Electroweak Precision Tests With Deep Inelastic Scattering at HERA

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Abstract: We investigate the potential of deep inelastic lepton proton scattering with polarized electrons or positrons for precision tests of the electroweak standard model. We interpret the measurements of cross sections as a measurement of the electroweak parameters m_W and m_t, assuming that m_H is a fixed external parameter. As a main result we find that measurements with left-handed polarized electrons give the highest precision. Provided the top mass is known within ± 5GeV, HERA can measure the W boson mass with a precision of ± 55MeV from 1000 pb^{-1} of data from electron scattering with 70 % left-handed longitudinal polarization.

1 Introduction

Deep inelastic lepton nucleon scattering has always played an important rôle in understanding the interplay of electromagnetic, weak and strong interactions. In fact, weak neutral currents were first observed in deep inelastic scattering. Neutrino scattering has established the structure of the neutral current at low momentum transfer, Q^2, and, for a long period, provided the most precise value of the weak mixing angle [1]. It is therefore an important question whether experiments at HERA are able to provide further insight into the structure of the electroweak interaction. The hope that this question will have a positive answer is based on the fact that HERA is extending the measurements to much larger values of the momentum transfer and that beams with both electrons and positrons and with longitudinal polarization of up to 70 % [2] could be utilized at a future upgrade of the machine.

The electroweak theory based on $SU(2) \times U(1)$ gauge symmetry and the mechanism of spontaneous symmetry breaking (the Standard Model, SM) successfully describes the wealth of existing data on the electroweak interactions almost perfectly. After having fixed the basic free parameters of the standard model, it allows for the derivation of unambiguous predictions which can be compared to experimental data. In order to quantify the agreement or disagreement of theoretical predictions with experimental results it is often convenient to generalize the theory either by treating certain quantities as free parameters (e.g., the vector and axial-vector

couplings of leptons and quarks) or by introducing additional free parameters, for example anomalous three-boson couplings. Experimental results interpreted as measurements of these free parameters can then be confronted with the predictions of the standard model. Another possibility is to interpret experimental data as measurements of the basic standard model parameters, i.e., the mass of the W boson, m_W, or the top-quark mass, m_t, for example. A test of the theory is then possible by comparing results obtained in different experiments. It is the aim of this article to investigate whether HERA can contribute to this strategy of testing the standard model. A companion contribution to these proceedings prefers the first way of testing the standard model by investigating the question to what extent experiments at HERA would be able to measure the neutral current vector and axial-vector couplings of quarks [3].

In the following we shortly describe the predictions of the standard model for deep inelastic lepton proton scattering, taking into account $O(\alpha)$ electroweak radiative corrections. We will describe in detail the analysis strategy and the observables we suggest to use for precision measurements at HERA. Finally, we discuss the quality of HERA experiments in terms of the precision of the W boson mass that can be achieved with high luminosity.

The standard model has 5 independent free parameters (α, m_Z, m_W, m_H, m_t, apart from fermion masses other than the top-quark mass and CKM matrix elements) which are essentially not constrained by theory. Since present day's experimental observables depend only logarithmically on the Higgs mass m_H (via loop corrections), it is legitimate to perform analyses of data with fixed m_H and study variations of its value separately. Therefore we have to consider 4 independent parameters and meaningful tests of the theory can be performed only by combining the experimental measurements of more than 4 observables. Evidently the very precise measurements of the finestructure constant α and the Z boson mass m_Z can be taken from experiment. Any additional measurement will then put a constraint on the remaining two parameters m_W and m_t. Two additional measurements are needed to determine both m_W and m_t.

In analyses of the LEP data it is suggestive to reduce the number of free parameters to one (plus one for the Higgs mass) by taking into account the constraint from the precise measurement of the μ-decay constant, G_μ, which is unexplored at LEP. Thus it was possible to obtain an indirect determination of the top-quark mass from Z-peak observables. Former analyses of the electroweak physics potential of HERA [4, 5, 6] adopted a strategy similar to that used at LEP and attempted to combine HERA data with the G_μ constraint, i.e. only variations with m_W were considered and m_t was determined from m_W with the help of G_μ. Owing to the fact that at HERA most of the data are still obtained at relatively small Q^2, the constraint resulting from these data on the parameters of the standard model is not very different from the G_μ constraint. As a consequence, these former analyses usually predicted rather large uncertainties for measurements of m_W, of the order of 600 MeV. In the present work, however, we will adopt a different approach and choose to combine HERA data not with the G_μ constraint, but rather with independent information on the value of the top-quark mass. By the time when HERA has collected enough luminosity to allow for precision analyses, the top-quark mass will be known from experiments at Tevatron and the LHC with a precision of a few GeV. Since this direct constraint on one of the standard model parameters is very different from the G_μ constraint, a combination of HERA data with a direct m_t measurement allows to obtain an indirect m_W measurement with a reasonably small uncertainty. Comparing the results from such an analysis with other experiments (indirect determinations of m_W from LEP 1 precision measurements, or direct measurements of m_W at LEP 2 or hadron colliders, for

example) will provide constraints that simultaneously have to be fulfilled within the standard model.

2 Precise Standard Model Predictions for HERA

2.1 Lowest-Order Cross Sections

We study deep inelastic scattering of electrons or positrons off protons

$$e(l) + p(p) \to e'(l') + X(p_X), \tag{1}$$

with $e' = e$ for neutral current scattering (NC) and $e' = \nu_e$ for charged current scattering (CC). The particle momenta are given in parentheses. A useful set of kinematic variables to describe the process (1) is given by

$$Q^2 = -(l - l')^2, \quad s = (l + p)^2, \quad x = \frac{Q^2}{2p \cdot (l - l')}, \quad y = \frac{Q^2}{xs}. \tag{2}$$

In the region of large x and Q^2 where the contribution of the longitudinal structure function F_L and proton mass effects can be neglected, the differential cross section for the neutral current process for left-handed (L) and right-handed (R) polarized electrons is given by

$$\frac{d^2\sigma^{NC}}{dx\,dQ^2}(e^-_{L,R}) = \frac{2\pi\alpha^2}{xQ^4}\left[\left(1 + (1-y)^2\right) F_2^{L,R} + \left(1 - (1-y)^2\right) x F_3^{L,R}\right]. \tag{3}$$

For positron scattering one has to replace $F_3^{L,R} \to -F_3^{R,L}$ and cross sections for beams with an arbitrary degree of longitudinal polarization P can be obtained by calculating the appropriate average of the cross sections for left- and right-handed leptons. The structure functions are derived in the quark-parton model from basic 4-fermion scattering cross sections:

$$\begin{aligned} F_2^{L,R} &= \sum_q [xq(x,Q^2) + x\bar{q}(x,Q^2)] \cdot A_q^{L,R}, \\ xF_3^{L,R} &= \sum_q [xq(x,Q^2) - x\bar{q}(x,Q^2)] \cdot B_q^{L,R}. \end{aligned} \tag{4}$$

They contain the quark (q) and anti-quark ($\bar{q}$) distribution functions as well as coupling constants and propagators corresponding to photon and Z-boson exchange ($L = +, R = -$):

$$\begin{aligned} A_q^{L,R} &= Q_q^2 + 2Q_eQ_q(v_e \pm a_e)v_q\chi_Z + (v_e \pm a_e)^2(v_q^2 + a_q^2)\,(\chi_Z)^2\,, \\ B_q^{L,R} &= \pm 2Q_eQ_q(v_e \pm a_e)a_q\chi_Z \pm 2(v_e \pm a_e)^2 v_q a_q\,(\chi_Z)^2\,, \end{aligned} \tag{5}$$

with

$$\chi_Z = \frac{1}{4s_W^2 c_W^2}\frac{Q^2}{Q^2 + M_Z^2}. \tag{6}$$

In Eq. (3), also Higgs-boson exchange is neglected because of the small couplings to light fermions. By using Q^2 dependent parton distribution functions, QCD corrections in the leading-logarithmic approximation are included.

