Towards Study of Color Transparency with Medium Energy Electron Beams

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Abstract: Interference between vector mesons electroproduced at different longitudinal coordinates leads within Glauber approximation to a Q^2-dependence of nuclear effects similar to what is expected to be an onset of color transparency (CT). We suggest such a mapping of the photon energies and virtualities, which eliminates the undesirable Q^2-variation and allows to measure a net CT effect.

We develop a multichannel evolution equation, which for the first time incorporates CT and the effects of the coherence length in the exclusive electroproduction of vector mesons.

1. Vector Meson production. Coherence length

Vector mesons electroproduced at different points separated by longitudinal distance Δz have a relative phase shift $q_c\Delta z$, where $q_c = (Q^2 + m_V^2)/2\nu$ is the longitudinal momentum transfer in $\gamma^* N \to VN$, Q^2 and ν are the virtuality and energy of the photon, respectively. Taking this into account one arrives at the well known formula for nuclear transparency, $Tr = \sigma_A/A\sigma_N$, for the coherent production of vector mesons off nuclei [1],

$$Tr_{coh} = \frac{(\sigma_{tot}^{VN})^2}{4A\sigma_{el}^{VN}} \int d^2b \left|\int_{-\infty}^{\infty} dz\ \rho(b,z)\ e^{iq_c z} \exp\left[-\frac{1}{2}\sigma_{tot}^{VN}\int_z^{\infty} dz' \rho(b,z')\right]\right|^2 \qquad (1)$$

In the low-energy limit $q_c \gg 1/R_A$ and destructive interference eliminates the coherent production. However, nuclear transparency (1) monotonically grows with energy and reaches value $Tr_{coh} = \sigma_{el}^{VA}/A\sigma_{el}^{VN}$. Numerical examples for nuclear transparency in coherent production can be found in [2, 3, 4].

Formula for nuclear transparency for incoherent quasielastic photoproduction of vector mesons in Glauber approximation derived only recently [4], reads

$$\begin{aligned} Tr_{inc} &= \frac{\sigma_{tot}^{VN}}{2A\sigma_{el}^{VN}}(\sigma_{in}^{VN} - \sigma_{el}^{VN}) \int d^2b \int_{-\infty}^{\infty} dz_2\ \rho(b,z_2) \int_{-\infty}^{z_2} dz_1\ \rho(b,z_1) \\ &\times\ e^{iq_L(z_2-z_1)}\ \exp\left[-\frac{1}{2}\sigma_{tot}^{VN}\int_{z_1}^{z_2} dz \rho(b,z)\right] \exp\left[-\sigma_{in}^{VN}\int_{z_2}^{\infty} dz\ \rho(b,z)\right] \\ &+\ \frac{1}{A\sigma_{in}^{VN}}\int d^2b\left[1 - e^{-\sigma_{in}^{VN}T(b)}\right] - Tr_{coh}\ , \end{aligned} \qquad (2)$$

In contrast to coherent production nuclear transparency (2) decreases with energy from $Tr_{inc} = \sigma_{in}^{VA}/A\sigma_{in}^{VN}$ at low energy ($q_c \gg 1/R_A$) down to value $Tr_{inc} = \sigma_{qel}^{VA}/A\sigma_{el}^{VN}$ at high energies ($q_c \ll 1/R_A$), where σ_{qel}^{VA} is the cross section of quasielastic VA scattering. Numerical examples are presented in [2, 3, 4].

Such an energy dependence is easily interpreted in terms of lifetime of the photon fluctuations, which propagate over the coherence length $l_c \sim 1/q_c$: at low energy ($l_c \ll R_A$) the vector meson appears deep inside the nucleus and covers then about a half of the nuclear thickness. At high energy the vector meson preexists as a photon fluctuation long time and propagates through the whole nuclear thickness, resulting in a more of absorption.

Variation of l_c may be caused by either its ν- or Q^2-dependence. In the latter case l_c decreases with Q^2 and the nuclear transparency grows, what is expected to be a signature of color transparency (CT) [5], a QCD phenomenon related to suppressed initial/final state interaction in a nucleus for a small-size colorless wave packet [6]. Examples for incoherent electroproduction of ρ-meson on xenon are shown in Fig. 1 (more examples are in [2, 3, 4]). The predicted Q^2-dependence is so steep that makes it quite problematic to observe an additional Q^2-dependence generated by the color transparency effects.

Note that the cross section of photoproduction of the vector mesons on nuclei is energy-dependent at low energy due to quark (Reggeon) exchanges. This is very easy to include in our calculations, but we neglect the energy-dependence for the sake of simplicity.

