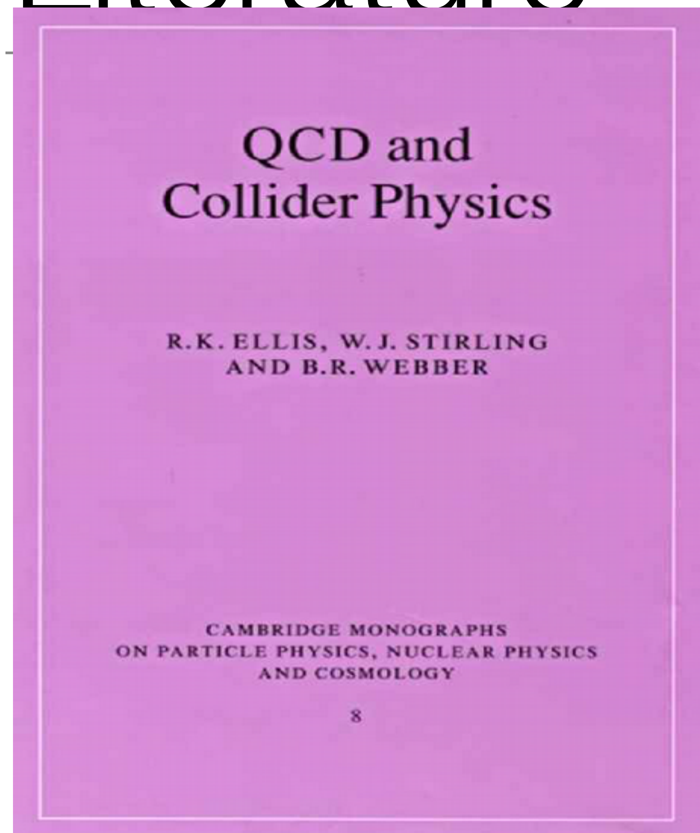
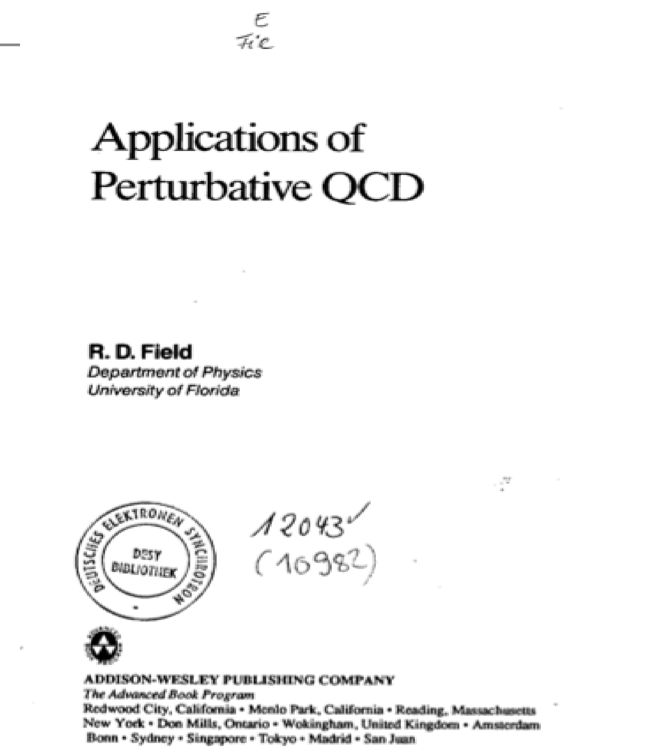
# QCD and Monte Carlo simulation III

H. Jung (DESY, University Antwerp)
hannes.jung@desy.de

http://www.desy.de/~jung/qcd_and_mc_2015/

# Literature



Applications of
Perturbative QCD

R. D. Field
*Department of Physics*
*University of Florida*

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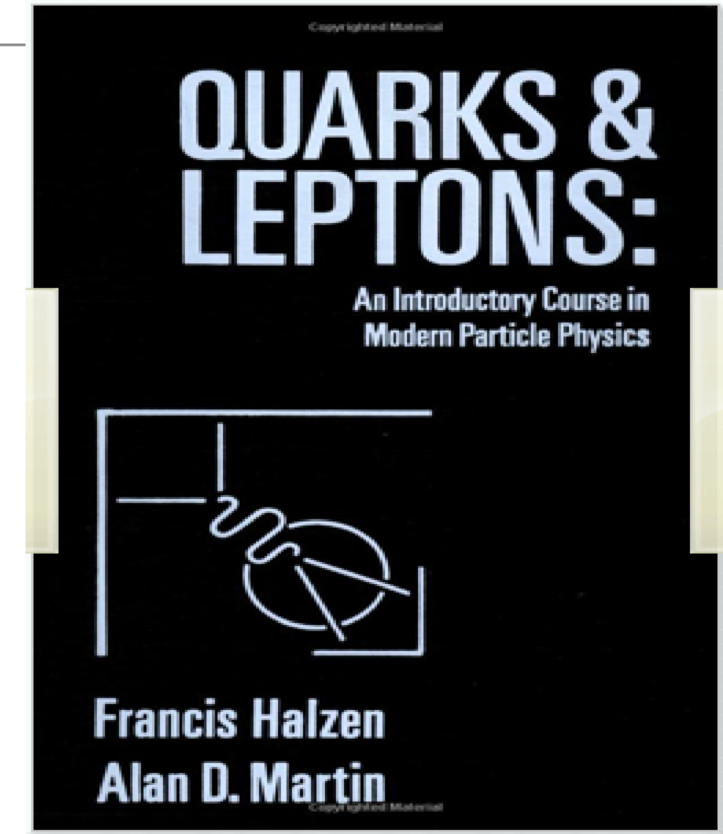


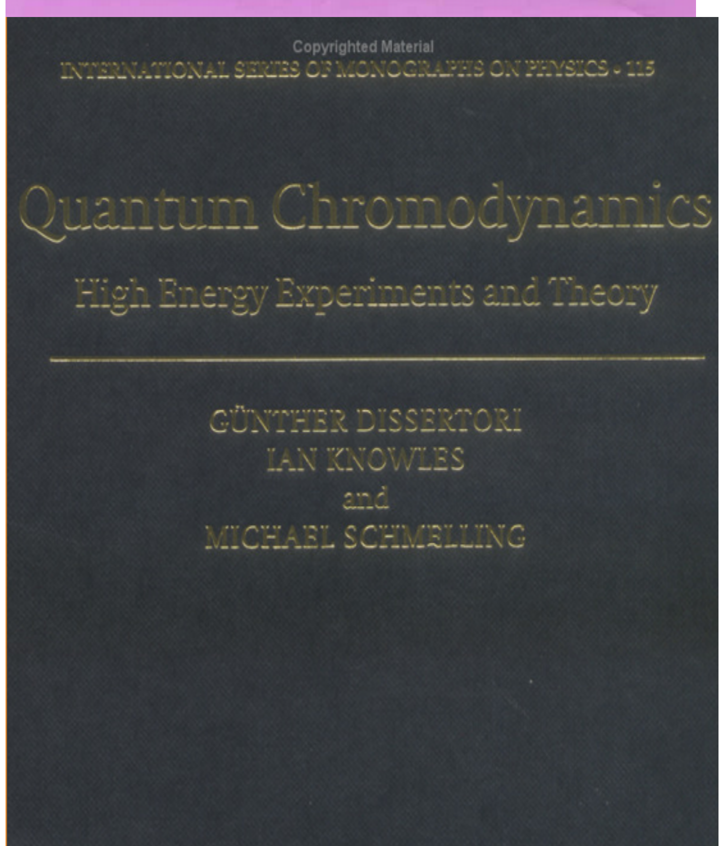


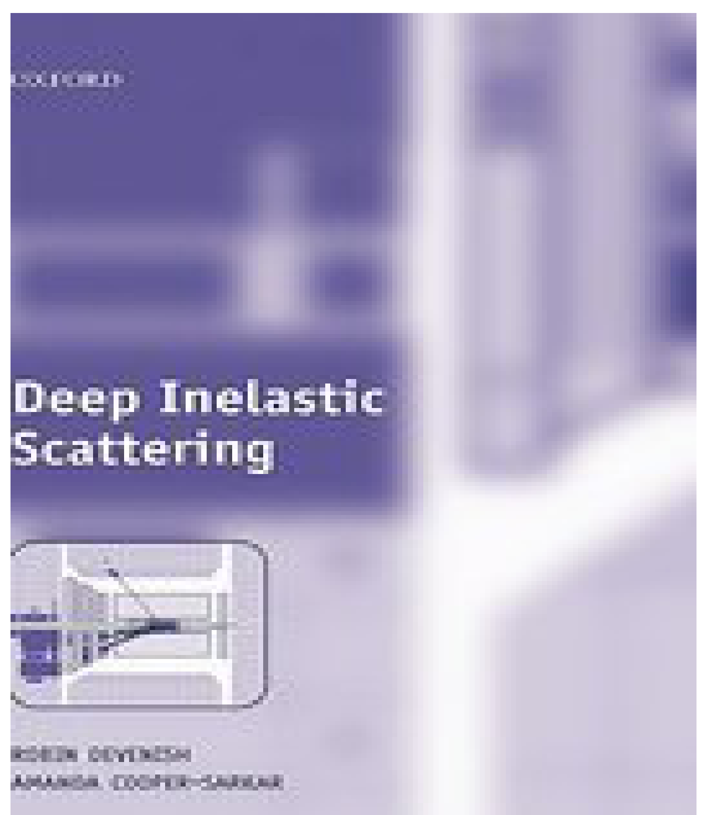


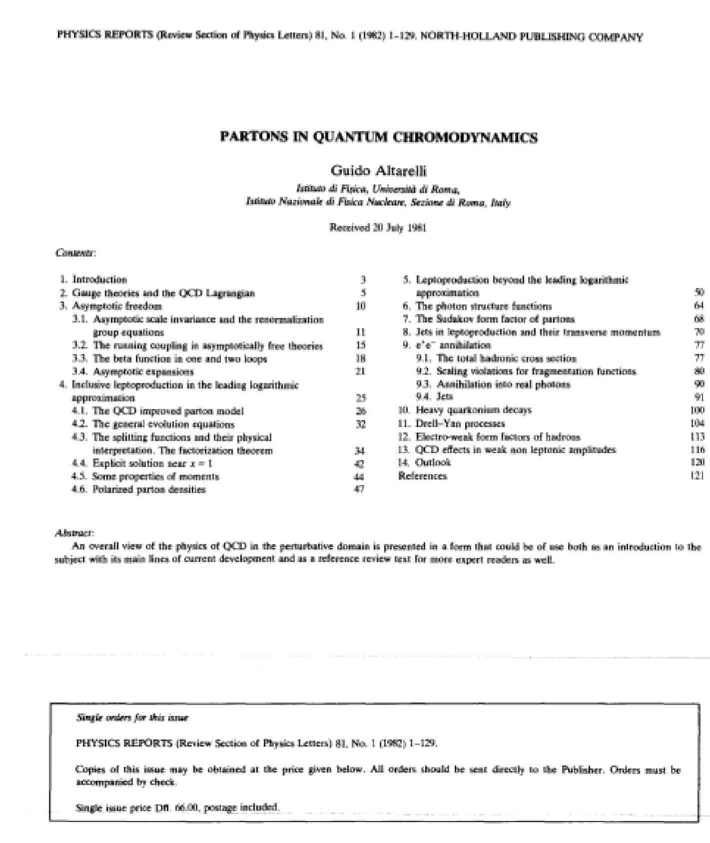

PHYSICS REPORTS (Review Section of Physics Letters) 81, No. 1 (1982) 1–129. NORTH-HOLLAND PUBLISHING COMPANY

**PARTONS IN QUANTUM CHROMODYNAMICS**

Guido Altarelli
*Istituto di Fisica, Università di Roma,*
*Istituto Nazionale di Fisica Nucleare, Sezione di Roma, Italy*

Received 20 July 1981

*Contents:*



*Abstract:*
An overall view of the physics of QCD in the perturbative domain is presented in a form that could be of use both as an introduction to the subject with its main lines of current development and as a reference review text for more expert readers as well.

