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Emergent Spacetime from Quantum Entanglement

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It from Qubit

Simons Collaboration

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1703.00456 + in preparation

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① Introduction

Why Quantum Entanglement ?

Quantum Entanglement = Measure of `Quantumness'

[`Quantumness'

⇒ We know the total system but not its part.]

= Structures of Quantum Matter

Holography (Gauge/Gravity duality, AdS/CFT)

= Structures of Spacetime

in Gravity (or String theory)

e.g. Surface Area = Entanglement Entropy (EE)

Perturbative Einstein eq. = First law of EE

What is the quantum entanglement ?

Consider the following states in **two spin systems**:

(i) A direct product state (unentangled state)

$$|\Psi\rangle = \frac{1}{2}\left[|\uparrow\rangle_A + |\downarrow\rangle_A\right] \otimes \left[|\uparrow\rangle_B + |\downarrow\rangle_B\right] = |\psi_1\rangle \otimes |\psi_2\rangle.$$

Independent

(ii) An entangled state (EPR pair)

$$|\Psi\rangle = \frac{1}{\sqrt{2}}\left[|\uparrow\rangle_A \otimes |\downarrow\rangle_B - |\downarrow\rangle_A \otimes |\uparrow\rangle_B\right] \neq |\psi_1\rangle \otimes |\psi_2\rangle.$$

One determines the other !

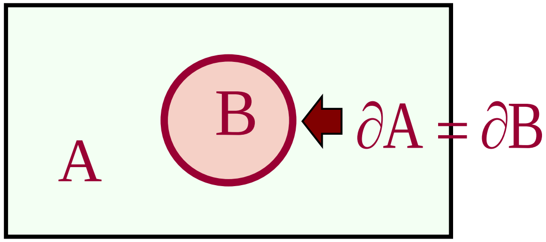
We know the state of A+B but not the state of A or B.

Entanglement entropy (EE)

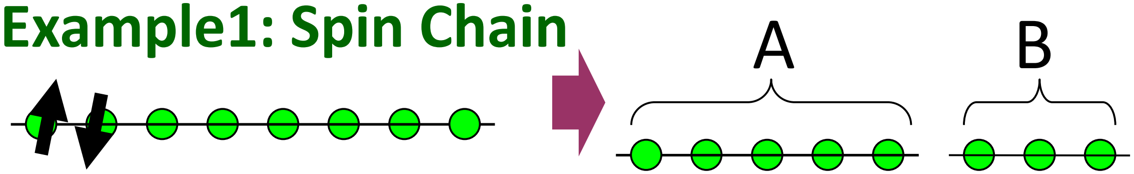
Divide a quantum system into two parts A and B.
The total Hilbert space becomes factorized:

$$H_{tot} = H_A \otimes H_B \quad .$$

Example2: QFT



Example1: Spin Chain



Define the reduced density matrix ρ_A for A by

$$\rho_A = \mathrm{Tr}_B \rho_{tot} \quad ,$$

Finally, the entanglement entropy (EE) S_A is defined by

$$S_A = -\mathrm{Tr}_A \, \rho_A \log \rho_A \quad .$$

(von-Neumann entropy)

Quantum Entanglement has recently been applied also to other topics in theoretical physics:

- **Condensed Matter Theory**

  Entanglement Entropy (EE), Entanglement Spectrum

  → Quantum Order Parameter

  ~ Required `Size' of numerical calculations

- **Quantum Field Theories (QFTs)**

  (Renyi) Entanglement Entropy (EE)

  → Universal quantities which characterize the degrees of freedom of QFTs

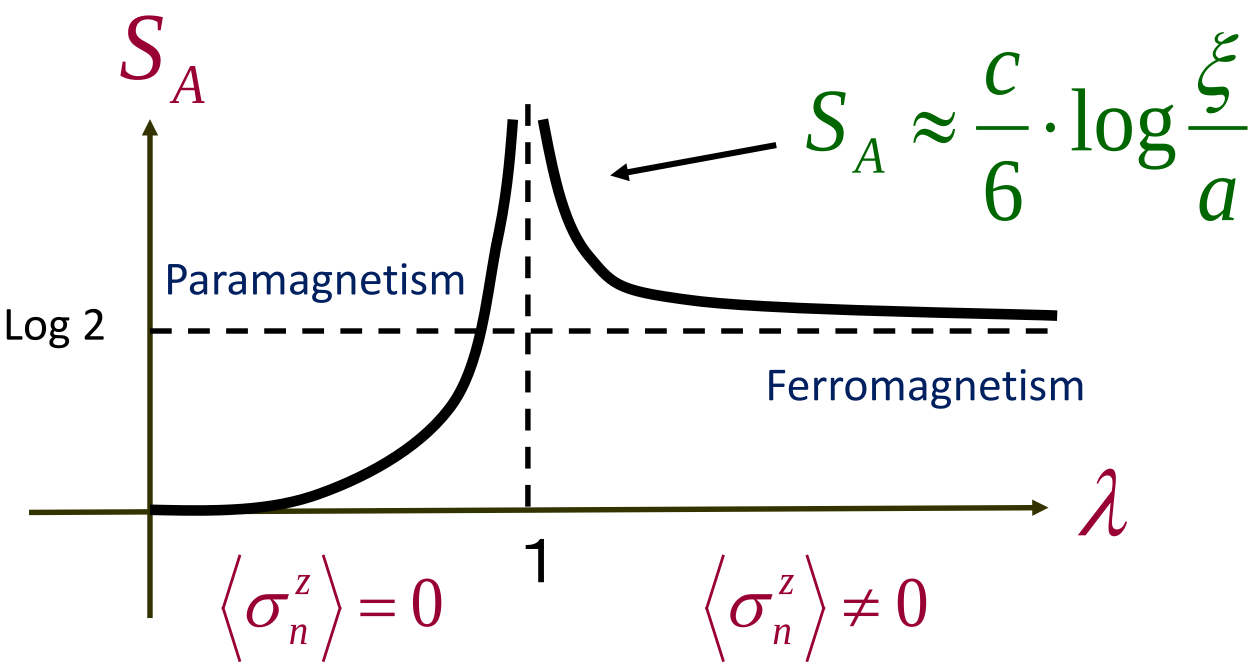
  → Proof of c-theorem, F-theorem etc.

