Heavy Flavour Production

R. Eichler, S. Frixione

ETH-Zürich, CH 8093 Zürich, Switzerland

Abstract: We discuss the impact of the planned upgrades of the HERA collider on the study of open heavy flavour and quarkonium production. New experimental techniques in charm physics are presented.

1 Introduction

The ep collider HERA offers new opportunities to study the production mechanism of heavy quarks and to test the predictions of the underlying theory. In the following, we will deal with the processes

$$e + p \longrightarrow Q + \overline{Q} + X, \tag{1}$$
$$\gamma + p \longrightarrow Q + \overline{Q} + X, \tag{2}$$
$$\gamma + p \longrightarrow \left(Q\overline{Q}\right) + X, \tag{3}$$

where $\left(Q\overline{Q}\right)$ indicates a generic heavy quarkonium state, like J/ψ. The dominant contribution to the $Q\overline{Q}$ cross section is due to those events in which the virtuality of the photon exchanged between the electron and the proton is very small. In this case, the electron can be considered to be equivalent to a beam of on-shell photons, whose distribution in energy (Weizsäcker-Williams function [1]) can be calculated in QED. The underlying production mechanism is therefore a photoproduction one (eqs. (2) and (3)), which has been studied extensively in fixed-target experiments. At HERA, the available center-of-mass energy is about one order of magnitude larger than at fixed-target facilities; this energy regime is totally unexplored in photoproduction, and several new features have to be taken into proper account. In particular, the large contribution of the resolved photon component introduces in the theoretical predictions a source of uncertainty which is totally negligible at fixed-target energies.

A complementary way of studying heavy flavour production at HERA is to retain only those events characterized by a large photon virtuality (eq. (1)). Although the total rates are much smaller than the photoproduction ones, the resolved component is completely eliminated and more reliable theoretical and experimental results can be obtained. Also, the dependence of the data upon the photon virtuality can be used as a further test of QCD predictions.

2 Theoretical summary

Fully exclusive, next-to-leading order perturbative QCD calculations [2, 3, 4] are now available for the processes of eqs. (1) and (2). In the framework of the factorization theorem of ref. [5], a next-to-leading order calculation for the direct color-singlet photoproduction of quarkonium has been presented in ref. [6]. Other contributions to the production of quarkonium, like the resolved photoproduction and the direct color-octet photoproduction, which are potentially

relevant in kinematical regions accessible at HERA, have been considered at the leading order in ref. [7].

In spite of these substantial progresses in understanding the production mechanism, the (fixed-order perturbative) results may become unreliable in certain kinematical regions, due to the appearance of potentially large logarithms which spoil the convergence of the perturbative expansion. In this case, a resummation to all orders of these large logarithms has to be performed.

When the available center-of-mass energy $\sqrt{S}$ gets large, the effective expansion parameter of the perturbative series becomes $\alpha_S \log(S/m_Q^2)$. The problem of resumming these terms (*small-x effects*) has been tackled by several authors [8], mainly in the context of b production in hadronic collisions. Specific studies for HERA [9] lead to the conclusion that the total photoproduction rates can be increased by the resummation of the $\log(S/m_Q^2)$ terms by a factor smaller than or equal to 40% (45%) with respect to the next-to-leading order prediction for the direct (resolved) contribution in charm production. In the case of bottom, the effect is much milder, being always smaller than 10%.

The transverse momentum distribution is in principle affected by the presence of $\log(p_T/m_Q)$ terms. These logarithms can be resummed by observing that, at high p_T, the heavy-quark mass is negligible, and by using perturbative fragmentation functions [10]. Remarkably enough, the fixed-order and the resummed results agree in a very wide range in p_T for charm production (in the HERA energy range, the contribution of the resummation of this kind of logarithms is expected to be negligible for bottom production); the effect of the resummation might be visible only for p_T larger than 50 GeV.