2. Beyond Glauber approximation. Formation length.

According to the uncertainty principle one needs (formation) time to resolve different levels, V, V'..., in the final state. Corresponding formation length $l_f \approx 2\nu/(m_{V'}^2 - m_V^2)$ is longer than the coherence length. It has a close relation to the onset of CT [7, 5], which is possible only if $l_f \geq R_A$. Note that a full CT effect at $Q^2 \gg m_V^2$ imposes a stronger condition $l_f m_V/\sqrt{Q^2} \gg R_A$. This is because a small size, $\sim 1/Q^2$, wave packet, decomposed over different V-meson states, includes all states up to $M^2 \sim Q^2$. Such a decomposition must be frozen by Lorentz time dilation up to the heaviest states, what leads to above condition within oscillatory model.

At medium energies and Q^2 one can expect only an onset of CT, i.e. a monotonic growth of $Tr(Q^2)$ with Q^2, which results mostly from interference of the two lowest states, V and V'. Inclusion of the second channel in the case of coherent production is quite simple and the results are presented in [3]. We concentrate on the incoherent electroproduction here. For the first time we present the evolution equation and its solution incorporating both the coherence and formation length effects, what is of a special importance in the intermediate energy range.

The solution of the general multichannel equation will be presented elsewhere. Here we consider the two-coupled-channel case. Since we have to sum over all final states of the nucleus, the propagation of the charmonium wave packet through the nucleus is to be described by density matrix $P_{ij} = \sum |\psi_i\rangle|\psi_j\rangle^+$. The wave function $|\psi_i\rangle$ has three components, γ^*, V and V'. The evolution equation reads,

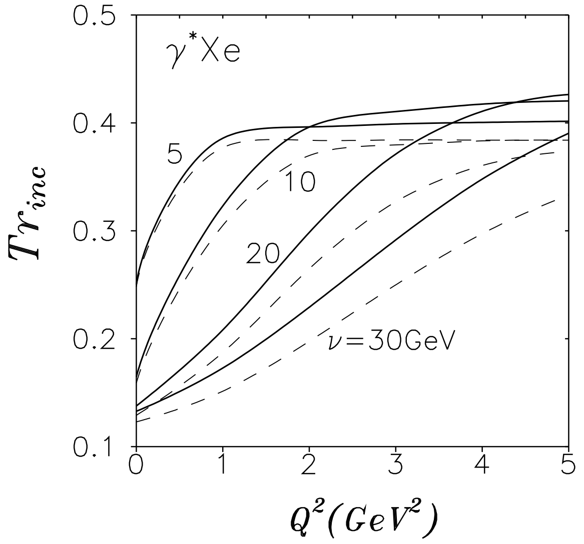
$$i\frac{d}{dz}\widehat{P} = \widehat{Q}\widehat{P} - \widehat{P}\widehat{Q}^+ - \frac{i}{2}\sigma_{tot}^{VN}\left(\widehat{T}\widehat{P} + \widehat{P}\widehat{T}^+\right) + i\sigma_{el}^{VN}\widehat{T}\widehat{P}\widehat{T}^+. \tag{3}$$

Here

$$\widehat{Q} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & q & 0 \\ 0 & 0 & q' \end{pmatrix} \;; \quad \widehat{T} = \begin{pmatrix} 0 & 0 & 0 \\ \lambda & 1 & \epsilon \\ \lambda R & \epsilon & r \end{pmatrix} \tag{4}$$

are the longitudinal momentum transfer and scattering amplitude operators, respectively, where $q(q') = (m^2_{V(V')} + Q^2)/2\nu$. For other parameters we use notations from [8], $r = \sigma^{V'N}_{tot}/\sigma^{VN}_{tot}$, $\epsilon = f(VN \to V'N)/f(VN \to VN)$ and $R = f(\gamma N \to V'N)/f(\gamma N \to VN)$. The value of parameter $\lambda = f(\gamma N \to V'N)/f(VN \to VN)$ is inessential, since it cancels in nuclear transparency. The boundary condition for the density matrix is $P_{ij}(z \to -\infty) = \delta_{i0}\delta_{j0}$. Note, the same eq. (3) reproduces Glauber approximation (2) when $\epsilon = 0$. It also calculates coherent production when one fixes $\sigma^{VN}_{el} = 0$.