  ``Geometrization of QFTs''

An example in Stat-Mech: Quantum Ising spin chain

Consider the Ising spin chain with a transverse magnetic field:

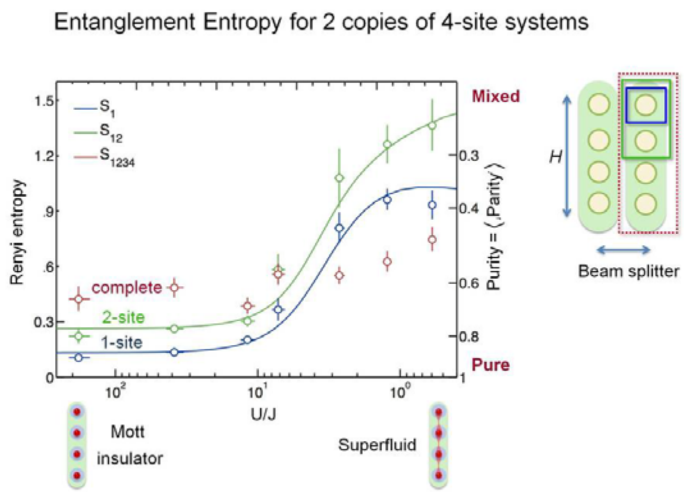
$$H = -\sum_n \sigma_n^x - \lambda \sum_n \sigma_n^z \sigma_{n+1}^z$$



[Vidal-Latorre-Rico-Kitaev 02, Calabrese-Cardy 04]

[1] It is recently reported that (2nd Renyi) EE was measured even experimentally in a cold atom system.

[Greiner et.al. 1509.01160]



[2] Recent Experimental realization of `holographic EE'

arXiv:1705.00365

Measuring Holographic Entanglement Entropy on a Quantum Simulator

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2017

Anti-de Sitter/conformal field theory (AdS/CFT) correspondence is one of the most promising realizations of holographic principle towards quantum gravity. The recent development of a discrete version of AdS/CFT correspondence in terms of tensor networks motivates one to simulate and demonstrate AdS/CFT correspondence

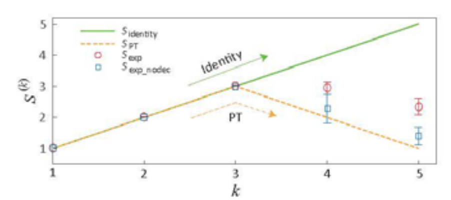


FIG. 3: Entanglement entropy $S^{(k)}$ of the k-qubit subsystem of the

PT—a build block of a complex
delity, which is already state-of-
some non-negligible decoheren
fore, our results successfully tes
coherence. Our results further s
processor would be able to sim
formulae and hence the AdS/CFT
of the two assumptions: the dec
ments can be tolerated, or the or
be *pure*. The latter has been in
detail in [28].

② Quantum Entanglement and Holography

(2-1) AdS/CFT

[Maldacena 97]

AdS/CFT

Quantum Gravity (String theory) on d+2 dim. AdS spacetime (anti de-Sitter space)	=	Conformal Field Theory (CFT) on d+1 dim. Minkowski spacetime
↓ Classical limit		↓ Large N limit, Strong coupling limit
General relativity with $\Lambda < 0$ (Geometrical)		Strongly interacting quantum many-body systems