Finally, multiple soft gluon emission makes the perturbative expansion unreliable close to the threshold or to the borders of the phase space, like for example the regions $p_T^{Q\bar{Q}} \simeq 0$ and $\Delta\phi^{Q\bar{Q}} \simeq \pi$. This problem has not been directly dealt with during the workshop, but a lot of theoretical work has recently been performed [11]. The resummation of soft gluons would be of great help in order to have a reliable prediction for J/ψ production at $p_T \simeq 0$; also, threshold effects are important for b production at HERA-B.

2.1 Quarkonium Photoproduction

Quarkonium photoproduction has been dealt with in ref. [12]. The benchmark process is in this case the production of J/ψ. With suitable cuts, typically $p_T > 1$ GeV and $z < 0.9$, where

$$z = \frac{k(p) \cdot k(J/\psi)}{k(p) \cdot k(\gamma)}, \tag{4}$$

the dominant contribution to the cross section is due to the direct color-singlet production of J/ψ; this cross section has been calculated at next-to-leading order in QCD [6]. The radiative corrections turn out to be sizeable in the HERA energy range, being of the same order of the leading-order contribution. The total rate is of the order of tens of nb, and a first comparison with data [13] has already been performed. Although there is a good agreement as far as the shape of (z and p_T) distributions is concerned, the theoretical predictions appear to slightly undershoot the data for the total cross section; this can be adjusted by properly tuning the input parameters of the calculation, since the normalization is affected by large uncertainties.

With a larger integrated luminosity, a more significant comparison will be possible, eventually helping in constraining the input parameters.

The luminosity upgrade is mandatory in order to assess the importance of the J/ψ production mechanisms neglected so far, namely the direct color-octet contribution and the resolved contribution. The former has been considered in refs. [7, 12] at the leading order in QCD. It has to be stressed that this contribution can not be calculated from first principles, and relies upon fits to Tevatron data on color-octet J/ψ production. The comparison with HERA data can therefore be regarded as a test of the underlying factorization picture [5]. Using the result of ref. [7], the color-octet contribution appears to be dominant over the color-singlet one in the large-z region; the first HERA data do not support this behaviour. Nevertheless, since both the data and the theoretical calculations need some refinement, no definite conclusion can be drawn at present. The resolved J/ψ production is expected to be dominant at very low-z values, being enhanced by the color-octet terms. Since the total rate is $\mathcal{O}(1\ nb)$ in that region, large integrated luminosity is needed to study this problem.

To further investigate the interplay between color-singlet and color-octet contributions, other possibilities were taken into account in ref. [12]. For example, the photoproduction cross section of χ should be dominated by color-octet terms; the total rate is however suppressed by two orders of magnitude with respect to the one of J/ψ, and therefore large luminosity is required. The $J/\psi + \gamma$ production has a very distinctive signature to color-octet contribution in the large-z region [14], but the cross section is very small ($\mathcal{O}(10\ pb)$). Finally, large luminosity is necessary also in order to investigate J/ψ production for $p_T > 10$ GeV, where the dominant contribution is expected to be due to charm fragmentation (the total rate is at most $\mathcal{O}(1\ pb)$).

2.2 Open Heavy Flavour Production

Several aspects of the deep-inelastic production of heavy flavours at HERA have been studied during the workshop. The work of ref. [15] basically falls into three parts. To begin with, the authors present a study of the heavy quark inclusive structure function $F_2(x, Q^2, m^2)$, its sensitivity to the input parameters of the calculation, and the size of the QCD corrections. The full next-to-leading order theoretical prediction [3, 4] suffers from a fairly little uncertainty, and the radiative corrections are not too large. The structure function is on the other hand significantly sensitive to the small-x behaviour of the gluon density in the proton. Therefore, it is concluded that F_2 in charm production is an excellent probe to infer the gluon density in the proton at small x.