The parametrization of the lowest-order cross section in terms of vector and axial-vector coupling constants $v_{e,q}$, $a_{e,q}$ is quite general and applies to any model where a vector boson with v and a couplings is exchanged together with the photon. In the standard model, the couplings of the fermions to the bosons are given by the electric charge (in units of e) for the photon exchange and, for the Z-boson exchange, by ($f = e, q$)

$$\begin{aligned} a_f &= I_3^f, \\ v_f &= I_3^f - 2Q_f \sin^2\theta_W \end{aligned} \tag{7}$$

where Q_f and I_3^f denote the charge and the third component of the weak isospin of the fermion f. In the standard model with an arbitrary Higgs sector, the mixing angle is related to the vector boson masses by

$$\sin^2\theta_W = 1 - \frac{m_W^2}{\rho_0\, m_Z^2}. \tag{8}$$

In the following we restrict ourselves to the minimal model with $\rho_0 = 1$ at the tree level and use as abbreviation

$$s_W^2 = 1 - \frac{m_W^2}{m_Z^2}. \tag{9}$$

In a similar way one can write for the charged current differential cross section

$$\frac{d^2\sigma^{CC}}{dx\,dQ^2}(e^-) = (1-P)\frac{\pi\alpha^2}{4s_W^4}\cdot\frac{1}{(Q^2+M_W^2)^2}\left[u + c + (1-y)^2(\bar{d}+\bar{s})\right], \tag{10}$$

with P denoting the degree of left-handed longitudinal polarization ($P = -1$ for left-handed electrons, $P = 1$ for right-handed electrons). For positron scattering one has to replace $u + c \to \bar{u} + \bar{c}$ and $\bar{d} + \bar{s} \to d + s$ and adjust the sign of P.

2.2 Higher-Order Corrections

A study of the sensitivity to electroweak parameters must be based on cross section formulae which incorporate their dependence on m_W, m_t, etc. as precisely as possible. It is, however, not necessary to include all kinds of corrections if these do not affect the electroweak parameter dependence. This is the case for the purely photonic (QED) corrections and for some parts of QCD corrections.

QED corrections are large and unavoidable, in particular for NC scattering at small x and large y. They constitute a well-understood (thus 'uninteresting') part of physics which has to be separated from genuine electroweak effects. Careful comparisons of different calculations during the HERA workshop in 1991 have shown good agreement at the permille level between results for the first order corrections obtained by various authors [7]. Also corrections of higher than one-loop order are known in the leading logarithmic approximation so that we can be confident that QED corrections can be taken into account in the analysis of experimental data at a level of precision that does not introduce severe uncertainties. Moreover, explicit calculations show that QED corrections do not change the sensitivity on electroweak parameters. An example is shown in Fig. 1 where the m_W dependence of the ratio R_+ of CC to NC cross sections for positron scattering at a fixed value of the momentum transfer Q^2 is compared for calculations without and including QED corrections. The corrections are large, of the order of 10 %, in

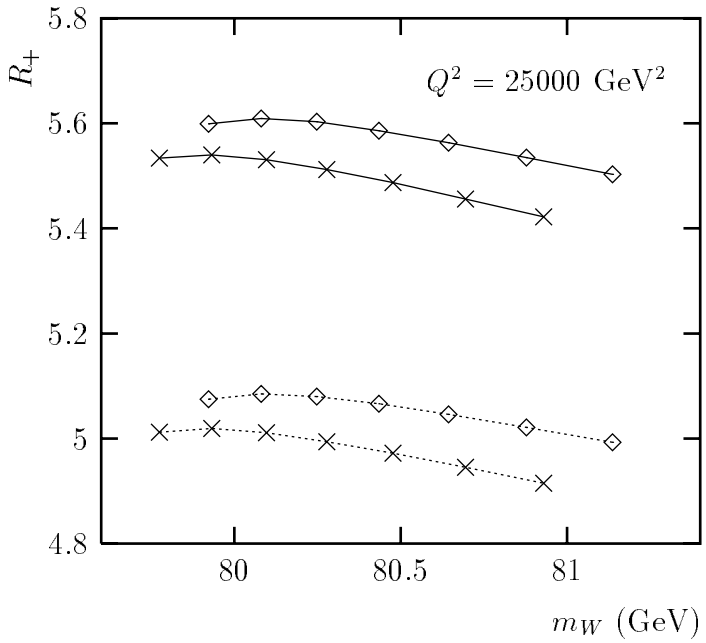

Figure 1: *The ratio R_+ of CC to NC cross sections $d\sigma/dQ^2$ for positron scattering at fixed $Q^2 = 25000$ GeV^2 with (full lines) and without (dashed lines) QED corrections. The curves with diamonds are for $m_H = 100$ GeV, those with crosses for $m_H = 800$ GeV. m_t has been fixed by the G_μ constraint.*

particular much larger than the variation of the results with m_W or m_H. However, it is obvious that QED corrections do not affect the dependence on these standard model parameters, but simply shift the theoretical predictions.

Since we are dealing with small effects, as we will see below, it will be indispensable to take also QCD corrections into account in the analysis of HERA data. QCD corrections needed for precise predictions of DIS cross sections can be classified according to three sources: *i*) there are universal corrections modifying the relations among standard model parameters, entering via the corrections to the μ-decay constant G_μ (see Δr in the formulae below). We included the dominating two-loop contributions of order $O(\alpha\alpha_s)$ [9] corresponding to shifts of m_W in the order of 60 MeV. Contributions of order $O(\alpha\alpha_s^2)$ are known [8], but not included in our calculations. *ii*) Additional corrections of $O(\alpha\alpha_s)$ modify the x and Q^2 dependence of the electroweak form factors. These contributions are only partly known and probably small. *iii*) Finally, corrections due to real and virtual emission of gluons from quarks in the hard lepton-quark scattering subprocesses modify the way the structure functions are expressed in terms of parton densities. In the DIS scheme, these corrections of order $O(\alpha_s)$ enter via F_L and affect F_3 in NC scattering and the CC cross section. They are known in next-to-leading order for all structure functions [10] and up to next-to-next-to-leading order for the photon-exchange contribution in NC scattering [11].