Basic Principle

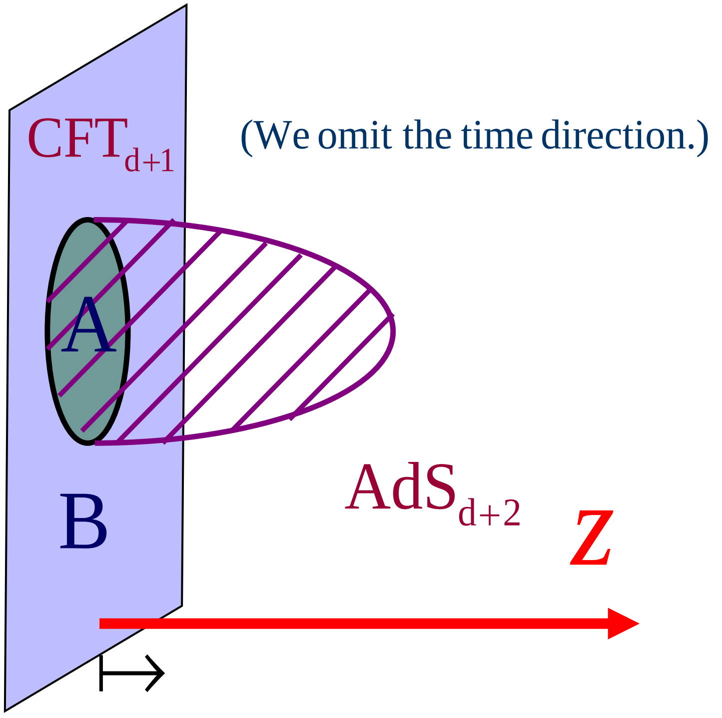
(Bulk-Boundary relation) :

$$Z_{Gravity} = Z_{CFT}$$

(2-2) Holographic Entanglement Entropy (HEE)

[Ryu-TT 06; derived by Lewkowycz-Maldacena 13]

$$S_A = \underset{\substack{\partial\gamma_A = \partial A \\ \gamma_A \approx A}}{\mathrm{Min}} \left[\frac{\mathrm{Area}(\gamma_A)}{4G_N} \right]$$



γ_A is the minimal area surface (codim.=2) such that

$$\partial A = \partial \gamma_A \text{ and } A \sim \gamma_A .$$

homologous

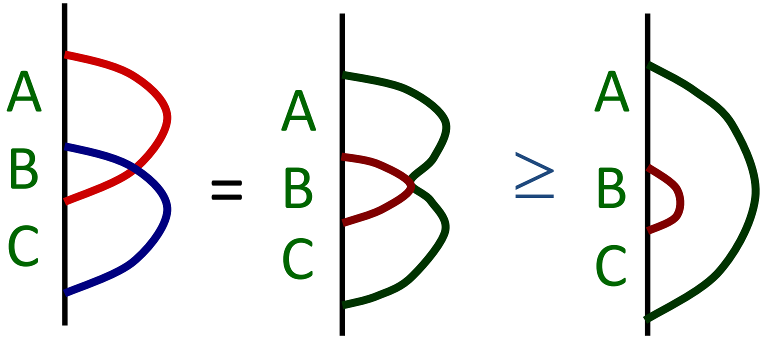
$z > \varepsilon$ (UV cut off)

Note: In time-dependent spacetimes, we need to take extremal surfaces.
[Hubeny-Rangamani-TT 07]

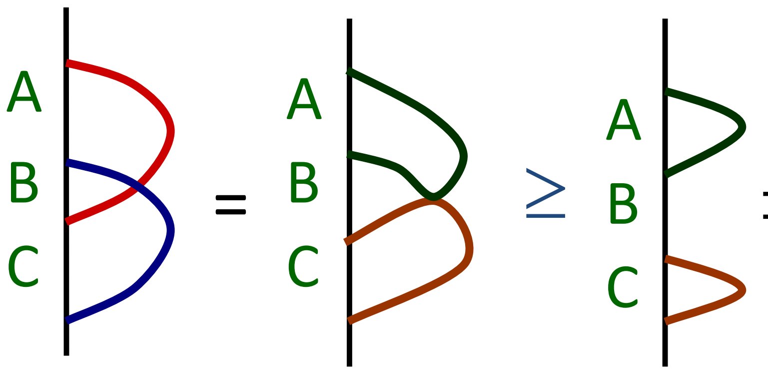
$$ds^2 = R^2 \cdot \frac{dz^2 - dt^2 + \sum_{i=1}^{d} dx_i^2}{z^2}$$

Holographic Proof of Strong Subadditivity

[Headrick-TT 07]



$$\Rightarrow S_{A\cup B} + S_{B\cup C} \geq S_{A\cup B\cup C} + S_{B}$$



$$\Rightarrow S_{A\cup B} + S_{B\cup C} \geq S_{A} + S_{C}$$

Algebraic relations in Quantum Information Theory

⇔ Geometric properties in Gravity

(2-3) Einstein Equation from Entanglement

A is a round ball (radius l) in 3d CFT. Its center is at $(t, \vec{x})$.