Next, the exclusive properties of the final state are investigated, since they allow a deeper understanding of the dynamics of the production process with respect to the inclusive structure function. The next-to-leading order corrections to $F_k,\ k = 2, L$, in a fully differential form have been recently calculated [4] using the subtraction method. The results were incorporated in a Monte-Carlo style program which allows one to study correlations in the laboratory frame. The authors of ref. [15] present some single-inclusive distributions and compare them with preliminary ZEUS data [16]. In general, the data follow the shape of the theoretical curves, but lie above them. This behaviour is consistent with the recent analysis by H1 [17] that, using a single-inclusive quantity, showed that the charm production mechanism is boson-gluon fusion, and not a consequence of charm quarks in the sea. Correlations between charm and anticharm are also presented. Clearly, all these issues will be further clarified as soon as data with better statistics will be available; a large integrated luminosity is highly desirable.

Finally, the authors of ref. [15] deal with the fact that, at truly large Q^2, a charm quark should be described as a light quark, i.e. as a constituent parton of the proton, whereas at Q^2 of order of m^2 it can only be produced through boson-gluon fusion (see ref. [18] for a detailed discussion). It is shown that, surprisingly, already at Q^2 of order 20-30 GeV^2, the asymptotic next-to-leading order formula (large Q^2) for the inclusive structure function differs from the exact result for a factor of 5% or less, indicating that at these not so large Q^2 values the charm quark behaves already very much like a parton. Since, as mentioned before, the boson-gluon fusion almost correctly accounts for experimental results on single-inclusive distributions, this points out that the scale at which the charm quark can be regarded as a parton actually depends upon the physical observables considered.

Open heavy flavour photoproduction has been considered in ref. [19]. With an integrated luminosity of 1000 pb^{-1}, next-to-leading order QCD predicts about 10^9 (10^6) charm (bottom) particles produced in ep collisions (Weizsäcker-Williams approximation). Therefore, taking into account the experimental efficiencies, the number of reconstructed particles will be comparable to or larger than the one obtainable by the fixed-target hadroproduction experiments of the new generation at Fermilab. This will provide the HERA experiments with the possibility of performing studies of charm and bottom physics at an excellent level of accuracy. In this respect, it is extremely interesting to compare the results at HERA with the results of fixed-target photoproduction experiments, which have a center-of-mass energy of about one order of magnitude smaller. In charm physics, high luminosity will allow to consider exclusive quantities, like correlations, which constitute the most stringent test for the underlying theoretical picture. One may also adopt a different point of view, namely to look at charm production as a useful tool to constrain the input parameters entering the calculations, like for example the quark mass. To this end, it is mandatory to have data with large statistics. The capability of HERA of producing bottom quarks is also very promising. In particular, the comparison of the QCD prediction for the p_T spectrum of bottom (which has been shown in ref. [20] to be only marginally affected by the uncertainties on the input parameters of the calculation) with the data could be of great help in understanding the origin of the discrepancy observed at the Tevatron for the same quantity.

2.3 Production at HERA-B

The study of the production mechanism of heavy flavours at HERA-B might prove to be extremely useful. The data on bottom production at fixed target have low statistics, and have been mainly obtained in πN collisions (only very recently, the first measurement of the total $b\bar{b}$ rate in pN collisions has been presented). A study of distributions would be interesting in order to test the QCD description of bottom quark production at low center-of-mass energies. In charm physics, the results of fixed-target experiments at CERN and Fermilab still leave plenty of open questions (see ref. [21] and references therein). In particular, new measurements may help in understanding the importance and the nature of non-perturbative contributions to charm cross section. The hypothesis of the intrinsic charm in the proton could be tested as well.