Combination of the two effects of coherence and formation (onset of CT) lengths may provide an unusual energy behaviour of nuclear transparency. An example is shown in Fig. 2 for real photoproduction of ρ and ρ' on lead predicted by eq. (3). We try two models for the ρ and ρ' wave functions, the nonrelativistic oscillator and a relativized version: i) $r = 1.5$, $\epsilon = -0.5$, $R^2 = 0.074$ ($Q^2 = 0$) for the nonrelativistic oscillator; ii) $r = 1.25$, $\epsilon = -0.14$, $R^2 = 0.22$ for the relativized wave function [3]. We calculate the Q^2-dependence of R^2 in accordance with [3]. Our predictions for energy dependence of nuclear transparency for incoherent photoproduction of ρ and ρ' are shown in Fig. 2 for the two sets of parameters. While both variants give similar results for the ρ, we expect a nontrivial energy dependence for the ρ', which results from interplay of the effects of coherence and formation lengths. It turns out to be extremely sensitive to the form of the wave function. Thus, such kind of measurement may bring unique information about the structure of vector mesons.



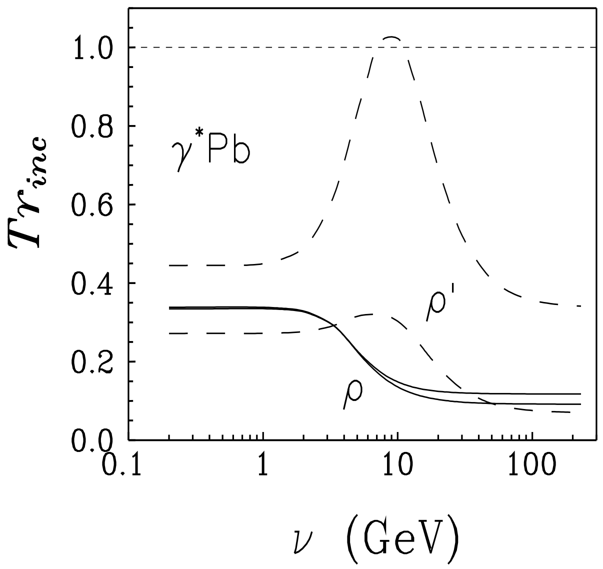
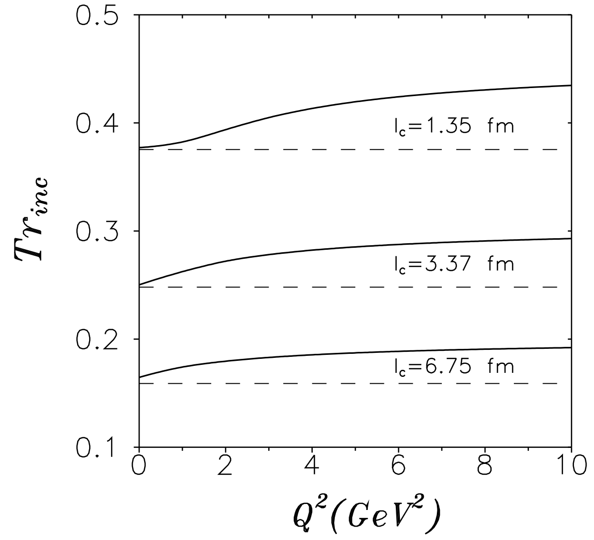


Figure 1: *Q^2-dependence of nuclear transparency for ρ-meson electroproduction on xenon at photon energies $\nu = 5,\ 10, 20$ and 30 GeV. Dashed curves correspond to Glauber approximation, solid curves are calculated with the evolution equation eq. (3).*

Figure 2: *Nuclear transparency versus ν for real photoproduction of ρ (solid curves) and ρ' (dashed curves) on lead. Upper curves correspond to the nonrelativistic oscillatory wave functions, while the bottom curves are calculated with relativized variant [9]*

Note that at low energy, where the eikonal approximation is exact, the nuclear transparency for the ρ' is about the same as for the ρ despite the bigger radius and stronger absorption of the ρ'. This is because the transitions $\rho \rightleftharpoons \rho'$ are possible even without interference, but their



