# Saturation effects at LHC energies

*E. Gotsman, E. Levin, U. Maor, E. Naftali*
HEP Department, School of Physics and Astronomy, Raymond and Beverly Sackler Faculty of Exact Science, Tel Aviv University, Tel Aviv, 69978, ISRAEL

**Abstract**
Within the framework of a modified Balitsky-Kovchegov equation, we calculate and provide estimates of non-linear saturation effects expected in the LHC range of energies. Both shadowing and anti-shadowing phenomena are encountered and evaluated.

The main objective of this letter is to provide estimates for high density QCD (saturation) effects at the LHC range of energies. Our estimates are based on the Balitsky-Kovchegov [1] equation, which is the mean field approximation to high density QCD [2–4]. In spite of the restricted theoretical accuracy of this equation [5–17] in the framework of the colour dipole approach [18], the physics underlying it is rather transparent, and has led to a successful description of the main features of the experimental data, both for deep inelastic scattering and ion-ion collisions [19–23]. Among the advantages of the Balitsky-Kovchegov (BK) equation is the theoretical understanding of its mathematical structure [2, 24, 25], analytical solutions in the restricted domains [25–27], as well as extensive experience in numerical solutions [28–36].

The BK equation, however, does not include the next-to-leading (NLO) corrections to the BFKL equation [39] which are large [40] and should be taken into account at higher energies [41]. In the estimates given here we rely on the modified version of the BK equation [42] which includes the correct re-summation of the NLO corrections to the BFKL equation suggested by the Florence group [43]. The modification is based on the important observation of the Durham group [44], that, in the framework of the BFKL approach, the main part of the NLO correction is the inclusion of the leading order anomalous dimension of the DGLAP equation [45]. For a complete calculation at the NLO, one needs to include additional corrections which we have neglected in the present treatment.

We are aware that QCD evolution should be performed with running $\alpha_s$. However, attempting to obtain a numerical solution to the nonlinear BK equation with a running coupling constant is extremely complicated and time consuming. In [42] we have assessed the influence of the running QCD coupling on the evolution process, the results obtained there should be true in the present approximation.

The modified BK equation[1] is:

$$\frac{\partial N\left(r,Y;b\right)}{\partial Y} \;=\; \frac{C_F\,\alpha_S}{\pi^2}\int\frac{d^2r'\,r^2}{(\vec{r}\,-\,\vec{r}\,')^2\,r'^2}\left(1\,-\,\frac{\partial}{\partial Y}\right)\left[2N\left(r',Y;\vec{b}\,-\,\frac{1}{2}\,(\vec{r}-\vec{r}\,')\right)\right. \tag{1}$$
$$\left.-\;N\left(r,Y;\vec{b}\right)\;-\;N\left(r',Y;\vec{b}-\frac{1}{2}\,(\vec{r}-\vec{r}\,')\right)\,N\left(\vec{r}-\vec{r}\,',Y;b-\frac{1}{2}\vec{r}\,'\right)\right]$$

Eq. (1) includes the full anomalous dimension of the DGLAP equation in leading order, in the approximate form proposed in Ref. [46], namely,

$$\gamma(\omega)\;=\;\bar{\alpha}_S\left(\frac{1}{\omega}\,-\,1\right) \tag{2}$$

this describes the exact $\gamma(\omega)$ behaviour with an accuracy $>\;95\%$.

[1] We have solved a slightly different equation (see Eq.(3.1) in Ref. [42]) The equation, that has been solved, is a simplified version of Eq. (1) due to numerical difficulties.

The main advatage of Eq. (1) is that it conserves energy, even when including the non-linear term. The imposition of energy conservation is crucial as it leads to the presence of anti-shadowing. The conventional treatment [20, 23, 34] do not demand energy conservation, and therefore miss the anti-shadowing corrections.