The calculations of electroweak higher order corrections to deep inelastic scattering at HERA have been reviewed in [7] (see also references therein). They are performed in the on-shell scheme, where m_W and m_Z are treated symmetrically as basic parameters together with m_t and m_H (besides the fine structure constant α and other fermion masses). Amplitudes are functions of these basic parameters. The electroweak corrections are collected in form factors that change the amplitudes in a simple form: the dressed photon-exchange amplitude can be

written in the following way:

$$\mathcal{M}_\gamma = \frac{e^2}{1-\hat{\Pi}_f^\gamma(Q^2)} \cdot \frac{1}{Q^2} \cdot Q_e\gamma_\mu \otimes Q_f\gamma^\mu. \tag{11}$$

$\hat{\Pi}_f^\gamma$ is the fermionic part of the photon vacuum polarization. Writing it in the denominator resums the leading logarithmic corrections according to the renormalization group equation and the first factor in Eq. (11) constitutes the running fine structure constant. Bosonic contributions to the vacuum polarization have to be combined with vertex and box corrections and added separately.

The weak one-loop corrections to the Z-exchange amplitude $\mathcal{M}_Z$ for $eq \to eq$ can be expressed in terms of four weak form factors (ρ_{eq}, κ_e, κ_q, and κ_{eq}) in the following way, making use of dressed vector couplings [12, 13]:

$$\mathcal{M}_Z(\hat{s},Q^2) \sim \frac{1}{Q^2+m_Z^2}\left[\frac{\pi\alpha}{4s_W^2c_W^2}\frac{\rho_{eq}(\hat{s},Q^2)}{1-\Delta r}\right]\Big[I_3^eI_3^q\gamma_\mu\gamma_5\otimes\gamma_\mu\gamma_5 + \bar{v}_eI_3^q\gamma_\mu\otimes\gamma_\mu\gamma_5 + I_3^e\bar{v}_q\gamma_\mu\gamma_5\otimes\gamma^\mu + \bar{v}_{eq}\gamma_\mu\otimes\gamma^\mu\Big], \tag{12}$$

$$\bar{v}_f = I_3^f\left[1-4|Q_f|s_W^2\,\kappa_f(\hat{s},Q^2)\right], \quad f=e,q\,, \tag{13}$$

$$\bar{v}_{eq} = I_3^e\bar{v}_q + \bar{v}_eI_3^q - I_3^eI_3^q\left[1-16|Q_eQ_q|s_W^4\,\kappa_{eq}(\hat{s},Q^2)\right], \tag{14}$$

with $\hat{s}=xs$ and s_W^2 from Eq. (9). The normalization factor in square brackets can be related to the μ-decay constant (taking into account the radiative corrections to the μ-decay, $(1-\Delta r)^{-1}$), $[\cdots] = (G_\mu m_Z^2/2\sqrt{2})\cdot\rho_{eq}$, showing that ρ_{eq} denotes the ratio of the NC and CC coupling strengths in eq scattering. In the Born approximation, $\rho=\kappa=1$, $\Delta r=0$ and $\bar{v}_{eq}=v_ev_q$. The above parametrization has the form of a Born-like expression except that at the Born level there is no parallel for the coupling $\bar{v}_{eq}$. The axial-vector couplings can be kept in their form $a_f=I_3^f$ by absorbing corresponding vertex corrections into the normalization factor $\rho_{eq}=\rho_{eq}(\hat{s},Q^2)$ which receives also contributions from self energies. The form factors κ_f and κ_{eq} combined with s_W^2 give rise to effective mixing angles which depend through the κ's on the fermion species and on the kinematic variables. The form factors can be separated into a universal part independent of the fermion species and a non-universal remainder term. The universal parts contain the dependence on the masses of the top quark and the Higgs boson. The dominating contribution to them is proportional to m_t^2 and with

$$\Delta\rho = \frac{3\alpha}{16\pi s_W^2c_W^2}\frac{m_t^2}{m_Z^2} \tag{15}$$

one has to one-loop precision

$$\rho_{eq} = 1+\Delta\rho+\Delta\rho_{eq}^{rem}, \tag{16}$$

$$\kappa_f = 1+\frac{c_W^2}{s_W^2}\Delta\rho+\Delta\kappa_f^{rem}. \tag{17}$$

Additional details, in particular concerning the inclusion of higher than one-loop order terms which are taken into account in our application, are described in [14].

The W-exchange amplitude including higher-order contributions can be written in the following way:

$$\mathcal{M}_W(\hat{s}, Q^2) = \frac{\pi\alpha}{8s_w^2} \frac{\rho_{eq}^W(\hat{s}, Q^2)}{1-\Delta r} \frac{1}{Q^2+m_W^2} \gamma_\mu \left[1-\gamma_5\right] \otimes \gamma^\mu \left[1-\gamma_5\right]. \tag{18}$$

Differently from the neutral current, only a single form factor ρ_{eq}^W for each parton scattering process is required to accommodate the higher-order contributions. Rewriting the normalization of the W-exchange amplitude with the help of the μ-decay constant using $\pi\alpha/8s_W^2(1-\Delta r) = G_\mu m_W^2/4\sqrt{2}$ shows explicitly that the measurement of the CC cross section at low Q^2 is, in fact, a measurement of G_μ. In this form, weak loop corrections are very small since ρ_{eq}^W deviates from 1 typically by a few permille with very little dependence on m_t and m_H.

The above formulae are implemented in the FORTRAN code `eprc93.f` [14] which has been used for all our numerical evaluations. We stress that the formulae given here are sufficient to study the *sensitivity* on electroweak parameters, but not for the determination of their central values from real data since QED corrections are not, and QCD corrections only partly, included. These corrections will have a non-negligible effect on the predicted values of observables. Consequently, using measurements of these observables for a determination of m_W, for example, these corrections will also have a non-negligible effect on the central value of m_W. In this work, however, we are interested in a determination of the *sensitivity* of such a measurement expressed in terms of the precision Δm_W of the measurement. Since it is completely sufficient to know the sensitivity Δm_W with a relative precision of $O(10\,\%)$, higher than one-loop corrections are not relevant here.

3 Precision Measurements at HERA

3.1 Analysis Strategy

We consider the well measured α and m_Z as fixed parameters of the theory ($1/\alpha = 137.036\cdots$ and $m_Z = 91.1884\,\mathrm{GeV}$). The Higgs boson enters via loops, so that there is only a weak logarithmic dependence on m_H which allows us to treat m_H as an external parameter. The numerical results presented below refer to $m_H = 100\,\mathrm{GeV}$ and variations of m_H are discussed separately. Hence, as discussed in the introduction, we are left with m_t and m_W as free parameters which are to be determined by experiment. A precision measurement of any observable constrains the allowed region in the (m_W, m_t) plane and a test of the theory consists in comparing the overlap of regions obtained from different experiments.