The perturbative Einstein equation is rewritten as follows:

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} + \Lambda g_{\mu\nu} = T_{\mu\nu}$$

Kinetic term **C.C.** **Matter field contributions**

$$\left(\partial_l^2 - \partial_l - \partial_{\vec{x}}^2 - \frac{3}{l^2} \right) \Delta S_A(t, \vec{x}, l) = \langle O \rangle \langle O \rangle$$

$$\phi \leftrightarrow O$$

[Nozaki-Numasawa-Prudenziati-TT 13, Bhattacharya-TT 13]

The perturbative Einstein eq. turns out to be equivalent to the first law of entanglement entropy. [Lashkari-McDermott-Raamsdonk 13, Faulkner-Guica-Hartman-Myers-Raamsdonk 13]

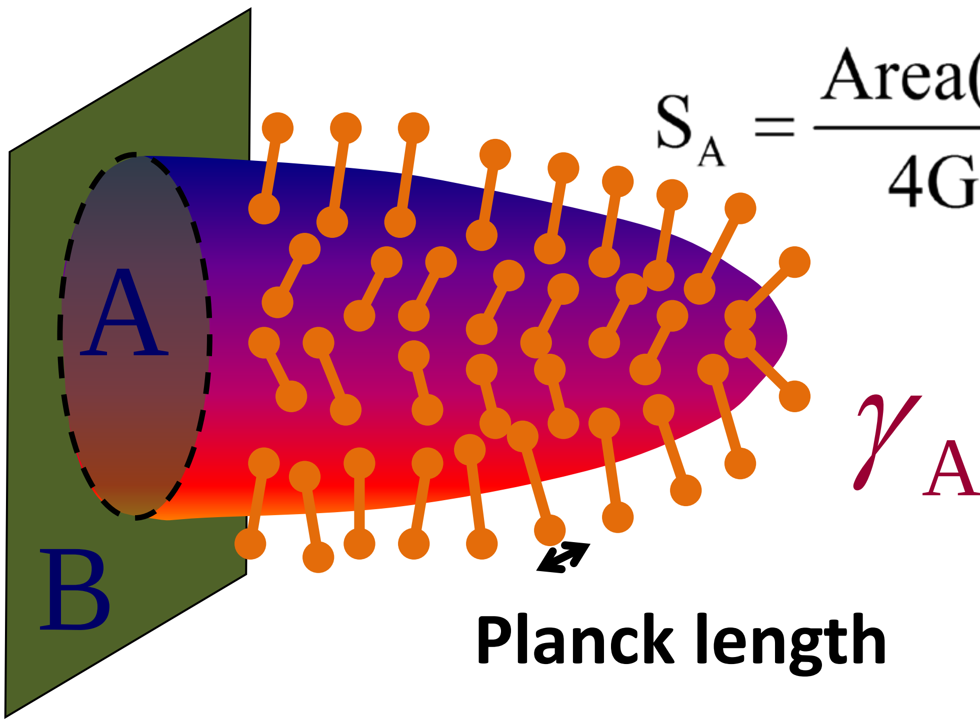
$$1st \text{ law of EE}: \; S(\rho_A + \Delta\rho_A \,||\, \rho_A) = \Delta S_A - \Delta H_A \approx 0, \quad (\rho_A \equiv e^{-H_A}).$$

(2-4) Emergent Spacetime from Quantum Entanglement

The HEE suggests the following novel interpretation:

A spacetime in gravity = Bits of quantum entanglement

$$S_A = \frac{\text{Area}(\gamma_A)}{4G_N} \approx \frac{\text{Area}(\gamma_A)}{l_{pl}^2}.$$



⇒ Manifestly realized in the recently argued connection between AdS/CFT and Tensor networks !

[Swingle 09, cf. Raamsdonk 09, Susskind-Maldacena 13, ….]

(2-5) Tensor Networks and AdS/CFT

Tensor Network [See e.g. Cirac-Verstraete 09(review)]

Tensor network = Geometrical description of wave function

⇒ Efficient variational ansatz for the ground state wave functions in quantum many-body systems.