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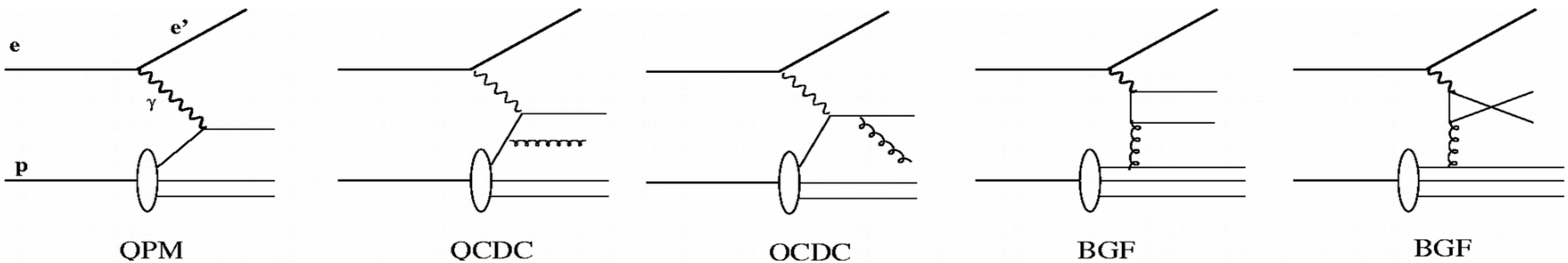
# Outline of the lectures

- 12. Oct   Intro to Monte Carlo techniques and structure of matter
- 13. Oct   DGLAP: solution with MCs
- 26. Oct   DGLAP/BFKL/CCFM: evolution for small x
- 27. Oct   W/Z production in pp and soft gluon resummation
- 16. Nov   Multiparton interactions
- 17. Nov   Latest LHC results: small x, multiparton interactions,
  QCD in high luminosity phase: Higgs as a gluon trigger
- Exercises
- 14 & 15 Oct
- 28 & 29 Oct
- 18 & 19 Nov

# Higher order corrections to DIS

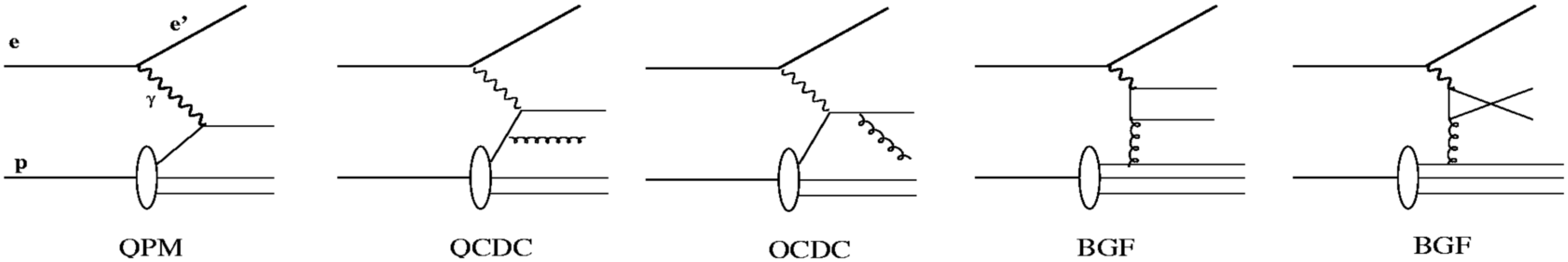



- lowest order: $e + q \to e' + q' \quad \mathcal{O}(\alpha_s^0)$
- higher order: $e + q \to e' + q' + g, \quad e + g \to e' + q + \bar{q} \quad \mathcal{O}(\alpha_s^1)$

- What is the dominant part of the x-section ?
  - Investigate full x-section of QCDC and BGF
  - dominant part comes from small transverse momenta ...
  - rewrite x-section in terms of $k_\perp$
  - use small *t* limit:

$$
\begin{aligned}
\frac{d\sigma}{dk_\perp^2} &= \frac{d\sigma}{dt}\frac{1}{(1-z)} = \frac{1}{(1-z)}\frac{1}{F} dLips|ME|^2 \\
&= \frac{1}{(1-z)}\frac{1}{16\pi}\frac{1}{\hat{s}+Q^2}\frac{1}{\hat{s}}|ME|^2
\end{aligned}
$$

# Adding up everything




$$\sigma^{\gamma * p} \sim \frac{4\pi^2\alpha}{Q^2} F_2(x,Q^2) = \frac{4\pi^2\alpha}{ys}\frac{F_2(x,Q^2)}{x}$$

$$\sigma^{\gamma * p} = \sigma_0 \frac{F_2(x,Q^2)}{x}$$

$\xi$ is parton momentum fraction

$$\sigma_0 = \frac{4\pi^2\alpha}{2qP}$$

- Connect with $F_2$:

$$\sigma^{QPM} = \sigma_0 e_q^2 \delta\left(1-\frac{x}{\xi}\right)$$

$$\sigma^{QCDC} = \hat{\sigma}_0 e_q^2 \otimes P_q(z) \otimes \log \ldots$$

$$\sigma^{BGF} = \hat{\sigma}_0 e_q^2 \otimes P_g(z) \otimes \log \ldots$$

# Collinear factorization (part 1)

- bare distributions $q_0(x)$ are not measurable (like the bare charges .... )

$$F_2 \quad = \quad x \sum e_q^2 \left[ q_0(x) + \int \frac{d\xi}{\xi} q_0(x) \frac{\alpha_s}{2\pi} P_{qq}\left(\frac{x}{\xi}\right) \log\left(\frac{Q^2}{\chi^2}\right) + C_q(z,..)\right]$$

- collinear singularities are absorbed into this bare distributions at a factorization scale $\mu^2 \gg \chi^2$ , defining renormalized distributions

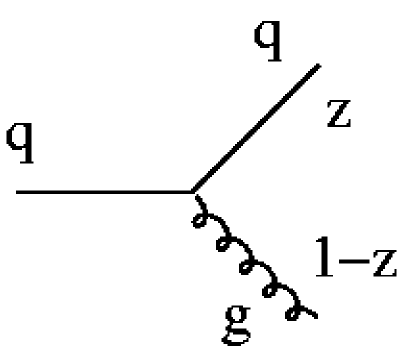
$$q_i(x,\mu^2) = q_i^0(x) + \frac{\alpha_s}{2\pi}\int_x^1 \frac{d\xi}{\xi}\left[q_i^0(\xi) P_{qq}\left(\frac{x}{\xi}\right)\log\left(\frac{\mu^2}{\chi^2}\right) + C_q\left(\frac{x}{\xi}\right)\right] + \ldots$$

- now $F_2$ becomes:

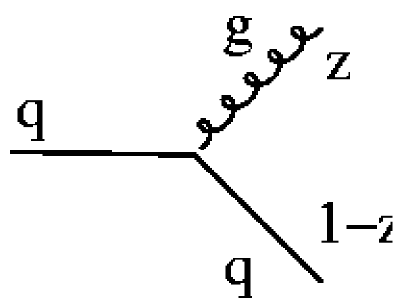
$$F_2 = x \sum e_q^2 \int \frac{d\xi}{\xi} q(\xi,\mu^2)\left[\delta\left(1-\frac{x}{\xi}\right) + \frac{\alpha_s}{2\pi} P_{qq}\left(\frac{x}{\xi}\right)\log\left(\frac{Q^2}{\mu^2}\right) + C\right]$$

- separating or factorizing the long distance contributions to structure functions is a fundamental property of the theory
- factorization provides a description for dealing with the logarithmic singularities, there is arbitrariness in how the finite (non-logarithmic) parts are treated.

# Splitting functions in lowest order

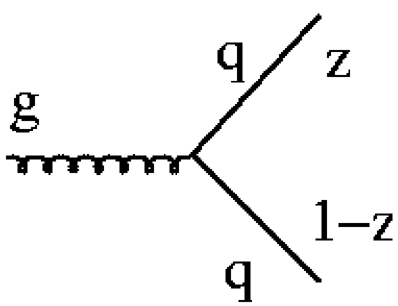



$$P_{qq} = \frac{4}{3}\left(\frac{1+z^2}{1-z}\right)$$

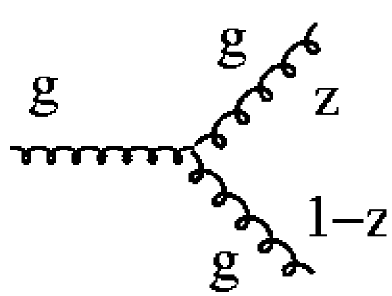



$$P_{gq} = \frac{4}{3}\left(\frac{1+(1-z)^2}{z}\right)$$

similarity to EPA...




$$P_{qg} = \frac{1}{2}\left(z^2+(1-z)^2\right)$$




$$P_{gg} = 6\left(\frac{1-z}{z}+\frac{z}{1-z}+z(1-z)\right)$$

# Collinear factorization: DGLAP

- introduce new scale $\mu^2 \gg \chi^2$ and include soft, non-perturbative physics into renormalized parton density:

$$q_i(x,\mu^2) = q_i^0(x) + \frac{\alpha_s}{2\pi}\int_x^1 \frac{d\xi}{\xi}\left[q_i^0(\xi)P_{qq}\left(\frac{x}{\xi}\right) + g^0(\xi)P_{qg}\left(\frac{x}{\xi}\right)\right]\log\left(\frac{\mu^2}{\chi^2}\right)$$

- Dokshitzer Gribov Lipatov Altarelli Parisi equation:

  V.V. Gribov and L.N. Lipatov Sov. J. Nucl. Phys. 438 and 675 (1972) 15, L.N. Lipatov Sov. J. Nucl. Phys 94 (1975) 20, G. Altarelli and G. Parisi Nucl.Phys.B 298 (1977) 126, Y.L. Dokshitser Sov. Phys. JETP 641 (1977) 46

$$\frac{dq_i(x,\mu^2)}{d\log\mu^2} = \frac{\alpha_s}{2\pi}\int_x^1 \frac{d\xi}{\xi}\left[q_i(\xi,\mu^2)P_{qq}\left(\frac{x}{\xi}\right) + g(\xi,\mu^2)P_{qg}\left(\frac{x}{\xi}\right)\right]$$

- BUT there are also gluons....