**Unintegrated structure function:**

Solving Eq. (1) we obtain the dipole scattering amplitude, which is related to the so-called unintegrated structure function $\phi(Y, k_t)$. Physically, $\phi(Y, k_t)$ can be understood as the probability for a hadron to emit or absorb a gluon with a fixed transverse momentum $k_t$. This function enters the calculation of the inclusive production for the gluon jet, as well as all other inclusive cross sections [2, 37, 38],

$$\frac{d^2\sigma}{dy\, d^2 p_t} \;=\; \frac{4\, N_c\, \alpha_S}{N_c^2 - 1}\, \frac{1}{p_t^2} \int \phi(Y - y, k_t)\, \phi(y, \vec{p}_t - \vec{k}_t) d^2 k_t\,. \tag{3}$$

$\phi$ is determined from the solution of Eq. (1) by the following equation [38]

$$\phi(Y, k_t) \;=\; \frac{N_c}{4\, \alpha_S(k_t^2)\, \pi^2}\; k_t^2 \int d^2 b\; \Delta_{k_t}\, N(Y; k_t, b) \tag{4}$$

where $\Delta_{k_t}$ is a two dimensional Laplacian and

$$N(Y; k_t, b) \;=\; \int \frac{d^2 r}{(2\,\pi)^2\, r^2}\; e^{i\vec{k}_t \cdot \vec{r}}\; N(Y; r, b)\,. \tag{5}$$

For very small x, the integrated gluon distribution $xG$ is related to $\phi$ via

$$\alpha_S(Q^2)\, xG(x, Q^2) \;=\; \int^{Q^2} d\, k_t^2\; \alpha_S(k_t^2)\; \phi(Y, k_t)\,. \tag{6}$$

**Suppression due to the saturation effects:**

To illustrate the role of the shadowing corrections, in Fig. 1 we plot the ratio

$$D\,(k_t; x) \;=\; \frac{\phi^{NL}\,(k_t, x)}{\phi^{L}\,(k_t, x)} \tag{7}$$

where $\phi^{NL}$ denotes the unintegrated structure function obtained using all terms of Eq.(1) , and $\phi^{L}$ is the unintegrated structure function obtained by excluding the non-linear part of Eq.(1).

Our initial condition for solving Eq. (1), is the dipole-proton amplitude from the CTEQ parameterization for the gluon structure function, $xG^{CTEQ(6)}(x, Q^2)$ . We use the following form of this initial distribution:

$$N\,(r, x; b) \;=\; 1 \;-\; exp\left( -\frac{\alpha_S \pi^2}{6}\, r^2\, xG^{CTEQ(6)}(x, 4/r^2)\, S(\sqrt{r^2 + b^2})/S(0) \right) \tag{8}$$

where $S(b)$ is a dipole-like profile, $R$ is the size of the proton, we took $R^2 \;=\; 10\, GeV^{-2}$.

In Fig. 1 we have plotted contours of fixed value of $D(k_t, x)$. Also shown in Fig. 1 is the saturation momentum $Q_s(x)$ as predicted analytically [42] (dotted line), and as calculated numerically from the solution to Eq. (1) (dot-dash line). The latter is defined as the value of $Q \;=\; \frac{2}{r}$ at which $2 \log[1 \;-\; N(Y, r; b \;=\; 0)] \;=\; -1$. Note that whereas $D$ is a $b$-integrated quantity, $Q_s^2$ is defined at the particular value of $b \;=\; 0$. The comparison of these two quantities is therefore strictly from the standpoint of energy-dependence. To facilitate a comparison the value of $Q_s^2$ is normalized such that at the left most point, it coincides with the saturation momentum of the GBW model [19].