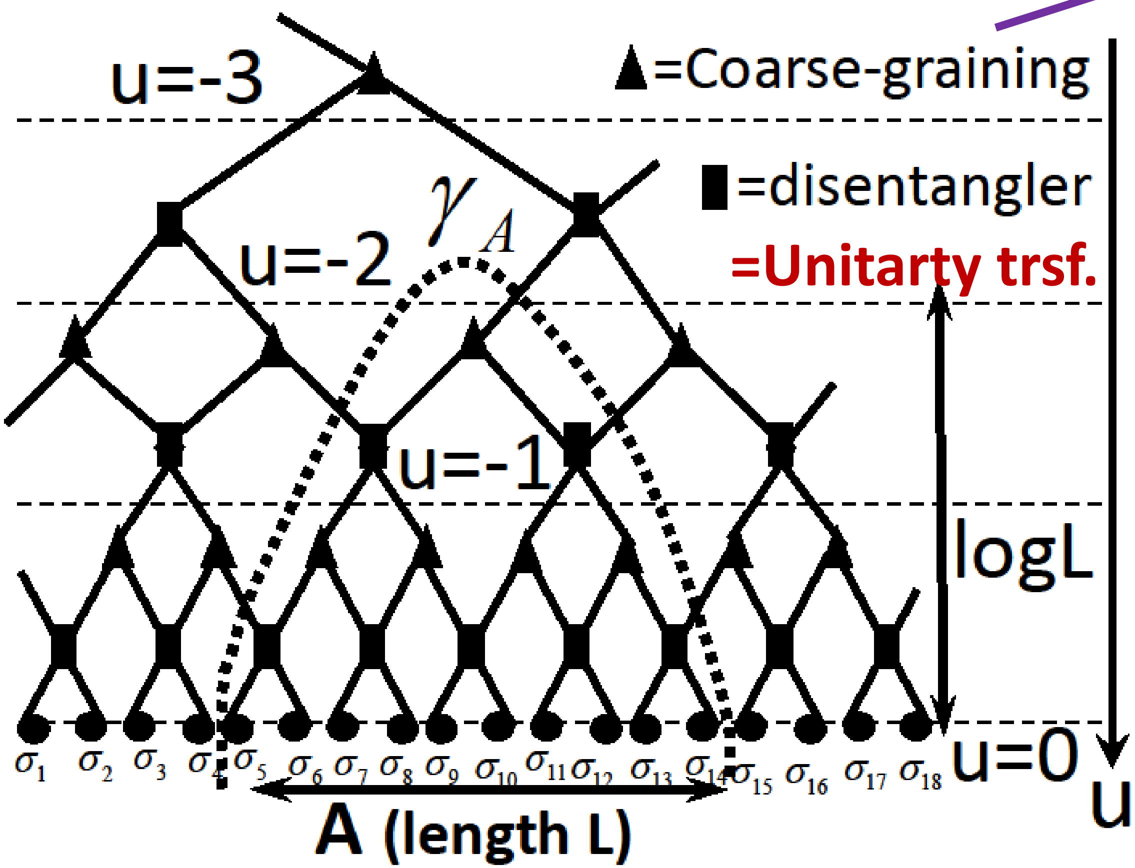
⇒ An ansatz should respect the correct **quantum entanglement** of ground state.

~Geometry of Tensor Network

MERA (Multiscale Entanglement Renormalization Ansatz):

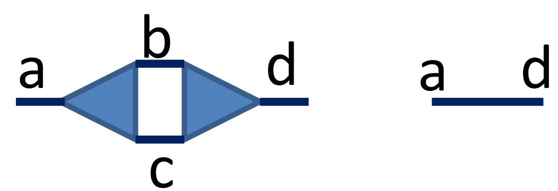
⇒ An efficient variational ansatz for CFT vacuum. [Vidal 05]

To increase entanglement in a CFT, we add (dis)entanglers.



Isometry

$$[T]^{\dagger}_{abc}[T]_{bcd} = \delta_{ad}$$



$$S_A \propto \mathrm{Min}[\#\,\mathrm{links}]$$

$$\propto \log L$$

⇒ agrees with results in 2d CFT !

The original idea: Tensor Network of MERA (∃ scale inv.)
= a time slice of AdS space

However, recent detailed studies show several problems:
(1) MERA does not have any isometry other than scale inv.
(2) Why EE bound is saturated ?
(3) Sub AdS scale locality

Some of these problems may be due to lattice artifacts.
We should consider CFTs in genuine continuum limit.

In our latest work, we gave a new approach which realizes continuous description **using path-integrals**.

③ Optimization of Path-Integral and AdS/CFT

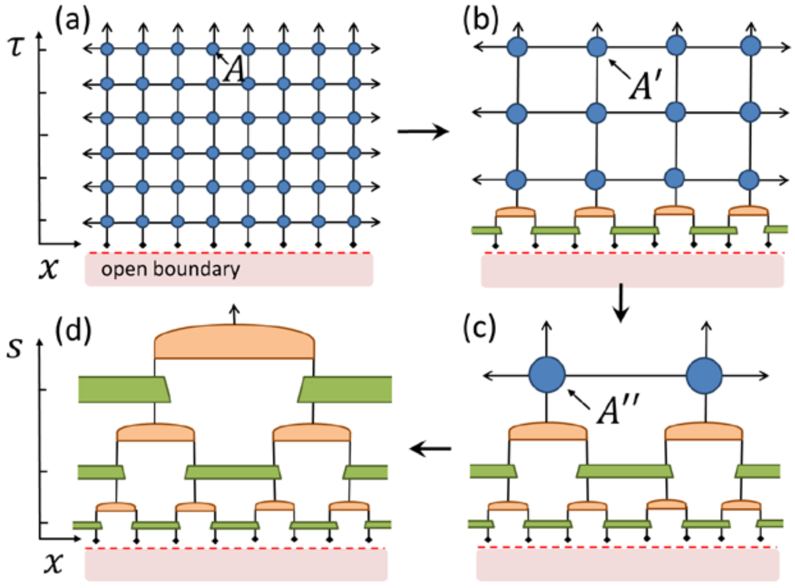
(3-1) Motivation

Remember that the MERA can be obtained from the `optimization' of tensor networks