$$\frac{dg(x,\mu^2)}{d\log\mu^2} = \frac{\alpha_s}{2\pi}\int_x^1 \frac{d\xi}{\xi}\left[\sum_i q_i(\xi,\mu^2)P_{gq}\left(\frac{x}{\xi}\right) + g(\xi,\mu^2)P_{gg}\left(\frac{x}{\xi}\right)\right]$$

- DGLAP is the analogue to the beta function for running of the coupling

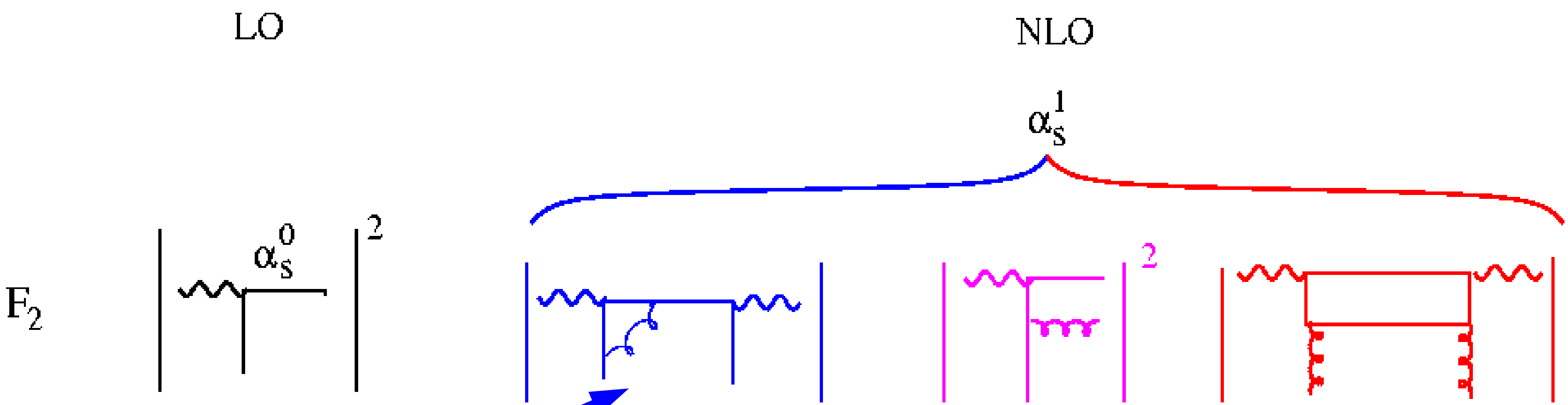
# Collinear factorization

see handbook of pQCD, chapter IV, B

$$F_2^{(Vh)}(x,Q^2) = \sum_{i=f,\bar{f},G} \int_0^1 d\xi C_2^{(Vi)}\left(\frac{x}{\xi}, \frac{Q^2}{\mu^2}, \frac{\mu_f^2}{\mu^2}, \alpha_s(\mu^2)\right) \otimes f_{i/h}(\xi, \mu_f^2, \mu^2)$$

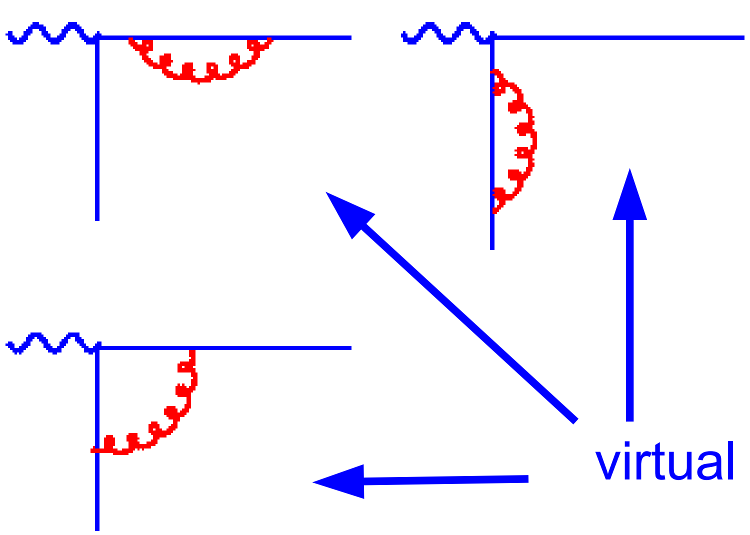
- Factorization Theorem in DIS (Collins, Soper, Sterman, (1989) in Pert. QCD, ed. A.H. Mueller, Wold Scientific, Singapore, p1.)
  - hard-scattering function $C_2^{(Vi)}$ is infrared finite and calculable in pQCD, depending only on vector boson *V*, parton *i*, and renormalization and factorization scales. It is independent of the identity of hadron *h*.
  - pdf $f_{i/h}(\xi, \mu_f^2, \mu^2)$ contains all the infrared sensitivity of cross section, and is specific to hadron h, and depends on factorization scale.
- Generalization: applies to any DIS cross section defined by a sum over hadronic final states .... but be careful what it really means....
- explicit factorization theorems exist for:
  - diffractive DIS (... see above....)
  - Drell Yan (in hadron hadron collisions)
  - single particle inclusive cross sections (fragmentation functions)

# NLO contributions to $F_2(x,Q^2)$




virtual corrections cancel collinear singularity

$$P_{qq} = \frac{4}{3}\left(\frac{1+z^2}{1-z}\right)$$



virtual corrections to QPM

$$P_{qg} = \frac{1}{2}\left(z^2 + (1+z)^2\right)$$

no collinear singularity

# Evolution kernels – splitting fcts

- some of the splitting functions are also divergent...

$$\frac{1}{1-z}$$

- use *plus-distribution* to avoid dangerous region:

$$\int_0^1 dz \frac{f(z)}{(1-z)_+} = \int_0^1 dz \frac{f(z)-f(1)}{1-z}$$

BLACKBOARD

- divergence cancelled by virtual corrections ...
- use splitting functions with plus-distribution

# Conservation rules with DGLAP

$$\int_0^1 dxx \left[ \sum_{i=-6}^{6} q(x,\mu^2) + g(x,\mu^2) \right] = 1$$

- use DGLAP

$$\frac{dq_i(x,\mu^2)}{d\log\mu^2} = \frac{\alpha_s}{2\pi}\int_x^1 \frac{d\xi}{\xi}\left[ q_i(\xi,\mu^2) P_{qq}\left(\frac{x}{\xi}\right) + g(\xi,\mu^2) P_{qg}\left(\frac{x}{\xi}\right)\right]$$

$$\frac{dg(x,\mu^2)}{d\log\mu^2} = \frac{\alpha_s}{2\pi}\int_x^1 \frac{d\xi}{\xi}\left[ \sum_i q_i(\xi,\mu^2) P_{gq}\left(\frac{x}{\xi}\right) + g(\xi,\mu^2) P_{gg}\left(\frac{x}{\xi}\right)\right]$$

➔to obtain:

$$\int_0^1 dx\ x\left[P_{qq}(x) + P_{gq}(x)\right] = 0$$

$$\int_0^1 dx\ x\left[P_{gg}(x) + 2n_f P_{qg}(x)\right] = 0$$

# Solution of DGLAP equation:

# What happens at small x ?

# What is happening at small x ?

- For $x \to 0$ only gluon splitting function matters:

$$P_{gg} = 6\left(\frac{1-z}{z} + \frac{z}{1-z} + z(1-z)\right) = 6\left(\frac{1}{z} - 2 + z(1-z) + \frac{1}{1-z}\right)$$

$$P_{gg} \sim 6\frac{1}{z} \text{ for } z \to 0$$

- evolution equation is then:

**BLACKBOARD**

$$\frac{dg(x,\mu^2)}{d\log\mu^2} = \frac{\alpha_s}{2\pi}\int_x^1 \frac{d\xi}{\xi} g(\xi,\mu^2) P_{gg}\left(\frac{x}{\xi}\right)$$

$$xg(x,t) = xg(x,\mu_0^2) + \frac{3\alpha_s}{\pi}\int_{t_0}^{t} d\log t' \int_x^1 \frac{d\xi}{\xi}\xi g(\xi,t')$$

with $t = \mu^2$

# Solving integral equations

- Integral equation of Fredholm type:

Weisstein, Eric W. "Integral Equation Neumann Series." From MathWorld--A Wolfram Web Resource. http://mathworld.wolfram.com/IntegralEquationNeumannSeries.htm

$$\phi(x) = f(x) + \lambda \int_a^b K(x,y)\phi(y)dy$$

- solve it by iteration (Neumann series):

**Blackboard**

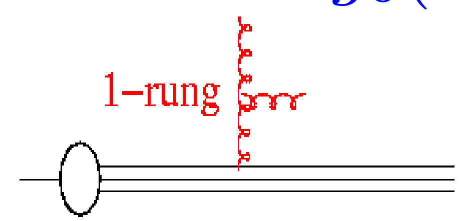
$$\begin{aligned}
\phi_0(x) &= f(x) \\
\phi_1(x) &= f(x) + \lambda \int_a^b K(x,y) f(y) dy \\
\phi_2(x) &= f(x) + \lambda \int_a^b K(x,y_1) f(y_1) dy_1 + \lambda^2 \int_a^b \int_a^b K(x,y_1) K(y_1,y_2) f(y_2) dy_2 dy_1 \\
\phi_n(x) &= \sum_{i=0}^{n} \lambda^i u_i(x) \\
u_0(x) &= f(x) \\
u_1(x) &= \int_a^b K(x,y) f(y) dy \\
u_n(x) &= \int_a^b \cdots \int_a^b K(x,y_1) K(y_1,y_2) \cdots K(y_{n-1},y_n) f(y_n) dy_1 \cdots dy_n
\end{aligned}$$

**with the solution:** $$\phi(x) = \lim_{n\to\infty} q_n(x) = \lim_{n\to\infty} \sum_{i=0}^{n} \lambda^i u_i(x)$$

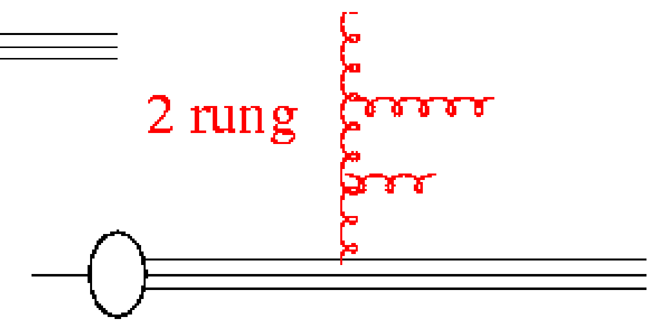
# Estimates at small x: DLL

$$xg(x,t) = xg(x,\mu_0^2) + \frac{3\alpha_s}{\pi}\int_{t_0}^{t} d\log t' \int_x^1 \frac{d\xi}{\xi}\xi g(\xi,t') \quad \text{with} \quad t = \mu^2$$