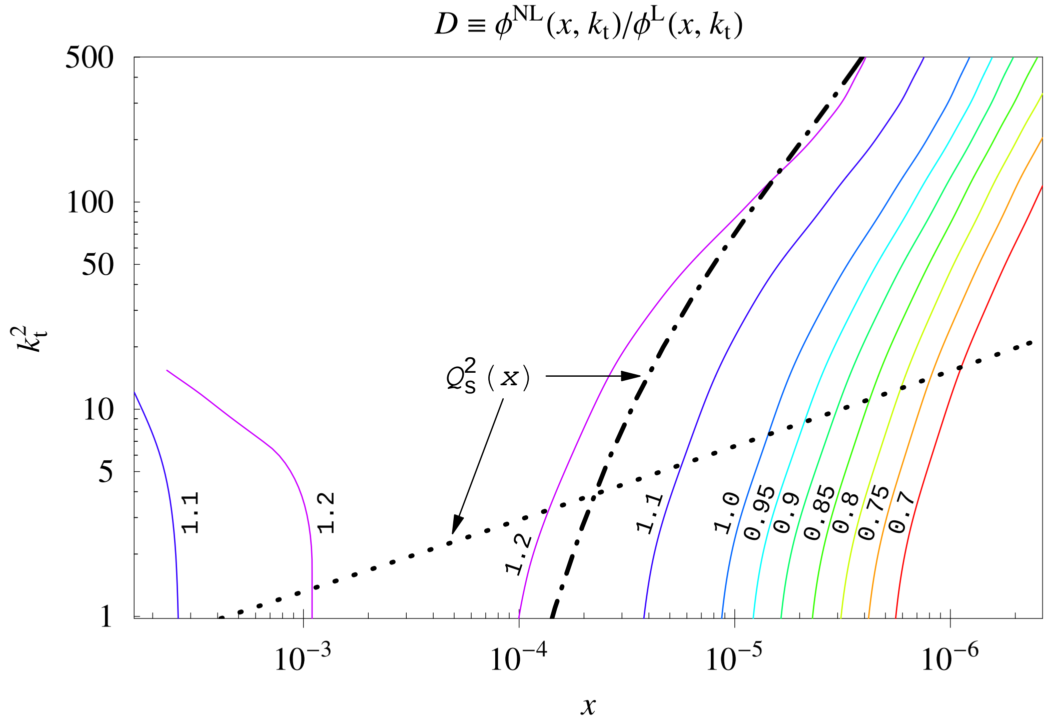


Fig. 1: The ratio given by Eq. (7) which shows the effect of the non-linear terms on the prediction for the dipole scattering amplitude at the LHC energies. The dotted curve for $Q_s^2(x)$ indicates the value obtained from the asymptotic analytic solution [42], while the dash-dot curve is from the numerical solution to Eqn.(1).

From Fig. 1 we see that both $D$ and $Q_s^2$ as obtained from the numerical solution grow rapidly with energy, compared to the analytical prediction. Referring to the $D$-contours, we note that the non-linear term in Eq. (1) is important in the LHC range of energy where it suppresses the value of $\phi$ by 30%. As the inclusive cross section is proportional to $\phi^2$, we expect an even larger suppression for the inclusive production (approximately twice as large). Note that $D(x, k_t) < 1$ for low $k_t^2$. For larger values of $k_t^2$, on the other hand, we observe an antishadowing effect on which we shall comment below.

Such behaviour of the ratio was expected theoretically (see for example Refs. [19–21, 27]), however, there are no reliable estimates in the approach which is based on the solution of the BK equation, and which describes all data at lower energies.

**Increase due to the saturation effects (antishadowing):**

Although the structure function $F_2(x, Q^2)$ is directly related to experimentally measured quantities, at high energies (low $x$), it is well approximated by the gluon distribution. In the following we will, therefore, examine the behaviour of $xG(x, Q^2)$, which is less complicated. Still, the comparison of the results of the present calculation with available gluon parameterizations, such as CTEQ and MRST, is not straightforward because we calculated the unintegrated gluon distribution, while CTEQ and MRST only present results for the structure functions integrated over $k_t$.

Fig. 2 shows the value of $xG(x, Q^2)$ for the non-linear equation (see Eq. (1)), together with the value for the solution of the linear equation (Eq.(1) without the non-linear terms), and CTEQ parameterization. Note that, for large values of $x$, the solution to the non-linear equation leads to a larger amplitude. This is not expected as the non-linear term has a negative sign, and manifests itself as the shadowing correction which decreases the value of the amplitude. The increase of the ratio $D$ (see Eq. (7) and Fig. 3, below) at low energies is significant, and cannot be explained by errors neither due to the approximate formula for the anomalous dimension of the DGLAP equation, nor to the numerical procedure.

