SURFACE ROUGHNESS WAKEFIELDS FOR THE

FOR THE

ILC POSITRON UNDULATOR VESSEL

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ABSTRACT

A simple, and probably pessimistic, estimation of the surface roughness specification for the undulator vessel can be made using an inductive impedance model. The results shows that a surface roughness of ~600nm should not induce an energy spread more than 10% of the nominal value for the ILC beam and 200m long 2 mm radius vessel. This specification, although quite stringent, can be met, according to several manufacturers catalogues. However, due to currently unknown factors such as the exact bunch charge distribution and to induce the lowest possible energy spread in the bunch the smoothest commercially available vessel should be used, which has a roughness less than 200nm.

INTRODUCTION

A number of models exist for surface roughness and have all been shown to be roughly equivalent [1]. The so called ‘inductive impedance model’ [2] shall be used which makes the following assumptions:

- The vessel is cylindrically symmetric;
- The charge distribution is smooth;
- The surface roughness is assumed to have no resistance;
- The length of the bunch is longer than the length of the roughness features;
- As the impedance is purely inductive only surface roughness effects that influence the energy spread of the beam are considered.
- The height of the surface roughness bumps is about the same as the length of the bump.

The model assumes a number of bumps located on the surface of height δ, the radius of the vessel is b. For a particular angular frequency, ω, the impedance, $Z(\omega)$, per unit length L, of a hemispherical bump is given by [3]:

$$\frac{Z(\omega)}{L} = -i\omega I = -i\omega \frac{Z_0 \delta^3}{4\pi c b},$$

where I is the inductance, Z_0 is the vacuum impedance and c the speed of light. The impedance is purely inductive with no resistive part. For different shape bumps the impedance has to be multiplied by a form factor that depends upon the shape of the bump. Different shaped bumps have been modelled in Mafia and various form factor ascribed to them [4]. Figure 1 shows some of the different shapes considered and Table 1 gives their form factors.

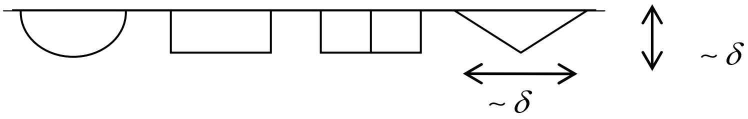

Figure 1: sample bump shapes, from left to right, hemisphere, half cube, rotated half cube and wedge.

Bump Shape	Form Factor, f
Hemisphere	1
Half Cube	2.6
Rotated Half Cube	0.6
Wedge	1.1

Table 1: form factors for various bump shapes from Figure 1.

As well as considering the shape of the surface bumps there is also the proportion of the surface area that is covered by the bumps to consider. For many bumps the total impedance is assumed to be the sum of the impedances of each bump. The total impedance per unit length is then:

$$\frac{Z(w)}{L} = -i\alpha f \omega \frac{Z_0 \delta}{4\pi c b}, \tag{1}$$

where f is the form factor relating to the bump shape and α is the packing factor, giving the fraction of the surface occupied by bumps.

The wake function is then simply the Fourier transform of the impedance. As the impedance is inductive it only has imaginary components and the Fourier Sine integral can be used. For a Gaussian bunch of rms length σ_z the wake potential, $W(s)$, is:

$$W(s) = -\frac{\alpha f Z_0 \delta c}{2\pi^2 b} \int_0^\infty k Exp\left[\frac{k^2\sigma_z^2}{2}\right] Sin(ks)\, dk = -\frac{\alpha f Z_0 \delta c}{\pi b \sqrt{2\pi}\sigma_z^3} s Exp\left[\frac{-s^2}{2\sigma_z^2}\right],$$

where $Exp\left[\frac{k^2\sigma_z^2}{2}\right]$ is the Fourier transform of the longitudinal charge distribution. The induced relative energy spread, σ_E, of a bunch of N electrons of charge e is:

$$\sigma_E = e^2 N \left| \int_{-\infty}^{\infty} \left(\frac{1}{\sigma_z\sqrt{2\pi}} Exp\left[\frac{-s^2}{2\sigma_z^2}\right] W^2 - k_{\|}^2 \right) ds \right|^{1/2},$$

where $k_{\|}^2$ is the square of the longitudinal loss factor and is equal to zero for purely inductive impedances. This gives, for the total induced energy spread of a round vessel L metres long:

$$\sigma_E = \alpha f \frac{e N Z_0 \delta c L}{3^{3/4} 2^{3/2} \pi^{3/2} b \sigma_z^2}.$$

A similar expression has also been derived [5]. The expression is given in terms of the inductance, I, and is:

$$\sigma_E = \frac{e N L c^2 I}{3^{3/4} 2^{1/2} \pi^{1/2} \sigma_z^2}, \tag{2}$$

where the inductance is determined from measurement and simulation. For the same form factor and packing factor the two expression for σ_E give the same result.

