember: W+jet and dijets

check on propagator:

CDF Collaboration (F. Abe et al.).Phys.Rev.Lett.73:2296-2300,1994.



Color Flow in pp

- quarks carry color
- anti-quarks carry anti-color
- gluons carry color anti-color
 - connect to color singlet systems
 - watch out pp or $p\bar{p}$





Color Flow in pp

Process: $gg \rightarrow q_i \ddot{q}_i$

Diagrams:







Amplitudes:

s:
$$g^{2}f^{abc}T^{c}_{\alpha\beta}\bar{u}^{\beta}_{i}(q_{4})\frac{\epsilon_{1}^{\kappa}\epsilon_{2}^{\lambda}\gamma^{\mu}}{\tilde{s}}C_{\kappa\lambda\mu}(q_{1},q_{2},-q_{1}-q_{2})v_{i}^{\alpha}(q_{3})$$

t: $-ig^{2}T^{b}_{\alpha\gamma}T^{n}_{\gamma\beta}\bar{u}^{\beta}_{i}(q_{4})\epsilon_{1}\frac{\dot{q}_{1}-\dot{q}_{4}}{i}\epsilon_{2}v_{i}^{\alpha}(q_{3})$
u: $-ig^{2}T^{a}_{\alpha\gamma}T^{b}_{\gamma\beta}\bar{u}^{\beta}_{i}(q_{4})\epsilon_{2}\frac{\dot{q}_{1}-\dot{q}_{3}}{\hat{\mu}}\epsilon_{1}v_{i}^{\alpha}(q_{3})$

The Lund Monte Carlo For High P(T) Physics H.U. Bengtsson Comput.Phys.Commun.31:323,1984.

Colour flows:

2



String configurations:



Colour factors: A: $T^{b}_{\alpha\gamma}T^{a}_{\gamma\beta}$; B: $T^{a}_{\alpha\gamma}T^{b}_{\gamma\beta}$ Amplitudes:

$$\begin{aligned} \mathbf{A}: & -\mathrm{i} g^2 \bar{u}_i^{\beta}(q_4) \bigg[\dot{q}_1 \frac{\dot{q}_1 - \dot{q}_4}{i} \dot{q}_2 - \frac{\epsilon_1^{\kappa} \epsilon_2^{\lambda} \gamma^{\mu}}{\hat{s}} C_{\kappa \lambda \mu}(q_1, q_2, -q_1 - q_2) \bigg] v_i^{\sigma}(q_3) \\ \mathbf{B}: & -\mathrm{i} g^2 \bar{u}_i^{\beta}(q_4) \bigg[\dot{q}_2 \frac{\dot{q}_1 - \dot{q}_3}{\hat{u}} \epsilon_1 + \frac{\epsilon_1^{\kappa} \epsilon_2^{\lambda} \gamma^{\mu}}{\hat{s}} C_{\kappa \lambda \mu}(q_1, q_2, -q_1 - q_2) \bigg] v_i^{\sigma}(q_3) \end{aligned}$$

Cross-sections:

$$\mathbf{A}: \frac{\pi \alpha_{s}^{2}}{\hat{s}^{2}} \frac{1}{6} \left(\frac{\hat{u}}{\hat{t}} - 2\frac{\hat{u}^{2}}{\hat{s}^{2}} \right); \quad \mathbf{B}: \frac{\pi \alpha_{s}^{2}}{\hat{s}^{2}} \frac{1}{6} \left(\frac{\hat{t}}{\hat{u}} - 2\frac{\hat{t}^{2}}{\hat{s}^{2}} \right)$$

$$A + B = \dots \left(\frac{\hat{u}}{\hat{t}} - 2\frac{\hat{u}^2 + \hat{t}^2}{\hat{s}^2} + \frac{\hat{t}}{\hat{u}}\right) = \frac{\hat{u}^2 + \hat{t}^2}{\hat{t}\hat{u}} - 2\frac{\hat{u}^2 + \hat{t}^2}{\hat{s}^2}$$

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Color Flow in pp



Colour flows:



Jet production at the LHC



Impressive agreement of theory predictions with measurements

at least in some regions of phase-space

is there still anything left ?

