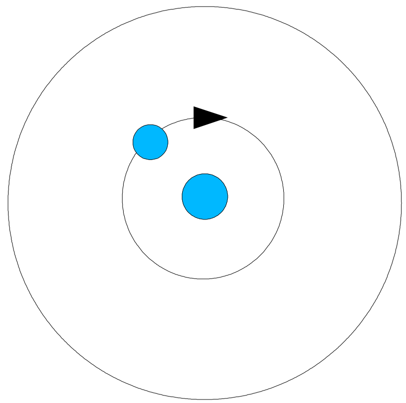
Spin Dynamics and Polarization

Gudrid Moortgat-Pick (CERN and IPPP, Durham)

- **Today: Spin introduction .... 'mathematics'**
  - History: Stern-Gerlach experiment and its interpretation
  - Group theory
  - Quantum numbers
  - Dirac equation
- **Tomorrow: Spin in experiments ..... 'phenomenology'**
  - Some applications (spread in the physics examples)
  - Physics at RHIC: spin crisis of the proton
  - Physics at HERA: protons crisis, right-currents and all that
  - Physics at the ILC: physics, sources and depolarization

Begin of last century

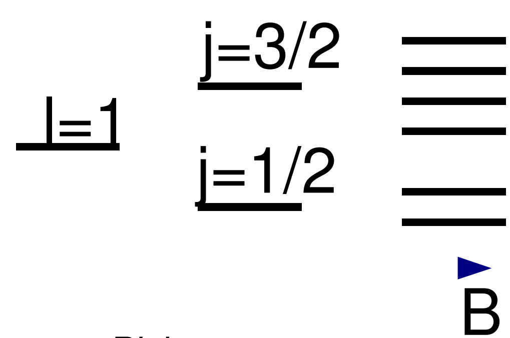
- Status of begin of last century: Bohr model



  - electron only at quantized orbits: 'space quantization'
  - many physicists had strong doubts, that 'spatial orientation of atoms is something physically true' (Debye, Pauli, etc.)

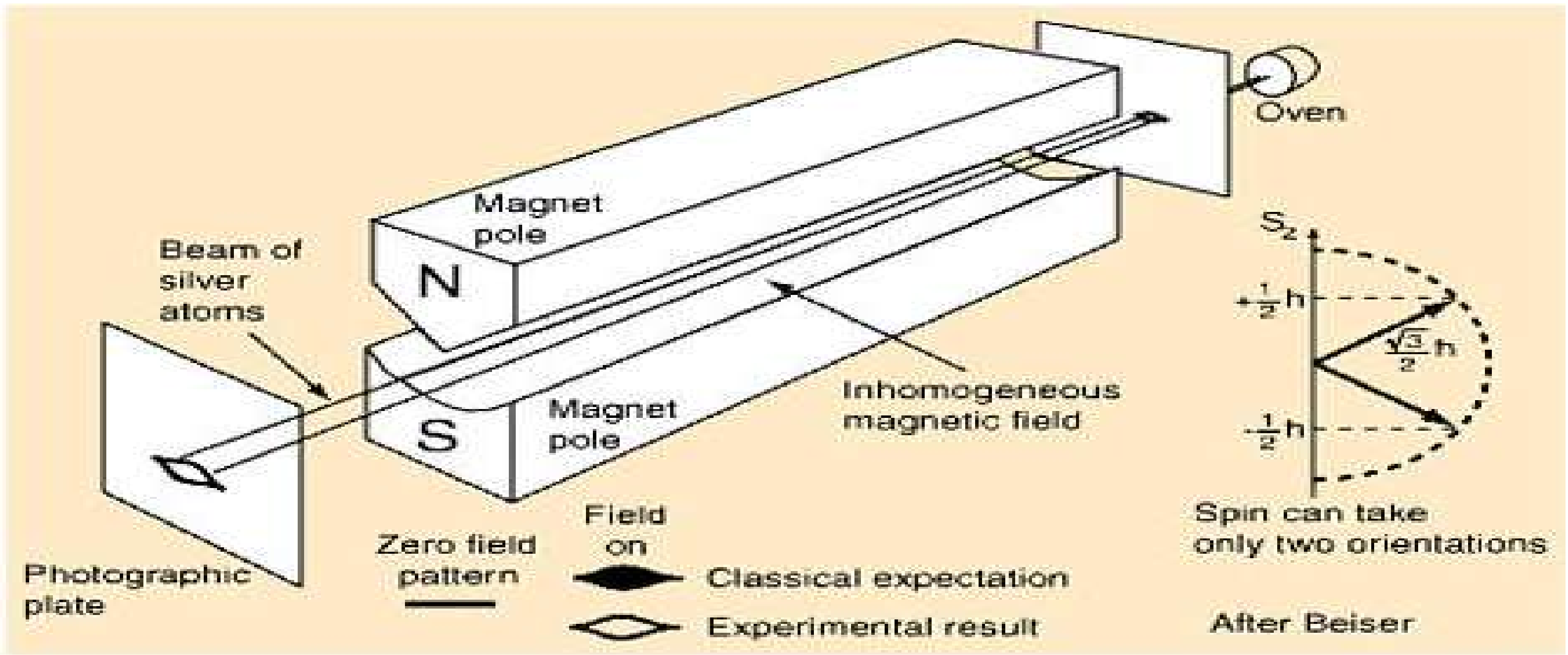
- Hydrogen excitation spectra: complex splitting patterns in a magnetic field not understandable (Zeeman effect)
  - $J^2 \Psi = j\,(j+1)\,\Psi$
  - $J_z \Psi = m\,\Psi$

J=L+S



Spin history 1

- **Proof of 'spatial quantization': Stern-Gerlach experiment (1922):**

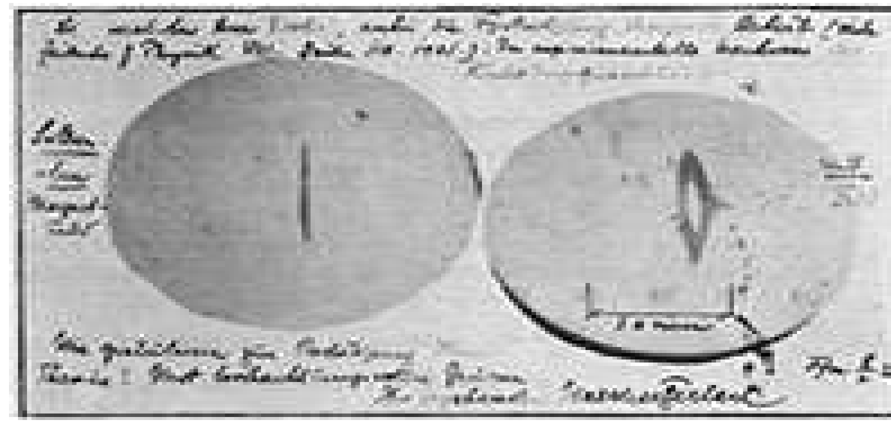


  - **beam of silver atoms in inhomogeneous magnetic field of 0.1 T and 10 T/cm**
  - **classical theory predicts two levels: splitting of silver beam was only 0.2 mm**
  - **misalignments of collimating slits by more than 0.01mm spoiled experiment!**

  (precise alignment needed to discover new physics .... ' same problem today' !)

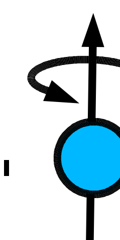
On the way to the spin

- **Stern-Gerlach continued:**
  - historical 'postcard' in 1922



  - still doubts from Einstein, Ehrenfest etc.: how could exact splitting occur when atoms entered field with random orientation?

- **Complete explanation (together with Zeeman effect) only with 'spin':**
  - 1924 Pauli: two-valued quantum degree of freedom
  - 1925 Pauli: exclusion principle
  - 1925 Kronig, Goudsmit and Uhlenbeck: electron self-rotation, has 'spin'
  - 1927 Pauli: theory of spin based on quantum mechanics, but non-relativistic



Spin history, cont.

- 1928 Dirac: published Dirac equation
  - description of relativistic electron spin via 'spinor'
  - connection between 'relativity' and 'quantum effect' !
- 1940 Pauli: spin-statistic-theorem
  - particle classification in fermions = anti-symmetric and bosons = symmetric states whose ensembles obey different statistics, Fermi-Dirac or Bose-Einstein
  - *breakthrough in description of particle phenomena !*
- Summary:
  - spin has no classical description *(only quasi-classical,....., see Des lecture)*
  - description as 'rotating particle' wrong, but helps understanding
  - behaves like a kind of 'angular momentum'
  - relativistic description via 'spinors'=vector

Group theory and Dirac equation

- **Questions:**
  - a) what are the kinematic properties of a particle? Only mass and spin?
  - b) how to describe the states of a relativistic particles with spin?
  - c) mathematical principle from which mass and spin follow deductively?