A possible result of a measurement is sketched in Fig. 2. The details of the correlation depend on the standard model and the particular choice of observable, i.e. its dependence on the particular electroweak parameter. The dependence on the parameters of the theory is smooth so that a linearization of the theoretical dependence around the value of the measurement is justified. The confidence levels are chosen such that for a single parameter test, i.e. a projection onto one of the axes, the usual 1 parameter confidence level for $1\,\sigma$ is obtained. The correlation between the two parameters can be exploited: using the value of a future direct top mass determination (vertical lines in Fig. 2) from other experiments yields the smaller tilted (shaded) ellipse in Fig. 2. Projecting this ellipse onto the m_W axis results in a tightly constrained interval for values of m_W. In quantifying results of the studies we have chosen, for

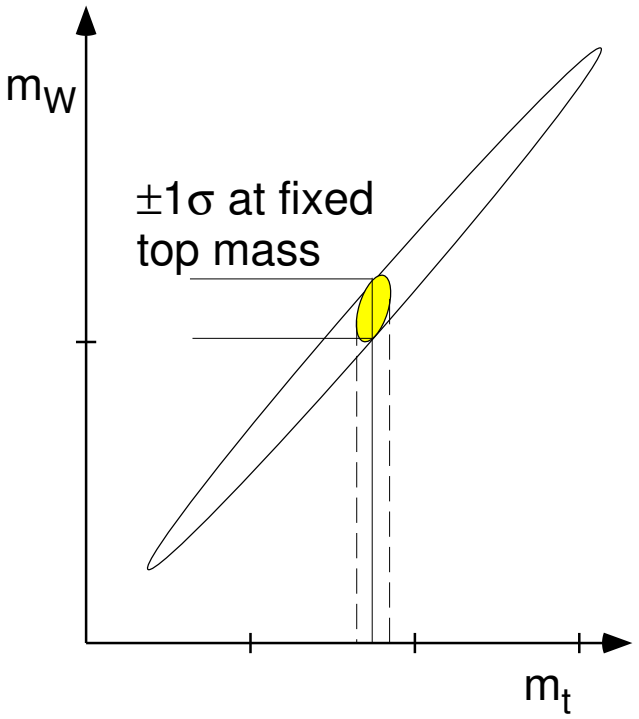

Figure 2: *Schematic view of the* $1\,\sigma$ *contour in the* (m_W, m_t) *plane, resulting from a measurement at HERA and its combination with a direct top-quark mass measurement (shaded ellipse)*

the sake of definiteness, to indicate the vertical extent of δm_W of the ellipse as the uncertainty of the W mass measurement. This corresponds to a scenario in which the top mass has been measured precisely elsewhere. In the end we shall indicate the extra uncertainty for a more realistic measurement of m_t for which we assume an uncertainty of 5 GeV[1].

The Higgs mass enters via one-loop contributions to the gauge boson self energies and modifies the interpretation of the measurement. Performing an analysis with different values for m_H leads to different contours (Fig. 3) which are shifted and have very similar slope.

3.2 Observables

We fix the beam energies at their nominal values $E_l = 27.5\,\mathrm{GeV}$, $E_p = 820\,\mathrm{GeV}$. All cross sections are evaluated with a cut on the transverse momentum of the scattered lepton (or missing transverse momentum) of $p_T \geq 15\,\mathrm{GeV}$. For scattering with polarized leptons we assume a degree of longitudinal polarization of $|P| = 0.7$. In order to exploit the information contained in the shape of the Q^2 distribution, we take the cross sections to be measured in 5 bins of Q^2, equidistant in $\log Q^2$, with lower bin limits (225.6, 747.2, 2475, 8199, 27160) GeV^2, the last bin extending up to the kinematical limit $Q^2_{max} \approx s$. The SM predictions, depending on m_W and m_t, are obtained by integrating over the Q^2 intervals:

$$\sigma_i = \sigma_i(m_W, m_t) = \int_{Q^2_{i,min}}^{Q^2_{i,max}} dQ^2 \int dx \frac{d^2\sigma}{dx\, dQ^2}\,. \tag{19}$$

[1]Presently, the precision for m_t is already as small as 7 GeV [15].

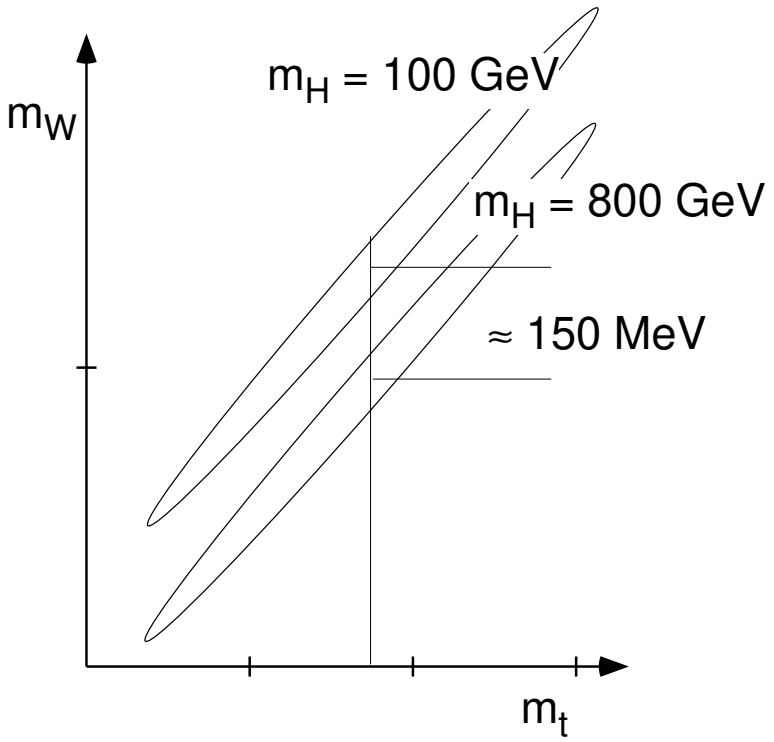

Figure 3: *The qualitative dependence of the* $1\,\sigma$ *confidence contour in the* (m_W, m_t) *plane from a measurement at HERA for two different values of the Higgs mass.*

The event numbers expected for an integrated luminosity of $\mathcal{L} = 1000\text{pb}^{-1}$ and $P = -0.7$ are shown in Fig. 4.

Future experiments measure cross sections σ_i^{exp} with errors $\Delta\sigma_i^{exp}$ for each Q^2 interval. To quantify the comparison of experimental data and theoretical predictions, we define a χ^2:

$$\chi^2 = \chi^2(m_W, m_t) = \sum_i \frac{(\sigma_i(m_W, m_t) - \sigma_i^{exp})^2}{(\Delta\sigma_i^{exp})^2}. \tag{20}$$

Lacking experimental data for the time being, we assume that the central value of the measurement coincides with the SM prediction for a set of reference values of the SM parameters. We choose

$$\sigma_i^{exp} = \sigma_i(m_W = 80.2\,\text{GeV}, m_t = 170\,\text{GeV}) \tag{21}$$

thus fixing the position of confidence regions in the (m_W, m_t) plane. Note again, that due to the weak dependence on the parameters the particular choice of reference values has a negligible influence on the following studies of sensitivity.