- use constant starting distribution at small *t*: $xg_0(x) = C$

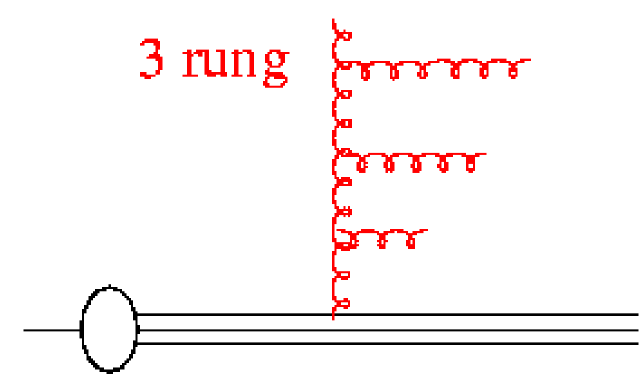



$$xg_1(x,t) = C + \frac{3\alpha_s}{\pi}\log\frac{t}{t_0}\log\frac{1}{x}C$$




$$xg_2(x,t) = C + \frac{1}{2}\frac{1}{2}\left(\frac{3\alpha_s}{\pi}\log\frac{t}{t_0}\log\frac{1}{x}\right)^2 C$$

$$\vdots$$




$$xg_n(x,t) = C + \frac{1}{n!}\frac{1}{n!}\left(\frac{3\alpha_s}{\pi}\log\frac{t}{t_0}\log\frac{1}{x}\right)^n C$$

$$xg(x,t) = \sum_n \frac{1}{n!}\frac{1}{n!}\left(\frac{3\alpha_s}{\pi}\log\frac{t}{t_0}\log\frac{1}{x}\right)^n C$$

$$xg(x,t) \sim C\exp\left(2\sqrt{\frac{3\alpha_s}{\pi}\log\frac{t}{t_0}\log\frac{1}{x}}\right)$$

double leading log approximation (DLL)

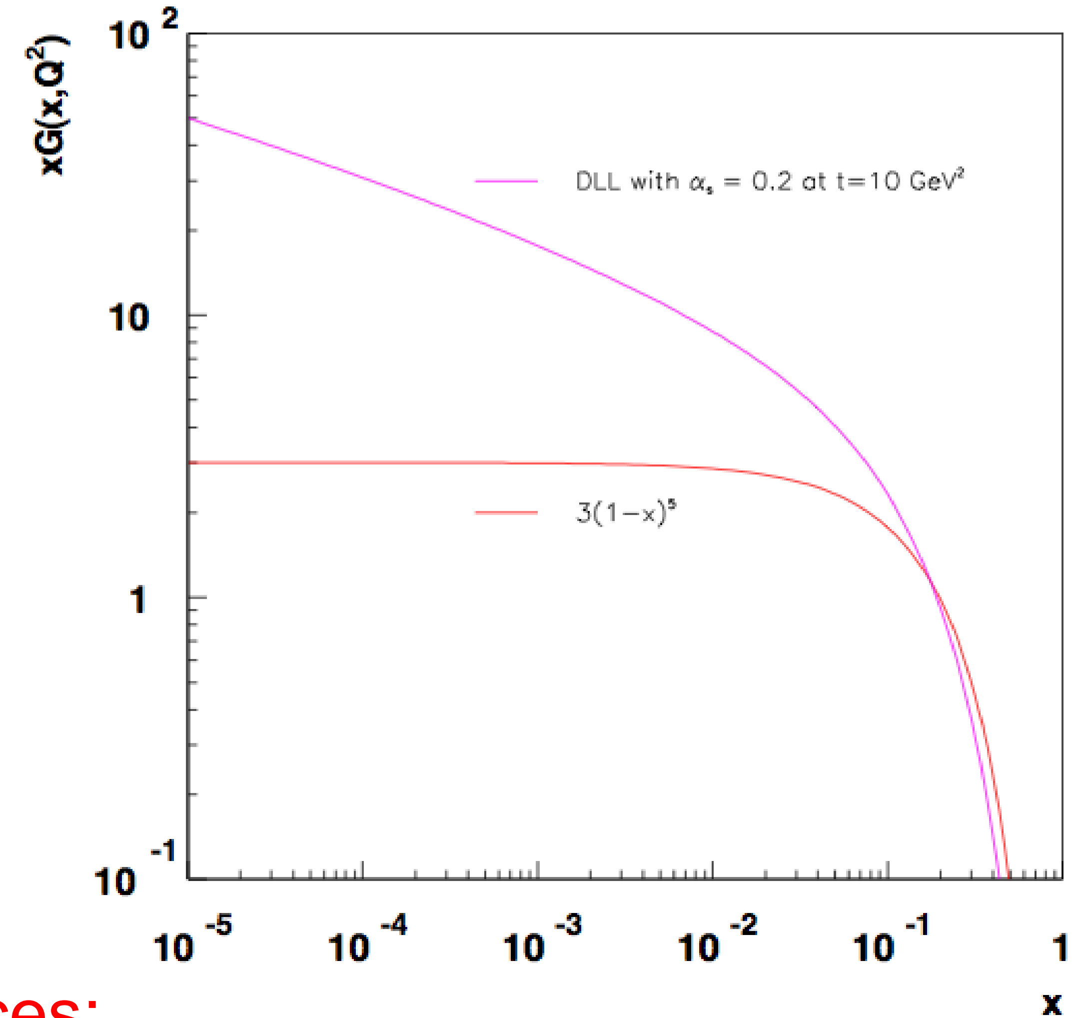
# Results from DLL approximation

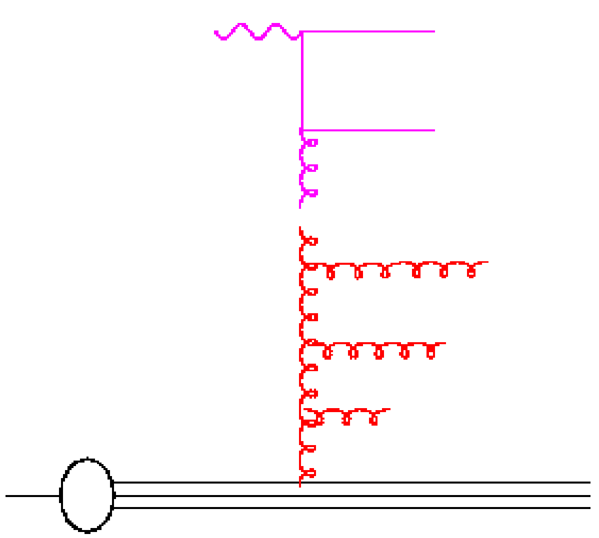
- DLL arise from taking small x limit of splitting fct:

$$xg(x,t) \sim C \exp\left(2\sqrt{\frac{3\alpha_s}{\pi}\log\frac{\mu^2}{\mu_0^2}\log\frac{1}{x}}\right)$$

  - $\log 1/x$ from small *x* limit of splitting fct
    $\log \mu^2/\mu_0^2$ from μ integration
  - strong ordering in *x* from small *x* limit
  - strong ordering in *t* from small *t* limit of ME...

- DLL gives rapid increase of gluon density from a flat starting distribution
- gluons are coupled to $F_2$... strong rise of $F_2$ at small x:



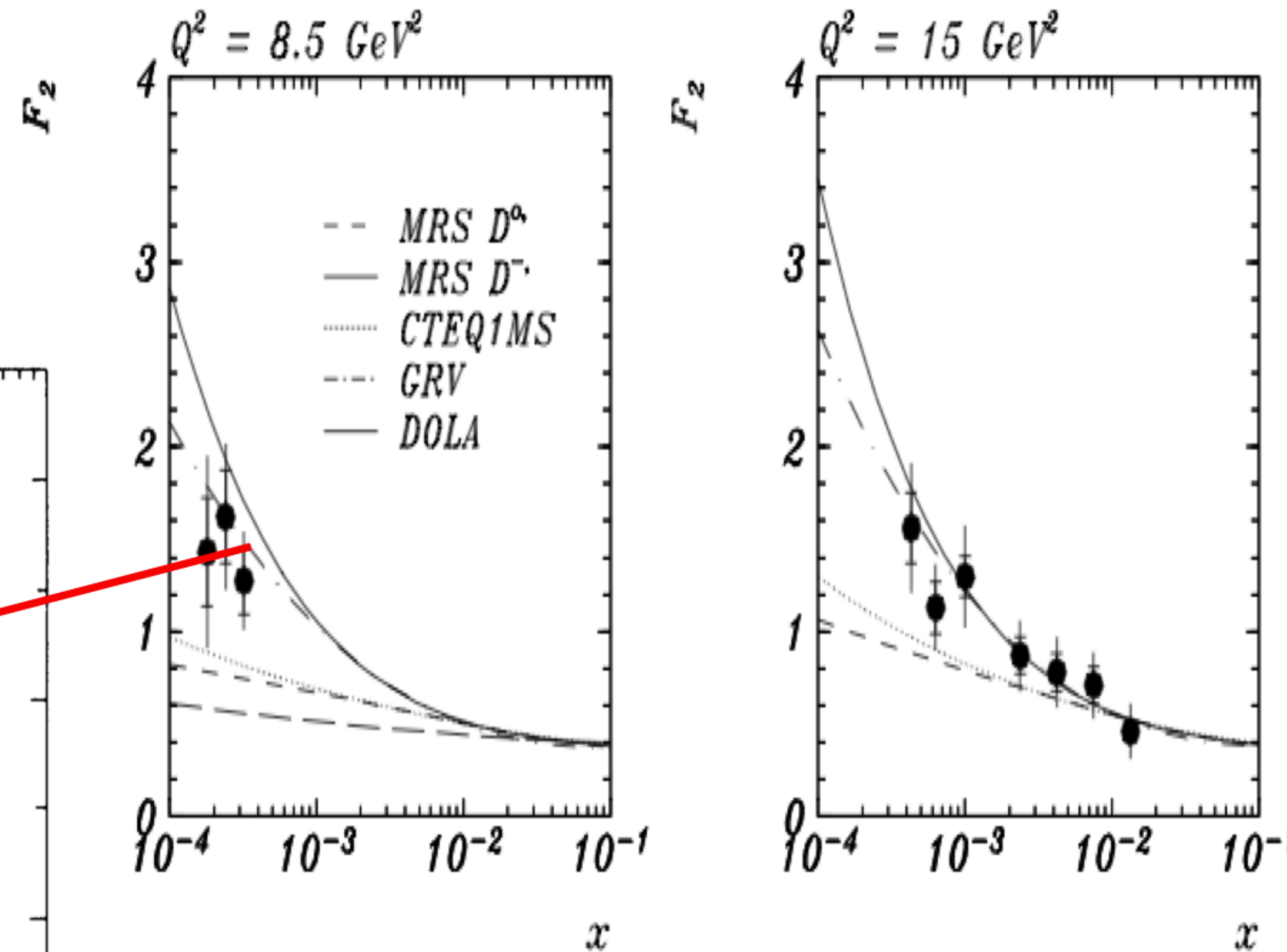
- consequences:
- rise continues forever ???
- what happens when too high gluon density ?