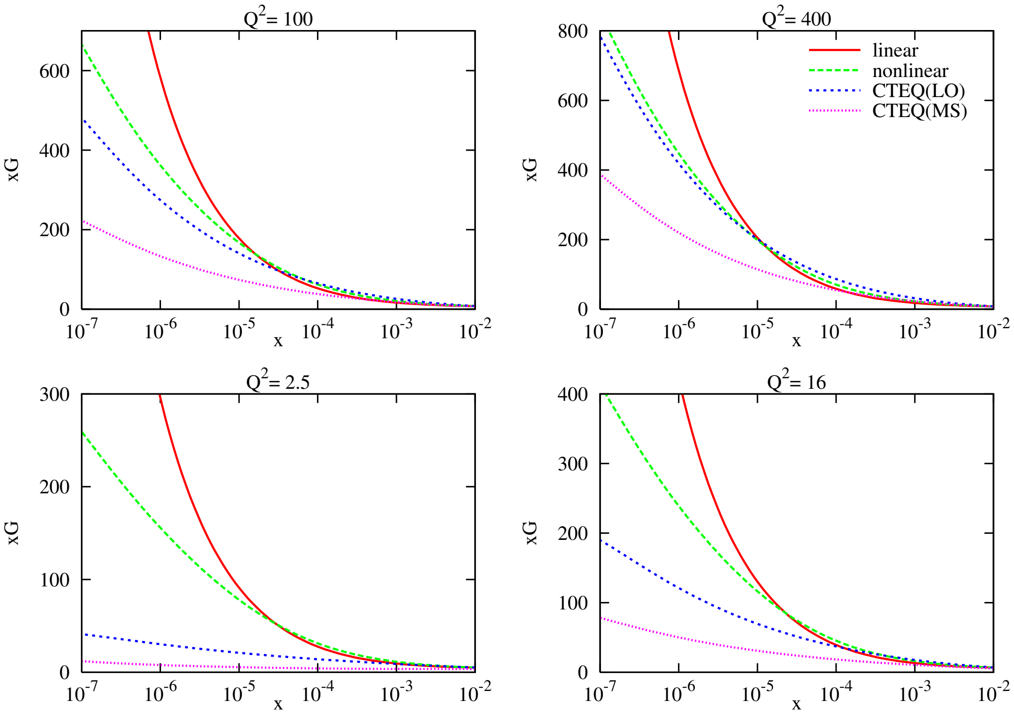


Fig. 2: The value of the gluon structure function as function of $x$ for different values of $Q^2$. Non-linear denotes the curve for the gluon structure function calculated using Eq. (6) from the solution of Eq. (1), while linear is the same without the non-linear term. CTEQ denotes the curve for the gluon structure function given by CTEQ parameterization.

The explanation for the anti-shadowing effect is clear. At large $k_t$, the non-linear term is small, and the linear evolution equation is a good approximation to the solution. Decreasing the value of $k_t$, the largest changes are induced in the slope of the amplitude (or/and gluon densities) with respect to $\ln(k_t)$ (see Refs. [2, 20, 21, 27]). The slope for the linear equation is larger than for the non-linear equation. For large values of $k_t$, the unintegrated structure function $\phi$ is equal to the solution of the linear equation, while for small values of $k_t$, $\phi$ becomes larger than the solution to the linear equation.

Another way to see the same effect is by comparing derivatives with respect to Y, see Fig. 2. For the solution to the linear equation at large $x$, the derivatives with respect to Y are much larger than the same derivative for the solution of the non-linear equation. Therefore, for the case of the non-linear equation, the linear term, which contains $-\partial/\partial Y$, is larger for the non-linear equation, leading to an increase in the value of the solution. The unintegrated structure function in Eq. (4) reproduces the same qualitative behaviour as the dipole scattering amplitude.

All these arguments can be obtained directly from energy-momentum sum rules, (a consequence of the conservation of energy and momentum), which hold for the modified BK equation, namely, that the integral over $x$ is independent of $Q^2$

$$\int dx \phi^{NL}\left(Y;Q^2\right) \;=\; Const(Q^2) \;=\; \int dx \phi^{L}\left(Y;Q^2\right) \,. \tag{9}$$

Note that this equation cannot be used together with Eq. (6) to obtain a sum-rule for the integrated gluon density, because Eq. (6), as stated, is correct only for very low $x$, whereas in our approach Eq. (9) holds for all values of $x$.