- **Solution:**
  - notion of mass is related with special relativity: Lorentz transformations
  - notion of spin is related with rotation ('angular momentum'): rotations

- **Group needed which embraces Lorentz transformations and rotations and allows a definition of mass and describes the spin ...**

Group theory, introduction

- Definition of a group G:

1. product ab belongs to G if a and b belong to G [$a \in G$, $b \in G \Rightarrow ab \in G$]
2. associativity: a (bc) = (ab) c for all a,b,c
3. unit element: $e \in G$, such that a e= e a = a for all a
4. every $a \in G$ has inverse $a^{-1} \in G$, such that $(a\,a^{-1}) = (a^{-1}\,a) = e$

- Order of a continuous group: number of parameters
  - e.g. for rotations: 2 for fixing direction of axis, 1 angle for rotation around axis
  - present rotation as matrix with 3 parameters
- Infinitesimal operations:
  - $A = \exp(\alpha) = I + \alpha + \alpha^2/2! + \ldots$ i.e. $\alpha = \lim_{n \to \infty} n\,(A^{1/n} - I)$
  - α has to fulfill the Lie algebra of the group G
  - g_n = basis matrices of algebra: $\alpha = c_1 g_1 + c_2 g_2 + \ldots + c_n g_n$ *'infinitesimal generators'*

Group theory in physics

- **SU(2): group of unitary 2 x 2 matrices with $U U^{+}=1$ and det U=1 ('S')**
  - 8 elements - (4 unitary +1 'special') conditions = 3 parameters
  - generators: U = exp(ih) with $h^{T} = h^{+}$ i.e. Lie algebra of SU(2) are hermitian matrices with the 3 Pauli matrices as basis: $h = c_1 \sigma_1 + c_2 \sigma_2 + c_3 \sigma_3$

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \qquad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

'spin matrices'

- **SU(3): group of unitary 3 x 3 matrices**
  - 18 elements - (9 unitarity +1 'special') conditions = 8 parameters
  - 8 hermitian matrices as generators

Rotation group

- R_3 = 'group of 3-dim rotations' : is subgroup of SU(3) but with real matrices
  - 3 only three parameters, i.e. 3 infinitesimal generators

in 3-dim

$$J_1 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{pmatrix} \qquad J_2 = \begin{pmatrix} 0 & 0 & -1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} \qquad J_3 = \begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

  - any hermitian 3 x 3 matrix with Tr=0 can be expressed via J_1, J_2, J_3
- Total angular momentum defined as $J^2 = J_1^2 + J_2^2 + J_3^2$
  - J^2 has the eigenvalues : $j\,(j+1)$
  - third component J_3 has (2j +1) eigenvalues $m \in [-j, -j+1, ..., +j]$
  - basis vectors denoted $| j\ m >$

Relation between SU(2) and rotations

- Connection between SU(2) and R_3: generators fulfill same 'commutation' relations
  - $[J_1, J_2] = J_1 J_2 - J_2 J_1 = i J_3$ $[J_2, J_3] = i J_1$ $[J_3, J_1] = i J_2$
  - same with σ_i, $i=1,2,3$
  - with every unitary matrix A of SU(2) there is an element of R_3 associated

$$X = \vec{x}\,\vec{\sigma} = \begin{pmatrix} x_3 & x_1 - i x_2 \\ x_1 + i x_2 & -x_3 \end{pmatrix}$$

  - transformation $X' = A X A^+$ is pure rotation!
  - rotation of any 3 dim vector $\vec{x}$ can be expressed via unitary 2 x 2 matrices A
- Representations of the rotation group
  - for j = spin ½ : $J_i = \frac{1}{2} \sigma_i$ Pauli- matrices = 'spin' matrices

Lorentz group

- Quantities characterizing an experiment, when referred to two frames S and S', are related by the Lorentz transformation, i.e. *'invariance under a change in description, not under a change in the experimental setup'.*

- Homogeneous Lorentz group: $x'_\mu = \Lambda_{\mu\nu} x_\nu$
  - with under Lorentz transformations conserved quantity $x^2 = g_{\mu\nu} x_\mu x_\nu$ and
  - metric

$$g = (g_{\mu\nu}) = \begin{matrix} +1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{matrix}$$

  - $x^2 = x_0^2 - \vec{x}^2$

Examples of the Lorentz group

- Lorentz transformation in x-direction with velocity v:
  - $$\Lambda_1(v) = \begin{pmatrix} \gamma & \gamma v & 0 & 0 \\ \gamma v & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$,with $\gamma = (1 - v^2)^{-1/2}$ (c=1 .... as usual ;-))
- Rotation around the z-direction with angle θ :
  - $$\Lambda_3(\theta) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta & 0 \\ 0 & \sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$
- Clearly that's a group, all 4 x 4 matrices conserve x^2 with $\Lambda^T g \Lambda = g$
  - 16 real elements - 10 conditions (symmetric matrix) = 6 parameters
  - 6 infinitesimal generators: $i K_1$, $i K_2$, $i K_3$ $\rightarrow$ $\Lambda_1(v)$, $\Lambda_2(v)$, $\Lambda_3(v)$ and
    $i J_1$, $i J_2$, $i J_3$ $\rightarrow$ $\Lambda_1(\theta)$, $\Lambda_2(\theta)$, $\Lambda_3(\theta)$

Lorentz group, cont.

- Done: generators for the homogeneous Lorentz group, have to fulfill the following commutation relations:

  - $[J_1, J_2] = i J_3$ → 'rotation group'
  - $[J_1, K_1] = 0$ → boost unchanged by rotation
  - $[J_1, K_2] = i K_3$ → K transform like 'vector'
  - $[K_1, K_2] = -i J_3$ → 'Thomas precession'

- What's still needed?
  - mass can only enter if energy and momentum are involved ....
  - energy $\hat{E} = i\,\partial / \partial t$ and momentum $\hat{p}_x = -i\,\partial / \partial x$ (p_y, p_z analoguous)
  - $\exp(i a \hat{p})\, f(x) = \exp(i a\, \partial / \partial x)\, f(x) = f(x) + a\, \partial / \partial x\, f(x) = f(x+a)$ 'Taylor series'
  - momentum = 'generator' of translations
  - inclusion of translations leads to the 'inhomogeneous Lorentz group'

Inhomogeneous Lorentz and Poincare group

- Inhomogeneous Lorentz group: $x'_\mu = \Lambda_{\mu\nu} x_\nu + a_\mu$ denoted by $\{a, \Lambda\}$
  - four more generators needed iP_0, iP_1, iP_2, iP_3
- Additional commutation relations:
  - $[P_i, P_{0,j}] = 0$ → 'Abelian'
  - $[J_1, P_2] = i P_3$ → 'P transforms like vector '
  - $[J_1, P_{0,1}] = 0$ → 'Rotation unchanged by translations'
  - $[K_1, P_0] = i P_1$ → ' energy changed by boost '
  - $[K_1, P_1] = i P_0$ → ' momentum changed under boost '
- Inhomogeneous Lorentz group with $\Lambda_{00} \geq 1$ is 'Poincare group'
  - Relativistic invariance := invariance under Poincare group
  - generators: P_0, P_i, J_i, K_i with i = 1, 2, 3
  - $P^2 = P_0^2 - P_i^2$ commutes with all generators
  - $P^2 = M^2$ conserved quantum number → that's the squared 'mass'

Casimir operators of Poincare group

- Casimir operators commute with all generators
  - 'conserved quantities'
- One Casimir operator of the Poincare group already known:
  - $P^2 = P_\mu P^\mu = M^2$
- The other one is the 'spin-operator' or 'Bargmann-Wigner-operator':
  - $W^2 = W_\mu W^\mu$ with $W_\mu = 1/2\ \varepsilon_{\mu\nu\rho\sigma}\ M^{\nu\rho}\ P^\sigma$
  - Matrix M = $\begin{pmatrix} 0 & K_1 & K_2 & K_3 \\ -K_1 & 0 & J_3 & -J_2 \\ -K_2 & -J_3 & 0 & J_1 \\ -K_3 & J_2 & -J_1 & 0 \end{pmatrix}$
  - $W_0 = \vec{P}\,\vec{J}$ and $\vec{W} = P_0\,\vec{J} - \vec{P} \times \vec{K}$

Status

- **What has been managed so far?**
  - describe boosts, rotations and translation within one group: Poincare group
- **What is still missing?**
  - relativistic description of spin
  - we know: spin ½ described via Pauli matrices (SU(2) group)
- **What's was relation between R_3 and SU(2)?**
  - any 3 dim vector can be expanded via hermitian 2 x 2 SU(2) matrices: $X = \vec{x}\,\vec{\sigma}$
  - now do a 3 dim rotation: $\vec{x}' = R\,\vec{x}$
  - 'rotated' 3 dim vector again expanded via Pauli matrices: $X' = \vec{x}'\,\vec{\sigma}$
  - transformation matrix X' obtained via unitary transformation: $X' = A\,X\,A^{+}$
  - 3 dim metric is conserved under this unitary transformation: $\vec{x}'^{\,2} = \vec{x}^{\,2}$

Relation: Lorentz Gruppe vs. SL(2C)