2.4 Determination of the Gluon Density in the Proton using Charm Data

The possibility has been considered of using charm data to constrain (or to measure) the gluon density in the proton, both in the unpolarized and the polarized case. Photoproduction of charm is in principle an ideal tool in order to perform this measurement, since the gluon density enters the cross section already at the leading order, and in a simpler way with respect to hadroproduction processes. It has been shown in ref. [22] that the use of correlations between the charm and the anticharm can give a direct measure of the unpolarized gluon density in the proton in a fairly large range in x ($10^{-3} \div 10^{-1}$), since the contribution of the resolved component can be suitably suppressed. Being necessary to reconstruct both the charm and the anticharm for this kind of measurements, a very large integrated luminosity is required. The measurement of the polarized gluon density in the proton has been considered in refs. [23, 24] (see also ref. [25]), in view of the possibility for HERA to operate in the polarized mode. The conclusion has been reached [23] that charm photoproduction data can be used to constrain this quantity, if an integrated luminosity of at least 100 pb^{-1} will be achieved.

2.5 Intrinsic Charm

The possibility of detecting signals due to intrinsic charm in the proton has been discussed during the workshop by G. Ingelman. An intrinsic charm contribution has been suggested long time ago [26] to explain an excess of data from fixed-target experiments with respect to theoretical predictions in the large-x_F region. Recently, this problem has been tackled in ref. [27], dealing directly with ep collisions at HERA. It turns out that the intrinsic charm contribution is non-negligible only in the very forward region, and can not be detected with the present experimental configuration; an upgrade in the very forward region would be necessary.

3 Heavy Quark Production Cross Sections and Reconstruction Efficiencies

In this section we give measured cross sections and expected reconstruction efficiencies for charmed mesons. As an example the capabilities of the H1 detector [28] are given. The effect of a double layer silicon vertex detector [29] is investigated.

The charm production cross section has been measured by both collaborations H1 and ZEUS [30, 31] and is close to one microbarn. The value for photoproduction ($Q^2 < 0.01$ GeV^2) amounts to [30]

$$\sigma(ep \to c\bar{c}X) = 941 \pm 160^{+142}_{-120}\ nb.$$

The first error is the statistical and the second the systematic error dominated by the extrapolation uncertainty to the unmeasured phase space. This rather large cross section makes HERA an ideal place for charm physics.

The b cross section has not been measured yet. An estimate [32] $\sigma(ep \to b\bar{b}X) = 6 \pm 2\ nb$ is of similar magnitude as on the Z^0- or the Υ-resonance and much lower than at hadron colliders.

The reconstruction of charm uses the central tracking chambers with an angular coverage of $-1.5 < \eta < 1.5$. The decay chain

$$ep \rightarrow D^* X \rightarrow D^0 \pi_s \rightarrow (K^- \pi^+) \pi_s$$

yields a signal to background ratio of 1:1 for the mass difference $\Delta M = M(K^- \pi^+ \pi^+) - M(K^- \pi^+)$ with a cut on the transverse momentum of $p_T(D^*) > 2.5\ GeV$. A total efficiency in photoproduction of trigger, acceptance, branching fraction, and reconstruction of charmed events via D^*-tagging of 10^{-4} has been achieved (table 1). This is only a tiny fraction of the total cross section. The yield can be enlarged by summing other decay channels of the D meson. A similar signal to background ratio can be achieved with the reconstruction of

$$D^0 \rightarrow K^- \pi^+ \ (3.83\%),\ K^0 \pi^+ \pi^- \ (5.4\%),\ K^- \pi^+ \pi^+ \pi^- \ (7.5\%),\ K^- \mu^+ \nu \ (3.23\%).$$

This is an improvement of a factor of 5 over the single $K^- \pi^+$ channel.

Besides the branching fractions into a specific decay channel the biggest event loss (order of magnitude) comes from the transverse momentum cut of the D^* which is necessary to reduce the combinatorial background. This can be partially compensated with the help of a vertex detector.

3.1 Charm Reconstruction with the H1 Vertex Detector

The main effect of a vertex detector is the assignment of charged particle tracks to a secondary vertex which results in a much improved signal to background ratio S/N, but with a corresponding loss of acceptance.