In the region of low $x$ we expect that $D(x, k_t) < 1$ (see Fig. 2), and based on Eq. (7) we anticipate that in some region of larger $x$, this ratio should be larger than 1.

Fig. 3 displays quantitative estimates for these antishadowing effects. The effect is rather large and has been discussed previously in Ref. [47]. In our approach it arises naturally [2].

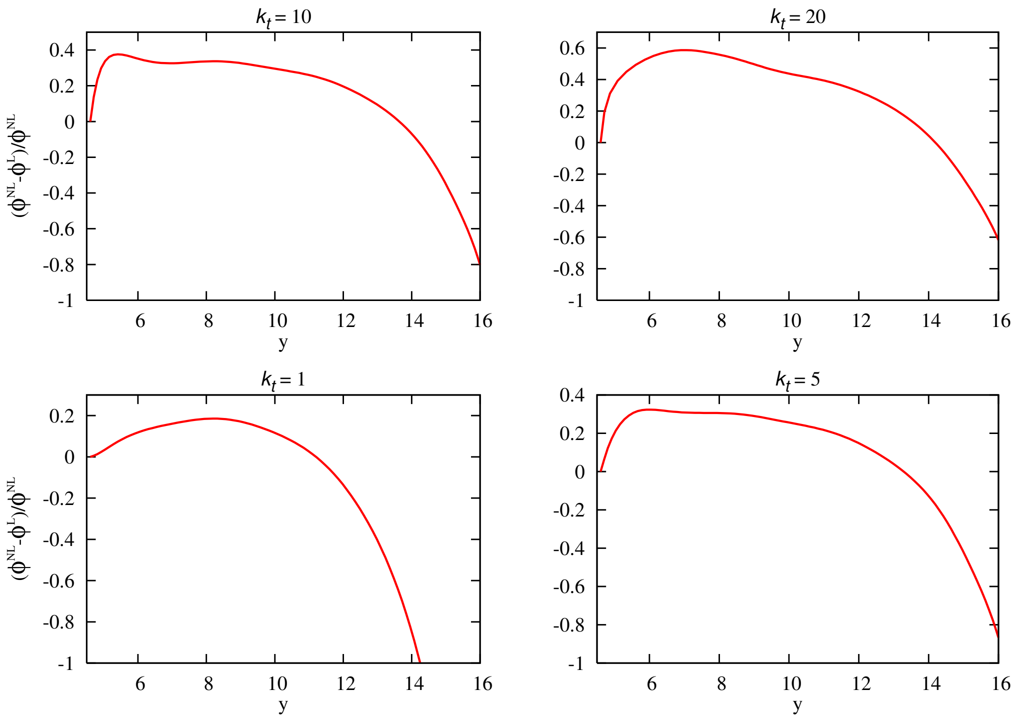


Fig. 3: The ratio $(\phi^{NL} - \phi^{L})/\phi^{L}$ for different values of $Q^2$ as function of $y = \ln(1/x)$.

In this paper we have obtained a numerical solution for a modified BK equation, where energy conservation is imposed. We found and evaluated both shadowing and anti-shadowing effects which appear in the solution, as a function of energy (including the LHC range). We hope that our estimates of the scale for the nonlinear effects at the LHC energies, will aid in producing more reliable calculations for the cross sections of 'hard' processes of interest at high energies.

**Acknowledgments:** We would like to thank Jochen Bartels, Markus Diehl, Hannes Jung, Michael Lublinsky and all participants of HERA-LHC workshop for fruitful discussions. This research was supported in part by the Israel Science Foundation, founded by the Israeli Academy of Science and Humanities.

## References

[1] Ia. Balitsky, Nucl.Phys. **B463** (1996) 99; Yu. Kovchegov, Phys. Rev. **D60** (2000) 034008.

[2] L. V. Gribov, E. M. Levin and M. G. Ryskin, Phys. Rep. **100** (1983) 1.

[3] A. H. Mueller and J. Qiu, Nucl. Phys. **B268** (1986) 427.