The precision of the measurement trivially improves with the statistical accuracy of the data,

$$\Delta\sigma_i^{stat} = \sqrt{\sigma_i^{exp}/\mathcal{L}}. \tag{22}$$

The systematic errors, $\Delta\sigma_i^{sys}$, encompass the uncertainties of the measurement (identification, efficiency, reconstruction of kinematics etc.) as well as uncertainties in the determination of the luminosity and the beam polarization. Hence, the $\Delta\sigma_i^{sys}$ will be correlated between different bins of Q^2 such as in the case of the luminosity and polarization measurement. Nevertheless,

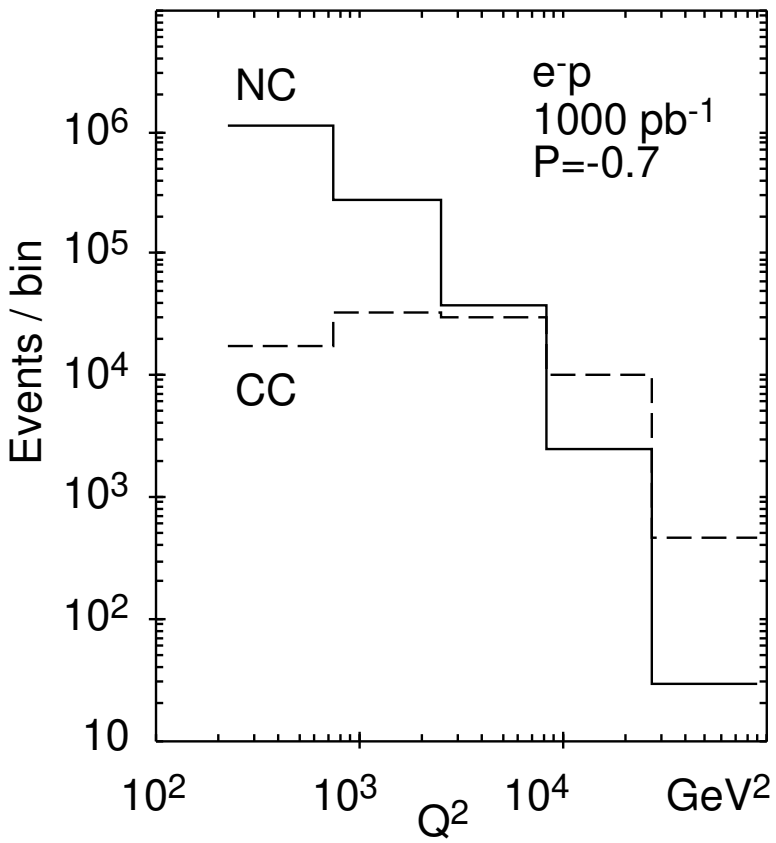

Figure 4: *Expected event numbers in the chosen Q^2 intervals for neutral (NC) and charged (CC) current e^-p cross sections at HERA for an integrated luminosity of 1000 pb^{-1} and longitudinal polarization $P = -0.7$.*

when quoting the dependence on the systematic error, we will assume a correlation coefficient $c = 0$ in most cases. However, we will also present a study on the transition from a result that is dominated by point-to-point errors $c = 0$ to a result that is dominated by overall normalization $c = 1$.

A precision of order 1 % has been achieved in present measurements of luminosity already and at high luminosity independent methods of measurements can be used for cross check. For the polarization measurement an accuracy of 2 % or better [2] is conceivable. (Note, that typically the polarization enters through a factor $(1+|P|)$ so that a precision of 2 % is adequate).

The largest experimental uncertainty arises from the incomplete knowledge of the absolute value of the calorimeter energy scale. The susceptibility to this uncertainty depends on the Q^2 variation of the cross section. Simplified, the photon propagator dominated NC cross section will be mostly affected in shape due to migration while the CC section will be predominantly affected in absolute normalization due to the lower Q^2 cut off. Typical experimental accuracies range around $1 - 2$ % for the reconstruction of the electron energy [16]. The hadronic energy measurement is presently considerably less certain [17]. However, the large statistics available from a high luminosity HERA will allow to impose considerable constraints on the reconstruction of the final state so that together with a good electron measurement a better precision at the level of per cent will be achieved. It will be important to cross check the CC channel with the more constrained data of the NC analysis.

Instead of fixing $\Delta\sigma_i^{sys}$ at some ad-hoc value, we will consider the dependence of the results on the relative systematic uncertainty $\epsilon = \Delta\sigma^{sys}/\sigma^{exp}$ ranging from 0 to 5 %. Since the theoretical uncertainties due to an imperfect knowledge of parton distribution functions can be absorbed in the $\Delta\sigma_i^{sys}$, the larger value of 5 % would become relevant if experimental measurements did not advance the precision beyond the present level, which is not expected. We consider the full range of variation.

The condition $\chi^2(m_W, m_t) = 1$ defines a 1σ-confidence region in the (m_W, m_t) plane. Since we are dealing with small variations of m_W and m_t, we assume that the parameter dependence can be linearly approximated. This simplifies the determination of confidence regions considerably. Our approach will become unreliable if these regions turn out to be large, corresponding to uncertainties for m_W of more than about 1 GeV.

3.3 Results

The results of the sensitivity studies are summarized in table 1. For two values of luminosity the sensitivity is shown for the different processes as a function of the systematic uncertainty ϵ. In the following discussion we compare the various options and will quote a value for the precision of the W mass measurement for a fixed value of the top mass as described above.

3.3.1 Charged vs. Neutral Current

Fig. 5 shows that a measurement of the charged current cross section for unpolarized electrons is superior to that of neutral current scattering. δm_W is of the order of 100 MeV and is only doubled when the relative systematic uncertainty is increased to 5 %. The neutral current measurement is worse by a factor of ≈ 3 and much more sensitive to systematic errors. Whereas for charged current scattering there is little improvement in precision for m_W when increasing

Electroweak Sensitivity for m_W								
	Charged Current				Neutral Current			
	e^-p		e^+p		e^-p		e^+p	
	$\mathcal{L}\,[\mathrm{pb}^{-1}]$		$\mathcal{L}\,[\mathrm{pb}^{-1}]$		$\mathcal{L}\,[\mathrm{pb}^{-1}]$		$\mathcal{L}\,[\mathrm{pb}^{-1}]$	
	250	1000	250	1000	250	1000	250	1000
$\epsilon\,[\%]$	δm_W [MeV] for $\lvert P\rvert = 0$							
0	68	34	101	51	223	112	363	181
1	80	53	111	68	347	237	699	526
2	105	84	136	101	475	364	1051	853
5	202	184	237	209	855	717	2032	1863
$\epsilon\,[\%]$	δm_W [MeV] for $\lvert P\rvert = 70\,\%$							
0	52	26	78	39	113	57	173	86
1	66	47	90	59	187	131	348	265
2	95	80	118	94	263	207	530	437
5	194	179	223	203	488	414	1048	973

Table 1: *The uncertainty δm_W in MeV for a given beam configuration and process as a function of the relative systematic uncertainty ϵ for fixed top mass. The sign of the longitudinal polarization is assumed to be chosen such that the cross sections are enhanced, i.e. $P = 0.7$ for e^+ and $P = -0.7$ for e^- beams.*

the luminosity from 250 pb^{-1} to 1000 pb^{-1}, there is an appreciable improvement for neutral current scattering.

3.3.2 Polarized vs. Unpolarized Lepton Beams

This trend persists for positron scattering and for polarized beams, as seen in Fig. 6 which shows the results for polarized electron scattering of degree of polarization $P = -0.7$. The gain in precision for m_W in this case is due to the increase of statistics (note that the charged current scattering cross section is proportional to $(1 - P)$). For a large systematic uncertainty, the gain in statistics is, of course, lost. This is in contrast to the case of neutral current scattering where polarization leads to a substantial increase of sensitivity even in the presence of large systematic uncertainties. This difference between NC and CC scattering is a consequence of the fact that for charged current scattering it is essentially the normalization of the cross section which determines m_W whereas in the neutral current case the shape of the Q^2 dependence contains the sensitivity. The influence of the weak interference terms can be enhanced by proper choice of polarization. Coarsely speaking, beams of $-70\,\%$ polarization are equivalent to a factor 4 larger integrated luminosity of unpolarized beams. Thus polarization is needed to render the NC data useful at all for precision measurements.