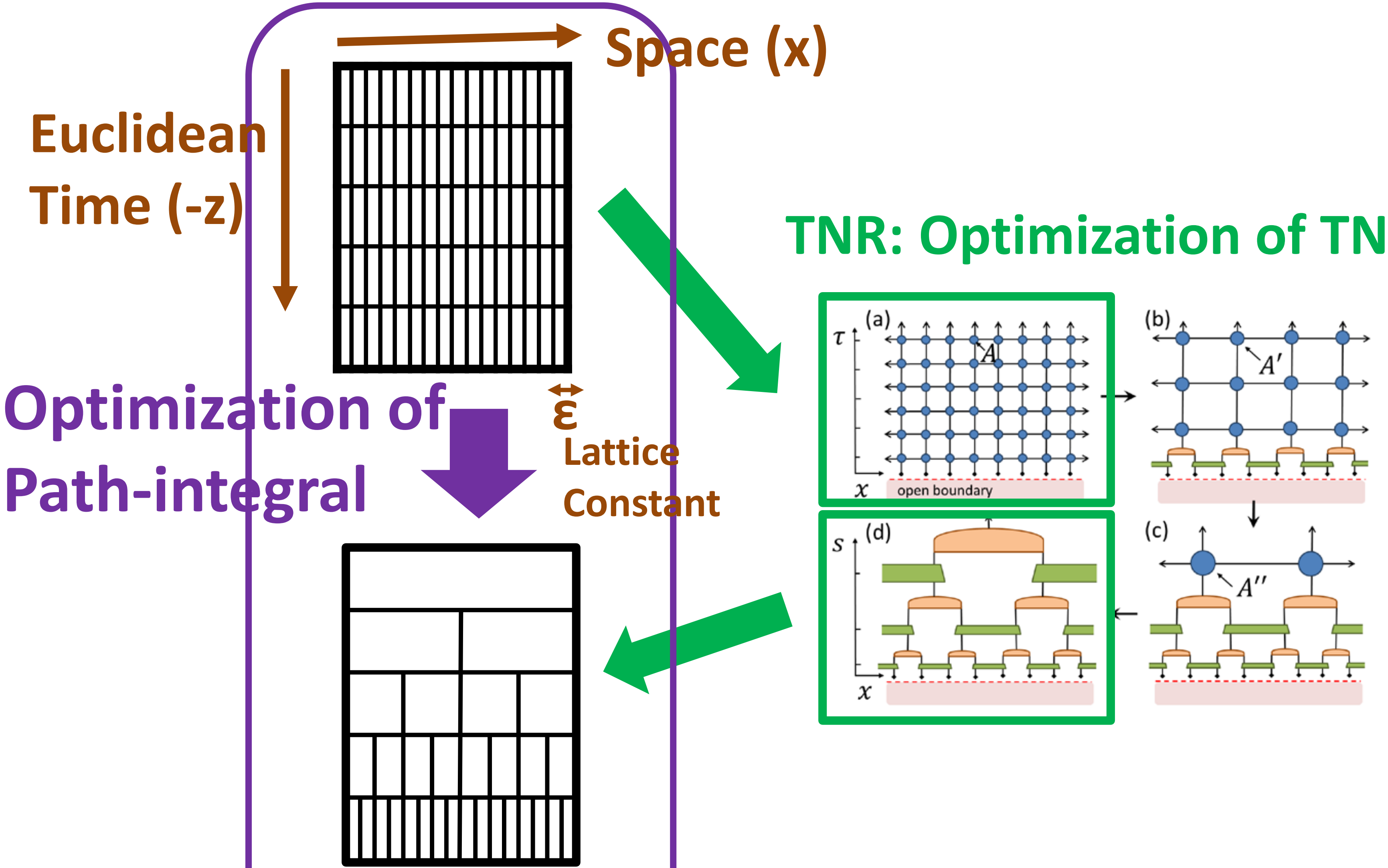
⇒ **Tensor network renormalization** [Evenbly-Vidal 14, 15]

Euclidean Path-Integral ⇒

MERA ⇐



Optimization of Path-Integral
[Miyaji-Watanabe-TT 2016]



Hyperbolic Space = Time slice of AdS3

(3-2) Formulating Optimization of Path-integral

[Caputa-Kundu-Miyaji-Watanabe-TT 2017]

A Basic Rule: Simplify a path-integral such that it produces the correct UV wave functional $\Psi_{UV}^{\mathrm{Flat}}[\Phi(x)]$.

Below we focus on 2d CFTs for simplicity.

Deformation of discretizations in path-integral = Curved metric such that one cell (bit) = unit length.

$$\Rightarrow \quad ds^2 = e^{2\phi(x,z)}(dx^2 + dz^2).$$

Note: The original flat metric is given as follows (ε is the UV cutoff):

$$ds^2 = \varepsilon^{-2} \cdot (dx^2 + dz^2).$$

Ground state UV wave function in curved space

$$\Psi_{UV}^{g}\left[\Phi(x)\right]=\int\prod_{\substack{0<z<\infty\\-\infty<x<\infty}}D\Phi(x,z)\,e^{-S_{CFT}(\Phi)}\cdot\delta\left(\Phi(x)-\Phi(x,z=0)\right)$$

In CFTs, owing to the Weyl invariance, we have

$$\Psi_{UV}^{g_{ab}=e^{2\phi}\delta_{ab}}\left[\Phi(x)\right]=N[\phi(x,z)]\cdot\Psi_{UV}^{\text{Flat}}\left[\Phi(x)\right].$$

Our Proposal (Optimization of Path-integral for CFTs):

Minimize $N[\phi(x,z)]$ w.r.t $\phi(x,z)$

with the boundary condition $e^{2\phi}\big|_{z=\varepsilon}=\varepsilon^{-2}$.