[4] L. McLerran and R. Venugopalan, Phys. Rev. **D 49** (1994) 2233, 3352; **D 50** (1994) 2225, **D 53** (1996) 458, **D 59** (1999) 09400.

[2] In the talk of K. Peters at HERA-LHC Workshop (19 January 2005), it was claimed that there is no antishadowing effect. As far as one can gather from his transparencies, he used the modification to the BK equation suggested in Ref. [48]. This modified version is quite different from ours, as energy is not conserved in [48], neither in the linear or non-linear terms, consequently no anti-shadowing effect was seen.

[5] J. Jalilian-Marian, A. Kovner, A. Leonidov and H. Weigert, Phys. Rev. **D59** (1999) 014014 [arXiv:hep-ph/9706377]; Nucl. Phys. **B504** (1997) 415 [arXiv:hep-ph/9701284];
J. Jalilian-Marian, A. Kovner and H. Weigert, Phys. Rev. **D59** (1999) 014015 [arXiv:hep-ph/9709432]; A. Kovner, J. G. Milhano and H. Weigert, Phys. Rev. D **62** (2000) 114005 [arXiv:hep-ph/0004014] ; E. Iancu, A. Leonidov and L. D. McLerran, Phys. Lett. **B510** (2001) 133 [arXiv:hep-ph/0102009]; Nucl. Phys. **A692** (2001) 583 [arXiv:hep-ph/0011241];
H. Weigert, Nucl. Phys. **A703** (2002) 823 [arXiv:hep-ph/0004044].

[6] E. Iancu and A. H. Mueller, Nucl. Phys. **A730** (2004) 460, 494, [arXiv:hep-ph/0308315],[arXiv:hep-ph/0309276].

[7] M. Kozlov and E. Levin, Nucl. Phys. **A739** (2004) 291 [arXiv:hep-ph/0401118].

[8] E. Levin and M. Lublinsky, Nucl. Phys. **A730** (2004) 191 [arXiv:hep-ph/0308279].

[9] A. H. Mueller and A. I. Shoshi, Nucl. Phys. **B692** (2004) 175 [arXiv:hep-ph/0402193].

[10] E. Iancu, A. H. Mueller and S. Munier, Phys. Lett. **B606** (2005) 342 [arXiv:hep-ph/0410018].

[11] E. Levin and M. Lublinsky, Phys. Lett. **B607** (2005) 131 [arXiv:hep-ph/0411121].

[12] E. Iancu and D. N. Triantafyllopoulos, *"A Langevin equation for high energy evolution with pomeron loops,"* arXiv:hep-ph/0411405.

[13] A. H. Mueller, A. I. Shoshi and S. M. H. Wong, *"Extension of the JIMWLK equation in the low gluon density region,"* arXiv:hep-ph/0501088.

[14] E. Levin and M. Lublinsky, *"Towards a symmetric approach to high energy evolution: Generating functional with Pomeron loops,"* arXiv:hep-ph/0501173.

[15] E. Iancu and D. N. Triantafyllopoulos, *"Non-linear QCD evolution with improved triple-pomeron vertices,"* arXiv:hep-ph/0501193.

[16] A. Kovner and M. Lublinsky, *"In pursuit of pomeron loops: The JIMWLK equation and the Wess-Zumino term,"* arXiv:hep-ph/0501198; *"Remarks on High Energy Evolution,"* arXiv:hep-ph/0502071; *"From target to projectile and back again: Selfduality of high energy evolution,"* arXiv:hep-ph/0502119;

[17] J. P. Blaizot, E. Iancu, K. Itakura and D. N. Triantafyllopoulos, *"Duality and Pomeron effective theory for QCD at high energy and large $N_c$,"* arXiv:hep-ph/0502221.

[18] A. H. Mueller, Nucl. Phys. **B 415** (1994) 373; *ibid* **B 437** (1995) 107.

[19] K. Golec-Biernat and M. Wüsthoff, Phys. Rev. **D59** (1999), 014017; *ibid.* **D60** (1999), 114023; Eur. Phys. J. **C20** (2001) 313.