PARAMETERS

The parameters given in Table 2 shall be used. The ILC “minimum” beam parameters [6] give the greatest effect and so only those are considered. Using these parameters the induced energy spread can be calculated for various different form factors and surface roughness values, these are shown in Figure 2 and Figure 3 for a vessel radius of 2 mm and 3 mm respectively. In Figure 4 the roughness required to produce an energy spread of 0.005% as a function of vessel radius is plotted for different form factors.

Parameter	**Unit**	**Value**
Energy	GeV	150
Nominal σ_E	%	0.05
Increase in σ_E from Nominal	%	10
Undulator Length	m	200
rms Bunch Length	mm	0.15
Bunch Charge	N	$1\ 10^{10}$
Packing Factor	α	0.5

Table 2: ILC and undulator parameters.

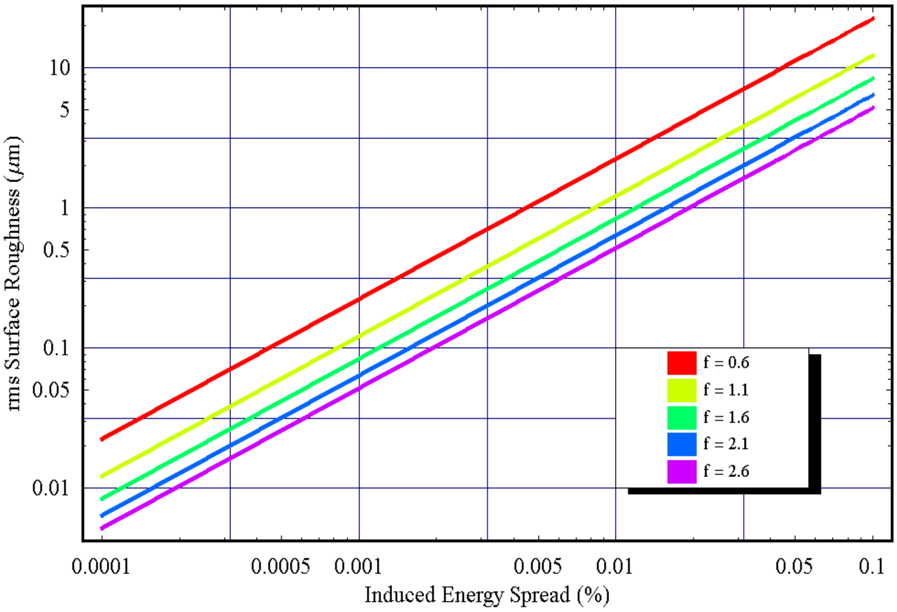

Figure 2: surface roughness v induced energy spread for different form factors and a 2mm radius vessel