- Goal: describe Lorentz transformations via 2 x 2 matrices possible?
  - would enable to embed spin within Lorentz transformations
- First step, take 4 dim Minkowski space, i.e. describe any 4 dim vectors:
  - any 4-vector can be expanded: $X = x_0 1 - \vec{x}\,\vec{\sigma} = x_\mu \sigma_\mu$ with $\sigma_\mu = (\sigma_0, \sigma_i)$
  - apply Lorentz transformation: $x'_\mu = \Lambda_{\mu\nu} x_\nu$
  - still valid: $X' = A X A^+$, with A = 2 x 2, but A no longer unitary! *reasonable?*
  - *yes,* since relativistic metric conserved: $x'^2 = x^2$
- The group corresponding to SU(2) (R_3 vs SU(2)), but applied for Lorentz transformations in Minkowski space: SL(2C)
  - 2 x 2 matrices A with det A =1 (keeps metric invariant), but not unitary!
  - $A^+ \neq A^{-1}$ (not unitary) which indicates that $X \neq X^+$ (not hermitian)

Last steps to the Dirac equation

- Lorentz group has 6 generators: $J_1, J_2, J_3, K_1, K_2, K_3$
- Generators of SL(2C), have to be 2 x 2 matrices: $J_i = \sigma_i / 2$ and $K_i = i\, \sigma_i / 2$
  - → fulfills all commutation relations we had for the Lorentz transformations
- Keypoint: same commutation relations also fulfilled by 2nd choice of generators, namely $J_i = \sigma_i / 2$ and $K_i = -i\, \sigma_i / 2$!
  - → two different 2 x 2 representations of the Lorentz group !
- Two different 'spin states' : $|\sigma, p, 1\rangle$ and $|\sigma, p, 2\rangle$
  - → these two spin states are not independent, but related !

And now we are there ...

- **Lorentz transformation on the two spin states:**
  - $|\sigma, p, 1\rangle = D(L(p))\ |\sigma', p'\rangle$
    $|\sigma, p, 2\rangle = \overline{D}(L(p))\ |\sigma', p'\rangle$
  - i.e. $|\sigma, p, 1\rangle = D(L(p))\ \overline{D}(L(p))\ |\sigma, p, 2\rangle$
- **With explicit Lorentz transformation:**
  - $|\sigma, p, 1\rangle = (p_0 - \vec{p}\,\vec{\sigma})/m\ \ |\sigma, p, 2\rangle$
    $|\sigma, p, 2\rangle = (p_0 + \vec{p}\,\vec{\sigma})/m\ \ |\sigma, p, 1\rangle$

    Dirac equation
- **Last step: take 4-dim denotation (with 4 x 4 γ-matrices)**
  - $(\gamma_\mu p_\mu - m)_{\alpha'\alpha}\ u_\alpha(p) = 0$ with the wave functions $u_1 = |1/2\ p\ 1\rangle$, $u_2 = |-1/2\ p\ 1\rangle$
    $u_3 = |1/2\ p\ 2\rangle$, $u_4 = |-1/2\ p\ 2\rangle$
- **Lorentz invariance indicates two spin states (of particle and antiparticle):**
  - **4-spinor wave function with mass and spin naturally from relativistic invariance!**

Little group for $m \neq 0$ and $m=0$

- **Definition: transformations that do not change p is called 'little group'**
  - transformation within Poincare group that do not change p
- **Case 1: particles with $m \neq 0$**
  - $P = (m, 0, 0, 0)$ and $P^2 = m^2$
  - Spin-operator $W = m\,(0, J_1, J_2, J3)$
  - $W_3 \mid m, p, s_3 > \; = \; m \; s_3 \mid m, p, s_3 >$
  - $W^2 \mid m, p, s_3 > \; = - m^2 \; s\,(s+1) \mid m, p, s_3>$
  - W_3 and W^2 do not change p, form elements of ' little group'
- **Case 2: particles with $m=0$**
  - Problem: $m=0 \longrightarrow P^2 = 0$ and $W^2 = 0$ and $W_\mu P^\mu = 0$
  - But all conditions are fulfilled if $W_\mu = \lambda \; P_\mu$ with λ = helicity = $\vec{s}\,\vec{p} \,/\, |\vec{p}|$

Conclusion, part 1

- Stern-Gerlach and Zeeman effect led to introduction of 'spin property'
- Kinematic properties of particles via mass and spin quantum numbers
- Total angular momentum J = L +S
  - angular momenta are generators of rotations
  - eigenvalues of $J^2 = j(j+1)$, $J_z = m \in [-j, \ldots, +j]$
- Fundamental principle: relativistic invariance
  - rotation, boosts and translations contained within Poincare group
  - relativistic description of spin leads to two different representations
  - Dirac equation expresses relativistic spin via 4-dim spinor wave function
- Group theory and commutation relations: SU(3) ⟶ R3 ⟶ SU(2) ⟶ SL(2C)
- 'Spin' defined for particles with m≠ 0 in their rest frame
  - if m=0 'no' boost into rest frame possible, but defined via 'helicity'

Outline, part 2

- **Part two: Spin in experiments ..... 'phenomenology'**
  - Some applications (addition of angular momenta, spin description, SUSY transformations , spread over the following physics topics)
  - Physics at RHIC: spin crisis of the proton
  - Physics at HERA: proton spin crisis and new physics searches
  - Physics at the ILC: new physics searches, polarized sources and possible depolarization effects in beam interactions

Addition of angular momenta

- **Short summary of known facts:**
  - angular momenta a axial-vectors
  - orbital angular momenta L are integers: l = 0,1,2,...
  - spin S is half-integer for fermions, integer for bosons
  - total angular momentum: $\vec{J} = \vec{L} + \vec{S}$
  - eigenvalues: $\vec{J}^2, \vec{L}^2, \vec{S}^2 = j\,(j+1)$, etc. and of $J_z, L_z, S_z = m$ ($\hbar = 1$, as usual ;-))
- **How to add angular momenta? Quantum numbers and eigenstates?**

1. take two states: $| j_1, m_1 >$, $| j_2, m_2 >$

  - $J_{1,z} | j_1, m_1> = m_1 | j_1, m_1>$ and $J_1^2 | j_1, m_1 > = j_1 (j_1+1) | j_1, m_1>$
  - $J_{2,z} | j_2, m_2> = m_2 | j_2, m_2>$ and $J_2^2 | j_2, m_2 > = j_2 (j_2+1) | j_2, m_2>$

Angular momenta, cont.

2. Algebra for angular momenta now applied on the sum: $\vec{J} = \vec{J}_1 + \vec{J}_2$

- $J^2 \mid j\, m > = j\,(j+1) \mid j\, m >$ and $J_z \mid j\, m > = m \mid j\, m >$
- with $-m \leq j \leq +m$ 'magnetic quantum number'

3. For a given j_1 and j_2 the total sum j can have the following values:

- $j = j_1 + j_2, (j_1 + j_2 - 1), \ldots, \mid j_1 - j_2 \mid$:= \` $j_1 \times j_2$ '

4. How to construct the corresponding eigenstates?

- $\mid j\, m > = \sum_{m2} c(m, m_2) \mid j_1\, m_1 > \mid j_2\, m_2 >$ with $m_1 + m_2 = m$
- $c(m, m_2)$ = 'Clebsch-Gordon-coefficient'
- denotation: $\mid j\, m> = \mid j_1\, j_2\, j\, m>$ and $\mid j_1\, m_1 > \mid j_2\, m_2 > = \mid j_1\, j_2\, m_1\, m_2>$
- obviously: $\mid j\, m>$ also eigenstate of J_1^2 and J_2^2, but not of $J_{1,z}$ and $J_{2,z}$!

Examples

- Let's sum orbital angular momentum with spin 1/2: $J = L + S$
  - $j = `l \times \frac{1}{2}' = l + \frac{1}{2}, l - \frac{1}{2}$ for $l \neq 0$
  - $j = \frac{1}{2}$ for $l = 0$
  - Number of states:
  - $J_z | j\, m > = \sum_{ms} c(m, m_s) (L_z + S_z) | l\, m_l > | \frac{1}{2}\, m_s >$

    $= \sum_{ms} c(m, m_s) (m_l + m_s) | l\, m_l > | \frac{1}{2}\, m_s >$

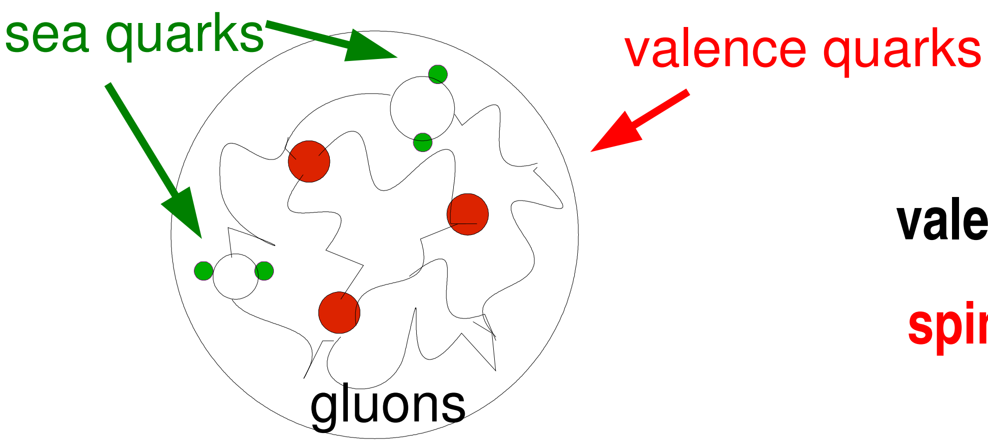
    $= (m_l + m_s) | j\, m > = m | j\, m >$
  - number of states conserved ✓ *sounds promising*
- Spin of the pions: (1) $\pi^+ \rightarrow \mu^+ + \nu_\mu$, (2) $\pi^0 \rightarrow 2\gamma$ in rest frame
  - (1) could have $s = ` \frac{1}{2} \times \frac{1}{2} ' = 1, 0$ and (2) could have $s = `1 \times 1' = 2, 0$
  - since angular distributions of μ 's and γ 's isotropic: only $s=0$ possible !