[20] D. Kharzeev, E. Levin and L. McLerran, Phys. Lett. **B561** (2003) 93 [arXiv:hep-ph/0210332];
D. Kharzeev and M. Nardi, Phys. Lett. **B507**, 121 (2001); D. Kharzeev, E. Levin, Phys. Lett. **B523**, 79 (2001); D. Kharzeev, E. Levin and M. Nardi, Nucl. Phys. **A730** (2004) 448 [arXiv:hep-ph/0212316]; R. Baier, A. Kovner and U. A. Wiedeman, Phys. Rev. **D68** (2003) 054009 [arXiv:hep-ph/0305265]; D. Kharzeev, Y. Kovchegov and K. Tuchin, Phys.Rev. **D68**, 094013 (2003) [arXiv:hep-ph/0307037]; Phys. Lett. **B599** (2004) 23 [arXiv:hep-ph/0405045].

[21] E. Iancu, K. Itakura and L. McLerran, Nucl. Phys. **A721** (2003) 293; **A708**, 327 (2002) [arXiv:hep-ph/0203137].

[22] E. Gotsman, E. Levin, M. Lublinsky and U. Maor, Eur. Phys. J. **C27**, 411 (2003) [arXiv:hep-ph/0209074]; [http://www.desy.de/ lublinm/]m

[23] E. Iancu, K. Itakura and S. Munier, Phys. Lett. **B590** (2004) 199 [arXiv:hep-ph/0310338].

[24] J. Bartels and E. Levin, Nucl. Phys. **B387** (1992) 617.

[25] S. Munier and R. Peschanski, Phys. Rev. **D70** (2004) 077503 [arXiv:hep-ph/0401215]. Phys. Rev. **D69** (2004) 034008 [arXiv:hep-ph/0310357]; Phys. Rev. Lett. **91** (2003) 232001 [arXiv:hep-ph/0309177].

[26] E. Levin and K. Tuchin, Nucl. Phys. **A693** (2001) 787, [arXiv:hep-ph/0101275] ; **A691** (2001) 779,[arXiv:hep-ph/0012167]; **B573** (2000) 833, [arXiv:hep-ph/9908317].

[27] A. H. Mueller and D. N. Triantafyllopoulos, Nucl. Phys. **B640** (2002) 331 [arXiv:hep-ph/0205167]; D. N. Triantafyllopoulos, Nucl. Phys. **B648** (2003) 293 [arXiv:hep-ph/0209121].

[28] N. Armesto and M. A. Braun, Eur. Phys. J. **C20**, 517 (2001) [arXiv:hep-ph/0104038].

[29] K. Golec-Biernat, L. Motyka and A. M. Stasto, Phys. Rev. **D65**, 074037 (2002) [arXiv:hep-ph/0110325].

[30] M. Lublinsky, E. Gotsman, E. Levin and U. Maor, Nucl. Phys. **A696**, 851 (2001) [arXiv:hep-ph/0102321].

[31] K. Rummukainen and H. Weigert, Nucl. Phys. **A739**, 183 (2004) [arXiv:hep-ph/0309306].

[32] K. Golec-Biernat and A. M. Stasto, Nucl. Phys. **B668**, 345 (2003) [arXiv:hep-ph/0306279].

[33] E. Gotsman, M. Kozlov, E. Levin, U. Maor and E. Naftali, Nucl. Phys. **A742**, 55 (2004) [arXiv:hep-ph/0401021].

[34] K. Kutak and A. M. Stasto, *"Unintegrated gluon distribution from modified BK equation,"* arXiv:hep-ph/0408117.

[35] J. L. Albacete, N. Armesto, J. G. Milhano, C. A. Salgado and U. A. Wiedemann, Phys. Rev. **D71**, 014003 (2005) [arXiv:hep-ph/0408216].

[36] T. Ikeda and L. McLerran, arXiv:hep-ph/0410345.

[37] E. Laenen and E. Levin, Ann. Rev. Nucl. Part. Sci. **44**, 199 (1994); Y. V. Kovchegov and D. H. Rischke, Phys. Rev. **C56**, 1084 (1997) [arXiv:hep-ph/9704201]; M. Gyulassy and L. D. McLerran, Phys. Rev. **C56**, 2219 (1997) [arXiv:nucl-th/9704034].