Observations so far

- parton densities increase at small x
 - leads to violation of unitarity
 - need saturation
 - high densities \rightarrow multi-partonic interactions
 - experimental observations
 - multiparton radiation \rightarrow multi-jet production
- parton densities at small x
 - require introduction of k_t
 - but effects are visible at large x
 - TMD parton densities
 - new complications
 - factorization breaking

parton densities at small xviolation of unitarity

High energy behavior of x section



From H. Abramowicz A. Levy hep-ph/9712415 $\sigma(\gamma^*p) = \frac{4\pi^2\alpha}{Q^2}F_2(x,Q^2)$ $= \frac{4\pi^2\alpha}{Q^2}\sum e_q^2 xq(x,Q^2)$ $x = \frac{Q^2}{W^2 + Q^2}$

- rising x-section with W^2
- at large energies can become larger than σ_{tot}
- mechanism needed which tames rise at large energies:

→Saturation

Why this happens ?

- when

 $E = xP \sim k_{\perp}$

new effects in evolution appear:

- k_{\perp} cannot be neglected
- strong k_{\perp} ordering broken
- high energy evolution (BFKL/CCFM)



Partonic Cross sections

Cross section

$$\sigma(p_1 + p_2 \to j_1 + j_2 + X) = f(x_1, \mu)$$



- $\iota^2) \otimes$ $\hat{\sigma}(x_1p_1 + x_2p_2 \rightarrow j_1 + j_2)$ $\otimes f(x_2,\mu^2)$
- partonic cross section diverges with p_{\perp}
- calculate x-section as function of $p_{\perp,min}$

 $\sigma_{\rm hard}(p_{\perp \rm min}^2) = \int_{p_{\perp \rm min}^2} \frac{d\sigma_{\rm hard}(p_{\perp}^2)}{dp_{\perp}^2} dp_{\perp}^2$

Partonic Cross sections



Partonic x-section exceeds total x-section !!! happens even for visible range !

Partonic Cross Section

Basic partonic perturbative cross section

$$\sigma_{\rm hard}(p_{\perp\rm min}^2) = \int_{p_{\perp\rm min}^2} \frac{d\sigma_{\rm hard}(p_{\perp}^2)}{dp_{\perp}^2} dp_{\perp}^2$$

• diverges faster than $1/p_{\perp,min}^2$ as $p_{\perp,min}$ inelastic (non-diffractive) cross section

and exceeds eventually total

- Interaction x-section exceeds total xsection
- happens well above λ_{QCD}
- in perturbative region



Multiparton Interaction

Basic partonic perturbative cross section

$$\sigma_{\rm hard}(p_{\perp \rm min}^2) = \int_{p_{\perp \rm min}^2} \frac{d\sigma_{\rm hard}(p_{\perp}^2)}{dp_{\perp}^2} dp_{\perp}^2$$

• diverges faster than $1/p_{\perp,min}^2$ as $p_{\perp,min}^2$ and exceeds eventually total inelastic (non-diffractive) cross section

HOW to solve this ?

Comparison with CMS measurement



- Saturation effects are clearly seen !
- What is the reason for this ?



Parton Distribution Functions

- number of gluons in long. phase space dx/x: $xg(x, \mu^2)dx/x$
- occupation area: nr of gluons x (trans size)²

$$g(x,\mu^2)\frac{1}{\mu^2}$$

• saturation starts when:

$$\frac{\alpha_s(\mu^2)}{\mu^2} x g(x,\mu^2) \frac{dx}{x} \ge \pi R^2$$

- gluon density is very large:~ 90 or 45 Gluons !!!!!
- with $R \sim 1 \text{ GeV}^{-1}$ we obtain:

$$\frac{0.2}{10 GeV^2} 90 \sim \pi R^2 \qquad \blacksquare$$

Parton saturation



Gluon densities

- recombination best seen as function of k_t
- density is largest at small $k_t \rightarrow$ largest suppression
 - density vanishes at small k_t



Toy Model for highest energies

 relation of diffraction – multiple scatterings – saturation (AGK Abramovsky-Gribov-Kancheli ...)