# Remember the pre-HERA times

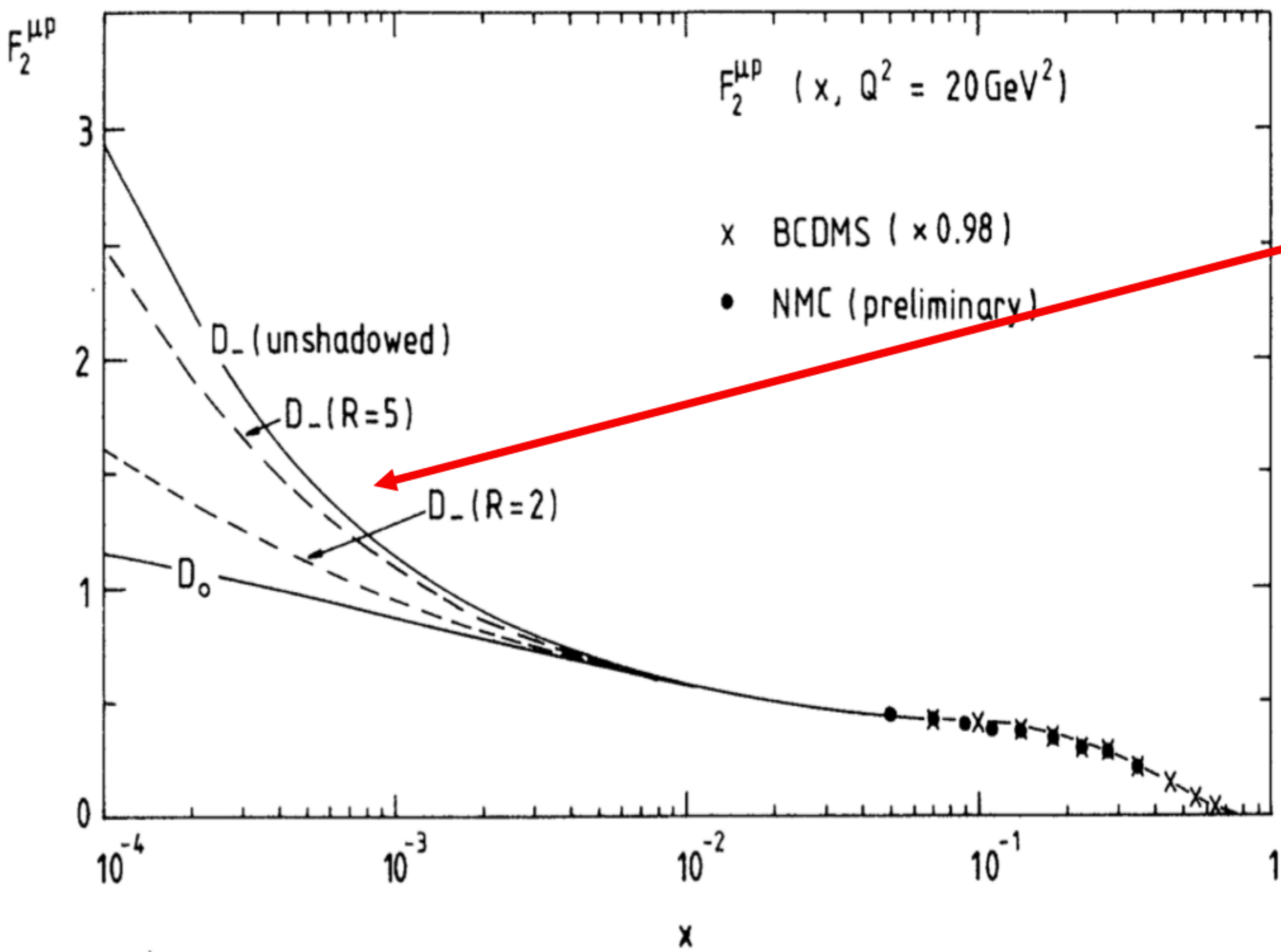
- Just before HERA started in 1992, new PDF fits (NLO DGLAP) were released, using all existing high precision data

- 1st HERA data 1992

H1 Nucl. Phys. B407 (1993) 515




Martin, Stirling, Roberts Apr 1992.
Phys.Rev.D47:867-882,1993.



From evolution equation to parton branching ...

How ?

# Divergencies again...

- collinear divergencies factored into renormalized parton distributions
- what about soft divergencies ?

  **treated with "plus" prescription**

**with**

$$\frac{1}{1-z} \to \frac{1}{1-z}_+ \qquad \int_0^1 dz \frac{f(z)}{(1-z)_+} = \int_0^1 dz \frac{f(z)-f(1)}{(1-z)}$$

- soft divergency treated with Sudakov form factor:

$$\Delta(t) = \exp\left[-\int_{t_0}^{t}\frac{dt'}{t'}\int^{z_{max}} dz \frac{\alpha_s}{2\pi}\tilde{P}(z)\right]$$

**Blackboard**

# DGLAP evolution again....

- differential form:

$$t\frac{\partial}{\partial t}f(x,t) = \int \frac{dz}{z}\frac{\alpha_s}{2\pi}P_+(z)\, f\left(\frac{x}{z},t\right)$$

$$\Delta_s(t) = \exp\left(-\int_x^{z_{max}} dz \int_{t_0}^{t} \frac{\alpha_s}{2\pi}\frac{dt'}{t'}\tilde{P}(z)\right)$$

- differential form using $f/\Delta_s$ with

$$t\frac{\partial}{\partial t}\frac{f(x,t)}{\Delta_s(t)} = \int \frac{dz}{z}\frac{\alpha_s}{2\pi}\frac{\tilde{P}(z)}{\Delta_s(t)}\, f\left(\frac{x}{z},t\right)$$

- integral form

$$f(x,t) = f(x,t_0)\Delta_s(t) + \int \frac{dz}{z}\int \frac{dt'}{t'}\cdot\frac{\Delta_s(t)}{\Delta_s(t')}\tilde{P}(z) f\left(\frac{x}{z},t'\right)$$

no – branching probability from $t_0$ to $t$

# Sudakov form factor: all loop resum...

$g \to gg$ Splitting Fct $\qquad \tilde{P}(z) = \frac{\bar{\alpha}_s}{1-z} + \frac{\bar{\alpha}_s}{z} + \ldots$

- Sudakov form factor .... all loop resummation

$$\Delta_S = \exp\left(-\int dz \int \frac{dq'}{q'} \frac{\alpha_s}{2\pi} \tilde{P}(z)\right)$$

$$\Delta_S = 1 + \left(-\int dz \int \frac{dq}{q} \frac{\alpha_s}{2\pi} \tilde{P}(z)\right)^1 + \frac{1}{2!}\left(-\int dz \int \frac{dq}{q} \frac{\alpha_s}{2\pi} \tilde{P}(z)\right)^2 \ldots$$

$$\tilde{P}(z)\left[1 - \int\int dz \frac{dq}{q} \frac{\alpha_s}{2\pi} \tilde{P}(z) + \frac{1}{2!}\left(-\int\int dz \frac{dq}{q} \frac{\alpha_s}{2\pi} \tilde{P}(z)\right)^2 - \ldots - \right]$$

# DGLAP re-sums leading logs...

$$f(x,t) = f(x,t_0)\Delta_s(t) + \int \frac{dz}{z} \int \frac{dt'}{t'} \cdot \frac{\Delta_s(t)}{\Delta_s(t')} \tilde{P}(z) f\left(\frac{x}{z}, t'\right)$$

- solve integral equation via iteration:

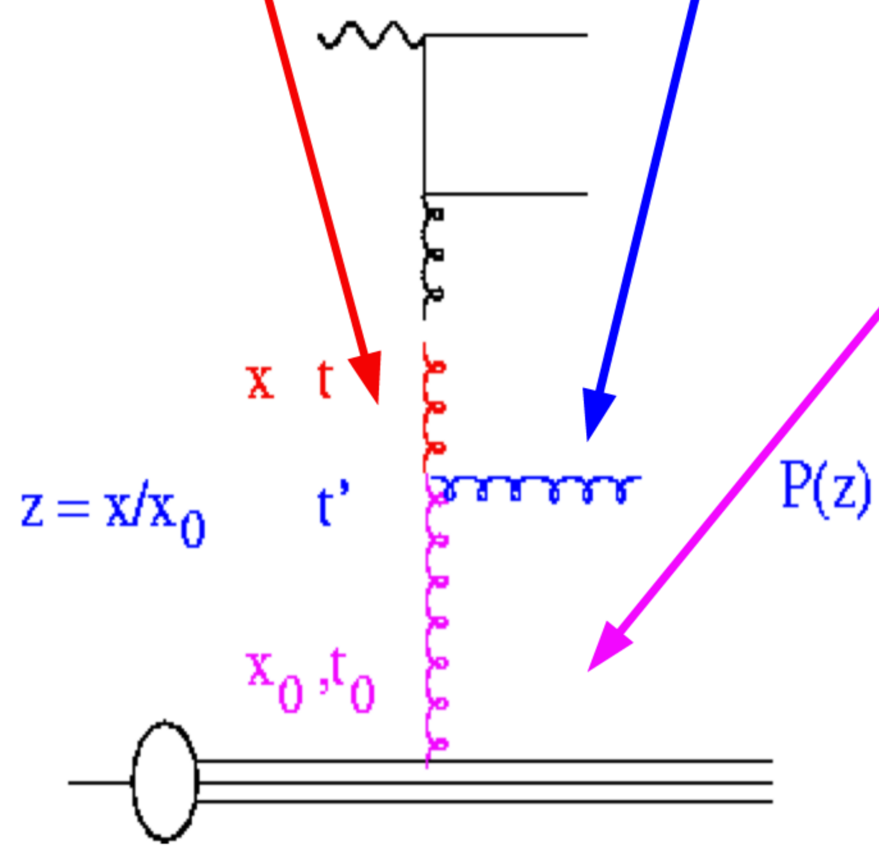
from $t'$ to $t$ w/o branching

branching at $t'$

from $t_0$ to $t'$ w/o branching

$$f_0(x,t) = f(x,t_0)\Delta(t)$$

$$f_1(x,t) = f(x,t_0)\Delta(t) + \int_{t_0}^{t} \frac{dt'}{t'} \frac{\Delta(t)}{\Delta(t')} \int \frac{dz}{z} \tilde{P}(z) f(x/z, t_0)\Delta(t')$$



# DGLAP re-sums leading logs...

$$f(x,t) = f(x,t_0)\Delta_s(t) + \int \frac{dz}{z} \int \frac{dt'}{t'} \cdot \frac{\Delta_s(t)}{\Delta_s(t')} \tilde{P}(z) f\left(\frac{x}{z}, t'\right)$$