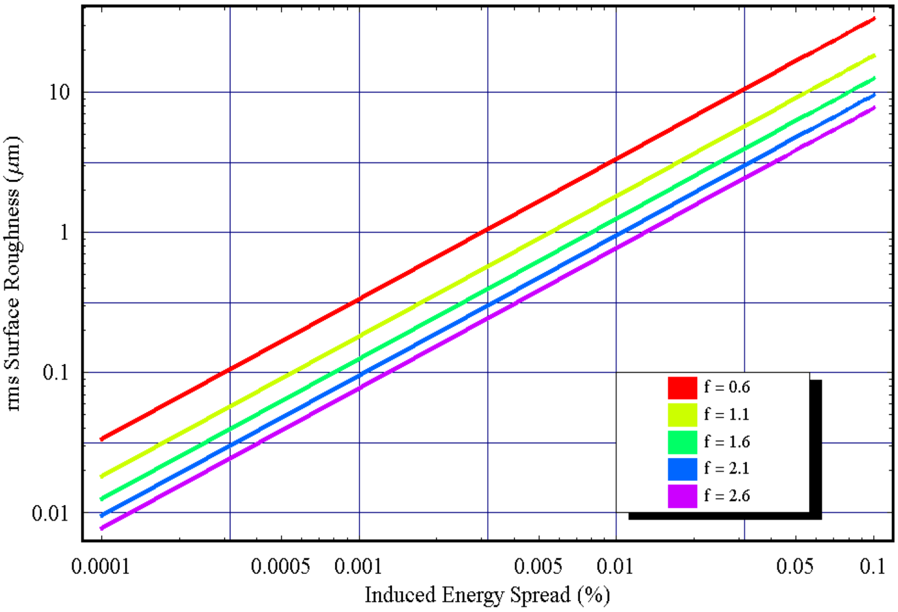

Figure 3: surface roughness v induced energy spread for different form factors and a 3mm radius vessel

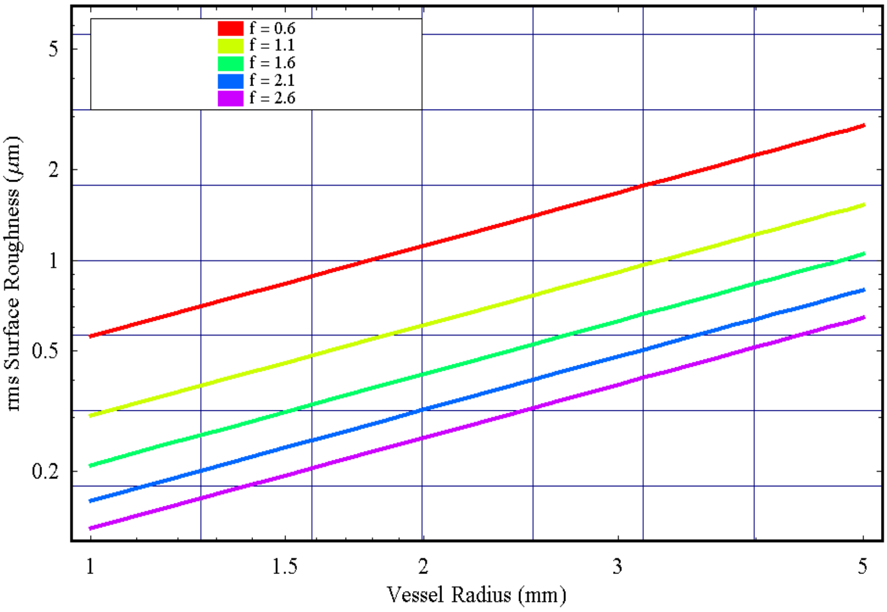

Figure 4: surface roughness required to produce an energy spread of 0.5 10^{-4} for different vessel radia and form factors.

CONCLUSIONS & DISCUSSION

The requirements on the surface roughness seem quite demanding. According to the model, to keep the increase in energy spread below 10% of the nominal value the surface roughness must be less than ~300nm, for the worst form factor. The surface roughness tolerance is not strongly dependant on the radius of the vessel, it decreases as the inverse of the radius. Other wakefield effects often scale as an inverse power of the radius.

The inductance of high quality smooth pipes from the VALEX Corporation [7] was measured in [5]. A 2.5mm radius vessel, gave an inductance of ~0.28 pH m^{-1}. The average roughness of the VALEX tube was measured to be ~125nm.

From Equation (2) the impedance budget for the ILC undulator is 33pH m^{-1} and if a similar tube to [5] were used then the inductance per meter would be ~0.35 pH. This indicates that the theory is pessimistic compared to measured inductances of real vessels. One of the reasons for this is that the assumption that the height of the bumps is approximately the same as the length of the bumps is not confirmed by the measurements [5].

If it is assumed that the measured inductance is correct for many metres of pipe then the corresponding form factor is ~0.06 and the height of the bumps required to keep the increase in energy spread less than 10% of the nominal is ~12 microns.

However even given all these factors the conclusions of [5] state that there is an active program underway to improve upon the ~125nm surface roughness of the VALEX vessels. There are other effects that have not been included, such as the true distribution of the bunch charge, which may have features that are approaching the scale of the surface roughness. Also misalignments of the vessels will have some effect. Given these unknowns it would seem prudent to use the smoothest vessel possible to help mitigate any future deleterious effects of the surface roughness.

REFERENCES

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