(3-3) AdS3/CFT2 (Vaccum State)

[Caputa-Kundu-Miyaji-Watanabe-TT 2017]

$$\text{Metric of Discretized Lattice : } \mathrm{ds}^2 = e^{2\phi}(dx^2 + dz^2)$$

$$\Rightarrow \quad \frac{\Psi_{g_{ab}=e^{2\phi}\cdot\delta_{ab}}}{\Psi_{g_{ab}=\delta_{ab}}} \propto e^{\frac{c}{24\pi}S_L},$$

$$S_L = \int dxdz\left[(\partial_x\phi)^2 + (\partial_z\phi)^2 + e^{2\phi}\right]$$

[Liouville Theory]

$$= \int dxdz\left[(\partial_x\phi)^2 + \left(\partial_z\phi + e^{\phi}\right)^2\right]$$

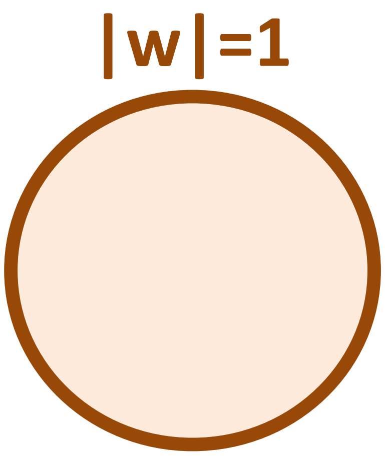
$$\Rightarrow \text{Minimum : } \quad e^{2\phi} = \frac{1}{z^2}.$$

⇒2dim. Hyperbolic space
= a time slice of AdS3

(3-4) Analysis of Excited States

Consider the vacuum state of 2d CFT on a circle dual to a global AdS3.

⇒ Optimize the path-integral on a disk with the unit radius.



The solution is simply given by (in the general expressions)

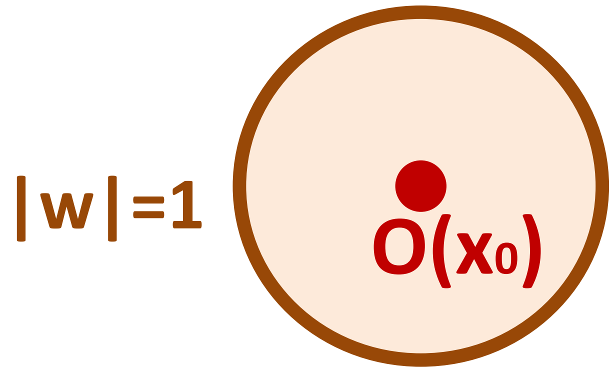
$$A(w) = w, \quad B(\overline{w}) = \overline{w}.$$

$$ds^2 = \frac{4dwd\overline{w}}{(1-|w|^2)^2}.$$

➡ Hyperbolic Disk H2

Now we insert an operator O(x) in the center of the disk x=x0.

O(x): conformal dim. $h_L=h_R=h$



$$\Rightarrow O(x) \sim e^{-2h\cdot\varphi}.$$

Thus we minimize $$\frac{\Psi_{g=e^{2\varphi}}}{\Psi_{\text{Flat}}} \propto e^{\frac{c}{24\pi}S_L} \cdot e^{-2h\varphi(x_0)}.$$

$$\Rightarrow \quad \boxed{\partial_w \partial_{\overline{w}} \varphi - \frac{1}{4} e^{2\varphi} + \frac{6\pi h}{c} \delta^2(w) = 0.}$$

Solution: $A(w) = w^a, \quad B(\bar{w}) = \bar{w}^a, \qquad (a \equiv 1 - 12h/c).$

Metric: $$ds^2 = \frac{4 d\zeta d\bar{\zeta}}{(1 - |\zeta|^2)^2}, \qquad \zeta \equiv w^a = r e^{i\theta}$$

⇒ Deficit angle geometry $\theta \sim \theta + 2\pi a.$

This agrees with the expected gravity dual if h/c<<1.