- solve integral equation via iteration:

from $t'$ to $t$ w/o branching

branching at $t'$

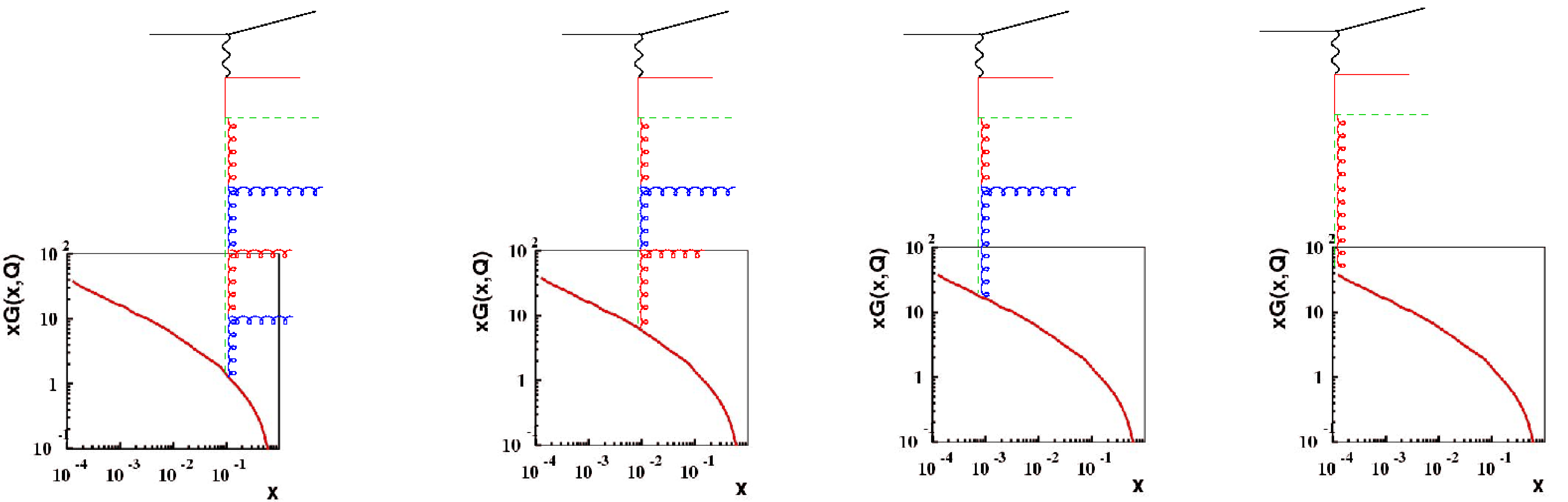
from $t_0$ to $t'$ w/o branching

$$\begin{aligned}
f_0(x,t) &= f(x,t_0)\Delta(t) \\
f_1(x,t) &= f(x,t_0)\Delta(t) + \int_{t_0}^{t} \frac{dt'}{t'} \frac{\Delta(t)}{\Delta(t')} \int \frac{dz}{z} \tilde{P}(z) f(x/z, t_0)\Delta(t') \\
&= f(x,t_0)\Delta(t) + \log\frac{t}{t_0} A \otimes \Delta(t) f(x/z, t_0) \\
f_2(x,t) &= f(x,t_0)\Delta(t) + \log\frac{t}{t_0} A \otimes \Delta(t) f(x/z, t_0) + \\
& \frac{1}{2}\log^2\frac{t}{t_0} A \otimes A \otimes \Delta(t) f(x/z, t_0) \\
f(x,t) &= \lim_{n\to\infty} f_n(x,t) = \lim_{n\to\infty} \sum_n \frac{1}{n!} \log^n\left(\frac{t}{t_0}\right) A^n \otimes \Delta(t) f(x/z, t_0)
\end{aligned}$$

**DGLAP re-sums $\log t$ to all orders !!!!!!!!!!!!!!!!!**

# DGLAP evolution equation... again...

- for fixed $x$ and $Q^2$ chains with different branchings contribute
- iterative procedure, spacelike parton showering



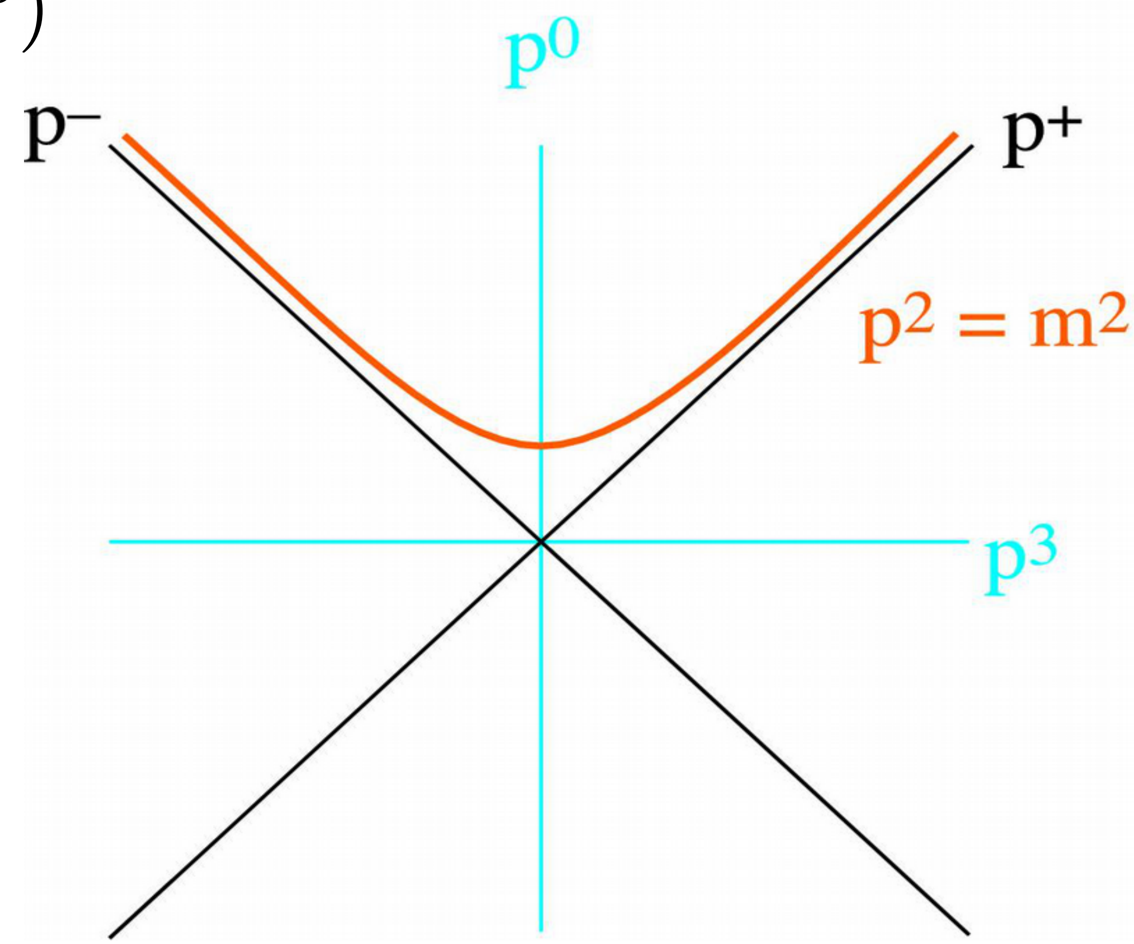

$$f(x,t) = f_0(x,t_0)\Delta_s(t) + \sum_{k=1}^{\infty} f_k(x_k,t_k)$$

# Light-cone variables

- Light Cone variables:

J. Collins hep-ph/9705393
D Soper, CTEQ 2001

$$V = (V^0, V^1, V^2, V^3) = (V^0, \mathrm{V}_t, V^3)$$
$$V^+ = \frac{1}{\sqrt{2}}(V^0 + V^3)$$
$$V^- = \frac{1}{\sqrt{2}}(V^0 - V^3)$$
$$V = (V^+, V^-, \mathrm{V}_t)$$
$$V.W = V^+W^- + V^-W^+ - \mathrm{V}_t\mathrm{W}_t$$
$$V^2 = 2V^+V^- - V_t^2$$




- lorentz boosts:

$$V'^0 = \frac{V^0 + vV^3}{\sqrt{1-v^2}}$$
$$V'^3 = \frac{vV^0 + V^3}{\sqrt{1-v^2}}$$
$$V'^+ = V^+ e^{\psi}$$
$$V'^- = V^- e^{-\psi}$$
$$\psi = \frac{1}{2}\ln\frac{1+v}{1-v}$$

**Blackboard**

# Sudakov form factor

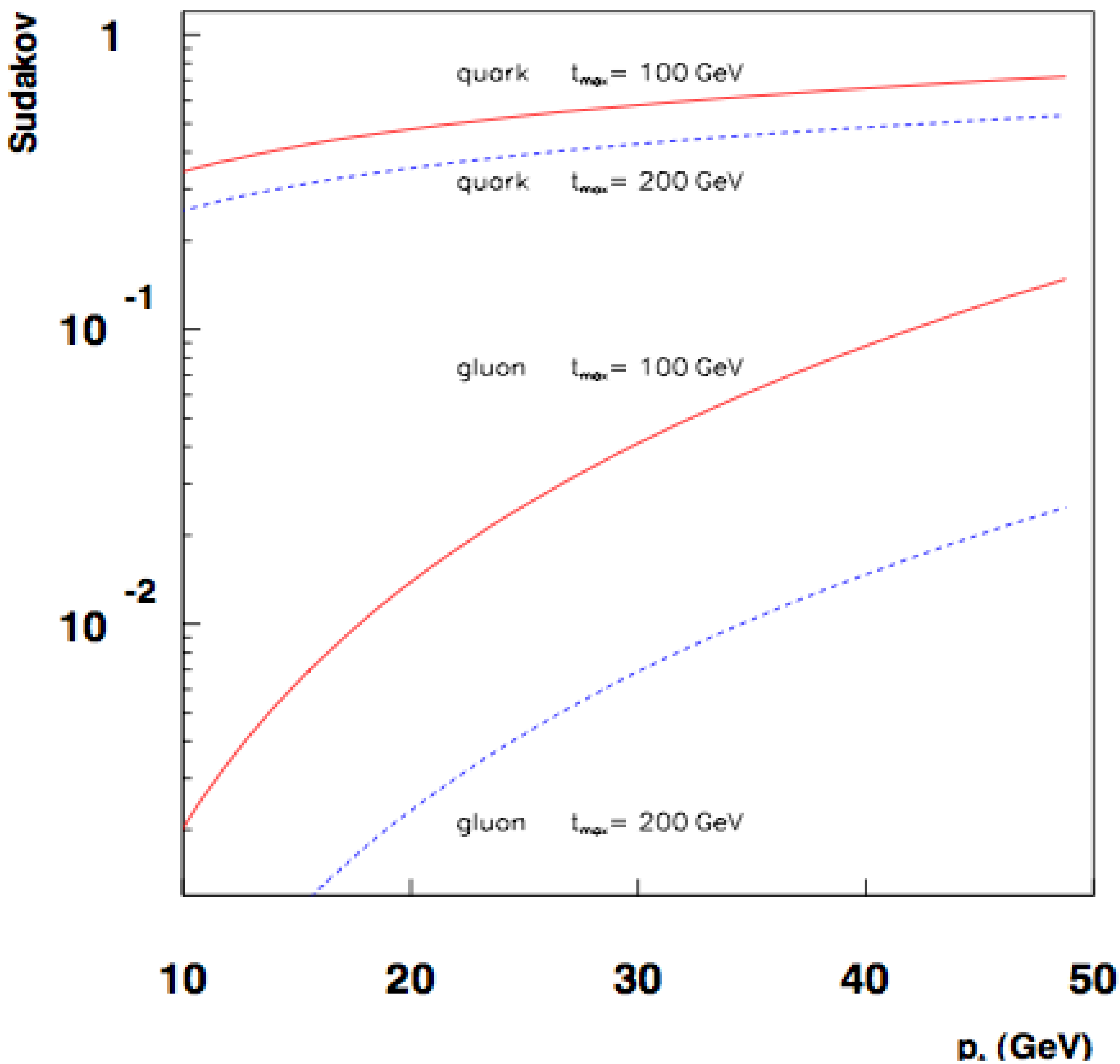
- what is the limit on z- integration ?
  - resolvable branching ?
  - $z < 1 - \frac{Q_0^2}{Q_b^2}$ with $Q_0$ a soft cutoff