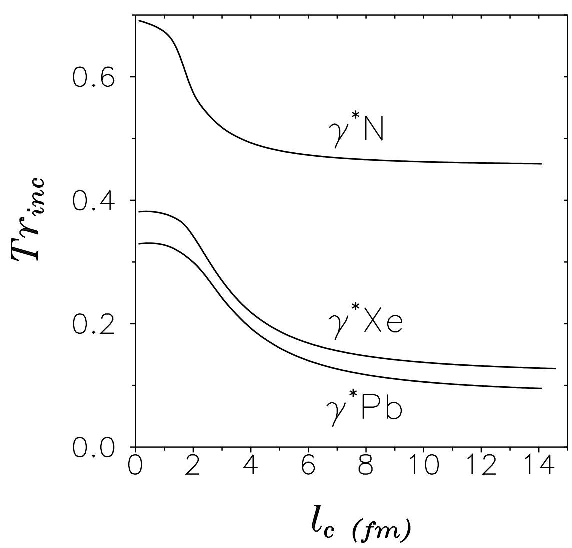


Figure 3: *Q^2-dependence of nuclear transparency at fixed $l_c = const$ for xenon. Glauber approximation predictions are shown by dashed curves. Solid curves demonstrate the effect of CT in the two coupled channel approximation*

Figure 4: *Nuclear transparency for nitrogen, xenon and lead versus the coherence length.*

probability is $\propto \sigma_{el}^{VN}$ rather than σ_{tot}^{VN}. For this reason it is very much suppressed for J/Ψ, but not for ρ.

We have arrived at the main point: how to search for a signal of CT? The two coupled channels are supposed to reproduce well the onset of CT, which is expected to manifest itself as an additional growth of nuclear transparency with Q^2. The results obtained with eq. 11 and relativized wave functions are shown in Fig. 1 by solid curves. We see that variation of the transparency with Q^2 due to the coherence length effect predicted by Glauber model is rather steep and makes it very difficult to detect a CT signal even with high-statistics measurements.

In order to single out the effect of CT one should remove the l_c-dependence of nuclear transparency. This can be done mapping values of Q^2 and ν in a way, which keeps $l_c = 2\nu/(m_V^2+Q^2)$ constant. Our predictions for Q^2-dependence of nuclear transparency at different values of l_c are plotted in Fig. 3 both for single- (Glauber) and double-channel approaches. The former, as we expected, provides a constant nuclear transparency, while the latter results in a rising $Tr(Q^2)$. Observation of such a dependence on Q^2 should be grated as an onset of CT. We show l_c-dependence of transparency in Glauber model in Fig. 4.

3. Onset of CT in $(e,e'p)$

The principal difference of $(e,e'p)$ reactions from the above discussed photoproduction of vector mesons is a strong correlation between the values of Q^2 and the energy of the recoil proton, $E_p \approx m_p + Q^2/2m_p$. Therefore, while one wants just to increase the proton energy in order to freeze its small size, one unavoidable has to increase Q^2, i.e. to suppress the cross section. This is why no CT effect is still detected in this reaction. As was demonstrated above, in order to freeze size $\sim 1/Q^2$ of the ejectile during propagation through the nucleus, its energy must be $\nu > R_A\sqrt{Q^2}\omega$, where $\omega \approx 0.3\ GeV$ is the oscillatory parameter. This demands unreachable values of Q^2 of a few hundred GeV^2. The same problem concerns $(p,2p)$ quasielastic proton scattering at large angles.

On the other hand, a growth of Q^2 causes an increase of the ejectile energy. It is known, however, since [10] that nuclear matter is more transparent at higher energies due to inelastic shadowing corrections. This effect is general and independent of color dynamics or whether the CT effect exists or not. The growth of nuclear transparency in $(e, e'p)$ with energy, and consequently with Q^2, was estimated in [11], assuming a simplest "anti-CT" scenario: a regular proton, rather than a small-size fluctuation, is knocked out in $(e, e'p)$ reaction. The predicted Q^2 dependence turns out to be similar to what one expects to be an onset of CT up to about $Q^2 \approx 20\ GeV^2$. Therefore, one should be very cautious interpreting the growth of $Tr(Q^2)$ as a signal of CT in this region. It can be safely done only at quite high, still unreachable Q^2.

4. Conclusion

We have developed for the first time a multichannel approach to incoherent exclusive electroproduction of vector mesons off nuclei, which incorporated effect of the coherent and formation lengths. It is based on a multichannel evolution equation for the density matrix of the produced wave packet. Variation of the coherence length with the photon energy and Q^2 causes substantial changes of the nuclear transparency even in Glauber approximation. This fact makes it very difficult to observe an onset of CT. We suggest such a mapping of ν and Q^2 values, which keeps the coherence length constant. in This case the nuclear transparency cannot change within Glauber approximation. We provide estimates of the onset of CT within two coupled channel model.

One faces even more problems searching for a signal of CT in $(e, e'p)$ quasielastic scattering, due to its specific kinematics. It is very difficult to interpret the growth of nuclear transparency with Q^2 as an onset of CT at $Q^2 < 20/GeV^2$.

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