This can be seen in figure 1a (from ref. [33]), where this ratio S/N is plotted versus the separation d/σ of primary and secondary vertex, with d the distance and σ its error. No lower momentum cut for the D^* was applied. The track finding efficiency of the slow pion in the $D^* \rightarrow D^0 \pi_s$ decay is non zero only above $p_T(\pi_s)$=120 MeV which implies an indirect transverse momentum cutoff for the D^* meson of about 1 GeV.

The signal to background grows rapidly with the vertex separation and reaches values above one for $d/\sigma > 2-2.5$. At the same time we observe a loss of the signal by a factor two compared to no vertex cut (see figure 1b). Therefore the net gain is not very large.

But with the vertex cut even more decay channels are reconstructable. The D^0 mesons can be found directly without the detour via the D^*. This gains a factor of more than two. Also the D^+ decays into $K^- \pi^+ \pi^+ (9.1\%)$, $K^0 \pi^+ \pi^+ \pi^-$ (7%), and $K^0 \pi^+$ (2.8%) will be possible. Therefore the net gain with a vertex detector to fully reconstruct charmed final states is a factor 5-10.

A vertex detector is also useful for tagging charm by looking for secondary vertices inside jets. A study [34] has been performed where two or more jets with $E_T^{jet} \geq 6\ GeV$, $|\eta^{jet}| < 2$, Q^2 <4 GeV^2, and 135 GeV $< W_{\gamma p} <$ 270 GeV were selected. The efficiency using very loose cuts (at least one track with an impact parameter displaced by d/σ >2) is 37% for charm and 17% for bottom with S/N=0.9 and also here the gain from a vertex detector is significant (order of magnitude).

Tables 1 and 2 summarize the efficiencies. For an integrated luminosity of 300 pb^{-1} we expect therefore of order 10^6 fully reconstructed D^0 and $D^{\pm}$ mesons with a S/N$\geq$1.

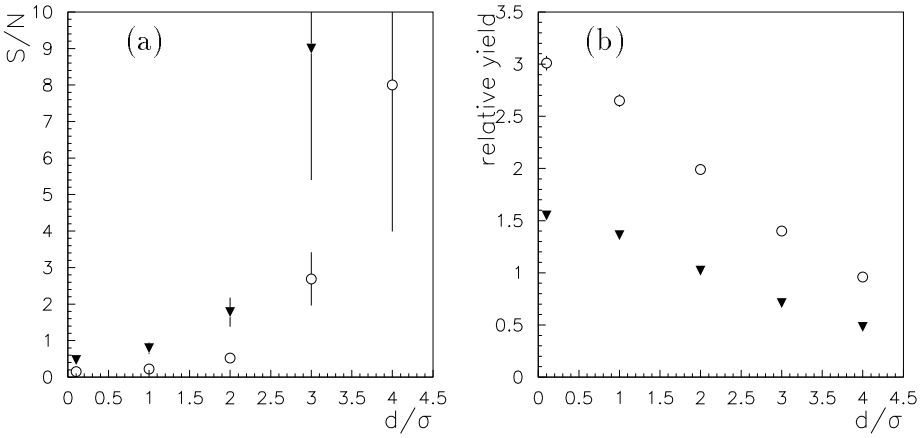

Figure 1: *a): Signal to background ratio S/N of the D^* signal versus the vertex separation d normalized to its error σ for a transverse momentum cutoff of 1 GeV (open circles) and 2 GeV (closed triangles) of the D^*. (b): Yield of the D^* signal with $p_T(D^*) > 1$ GeV (open circles) and > 2 GeV (closed triangles) respectively relative to a momentum cutoff of $p_T(D^*) > 2.5$ GeV.*

3.2 Charm-Anticharm Correlations

For many applications and QCD studies the two-particle inclusive distribution is of interest. We investigated the probability to fully reconstruct two charmed particles in H1.