Spin physics at RHIC

- **RHIC polarizes protons (see Desmonds lecture) :**
  - polarization = fix orientation of spin, i.e. fix magnetic spin quantum number
  - $$P = \frac{\# N_{up} - \# N_{down}}{\# N_{up} + \# N_{down}}$$ At RHIC at about P ~ 60%
- **Why polarized protons wanted?**
  - Still one serious problem: proton spin ½ not explainable
  - 'naive': proton = 3 quarks, \` ½ x ½ x ½ ' = \`1x ½ ' and \`0 x ½' = 3/2 and ½
  - *Spin = ½ should work, if two quarks 'up' and the third has 'down' orientation !*
  - In 1980' : 'naïve' picture is wrong! .... spin crises ....
  - experiments showed that 'quarks' provide only minor part of the spin !
- **Goal at RHIC: unravel the structure of the nucleons and explain the origin of the proton spin**

Polarized protons at RHIC

- **What's the proton picture today?**



valence and sea quarks and gluons have

spin and orbital momentum !

complicated case to sum up all spin and orbital momenta

➾ **parton distribution function of the quarks and gluons describe the structure**

- **Idea: with polarized protons one gets information about the spin dependent distribution functions**

➾ **gives mainly information about the gluon distribution and polarization since the polarized protons directly interact with the gluons**

➾ **from W production also information about quark polarization**

Spin Physics at HERA

- HERA = e p collider: use polarized electrons and positrons
  - reached 1995 (HERMES) at about 70% e^+ polarization (see Desmonds lecture)
  - polarization degree depends on number of spin rotators
  - in 2003: 6 rotators= 2 at each experiment (HERMES, ZEUS, H1) worked → 50% !
- Why are the rotators needed?
  - for physics the electrons beams have to be longitudinally polarized, since the electron mass is very small, remember spin definition for m=0
  - definition of polarization via 'helicity':

$$P = \frac{\# N_{left} - \# N_{right}}{\# N_{left} + \# N_{right}}$$

  - 'automatic' polarization in a storage ring gives only transverse polarization
- What are the physics goals?
  - again proton spin crisis but also searches for new physics

Polarized e^+/e^- at Hermes experiment

- **HERMES: target experiment**
  - target also polarized with about 95% (polarized H atoms)
  - electron can only interact with quarks that have opposite polarization:

    interaction occurs via photon (Spin ± 1) and conserves helicity: if incoming e^- right-handed, q has to be left-handed ...... $\hbar\,(m_e + m_q) = \hbar\, m \;\; = +1$

  - incoming (polarized) electrons are deflected / scattered at target
  - change target polarization, change in scattering leads to asymmetry
  - such an asymmetry gives information about the spin structure of the quarks
- **One HERMES result as example:**
  - valence quarks contribution to the proton spin only about 28% !
  - sea quarks probably only minor important
  - largest contribution comes from the gluons !

Polarized e^+/e^- at ZEUS and H1

- What is the physics at the other HERA experiments?

  a) nucleon structure: measure the (quark) parton distribution functions

  - longitudinal lepton polarization affects the neutral (Z-boson exchange) and charged (W-boson exchange) currents
  - with different choices for the lepton charge as well as the polarization, one gets complementary information about the parton distribution function

  b) searches for physics beyond the Standard Model

  - test/ check whether right-handed charged currents exist (' W_R ')
  - search for scalar leptoquarks, R-parity violating supersmmetry
  - search for excited leptons, in particular neutrinos

- Polarized leptons, e^+ and e^- would enlarge the physics potential and improve the results

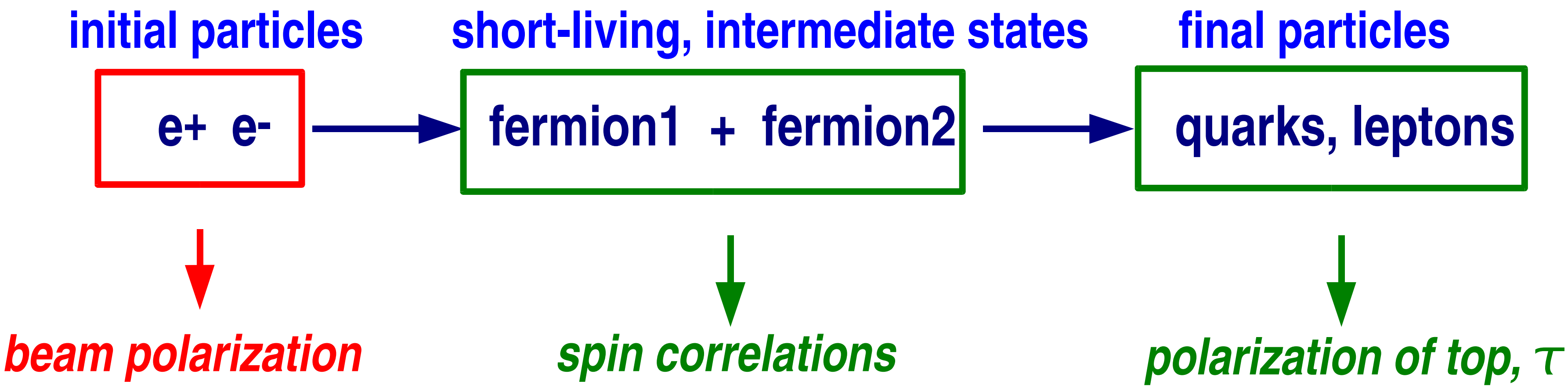
Spin physics at the ILC

What are the goals of the ILC?

- Discovery of New Physics (NP)
  - ➾ complementary to the LHC
  - ➾ large potential for direct searches and indirect searches
- Unraveling the structure of NP
  - ➾ precise determination of underlying dynamics and the parameters
  - ➾ model distinction through model-independent searches
- High precision measurements
  - ➾ test of the Standard Model (SM) with unprecedented precision
  - ➾ even smallest hints of NP could be observed

→ **Beam polarization = decisive tool for direct and indirect searches!**

Where are spin effects at the ILC?

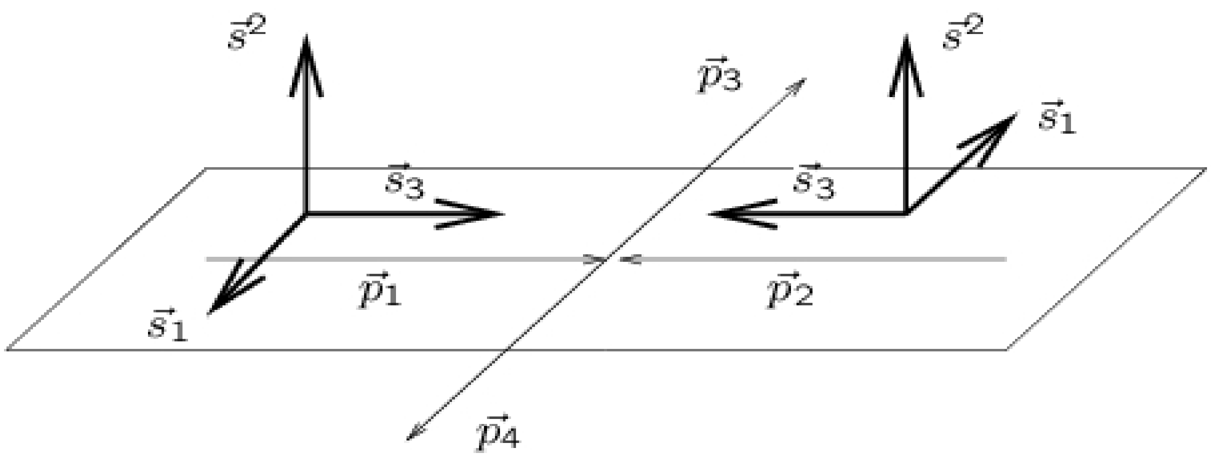
- 

- **Spin in physics processes:**
  - spin formalism to include all spin and polarization effects
  - effects of beam polarization for the physics analysis
- **How to polarize the beams at the ILC?**
  - spin effects at the source
- **Theoretical aspects to match the required precision**
  - depolarization effects at the beams

Spin formalism

- **1. Polarized beams: introduce 'dreibein' (better 'vierbein' (p, s^a, a=1,2,3))**