• 2-parton exchange:



Small p₊ behavior of x-section

- $\sigma(p_t) \sim \alpha_s(p_T^2)/p_T^4$
- simple ansatz for taming:

$$\frac{\alpha_s^2(p_{T0}^2 + p_T^2)}{\alpha_s^2(p_T^2)} \frac{p_T^4}{(p_{T0}^2 + p_T^2)^2}$$

 or change kt dependence of uPDF at small kt



A. Grebenyuk, F. Hautmann, H. Jung, P. Katsas, and A. Knut sson. Jet production and the inelastic pp cross section at the LHC. Phys.Rev., D86:117501, 2012.



Multiparton Interaction

Basic partonic perturbative cross section

$$\sigma_{\rm hard}(p_{\perp\rm min}^2) = \int_{p_{\perp\rm min}^2} \frac{d\sigma_{\rm hard}(p_{\perp}^2)}{dp_{\perp}^2} dp_{\perp}^2$$

- diverges faster than 1/p²_{⊥,min} as p_{⊥,min} and exceeds eventually total inelastic (non-diffractive) cross section eventually total inelastic (non-diffractive) cross section, resulting in more than 1 interaction per event (multiple interactions, MI).
- Average number of interactions per event is given by:

$$\langle n
angle = rac{\sigma_{
m hard}(p_{\perp
m min})}{\sigma_{nd}}$$

 It depends on how soft interactions are treated, **BUT** also on the parton densities and factorization scheme, parton evolution (DGLAP/BFKL) !!!!!!!

Models for Multi-Parton Interaction



$$\mu = \langle n \rangle = \frac{1}{\sigma_{nd}} \int_{p_{\perp \min}}^{p_{\perp \max}} \frac{d\sigma_{\text{hard}}}{dp'_{\perp}} dp'_{\perp}$$

Formalism for MI (Sjostrand, Zijl) I

• define: $p(x_t$

$$p(x_t) = \frac{1}{\sigma_{nd}} \frac{d\sigma}{dx_t}$$

T. Sjostrand, M. Zijl PRD 36 (1987) 2019

probability for hardest scattering at x₁₁:

$$P_1 = p(x_{t1}) \exp\left(-\int_{x_{t1}}^1 p(x')dx'\right)$$

probability for second hardest scattering at x_{t2}:

$$P_2 = \int_{x_{t2}}^1 dx_{t1} p(x_{t1}) \exp\left(-\int_{x_{t1}}^1 p(x') dx'\right) p(x_{t2}) \exp\left(-\int_{x_{t2}}^{x_{t1}} p(x') dx'\right)$$

• probability for 3^{rd} hardest scattering at x_{t3} :

$$P_{3} = \int_{x_{t2}}^{1} dx_{t1} \int_{x_{t3}}^{x_{t1}} dx_{t2} p(x_{t1}) \exp\left(-\int_{x_{t1}}^{1} p(x') dx'\right)$$
$$\times p(x_{t2}) \exp\left(-\int_{x_{t2}}^{x_{t1}} p(x') dx'\right) p(x_{t3}) \exp\left(-\int_{x_{t3}}^{x_{t2}} p(x') dx'\right)$$

• for nth scattering:

$$P_N = p(x_{tN}) \frac{1}{(N-1)!} \left(\int_{x_{tN}}^1 p(x') dx' \right)^{N-1} \exp\left(-\int_{x_{tN}}^1 p(x') dx' \right)$$

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Formalism for MI (Sjostrand, Zijl) II

compare to Poisson distribution:

$$p_r = \frac{\mu^r}{r!} \exp\left(-\mu\right) \tag{19}$$

T. Sjostrand, M. Zijl PRD 36 (1987) 2019

total probability to have any scattering at x₁₁:

$$\sum_{N} P_{N} = \sum_{N} p(x_{t}) \frac{1}{(N-1)!} \left(\int_{x_{t}}^{1} p(x') dx' \right)^{N-1} \exp\left(-\int_{x_{t}}^{1} p(x') dx' \right)$$

= $p(x_{t})$

preserving total probability

$$\mu = \int_{x_t}^1 p(x')dx' = \frac{1}{\sigma_{nd}} \int_{x_t}^1 \frac{d\sigma}{dx'_t} dx'_t$$

- recovering parton model result
 - Poisson distribution with mean
- recover AGK rules...