**Blackboard**

- probability of no -radiation between $Q_a$ and $Q_b$
  -

$$\Pi(Q_a^2, Q_b^2) = \frac{\Delta(Q_a^2)}{\Delta(Q_b^2)}$$
$$= \exp\left[ - \int_{Q_b^2}^{Q_a^2} \frac{dq^2}{q^2} \int_0^{z_{cut}} dz \frac{\alpha_s}{2\pi} P(z) \right]$$

# Sudakov form factors



# Sudakov form factors

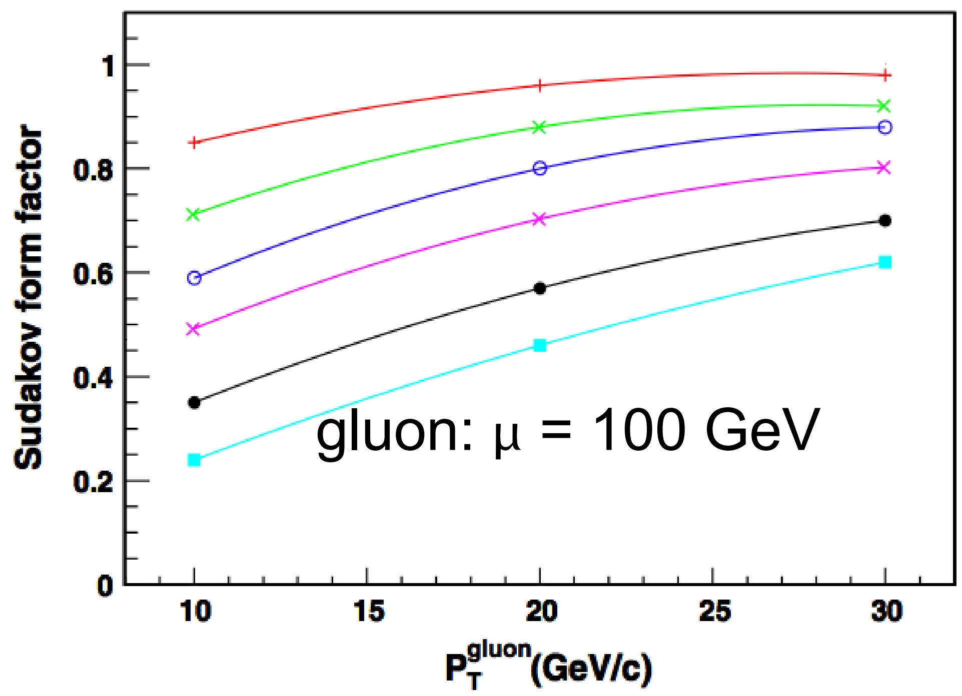



**Figure 21.** The Sudakov form factors for initial-state gluons at a hard scale of 100 GeV as a function of the transverse momentum of the emitted gluon. The form factors are for (top to bottom) parton $x$ values of 0.3, 0.1, 0.03, 0.01, 0.001 and 0.0001.

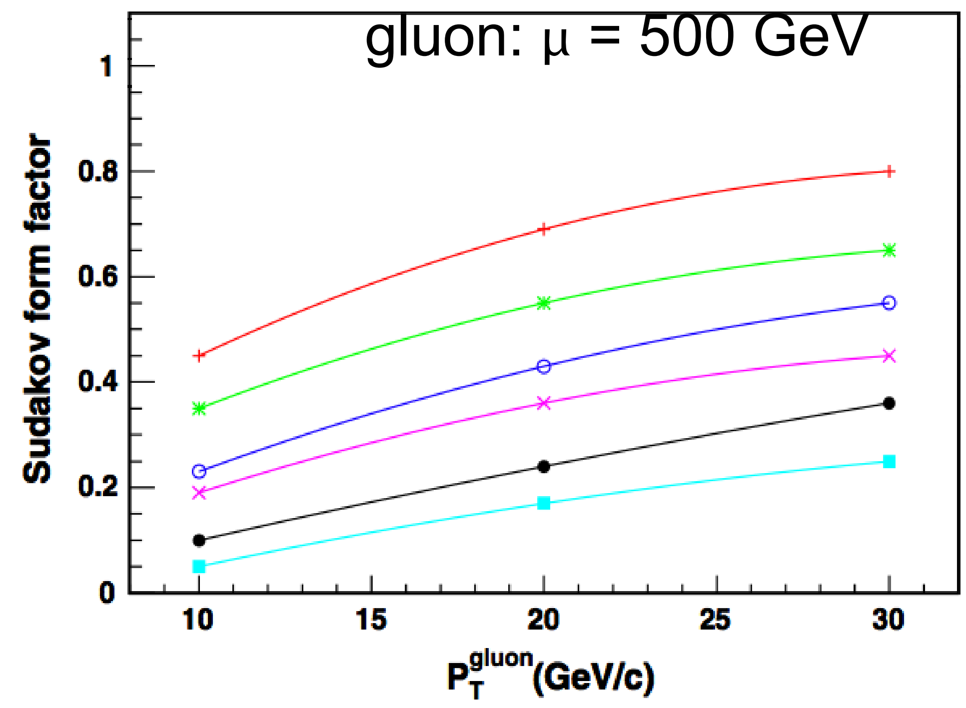



**Figure 22.** The Sudakov form factors for initial-state gluons at a hard scale of 500 GeV as a function of the transverse momentum of the emitted gluon. The form factors are for (top to bottom) parton $x$ values of 0.3, 0.1, 0.03, 0.01, 0.001 and 0.0001.

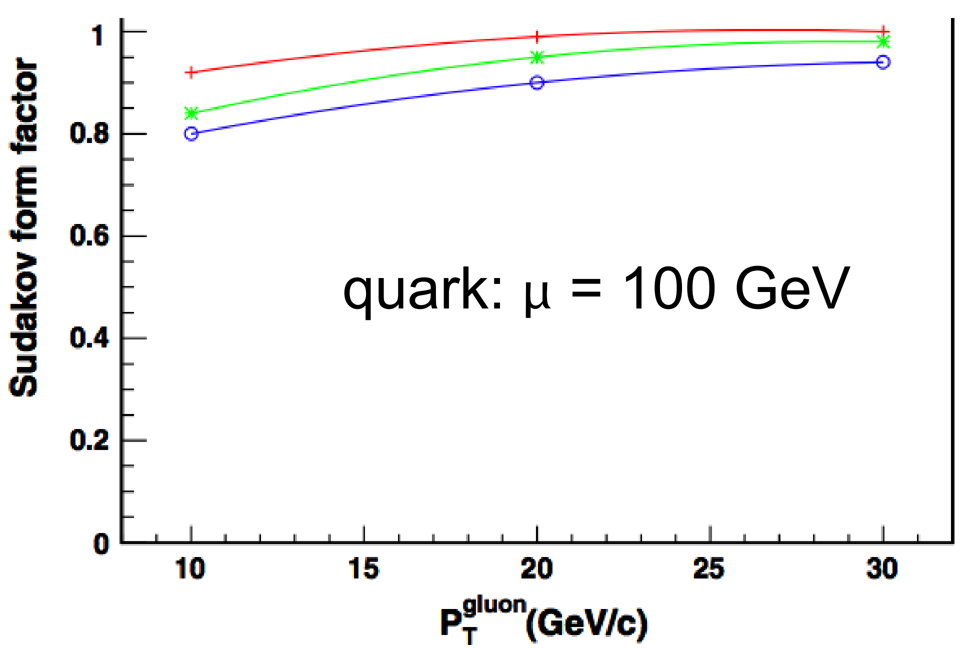



**Figure 23.** The Sudakov form factors for initial-state quarks at a hard scale of 100 GeV as a function of the transverse momentum of the emitted gluon. The form factors are for (top to bottom) parton $x$ values of 0.3, 0.1 and 0.03.

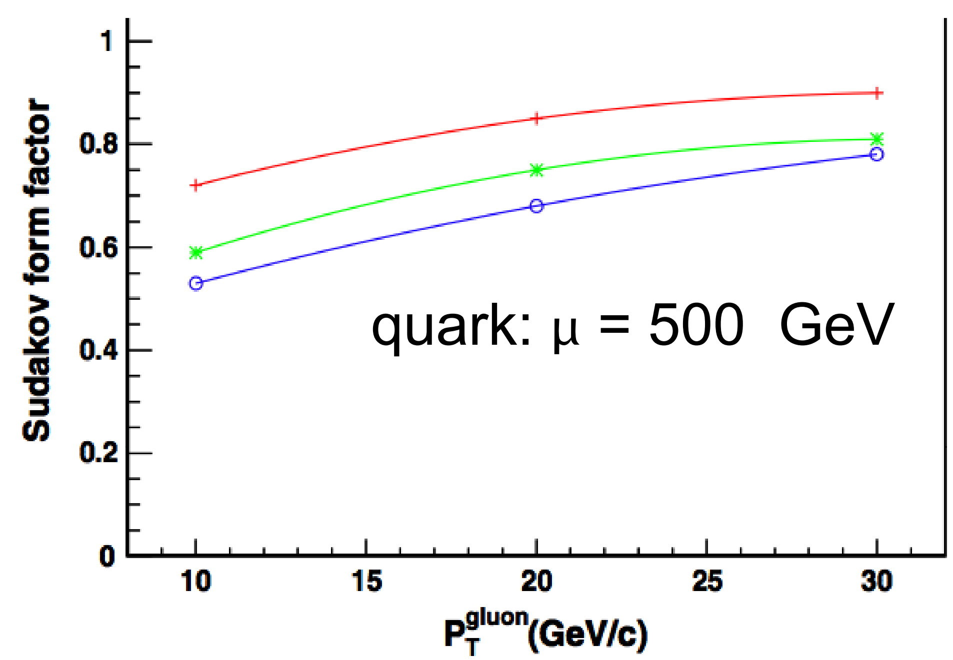
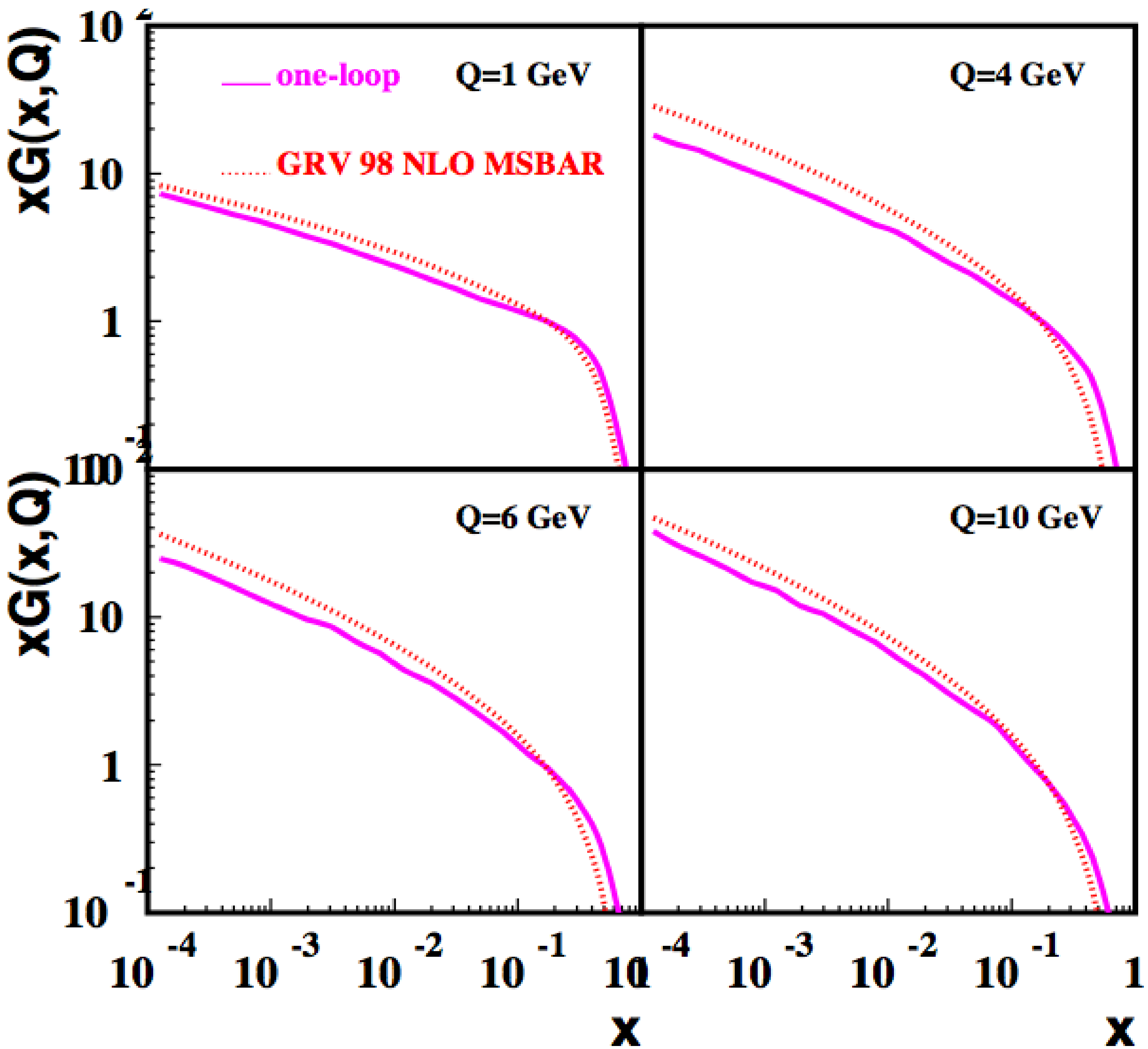