The acceptance that both charm quarks fall inside the acceptance of the tracking system ($|\eta| < 1.5$, $p_T > 1$ GeV) is 3%. Using all the above decay channels and a vertex detector we can reach a double tagging efficiency of order 2.5×10^{-5}, which translates into 7000 charm pairs for 300 pb^{-1}. Without a vertex detector this number will be an order of magnitude less.

If we do not require the full reconstruction of both charmed mesons, but ask only for one reconstructed D meson plus a lepton of momentum $p_T > 1.5$ GeV which does not come from the primary vertex, we reach similar efficiencies as above, therefore doubling the number of double charm tags. Relaxing the quality of the double tagged events even more one would ask for one fully reconstructed charmed meson and a well separated jet. Efficiencies depend very much on chosen cuts and no numbers are given here.

4 Summary and Comparison with other Machines

At HERA, a rich physics program can be achieved with an integrated luminosity $\mathcal{L} = 300$ pb^{-1}. The inclusive properties of open charm photo- and electroproduction can be studied at an excellent level of accuracy. The next-to-leading order QCD predictions for the direct color-singlet photoproduction of J/ψ can be thoroughly tested. Correlations between charm and anticharm, open bottom production, resolved and direct color-octet quarkonium photoproduction could be studied with $\mathcal{L} = 300$ pb^{-1} as well, although larger luminosity would allow a more significant

no vertex detector	acceptance $p_T > 2.5$ GeV	reconstr. efficiency	$c \to D^* \to D^0\pi$	D^0 decay mode	total efficiency
$D^0 \to K^-\pi^+$	0.03	0.5	0.18	0.038	$1\ 10^{-4}$
$D^0 \to K^-\pi^+\pi^+\pi^-$				0.075	
$D^0 \to K^-\mu^+\nu$				0.032	
$D^0 \to K^0\pi^+\pi^- \to 4\pi$				0.019	
sum (D^0)	0.03	0.5	0.18	0.164	$4\ 10^{-4}$
with vertex detector	acceptance $p_T > 1.0$ GeV $d/\sigma > 2.3$	reconstr. efficiency	$c \to D^0/D^+$	D^0/D^+ decay mode	total efficiency
$D^0 \to K^-\pi^+$	0.05	0.3	0.55	0.038	$3\ 10^{-4}$
$D^0 \to K^-\pi^+\pi^+\pi^-$				0.075	
$D^0 \to K^-\mu^+\nu$				0.032	
$D^0 \to K^0\pi^+\pi^- \to 4\pi$				0.019	
sum (D^0)	0.05	0.3	0.55	0.164	$1\ 10^{-3}$
$D^+ \to K^0\pi^+ \to 3\pi$	0.05	0.3	0.21	0.019	$6\ 10^{-5}$
$D^+ \to K^-\pi^+\pi^+$				0.091	
$D^+ \to K^0\pi^+\pi^+\pi^- \to 5\pi$				0.024	
sum ($D^\pm$)	0.05	0.3	0.21	0.134	$4\ 10^{-4}$
sum (all D-mesons)					$2\ 10^{-3}$

Table 1: *Summary of various reconstruction efficiencies for D^0 mesons in photoproduction with and without a vertex detector. Charged particles have been confined to $|\eta| < 1.5$, and a vertex separation of $d/\sigma > 2.3$ has been assumed for the lower example with a vertex detector. Acceptance and reconstruction efficiencies depend on details of cuts and only rough numbers using cuts similar to ref. [30] are given.*

tagging method	charm-tagging	bottom-tagging
D^*-meson	0.014	0.09
identified muon	0.073	0.20
non-vertex track, $d/\sigma > 3$	0.10	0.33
non-vertex track, $d/\sigma > 2$	0.37	0.17