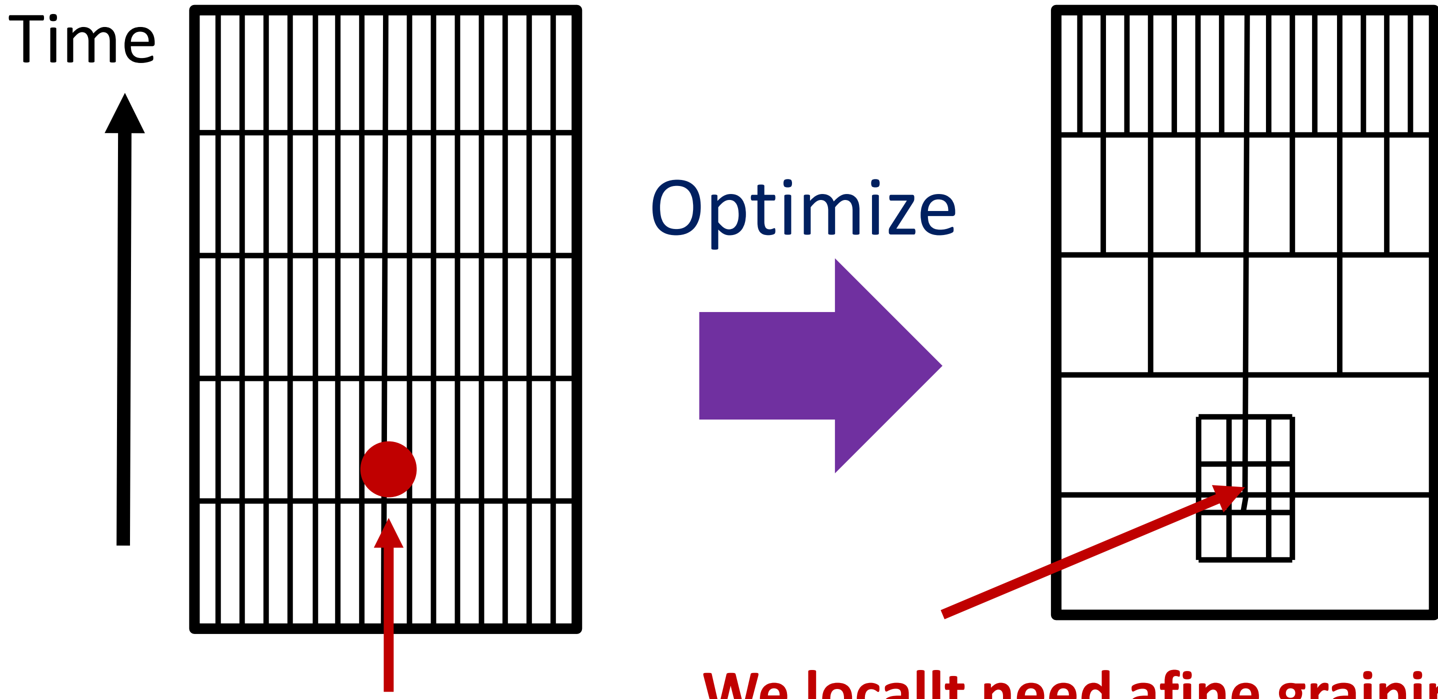
Note: the AdS/CFT predicts $a = \sqrt{1 - 24h/c}.$

Interestingly, if we consider **the quantum Liouville CFT**,

then $$h = \frac{\gamma\alpha}{4}(Q - \alpha\gamma/2.), \quad c = 1 + 3Q^2, \quad (Q \equiv 2/\gamma + \gamma).$$

⇒ We get $a = \sqrt{1 - 24h/c}.$

If we insert a local operator in the path-integral :

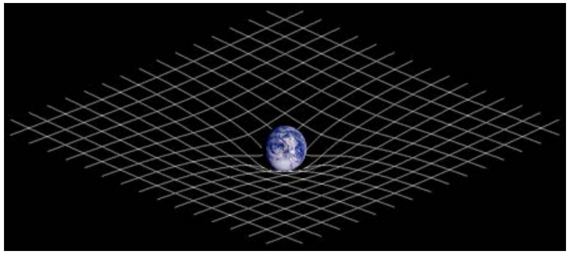


Local excitation (energy source)

We locallt need afine graining

⇒ The metric gets larger !

This agrees with general relativity !



④ Conclusions

- Holography tells us that gravitational dynamics is dual to that of quantum entanglement in QFTs.
  ⇒ Emergent spacetime from quantum entanglement !

- We argue that the optimization of Euclidean path-integral is an interesting candidate of emergent spacetime in AdS/CFT. This works even in continuum limit.
  ⇒Many future problems: Higher dim. version ?
  Time-dependent b.g. ?
  dS/CFT version ?
  :

Motivation of Our Proposal

The normalization N estimates repetitions of same operations of path-integration. → **Minimize this !**

⇒ **Our conjecture:** $N[\varphi(x,z)] \approx \exp[C[\varphi]]$

$C[\varphi] \equiv$ computational complexity of TN state

$\sim$ # of Tensors in TN

[Relevance of Complexity in holography: Susskind et.al. See also Czech 1706.00965]

Our Hope

Optimization of Tensor Network (~ Path-integral)

||

Minimizing Computational Complexity

||

Solving Einstein equation

Thank you !

Your visit to Kyoto U is very welcome !



Strings 2018 in OIST, Okinawa: June 25-29



Post Strings WS in YITP, Kyoto: July 2- Aug 3