3.3.3 Electron vs. Positron Beams

Fig. 7 shows a comparison of the potential of unpolarized charged current electron and positron scattering. Positron scattering always yields a lower sensitivity than electron scattering simply

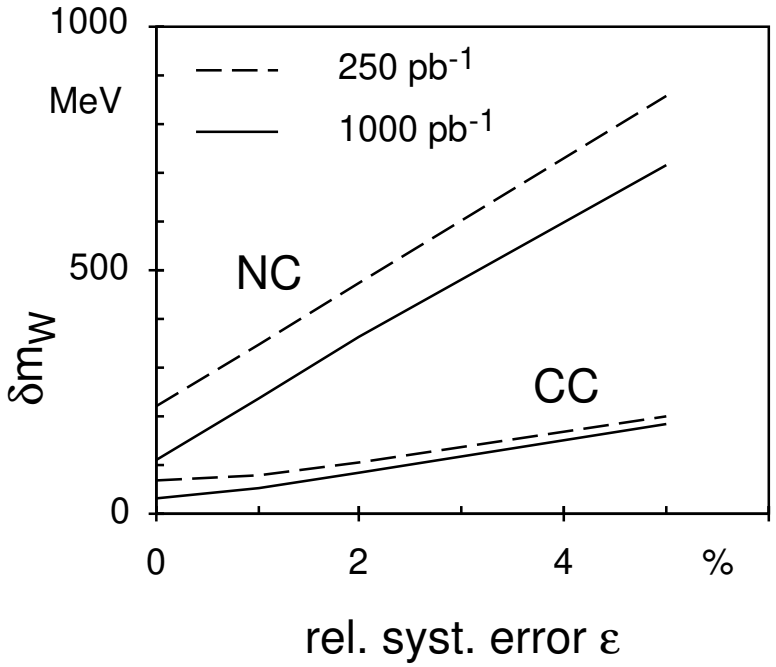

Figure 5: *Precision for m_W from measurements of the HERA NC and CC cross sections for unpolarized electrons at fixed top mass as a function of the relative systematic uncertainty ϵ.*

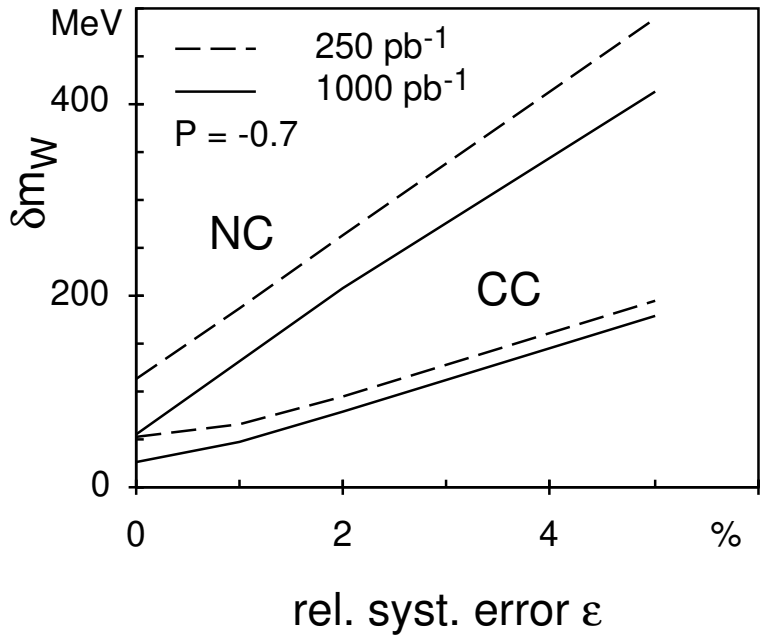

Figure 6: *Same as Fig. 5 for scattering with longitudinally polarized electrons of degree $P = -0.7$.*

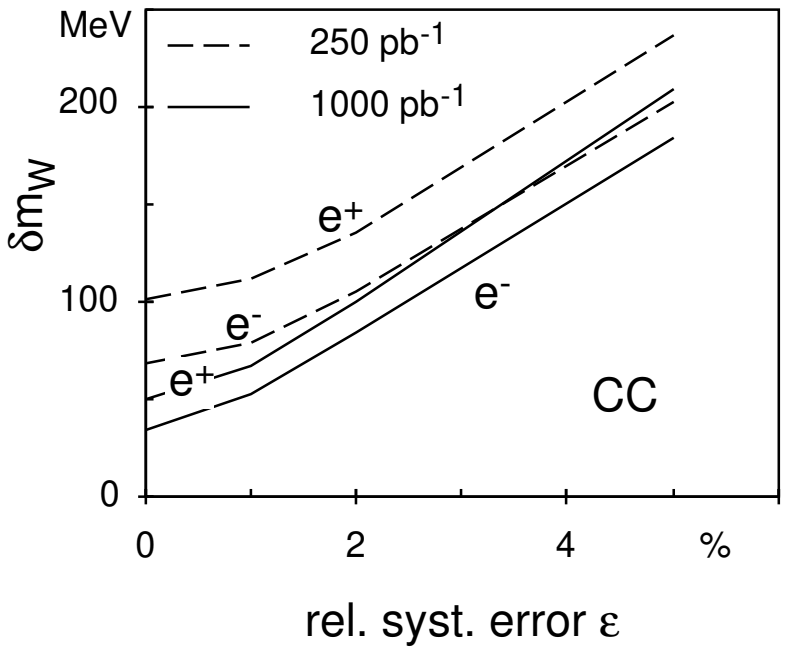

Figure 7: *Comparison of the precision for m_W from unpolarized electron and positron charged current scattering at HERA at fixed top mass.*

because of the smaller cross section. The same comparison for polarized neutral current scattering is made in Fig. 8. Again electron scattering is preferred since both the absolute values of δm_W as well as the sensitivity to systematic errors is more favourable in this case than for positron scattering.

3.3.4 Overall vs. Point-to-Point Error

Fig. 9 addresses the importance of the Q^2 dependence for an experimental determination of m_W. The sensitivity is depicted as a function of the correlation coefficient[2] c for the systematic error in the five chosen Q^2 bins (for the case of definiteness we have assumed a global value of the correlation coefficient c). A correlation coefficient of 0 describes a situation where the normalization of the cross section measurement has no systematic uncertainty, whereas a coefficient of 1 applies to an experiment where only normalization is affected by systematics. In turn, a coefficient of 0 ascribes the complete systematic error to relative uncertainties in different Q^2 bins and the value 1 for the correlation coefficient assumes no relative bin-to-bin uncertainties apart from statistical ones. The gently rising curve for the charged current measurement demonstrates that the normalization of the cross section measurement contains the information about the value of m_W. In this case, bin-to-bin uncertainties do not strongly increase the uncertainty δm_W provided the normalization can be well measured. In contrast to this, for neutral current scattering, uncertainties on the relative normalization result in a large

[2] If $\vec{\sigma} = \epsilon(\sigma_1^{exp}, \sigma_2^{exp} \cdots \sigma_5^{exp})$ describes the systematic uncertainties in the five Q^2 intervals the 5×5 variance matrix is calculated as $\mathbf{V} = \vec{\sigma}((1-c)\mathbf{I} + c\mathbf{1})\vec{\sigma}^T$. Here $\mathbf{I}$ and $\mathbf{1}$ refer to the unit matrix and the matrix containing ones in all positions respectively. The diagonal variance matrix of the statistical error has to be added.