Table 2: *Summary of charm and bottom tagging efficiency inside jets with $E_T^{jet} \geq 6$ GeV, $|\eta^{jet}| < 2$, $Q^2 < 4$ GeV^2, and 135 GeV $< W_{\gamma p} <$ 270 GeV. Non-vertex track means a track with an impact parameter d larger than 2 resp. 3 times its error σ (from ref. [34]).*

comparison between theory and experiments. Charm correlations can be exploited in order to measure the gluon density in the proton. In polarized ep collisions, charm data are useful to constrain the *polarized* gluon density in the proton. Very large luminosity is mandatory in order to study the production of χ, $J/\psi + \gamma$ and to investigate the charm fragmentation into J/ψ. Intrinsic charm signals are not detectable without upgrading the detectors in the very forward region. A vertex detector is essential to reach a high tagging efficiency for open charm and should be taken into account in the design of the luminosity upgrade program.

A figure of merit of the potential for charm physics is the total number of reconstructed open charmed particles. The biggest competition comes from fixed target experiments at Fermilab. A program to reach 10^6 reconstructed open charm is in place which corresponds to 300 pb^{-1} at HERA. Progress beyond 10^6 will depend on commitment to the physics and a potential to get 10^8 at Fermilab is enticing [35].

Acknowledgements: One of the authors (S.F.) wishes to thank the Swiss National Foundation for financial support.

References

[1] C. F. Weizsäcker, *Z. Phys.* **88**(1934)612;
E. J. Williams, *Phys. Rev.* **45**(1934)729.

[2] R. K. Ellis and P. Nason, *Nucl. Phys.* **B312**(1989)551;
J. Smith and W. L. van Neerven, *Nucl. Phys.* **B374**(1992)36;
S. Frixione, M. Mangano, P. Nason and G. Ridolfi, *Nucl. Phys.* **B412**(1994)225;
P. Nason, S. Dawson and R. K. Ellis, *Nucl. Phys.* **B303**(1988)607; **B327**(1988)49;
W. Beenakker et al., *Phys. Rev.* **D40**(1989)54; *Nucl. Phys.* **B351**(1991)507;
M. Mangano, P. Nason and G. Ridolfi, *Nucl. Phys.* **B373**(1992)295.

[3] E. Laenen, S. Riemersma, J. Smith and W. L. van Neerven, *Nucl. Phys.* **B392**(1993)162;
Phys. Lett. **B291**(1992)325.

[4] B. W. Harris and J. Smith, *Nucl. Phys.* **B452**(1995)109; *Phys. Lett.* **B353**(1995)535.

[5] G. T. Bodwin, E. Braaten and G. P. Lepage, *Phys. Rev.* **D51**(1995)1125.

[6] M. Krämer, J. Zunft, J. Steegborn and P.M. Zerwas, *Phys. Lett.* **B348**(1995)657;
M. Krämer, *Nucl. Phys.* **B459**(1996)3.

[7] M. Cacciari and M. Krämer, *Phys. Rev. Lett.* **76**(1996)4128;
J. Amundson, S. Fleming and I. Maksymyk, UTTG-10-95, hep-ph/9601298;
P. Ko, J. Lee and H. S. Song, SNUTP-95-116, hep-ph/9602223.

[8] R. K. Ellis and D. A. Ross, *Nucl. Phys.* **B345**(1990)79;
S. Catani and L. Trentadue, *Nucl. Phys.* **B327**(1989)323;
S. Catani, M. Ciafaloni and F. Hautmann, *Phys. Lett.* **B242**(1990)97;
J. C. Collins and R. K. Ellis, *Nucl. Phys.* **B360**(1991)3.

[9] S. Catani, M. Ciafaloni and F. Hautmann, in *Proceedings of the Workshop: "Physics at HERA"*, eds. W. Buchmüller and G. Ingelman, DESY, Hamburg, 1992;
S. Frixione, M. Mangano, P. Nason and G. Ridolfi, *Phys. Lett.* **B348**(1995)633;
V. A. Saleev, N. P. Zotov, hep-ph/9505279 (May 1995).