  → **longitudinal $s^3 = 1/m\,(|\vec{p}_1|, E_1\,\hat{p}_1)$, transverse $s^2 = (0, \vec{p}_1 \times \vec{p}_3)$, $s^1 = (0, \vec{p}_1 \times \vec{s}_2)$**



- **2. To include all spin and polarization effects also from the intermediate states: calculate amplitude squared $|T|^2$ with complete spin correlations**

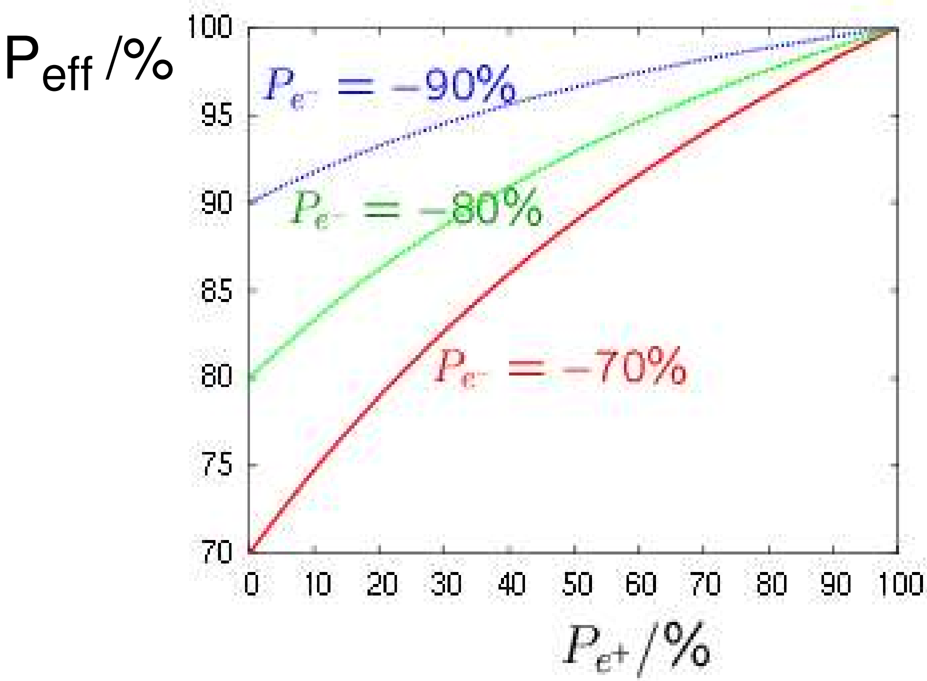
$$|T|^2 = |\Delta_{f_3}|^2 |\Delta_{f_4}|^2 \sum_{spins} \overbrace{(P^{\lambda_{f_3}\lambda_{f_4}} P^{*\lambda'_{f_3}\lambda'_{f_4}})}^{\text{spin-density matrix}} \times \overbrace{(Z_{\lambda_{f_3}} Z^*_{\lambda'_{f_3}})}^{\text{decay matrix}} \times \overbrace{(Z_{\lambda_{f_4}} Z^*_{\lambda'_{f_4}})}^{\text{decay matrix}}$$

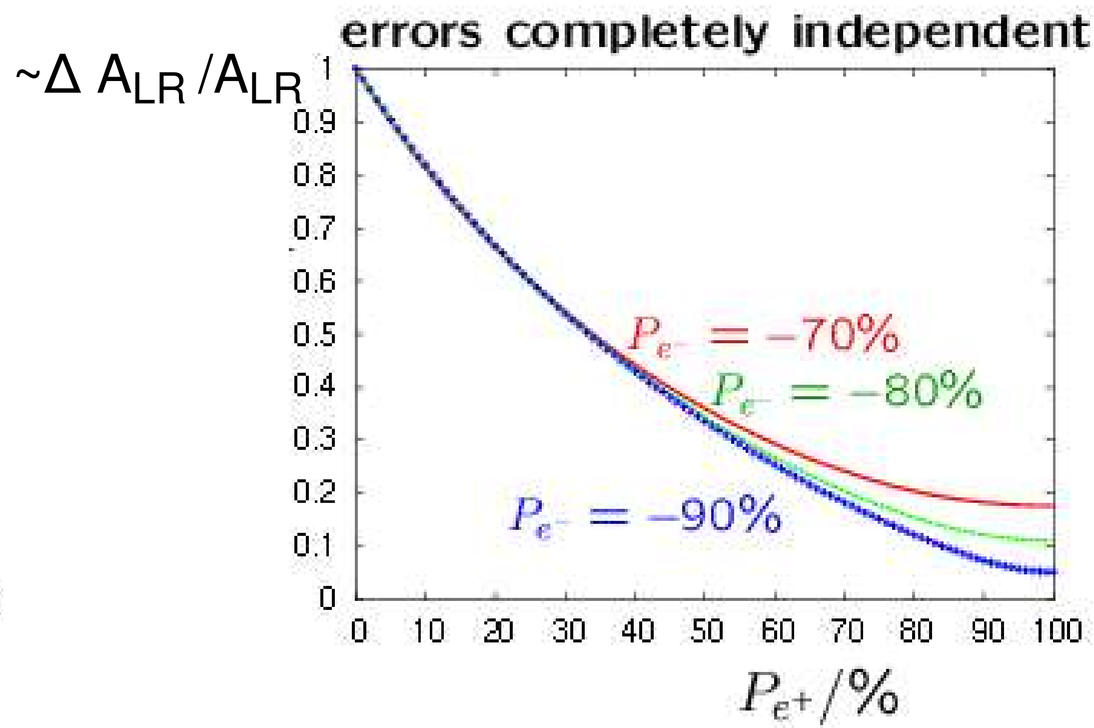
→ **production and decay processes are coupled by interference terms between various polarization states of the decaying fermions**

Well-known statistical examples

- **A warm-up: gain effective polarization P_{eff} and A_{LR}**
  - For many processes (V, A interactions) the cross section is given by:

$$\sigma(P_{e-}\, P_{e+}) = (1 - P_{e-}\, P_{e+})\, \sigma_0\, [1 - P_{eff}\, A_{LR}] \quad \text{with} \quad P_{eff} = (P_{e-} - P_{e+}) / (1 - P_{e-}\, P_{e+})$$





- **P(e+) = strong 'lucrative' factor ⟶ both beams should be polarized!**
  - P(e+) of about 60% sufficient with $\Delta P / P = 0.5\%$ for physics studies

High precision measurements of the SM

- Process: $e^+e^- \longrightarrow Z \longrightarrow f\bar{f}$ at the Z-pole

  measurement of the mixing angle $\sin\theta_{eff}$ via left-right asymmetry A_{LR}

$$A_{LR} = \frac{2(1-4\sin^2\Theta^{\ell}_{eff})}{1+(1-4\sin^2\Theta^{\ell}_{eff})^2}$$

- requested by physics: $\Delta\sin\theta_{eff} < 1.3\mathrm{x}10^{-5}$

  sensitive to tiny traces of new physics !

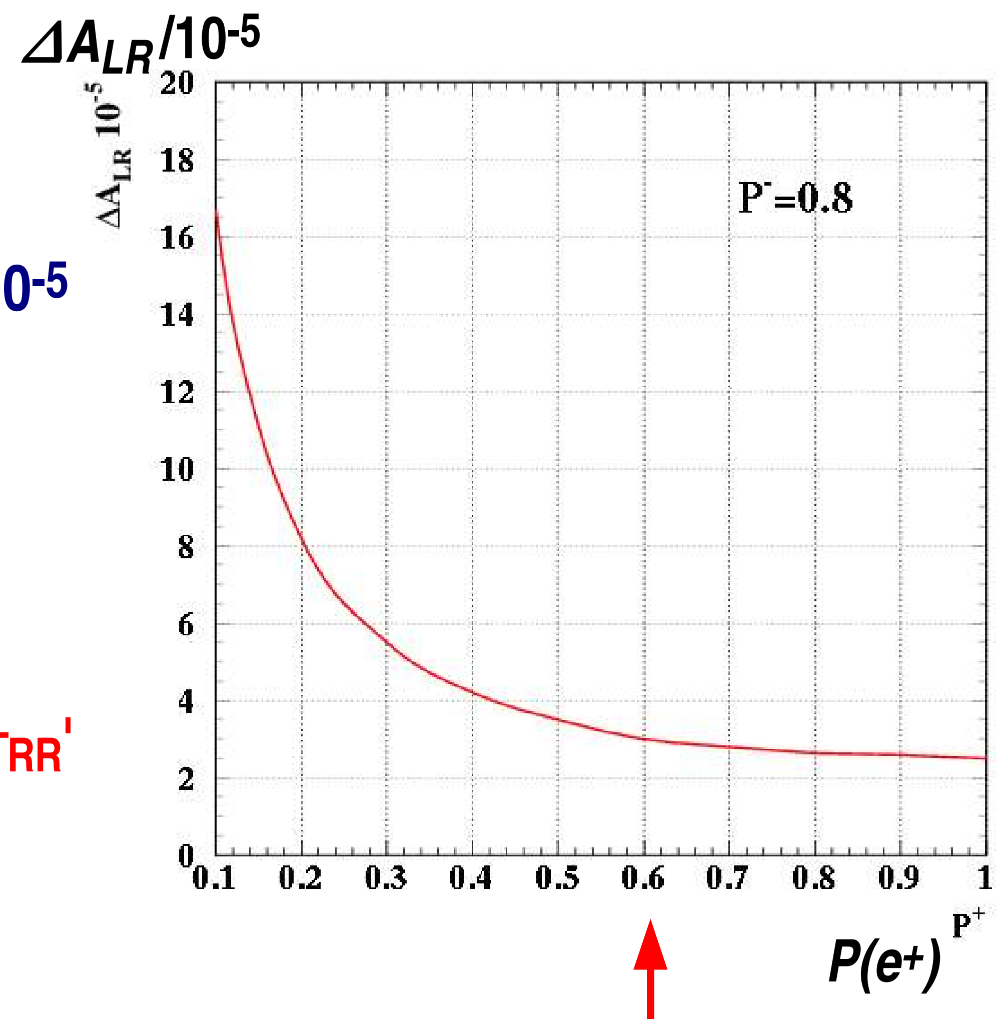
→ $\Delta A_{LR} < 10^{-4}$ needed

- only via Blondel scheme possible:

  \`express A_{LR} via polarized $\sigma_{LR}, \sigma_{LL}, \sigma_{RL}, \sigma_{RR}$'

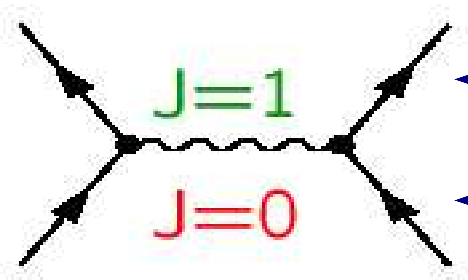
→ polarized e^- and e^+ required

→ $P(e^+)$ of 40% up to 60% sufficient !



General remarks about the coupling structure

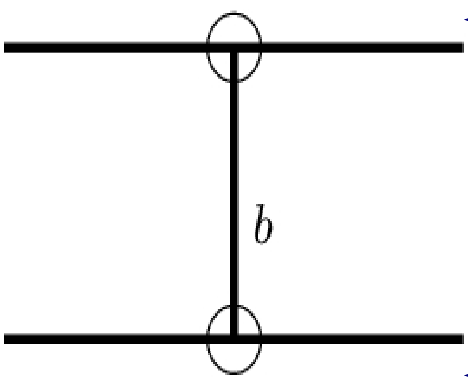
- Def.: left-handed = P(e±)<0 'L' right-handed= P(e±)>0 'R'
- Which configurations are possible in annihilation channels?



← contributions from LR, RL: SM and(?) NP (γ, Z)

← contributions only from LL, RR: NP !

- Which configurations are possible in scattering channels?



← depends on P(e+)!

→ helicity of e- not coupled with helicity of e+ !

← depends on P(e-) !

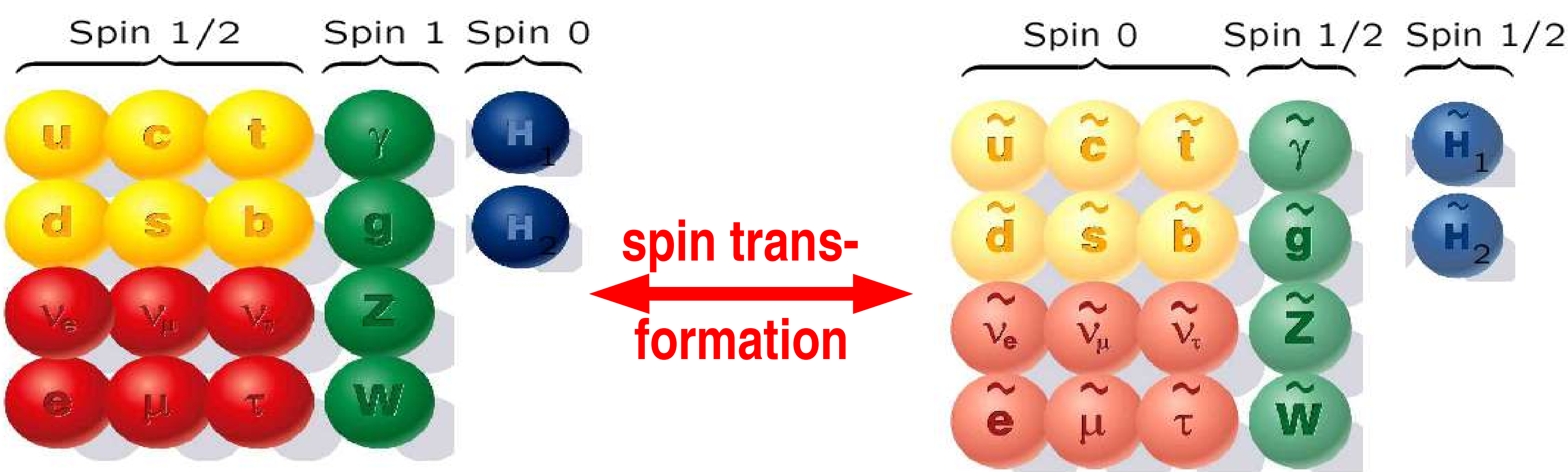
Supersymmetry (SUSY)

- **Why SUSY today and here ? → unique extension of Poincare group !**
  - Q | boson > = | fermion > and Q | fermion > = | boson >
  - SUSY operator Q changes spin (behaviour under spatial rotations) by ½
  - invariance under local coordinate change: general relativity can be included!
  - SUSY transformation influences in general both space-time and internal quantum numbers!

$$|\underbrace{m, s; \vec{p}, s_3;}_{\text{space-time}} \underbrace{Q, I, I_3, Y, \ldots}_{\text{internal}}\rangle$$

space–time internal quantum numbers

- **Particle spectrum -- compared to the Standard Model:**



Polarized e^+ for SUSY

- **Supersymmetry (SUSY): gives fermions a bosonic partner and vice versa**
  - all quantum numbers except the spin identical for SM ⟷ SUSY partner !
  - i.e. left-, right-handed electrons need scalar SUSY partners, the left and right selectrons:

$$e^-_{L,R} \longrightarrow \tilde{e}^-_{L,R} \quad \text{and} \quad e^+_{L,R} \longrightarrow \tilde{e}^+_{R,L}$$

  - Problem: scalar particles have spin zero and have no helicity .....

    but they should carry the L,R quantum number in their couplings

- **How to prove experimentally that such SUSY partners exist?**

  Strategy: a) study pair production with polarized beams

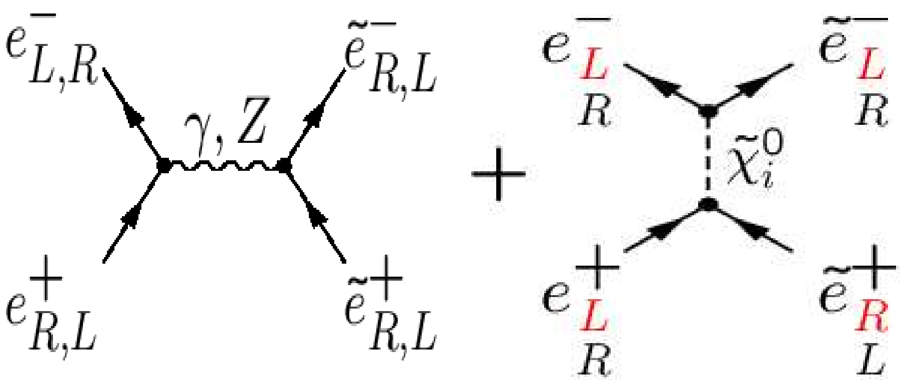
  b) extract the pair $\tilde{e}^-_R \tilde{e}^+_L$ from other produced particles