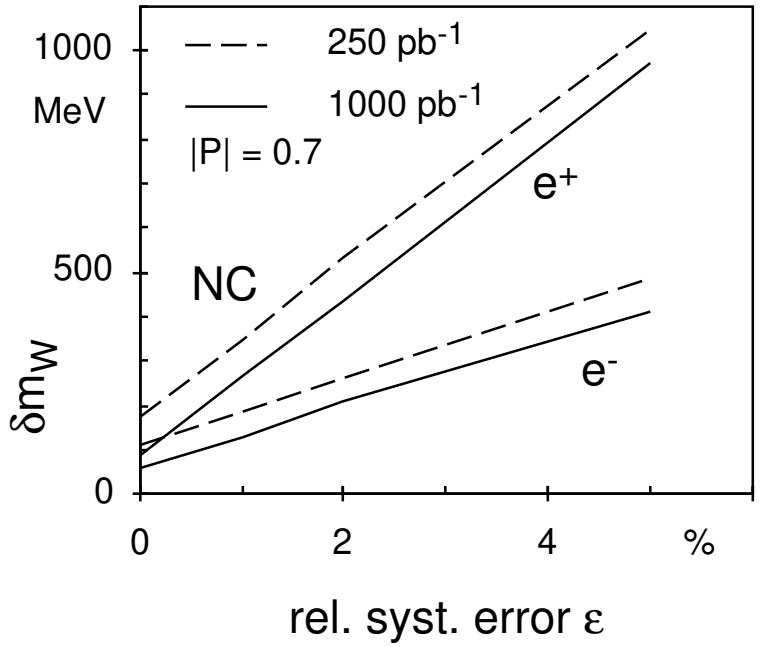

Figure 8: *Same as Fig. 7 for neutral current scattering of polarized electrons ($P = -0.7$) or positrons ($P = 0.7$).*

uncertainty for m_W and a good knowledge of the relative bin-to-bin normalization is essential for neutral current scattering to contribute significantly to an m_W determination. In the limit of vanishing point-to-point errors the NC measurement would compete or even be superior to the CC current measurement. The different dependence can thus be exploited in the interpretation of the experimental data in a favourable way.

3.3.5 The High Sensitivity Measurement at HERA

Finally, Fig. 10 indicates the result for a scenario with 1000 pb^{-1} of data from polarized neutral and charged current electron scattering. The corresponding 1σ-contour is represented by the shaded ellipse. Projecting it onto the axes results in precisions of $\delta m_t = \pm 50\,\mathrm{GeV}$ and $\delta m_W = \pm 300\,\mathrm{MeV}$. These values are more than a factor of 2 better than what can be obtained from charged current scattering alone with the smaller luminosity of 250 pb^{-1} (see the large ellipse in the figure). Combining the neutral and charged current data from HERA with a realistic top mass uncertainty (assuming a value of $\pm 5\,\mathrm{GeV}$) results in the small full ellipse in Fig. 10. The resulting precision on m_W is $\delta m_W = \pm 55\,\mathrm{MeV}$. This scenario assumes a 1 % relative systematic uncertainty, a serious experimental challenge. The accuracy decreases to $\delta m_W = \pm 81\,\mathrm{MeV}$ for a systematic error of 2 %. With a very precise measurement of m_t the corresponding values are 45 and 75MeV respectively. The figure also shows the relation between m_W and m_t following from the G_μ constraint. The upper full line refers to a Higgs mass of $m_H = 100\,\mathrm{GeV}$, the lower dashed one to $m_H = 800\,\mathrm{GeV}$. Note that the confidence contours for the HERA measurements have to be associated with a fixed Higgs mass of 100 GeV. They have to be shifted downwards much the same as the lines describing the G_μ constraint for a Higgs

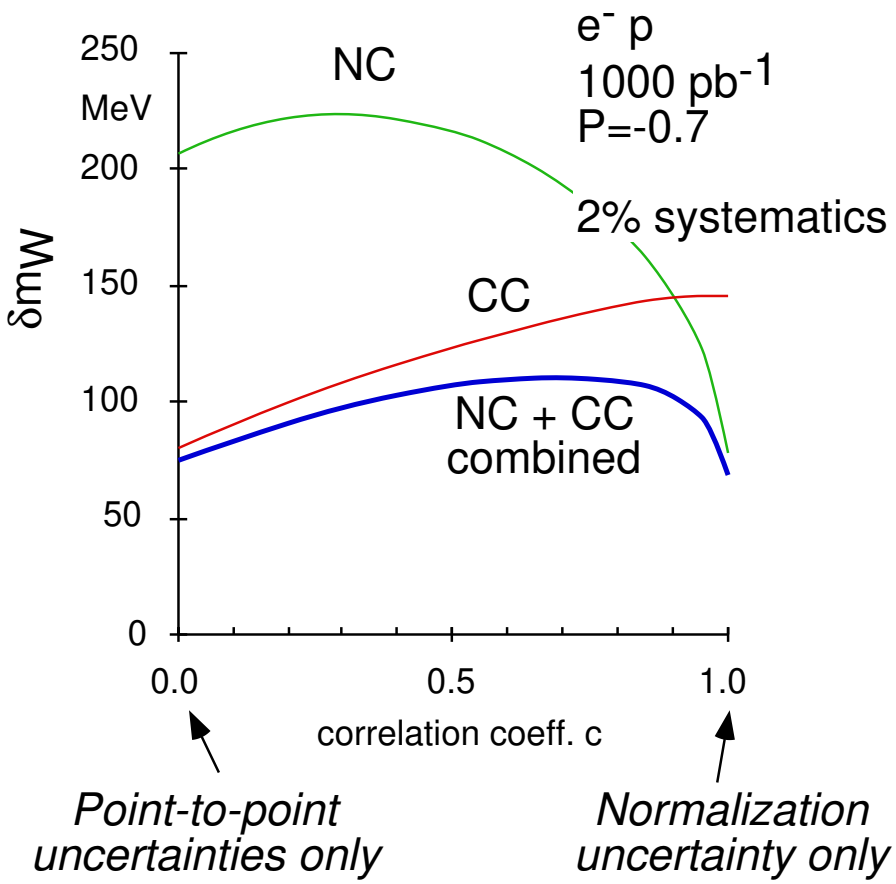

Figure 9: *Precision for m_W from neutral and charged current scattering of polarized electrons ($P = -0.7$) at fixed top mass as a function of the correlation coefficient c for the systematic error (see text). The overall systematic uncertainty for this example is assumed to be 2 %.*

mass of $m_H = 800\,\mathrm{GeV}$. It is obvious from this figure that with such precision one would be able to constrain the allowed range of Higgs masses in conjunction with independent measurements of both m_t and m_W.