Factorization

• factorization means:

$$\frac{d\sigma}{dy} = \sum_{a,b} \int_{x_A}^1 d\xi_A \int_{x_B}^1 d\xi_B f_A^a(\xi_A,\mu) f_B^b(\xi_B,\mu) \frac{d\hat{\sigma}_{ab}(\mu)}{dy} + \mathcal{O}\left(\left(\frac{m}{P}\right)^p\right)$$

• MPI approach reproduces inclusive jet x-section with

$$\langle n \rangle = rac{\sigma_{
m hard}(p_{\perp
m min})}{\sigma_{nd}}$$

- Similar in Drell-Yan with initial state interactions...
- factorization here does not hold graph-by-graph but only for all



Once upon a time ...



Multiparton interaction help to describe particle multiplicities in minimum bias events
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But, there are also other challenges!



Particle spectra at LHC



- It's not at all trivial to describe particle multiplicities
- MPI is needed, but even then the spectra are not perfectly described !
- Place for further Tuning !!!

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and at 13 TeV

• one of the very first results from LHC at $\sqrt{s} = 13 \, TeV$



Evidence for Multi Parton Interaction

• Study of the underlying event structure:

trigger on high p_t jets, observe additional hadron activity in the transverse regions



Underlying event study in DY



Adding back what was neglected before ...

• The standard ansatz: collinear factorization

$$\sigma = f_i^A(x_1, \mu^2)\hat{\sigma}(i+j \to X)f_j^B(x_2, \mu^2)$$

or in detail:

$$\frac{d\sigma}{dy} = \sum_{a,b} \int_{x_A}^1 d\xi_A \int_{x_B}^1 d\xi_B f_A^a(\xi_A,\mu) f_B^b(\xi_B,\mu) \frac{d\hat{\sigma}_{ab}(\mu)}{dy} + \mathcal{O}\left(\left(\frac{m}{\mu}\right)^n\right)$$

- terms which are suppressed by powers 1/1 are not covered by factorization theorem.
- The limitations:

$$\sigma = c_1 f_g f_g + c_2 f_q f_q + c_3 f_g F + c_4 F F$$

- Iast terms are suppressed by powers of ¹
- leading to double parton scattering





Some power suppressed terms

 In parton evolution with k_t dependent splitting functions we have some of the power suppressed terms:

$$\mathcal{P}_{gq}(z, q_t, k_t) = P_{qg, DGLAP}(z) \left(1 + \sum_{n=0}^{\infty} b_n(z) (k_t^2/q_t^2)^n \right)$$

- Can at least hard perturbative contributions simulated by a more advanced parton evolution ?
 - yes to some extend....

Double-Parton Interactions at LHC

• xsection for $p + p \rightarrow b\overline{b}b\overline{b}$

single parton exchange (SP) $\sigma^{SP} \sim f^2 \hat{\sigma} (2 \rightarrow 4)$

double parton exchange (DP)

 $\sigma^{DP} \sim f^4 \hat{\sigma}^2 (2 \to 2)$



• PYTHIA predictions:

 $\sigma^{DP} = 0.8 \cdots 11.1 \ \mu b$

Depending on model for underlying event/multi-parton interactions...

Multi-Parton Interactions at LHC



Hard multi - parton Interactions



Multiparton interaction at the LHC



parton densities at small x

transverse momenta

Inconsistency: example from DIS



Collinear approach: incoming/outgoing partons are on mass shell

$$(\gamma + q)^2 = q^2, -Q^2 + xys = 0 \rightarrow x = Q^2/(ys)$$

• **BUT** final state radiation:

$$(\gamma + q)^2 = q'^2, -Q^2 + xys = m^2 \rightarrow x = (Q^2 + m^2)/(ys)$$

AND initial state radiation:

$$(\gamma + q)^2 = q'^2, -Q^2 + xys + k^2 = 0 \rightarrow x = (Q^2 - k^2)/(ys)$$

Collinear approach: $q'^2 = k^2 = 0$, order by order NLO corrections... better treatment of kinematics... but still not all....

Transverse momentum effects in pp

S. Dooling, et al. Longitudinal momentum shifts, showering and nonperturbative corrections in matched NLO-shower event generators. Phys.Rev., D87:094009, 2013.

