

## Exercises 1 (10. April 2018)

### Monte Carlo technique

1. construct a uniform random number generator from the congruential method:

$$I_{i+1} = \text{mod}(a \cdot I_i + c, m)$$

$$R_{i+1} = \frac{I_{i+1}}{m}$$

with  $I_0 = 4711$ ,  $a = 205$ ,  $c = 29573$  and  $m = 139968$

Compare the correlation of 2 random numbers. Compare this with RANLUX.

2. construct a Gaussian random number generator from a uniform random number generator
3. write a small program that integrates (with Monte Carlo method) the function  $f(x) = 3x^2$  for  $\int_0^1 f(x)dx$ , and calculate the uncertainty.
4. write a small program that integrates (with Monte Carlo method)  $\int_0^1 \int_0^x dx dy$  with  $0 < x, y < 1$ .
5. write a small program to integrate a simple function in one dimension:  
 $\int_{x_{min}}^1 g(x)dx = \int_{x_{min}}^1 (1-x)^5 \frac{dx}{x}$ , using Monte Carlo integration, with  $x_{min} = 0.0001$   
Improve the above integration by using importance sampling.

If you have time, you can do some more exercises:

- use the LHAPDF library to calculate the flavor sum rules:

$$\int_0^1 dx u_V(x, Q^2) = 2$$

$$\int_0^1 dx d_V(x, Q^2) = 1$$

use the LHAPDF library and calculate the momentum sum rule:

$$\int_0^1 dx \sum_{i=-6}^6 x p_i(x, Q^2)$$

use the MRST(MRST2004nlo) set and the LO\* (MRST2007lomod) set. How much is the momentum sum rule violated in the LO\* set ? Is the momentum sum rule satisfied (or violated in the same way) for different  $Q^2$  values (use  $Q^2 = 5, 10, 100, 1000 \text{ GeV}^2$ ).

- use TMDlib to calculate the integral over  $k_t$  of the TMD parton density for a gluon for  $0.0001 < x < 0.9$  at  $q = 10$  GeV. Integrate in the range  $0.01 < k_t^2 < 100$  GeV<sup>2</sup>. Use TMD set: **PB-NLO-HERAI+II-2018-set2**.

Plot the integrate TMD in a histogram for  $0.0001 < x < 0.9$ .

Compare your results with the ones you can obtain from TMDplotter:

[tmdplotter.desy.de](http://tmdplotter.desy.de).

The integral is:

$$\int_{k_t^2 min}^{k_t^2 max} dk_t^2 \mathcal{A}(x, k_t, q) = xf(x, q)$$