3.4 Uncertainties from parton distributions

The DIS cross sections depend not only on electroweak parameters but also on QCD input given by parton densities and the strong coupling constant $\alpha_s(\Lambda_{QCD})$. A final analysis will have to determine all parameters simultaneously in a global fit to all DIS cross sections measured at HERA. The study of such a global fit is beyond the scope of the present work. The preceding discussions assumed that parton densities are a well-known independent input. Ignoring the possibility that electroweak parameters and QCD input might be correlated, we now address uncertainties of parton densities as an additional source of uncertainty for the precision of an m_W measurement. We have seen that the normalization of the CC cross section provides an important constraint to the measurement of m_W. Therefore we expect that uncertainties of the overall normalization of parton densities will have the largest impact on the precision of m_W.

Since no parton distribution functions exist which parametrize their errors, we are bound to study variations of the predictions for DIS cross sections obtained when using parametrizations given by different authors. A comparison of recent parton densities of GRV (94HO) [18], CTEQ (3D and 3M) [19] and MRS(H,A) [20] gives a variation in the CC cross section for left-handed electrons of up to 5pb. This can be interpreted as a contribution to the systematic uncertainty of about 5 %. Considering most recent parton densities which include 1994 HERA data (CTEQ3M and MRS(A')) yields already now only 2 % additional systematic uncertainty. It seems reasonable to expect that the uncertainties in our knowledge of the parton densities will further diminish with more data being available and that, consequently, the imperfect knowledge of parton densities will eventually not be the limiting source of uncertainties for the measurement of electroweak parameters.

An additional uncertainty is due to the error on α_s, viz. Λ_{QCD}. Since the extraction of parton densities from experimental data is usually performed for a fixed value of α_s, uncertainties on the latter induce uncertainties on the former. The effect on DIS cross sections can be studied using the parametrizations of MRS [21] which are given for a number of different values of Λ_{QCD}. Again concentrating on CC scattering, we observe an increase of the cross section by 0.7 pb (i.e., less than 1 %) when increasing α_s by 0.005. In addition, uncertainties on α_s would affect QCD corrections which modify the relation of structure functions to parton distributions. This direct effect is expected to be much smaller since QCD corrections themselves are small so that uncertainties of α_s will not contribute much to the uncertainties of electroweak precision measurements.

4 Conclusions

Precise HERA measurements of the differential NC and CC cross sections complete the picture of successful tests of the Standard Model in the region of large space-like momentum transfers. This study has shown that, given sufficient luminosity, a test of the model to a level of accuracy hitherto only achieved at LEP 1 can be reached at HERA. Consistent values of m_W and m_t

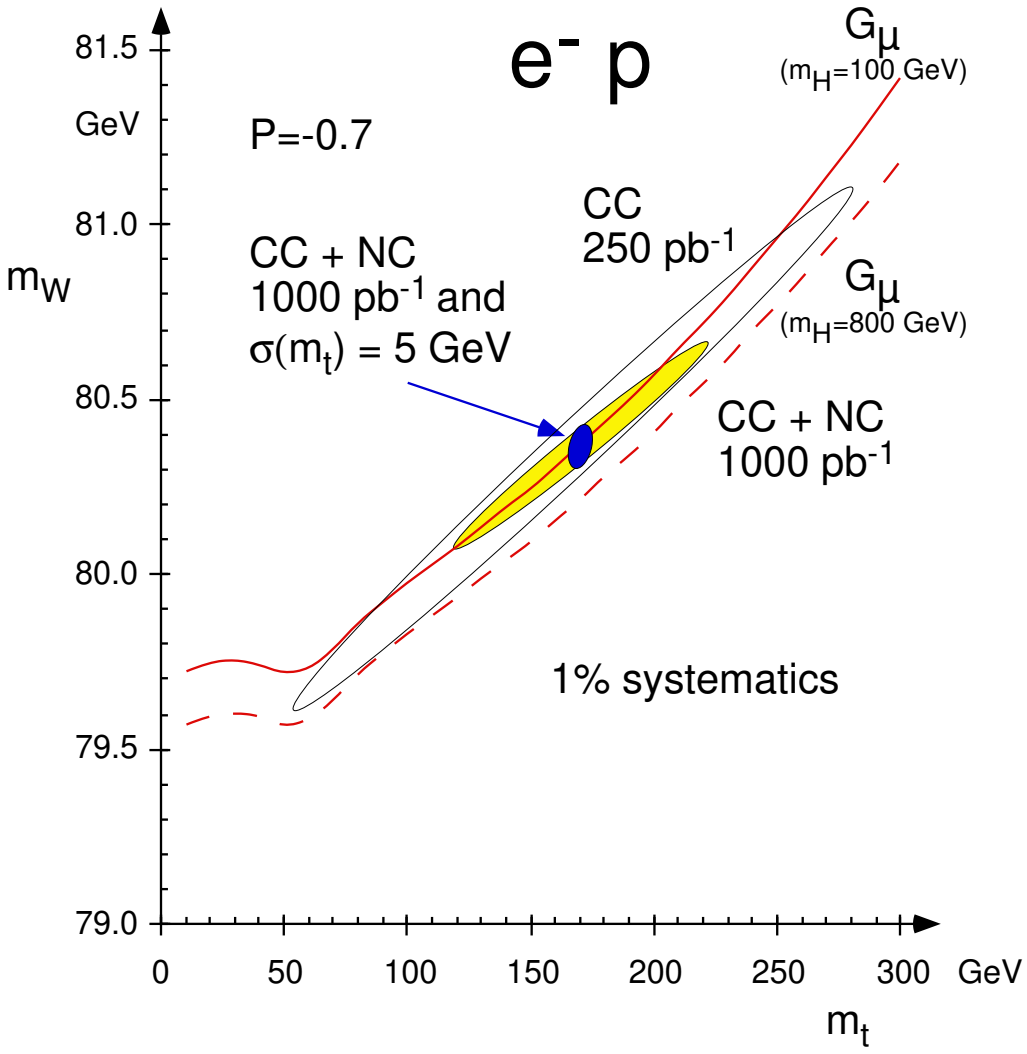

Figure 10: *1σ-confidence contours in the (m_W, m_t) plane from polarized electron scattering ($P = -0.7$), utilizing charged current scattering at HERA alone with an integrated luminosity of 250 pb^{-1} (large ellipse), neutral and charged current scattering at HERA with 1000 pb^{-1} (shaded ellipse), and the combination of the latter HERA measurements with a direct top mass measurement with precision $\sigma m_t = 5$ GeV (full ellipse). The m_W-m_t relation following from the G_μ constraint is also shown for two values of m_H.*

have to be obtained between the HERA measurements, the G_μ constraint and the direct mass measurements from colliders.

A luminosity of 250 pb^{-1} can be safely exploited at HERA for a given electroweak process. Admittedly, the uncertainty arising from the energy scale of the hadronic calorimeter imposes the largest experimental challenge. On the other hand sufficient NC data will be available to constrain the final state and hence the scale of the calorimeter energy. One can speculate that this uncertainty becomes dominating only well above the 250 pb^{-1} limit.

The most stringent tests can be performed with electrons rather than positrons. Both CC and NC cross sections are largest and the electroweak effects are enhanced for the latter. A scenario with nonzero longitudinal polarization will improve the sensitivity. In fact, NC data start to be useful once polarization is available which allows to enhance the γZ interference terms. Roughly speaking, a polarization of 70 % in the NC measurement is worth a factor 4 in luminosity.

In an optimistic scenario a value for m_W with an accuracy of 55 MeV can be extracted from each of the HERA experiments; this compares well with the projected precision from LEP 2 for all 4 LEP experiments combined.

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