- Transverse momentum effects are relevant for many processes at LHC
- parton shower matched with NLO (POWHEG) generates additional k_t, leading to energy-momentum mismatch
- Transverse momentum effects are visible in high pt processes, not only at small x



TMDs and the general pp case



But even this is not the full story...

• factorization breaking in $pp \rightarrow j_1 j_2 X$ J. Collins, J.W. Qiu hep-ph 0705.2141



FIG. 8 (color online). The exchange of two extra gluons, as in this graph, will tend to give nonfactorization in unpolarized cross sections.

- factorization breaking also in tt -production at large pt^{top}
 - S. Catani, M. Grazzini, and A. Torre. Transverse-momentum resummation for heavy-quark hadropro- duction. arXiv 1408.4564



Small pt in heavy quark prod.

Frixione et al, hep-ph/035252



... and the measurements

CMS coll.

Measurement of differential top-quark pair production cross sections in pp collisions at sqrt(s) = 7 TeV arXiv:1211.2220 EPJ C73 (2013) 2339



Imagine, would could probe gluons directly



Imagine ...

• all standard electro-weak currents couple to quarks:

 γ, Z_0, W

- structure function of quarks are well measured in DIS scattering, as well as in DY production
- structure function of gluons, as well as properties of gluons are measured only indirectly via quark

Imagine ...

• all standard electro-weak currents couple to quarks:



• Higgs is special:

in heavy top limit,

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QCD options at high luminosity LHC

With Higgs, we have a new and exciting result, which opens up a completely new world for QCD studies:

• gluon process with color singlet final state at large masses:



CMS Coll., PRD 85, 032002 (2012)

Differential cross sections of the higgs boson measured in the diphoton decay channel using 8 TeV pp collisions. ATLAS arXiv 1407.4222



Higgs as a gluon trigger

Start new QCD program with Higgs as gluon trigger

P. Cipriano, S. Dooling, A. Grebenyuk, P. Gunnellini, F. Hautmann, H. Jung, and P. Katsas. Higgs boson as a gluon trigger. Phys. Rev. D, 88:097501, Nov 2013. arXiv:1308.1655

- comparison with DY production at same mass range
- p_T spectrum of DY and Higgs:



Higgs as a gluon trigger

Start new QCD program with Higgs as gluon trigger

P. Cipriano, S. Dooling, A. Grebenyuk, P. Gunnellini, F. Hautmann, H. Jung, and P. Katsas. Higgs boson as a gluon trigger. Phys. Rev. D, 88:097501, Nov 2013. arXiv:1308.1655

- comparison with DY production as same mass range
- jet + DY / Higgs: in rest-frame sensitivity to spin-coupling to gluons vanishing effect of guark vrs gluon propagator



Challenge in QCD – another example

- Higgs + jet production
 - as fct of Δy jet multiplicity must increase
 - similar to dijet case
- Measure at fixed $m = 125 \, GeV$: $\frac{dn}{d\Delta y}_{Higgs} - \frac{dn}{d\Delta y}_{DY}$

- pileup and UE effects cancel
- isolate gluon contribution

High Energy Description of Processes with Multiple Hard Jets Jeppe R. Andersen. Jennifer M. Smillie. Nucl.Phys.Proc.Suppl. 205-206 (2010) 205-210, 1007.4449

$$pp \rightarrow h+2$$
 jets (+ n jets)
 $\sqrt{s} = 10$ TeV. $p_T > 40$ GeV



There are many open questions in QCD at highest energies and at highest scales

Your ideas, imagination and help is very much needed !

The End

.... what you should have learned

- Lecture:
 - QCD is still a interesting and challenging field
 - basic QCD calculations
 - important role of gluons, which comes from gluon self-coupling
 - understanding of parton evolution equations in terms of parton radiation
 - importance of "soft" gluon resummation
- Exercises:
 - basic Monte Carlo techniques
 - MC integration and generation of variables according to distributions
 - solution of parton evolution equation with MC technique
 - advantage of MC technique to solve complex problems like multiparton radiation (example pt spectrum of Drell Yan)

.... what you should have learned

Have a good time, and enjoy when gambling with Monte Carlos