[10] M. Cacciari and M. Greco, *Z. Phys.* **C69**(1996)459.

[11] E. Laenen, J. Smith and W. L. van Neerven, *Nucl. Phys.* **B369**(1992)543; *Phys. Lett.* **B321**(1994)254;
E. L. Berger and H. Contopanagos, *Phys. Lett.* **B361**(1995)115;
Phys. Rev. **D54**(1996)3085;
S. Catani, M. Mangano, P. Nason and L. Trentadue, CERN-TH/96-86, hep-ph/9604351;
N. Kidonakis and G. Sterman, ITP-SB-96-7, hep-ph/9604234;
E. L. Berger and R. Meng, *Phys. Rev.* **D49**(1994)3248.

[12] M. Cacciari and M. Krämer, these proceedings.

[13] S. Aid et al., H1 Coll., *Nucl. Phys.* **B472**(1996)3;
ZEUS Coll., pa-02-47, *XXVIII Int. Conf. on HEP*, Warsaw (1996).

[14] M. Cacciari, M. Greco and M. Krämer, DESY-96-147, in preparation.

[15] E. Laenen et al., these proceedings.

[16] ZEUS Coll., *XXVIII Int. Conf. on HEP*, Warsaw (1996).

[17] C. Adloff et al., H1 Coll., DESY-96-138, hep-ex/9607012, to appear in *Z. Phys.* **C**.

[18] S. Riemersma and F. I. Olness, *Phys. Rev.* **D51**(1995)4746;
M. A. G. Aivazis et al., *Phys. Rev.* **D50**(1995)3085; *Phys. Rev.* **D50**(1995)3102;
M. Buza et al., *Nucl. Phys.* **B472**(1996)611.

[19] S. Frixione, these proceedings.

[20] S. Frixione, P. Nason and G. Ridolfi, *Nucl. Phys.* **454**(1995)3.

[21] S. Frixione, M. Mangano, P. Nason and G. Ridolfi, *Nucl. Phys.* **B431**(1994)453.

[22] S. Frixione, M. Mangano, P. Nason and G. Ridolfi, *Phys. Lett.* **B308**(1993)137.

[23] S. Frixione and G. Ridolfi, hep-ph/9605209, to appear in *Phys. Lett.* **B**.

[24] M. Stratmann and W. Vogelsang, DO-TH-96/10, RAL-TR-96-033, hep-ph/9605330.

[25] M. Glück and E. Reya, *Z. Phys.* **C39**(1988)569;
M. Glück, E. Reya and W. Vogelsang, *Nucl. Phys.* **B351**(1991)579;
W. Vogelsang, in *Proceedings of the Workshop: "Physics at HERA"*, eds. W. Buchmüller and G. Ingelman, DESY, Hamburg, 1992.

[26] B. L. Combridge, *Nucl. Phys.* **B151**(1979)429;
S. J. Brodsky et al., *Phys. Lett.* **B93**(1980)451.

[27] G. Ingelman and M. Thunman, TSL/ISV-96-0137, hep-ph/9604289.

[28] S. Aid et al., H1 Coll., DESY 96-01, accepted by *Nucl. Instr. & Meth.* (1996).

[29] W. Erdmann et al., *Nucl. Instr. & Meth.* **A372**(1996)188.

[30] S. Aid et al., H1 Coll., *Nucl. Phys.* **B472**(1996)32.

[31] M. Derrick et al., ZEUS Coll., *Phys. Lett.* **B349**(1995)225.

[32] R. Eichler and Z. Kunszt, *Nucl. Phys.* **B308**(1988)791;
S. Frixione, M. Mangano, P. Nason and G. Ridolfi, *Phys. Lett.* **B348**(1995)633.

[33] S. Prell, Dissertation (1996), DESY FH1-96-02.

[34] G. Abbiendi, J.M. Butterworth and R. Graciani, these proceedings.

[35] J. A. Appel, contribution to this workshop.