[38] M. Braun, Eur. Phys. J. **C16** (2000) 337 [arXiv:hep-ph/0001268].

[39] E. A. Kuraev, L. N. Lipatov, and F. S. Fadin, Sov. Phys. JETP **45** (1977) 199 ;
Ya. Ya. Balitsky and L. N. Lipatov, Sov. J. Nucl. Phys. **28** (1978) 22 .

[40] V.S. Fadin and L.N. Lipatov, Phys. Lett. **B429** (1998) 127
G. Camici and M. Ciafaloni, Phys. Lett. **B430** (1998) 349.

[41] M. Ciafaloni, D. Colferai, G.P. Salam and A.M. Stasto, Phys. Rev. **D68** (2003) 114003; M. Ciafaloni, D. Colferai, G. P. Salam and A. M. Stasto, Phys. Lett. **B541**, 314 (2002) [arXiv:hep-ph/0204287]; Phys. Rev. **D66**, 054014 (2002) [arXiv:hep-ph/0204282]; J. R. Forshaw, D. A. Ross and A. Sabio-Vera, Phys. Lett. **B498**, 149 (2001) [arXiv:hep-ph/0011047]; M. Ciafaloni, M. Taiuti and A. H. Mueller, Nucl. Phys. **B616**, 349 (2001) [arXiv:hep-ph/0107009];

S.J. Brodsky, V.S. Fadin, V.T. Kim, L.N. Lipatov and G.B. Pivovarov, JETP Lett. **70** (1999) 155; D. A. Ross, Phys. Lett. **431**, 161 (1998) [arXiv:hep-ph/9804332]; E. Levin, Nucl. Phys. **B545** (1999) 481. [arXiv:hep-ph/9806228]; N. Armesto, J. Bartels and M. A. Braun, Phys. Lett. **442**, 459 (1998) [arXiv:hep-ph/9808340]; Y. V. Kovchegov and A. H. Mueller, Phys. Lett. **B439** (1998) 428 [arXiv:hep-ph/9805208].

[42] E. Gotsman, E. Levin, U. Maor and E. Naftali, *"A modified Balitsky-Kovchegov equation,"* arXiv:hep-ph/0411242.

[43] G.P. Salam, JHEP **9807** (1998) 019; Acta Phys. Pol. **B30** (1999) 3679;
M. Ciafaloni, D. Colferai and G.P. Salam, Phys. Lett. **B452** (1999) 372; Phys. Rev. **D60** (1999) 114036.

[44] V. A. Khoze, A. D. Martin, M. G. Ryskin and W. J. Stirling, Phys. Rev. **D70** (2004) 074013 [arXiv:hep-ph/0406135].

[45] V. N. Gribov and L. N. Lipatov, Sov. J. Nucl. Phys **15** (1972) 438;
G. Altarelli and G. Parisi, Nucl. Phys. **B 126** (1977) 298;
Yu. l. Dokshitser, Sov. Phys. JETP **46** (1977) 641.

[46] R. K. Ellis, Z. Kunszt and E. M. Levin, Nucl. Phys. **B420** (1994) 517 [Erratum-ibid. **B433** (1995) 498].

[47] K. J. Eskola, V. J. Kolhinen and R. Vogt J. Phys. **G30** (2004) S1171 [arXiv:hep-ph/0403130]; A. Dainese, R. Vogt, M. Bondila, K. J. Eskola and V. J. Kolhinen, J. Phys. **G30** (2004) 1787 [arXiv:hep-ph/0403098]; K. J. Eskola, V. J. Kolhinen and R. Vogt, Phys. Lett. **B582** (2004) 157 [arXiv:hep-ph/0310111]; K. J. Eskola, H. Honkanen, V. J. Kolhinen, J. W. Qiu and C. A. Salgado, Nucl. Phys. **B660** (2003) 211 [arXiv:hep-ph/0211239].

[48] K. Kutak and J. Kwiecinski, Eur. Phys. J. **C 29**, 521 (2003)[arXiv:hep-ph/0303209]