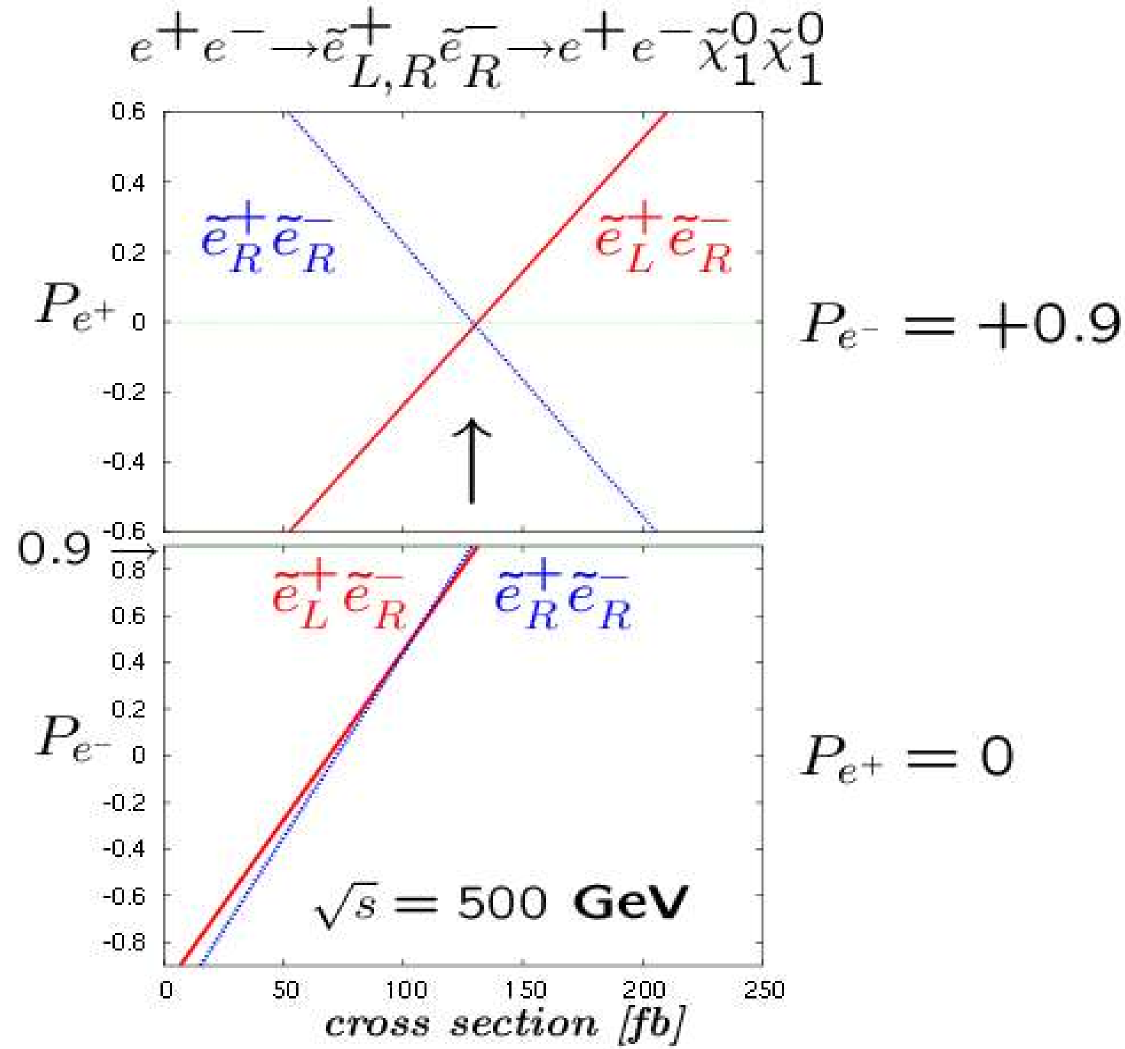
  c) this pair reflects the unique relation between SM and SUSY partner !

Test of SUSY quantum numbers

- **Association of chiral electrons to scalar partners $e^-_{L,R} \leftrightarrow \tilde{e}^-_{L,R}$ and $e^+_{L,R} \leftrightarrow \tilde{e}^+_{R,L}$:**

s-channel **t-channel**



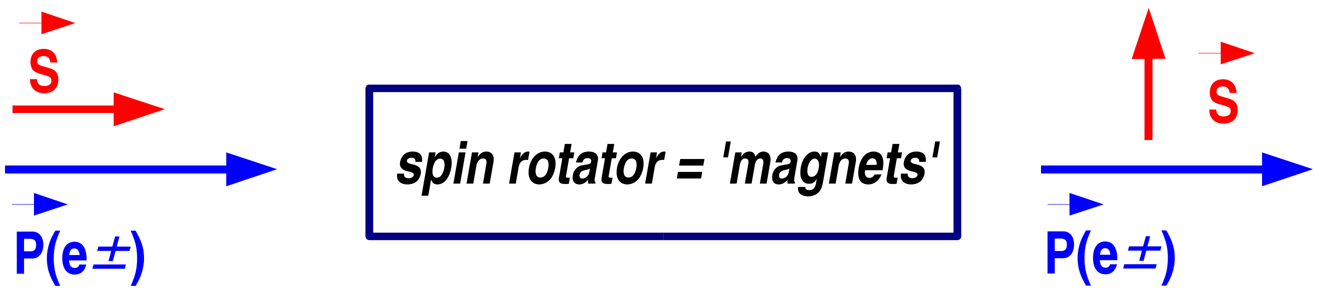


1. **separation of scattering versus annihilation channnel**
2. **test of 'chirality': only $\tilde{e}^+_L \tilde{e}^-_R$ may survive at P(e-) > 0 and P(e+) > 0 !**

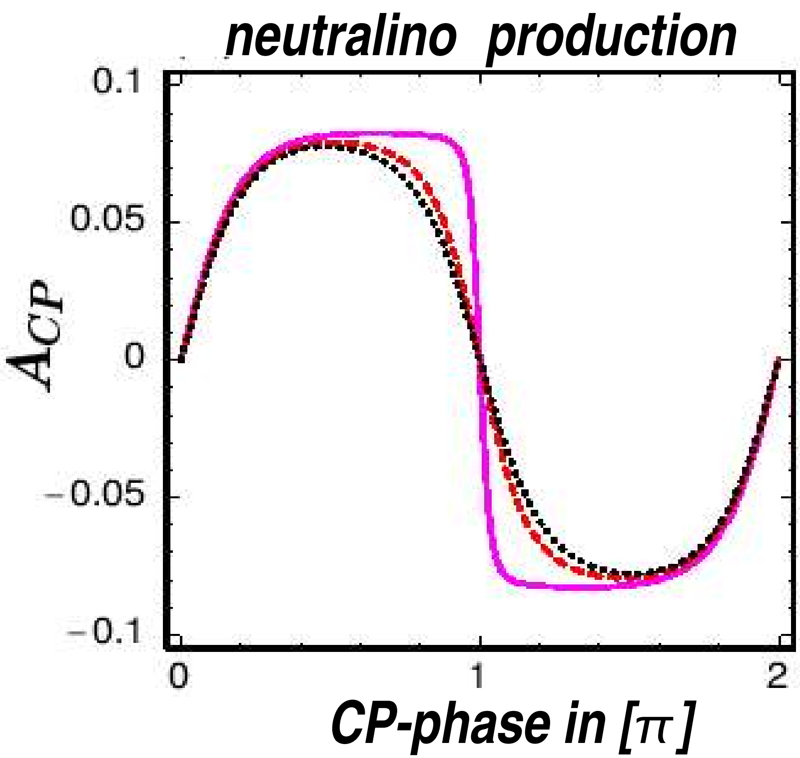
- ***Even high P(e-) not sufficient, P(e+) is substantial!***

Transversely polarized beams

- **Use spin rotators to rotate longitudinally into transversely polarized beams**



- **Only effects if polarized e- and e+ !**
  (effect proportional to [P(e-) P(e+)])
  - **sensitive to CP-violating effects**
  - **CP-asymmetries w.r.t. azimuthal angle**
  - **rather large values even for small CP-phases !**



- **Further examples in the polarization POWER report (hep-ph/0507011) !**

Beam polarization at linear colliders

Polarized beams at linear e^-e^+ colliders:

- synchrotron radiation due to longitudinal acceleration negligible
- beams have to be polarized at the source !

Polarized e^- source:

- at the SLAC Linear Collider (SLC): excellent e^- polarization of about 78%
- led to precision measurement of the weak mixing angle:

  $\sin\theta_{eff}$=0.23098±0.00026 (SLD) (LEP: 0.23221 ±0.00029)

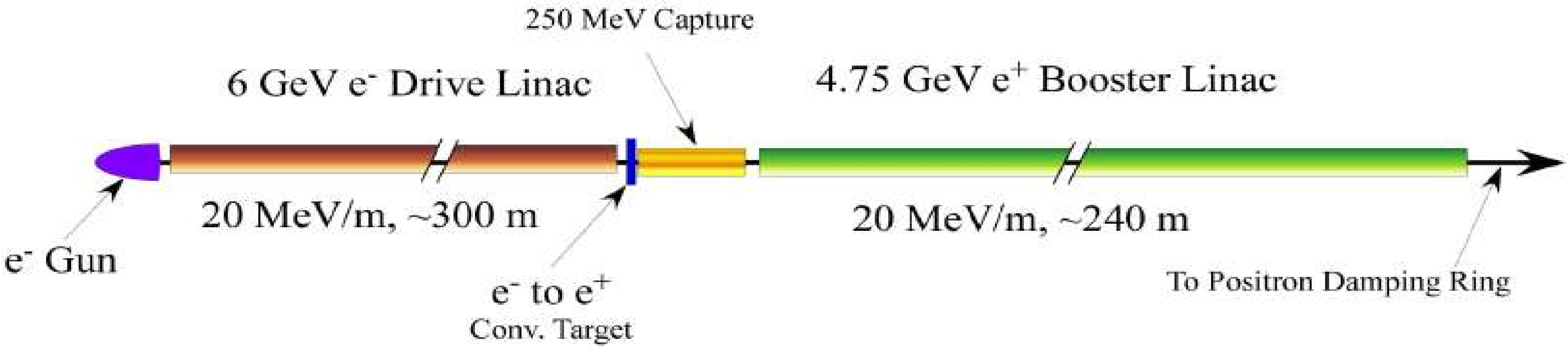
Polarized sources at the ILC:

- expected e^- polarization between 80% and 90%
- e^+ polarization as an absolute technical innovation:

  expected polarization about 60% with full intensity

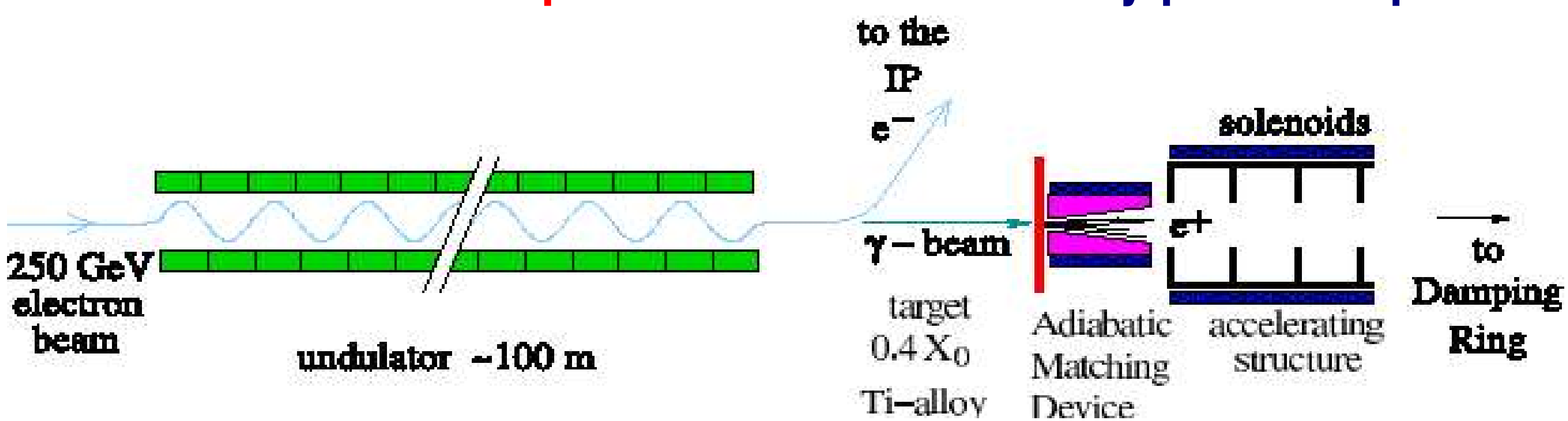
How to produce (polarized) e+ at the ILC?

- **Conventional scheme → only unpolarized e+:**



  - **Challenges: thermal stress in target → thick rotating target needed (360m/s)**

- **Undulator-based scheme: polarized e+ via circularly polarized photons**



  - **thin target sufficient, e+ polarization depends on undulator length**
    (alternative scheme: instead of undulator use multiple laser scattering)

Prototypes of e+ sources for the ILC

- **Polarized schemes are absolute innovations, do prototypes exist?**

  - proof of principle: experiment at *SLAC 'E166'*

  use 50 GeV e- beam at SLAC in conjunction with 1m long helical undulator
  → photons on target → analyze polarization of γ's and e+

  - *Institutes: SLAC, DESY, Daresbury, etc.*

  - physics runs in 2005: polarized e+ successfully verified !

- **Prototype of a helical undulator for ILC beam parameters**

  - currently under construction at RAL and Daresbury

  - collaboration within heLiCal group (*working group of Cockcroft Institute: CCLRC, DESY, Durham, Liverpool -- 'All aspects of the e+ production for the ILC'*)

- **ILC baseline design for the e+ source**

  undulator-based source!
  'the polarized source'

What is the required precision?

- Precision for polarization required by physics: $\Delta P/P = 0.5\%$ ➔ 0.25%

- Possible sources of depolarization at the ILC:

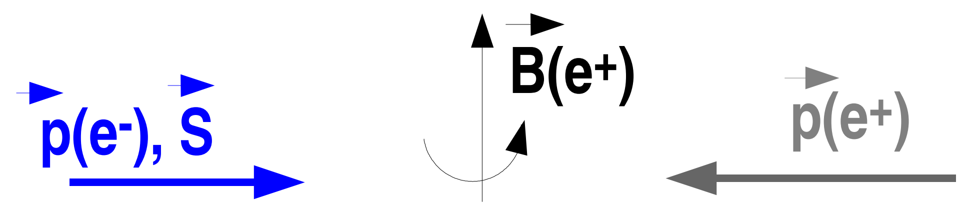
source ➔ damping ring ➔ bunch compresser ➔ linac ➔ final focus ➔ IP

➔ cradle-to-grave analysis of depolarization effects: task of the heLiCal group !

➔ probably largest depolarization during beam-beam interaction at the IP !

- Depolarization effects during beam-beam interaction:

a) spin precession:



Thomas -- Bargman-Michel-Telegdi (T-BMT) equation:

$$\dot{\vec{S}} = \vec{\Omega} \times \vec{S}$$

b) spin-flip processes due to synchrotron radiation (Sokolov-Ternov effect):
in case of colliding beams known as: beamstrahlung

Beam-beam interactions: higher-order processes

- **Absolutely needed for precision measurements at linear colliders:**
  - precise knowledge about all possible *depolarization effects*
  - only one analytically-based simulation code exists: CAIN
  - processes included so far in approximations and not with complete spin effects

- **Higher-order QED effects exactly with full spin correlations needed**
  - apply the spin density formalism, calculate these higher-order processes and provide precise theoretical predictions for the depolarization effects
  - relevant for both spin precession process (*T-BMT in strong fields*) and synchrotron radiation processes *(so far only in virtual γ approximation)*

within the UK heLiCal collaboration

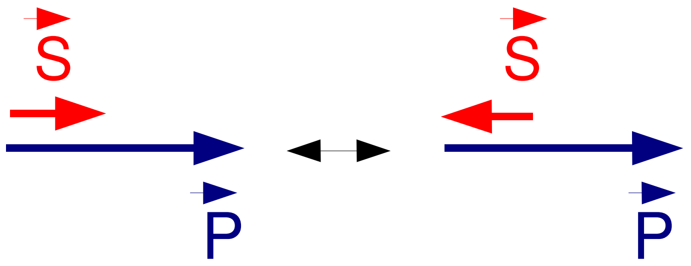
→ **Important for all linear collider designs (ILC, CLIC)**

Expected size of depolarization effects

- Polarimeters measure polarization before or after the IP
  - different from the actual polarization in the physics process
- Expected size of depolarization at the ILC with 500 GeV:
  - measurement before IP: $\Delta P \sim +0.3\%$ and measurement after IP: $\Delta P \sim -0.4\%$
- Estimates for depolarization at CLIC with 3 TeV:
  - CLIC (2001): total depolarization about 25%, effective lumi-weighted about 7%
- To match the required precision: update of CAIN program absoutely needed!
  - comparison with existing Monte-Carlo-based codes Guinea-Pig / Merlin

How to flip the helicity of the e+?

- Precision requirements e.g. at the Z-pole ('GigaZ')
  - Need of e- and e+ polarization with both helicities
  - to fulfill the high precision: fast flipping needed!
  - e- flipping no problem, only change laser polarity
    can be done bunch-by-bunch



- Helicity flip of e+ ?
  - current proposal for flipping $\lambda(e^+)$: two parallel spin rotators in damping rings,
    fast kickers change between them
    can be done pulse-by-pulse (sufficient for physics requirements)
  - depolarization due to spin rotators about 3%
- Alternative idea for helicity flipping of the e+
  - tricky combination of different undulator sectors..... more elegant, cheaper
  - *Under work*

Conclusions, part 2

- Sum of angular momenta: eigenvalues + states
- Spin physics at RHIC: polarized p to unravel the proton spin crisis
- Spin physics at HERA: polarized e^-/e^+ to unravel the proton spin crisis and new physics
- Spin effects at the ILC: polarized e^- and e^+
  - precise environment at the ILC allows to exploit spin effects of intermediate particles (spin correlations)
  - beam polarization at the ILC: crucial to determine the structure of new physics
- Undulator -based (polarized) positron source for the ILC baseline design !
- `Cradle-to-grave' spin tracking needed to get all possible 'depolarization' effects under control
  - needed to match the required precision for physics!

Some literature ...

- **Derivation of the Dirac equation with group theory:**
  - R. Omnes, Introduction to Particle Physics
  - Ryder, Relativistic Quantum Mechanics
- **'Historical' derivation of the Dirac equation:**
  - Bjorken/Drell, Relativistic Quantum Mechanics
  - Bjorken/Drell, Relativistic Quantum Field Theory
- **Group theory:**
  - Wu-Ki Tung, Group Theory in Physics
- **Phenomenology:**
  - H. Haber, `Spin Formalism', Proc. of 21th SLAC Summer Inst. , Stanford 1993
  - TESLA TDR, HERA TDR
  - Polarization at the ILC: POWER report, hep-ph/0507011