**Figure 24.** The Sudakov form factors for initial-state quarks at a hard scale of 500 GeV as a function of the transverse momentum of the emitted gluon. The form factors are for (top to bottom) parton $x$ values of 0.3, 0.1 and 0.03.

# Gluon from evolution

- comparison of gluon density obtained from standard solution of DGLAP with solution using integral equation and Sudakov form factor:

  - use same starting distribution
  - evolve using only gluons
  - evolve with simplified gluon splitting function




Good agreement, given the simplifications

# Solving evolution equation with Monte Carlo

# Evolution equation and Monte Carlo

F. Hautmann, H. Jung, and S. T. Monf
ared. The CCFM uPDF evolution uPDFevolv. Eur. Phys. J., C74:3082, 2014.

$$f(x,t) = f(x,t_0)\Delta_s(t) + \int \frac{dz}{z} \int \frac{dt'}{t'} \cdot \frac{\Delta_s(t)}{\Delta_s(t')} \frac{\alpha_s}{2\pi} \tilde{P}(z) f\left(\frac{x}{z}, t'\right)$$

$$= f(x,t_0)\Delta_s(t) + \int dz \int \frac{dt'}{t'} \cdot \frac{\Delta_s(t)}{\Delta_s(t')} \frac{\alpha_s}{2\pi} \tilde{P}(z) f\left(\frac{x}{z}, t'\right) \delta(x - zx')dx'$$

- use Sudakov: $$\Delta(t) = \exp\left[-\int_{t_0}^{t} \frac{dt'}{t'} \int^{z_{max}} dz \frac{\alpha_s}{2\pi} \tilde{P}(z)\right]$$
- generate *t* according to Sudakov

$$\frac{\partial}{\partial t'} \frac{\Delta_s(t)}{\Delta_s(t')} = \frac{\Delta_s(t)}{\Delta_s(t')} \left[\frac{1}{t'}\right] \int^{z_{max}} dz \frac{\alpha_s}{2\pi} \tilde{P}(z)$$

➔solve it for *t:* $\log \Delta_s(t,t') = \log R$

- generate z according to $$\int_{\epsilon}^{z} dz \frac{\alpha_s}{2\pi} P(z) = R \int_{\epsilon}^{1-\epsilon} dz \frac{\alpha_s}{2\pi} P(z)$$
- use momentum sum rule for normalization

# Evolution equation and MC

$$f(x,t) = f(x,t_0)\Delta_s(t) + \int \frac{dz}{z}\int \frac{dt'}{t'} \cdot \frac{\Delta_s(t)}{\Delta_s(t')}\tilde{P}(z) f\left(\frac{x}{z},t'\right)$$

- solve integral equation via iteration:

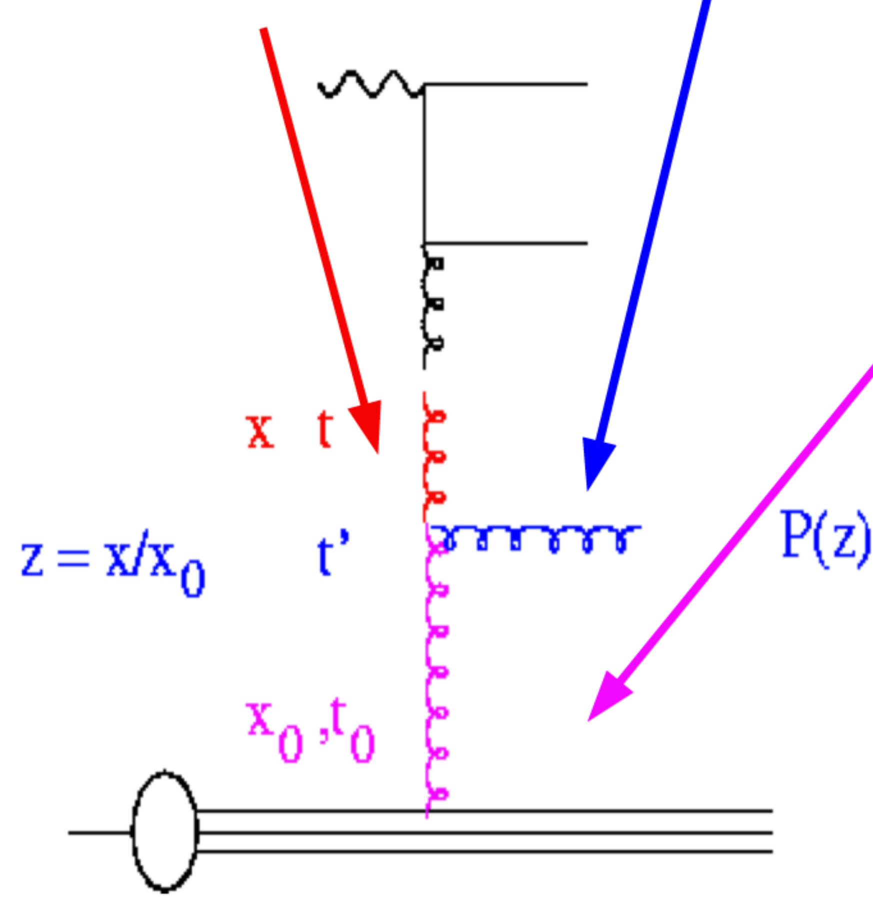
from $t'$ to $t$ w/o branching

branching at $t'$

from $t_0$ to $t'$ w/o branching

$$f_0(x,t) = f(x,t_0)\Delta(t)$$

$$f_1(x,t) = f(x,t_0)\Delta(t) + \int_{t_0}^{t}\frac{dt'}{t'}\frac{\Delta(t)}{\Delta(t')}\int\frac{dz}{z}\tilde{P}(z) f(x/z,t_0)\Delta(t')$$



# Evolution equation and MC

$$f(x,t) = f(x,t_0)\Delta_s(t) + \int \frac{dz}{z}\int \frac{dt'}{t'} \cdot \frac{\Delta_s(t)}{\Delta_s(t')}\tilde{P}(z) f\left(\frac{x}{z}, t'\right)$$

- solve integral equation via iteration:

from $t'$ to $t$ w/o branching

branching at $t'$

from $t_0$ to $t'$ w/o branching

$$\begin{aligned}
f_0(x,t) &= f(x,t_0)\Delta(t) \\
f_1(x,t) &= f(x,t_0)\Delta(t) + \int_{t_0}^{t} \frac{dt'}{t'}\frac{\Delta(t)}{\Delta(t')}\int \frac{dz}{z}\tilde{P}(z) f(x/z,t_0)\Delta(t') \\
&= f(x,t_0)\Delta(t) + \log\frac{t}{t_0} A \otimes \Delta(t) f(x/z,t_0) \\
f_2(x,t) &= f(x,t_0)\Delta(t) + \log\frac{t}{t_0} A \otimes \Delta(t) f(x/z,t_0) + \\
& \frac{1}{2}\log^2\frac{t}{t_0} A \otimes A \otimes \Delta(t) f(x/z,t_0) \\
f(x,t) &= \lim_{n\to\infty} f_n(x,t) = \lim_{n\to\infty}\sum_n \frac{1}{n!}\log^n\left(\frac{t}{t_0}\right) A^n \otimes \Delta(t) f(x/z,t_0)
\end{aligned}$$

**summing up all contribution up to t … advantage of importance sampling....**

# Monte Carlo solution of evolution

updfevolv is hosted by Hepforge, IPPP Durham

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- Contact
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- TMDplotter

## uPDFevolv 1.0.0

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### uPDFevolv manual

uPDFevolv is an evolution code for TMD parton densities using the CCFM evolution equation.

**Authors**

F. Hautmann
Dept. of Physics and Astronomy, University of Sussex,
Rutherford Appleton Laboratory, Dept. of Theoretical Physics,
University of Oxford, UK

H. Jung
DESY, Hamburg, FRG,
University of Antwerp, Belgium

S. Taheri Monfared
School of Particles and Accelerators, Institute for Research in Fundamental Sciences (IPM), Tehran, Iran

**Version**

1.0.0

**Date**

2014

Table of Contents

# TMDlib and TMDplotter

- combine and collect different ansaetze and approaches:
  http://tmd.hepforge.org/ and
  http://tmdplotter.desy.de

➔ TMDlib: a library of parametrization of different TMDs and uPDFs ( similar to LHApdf)

TMDlib and TMDplotter: library and plotting tools for transverse-momentum-dependent parton distributions, *F. Hautmann et al.* arXiv 1408.3015, submitted to EPJC.