

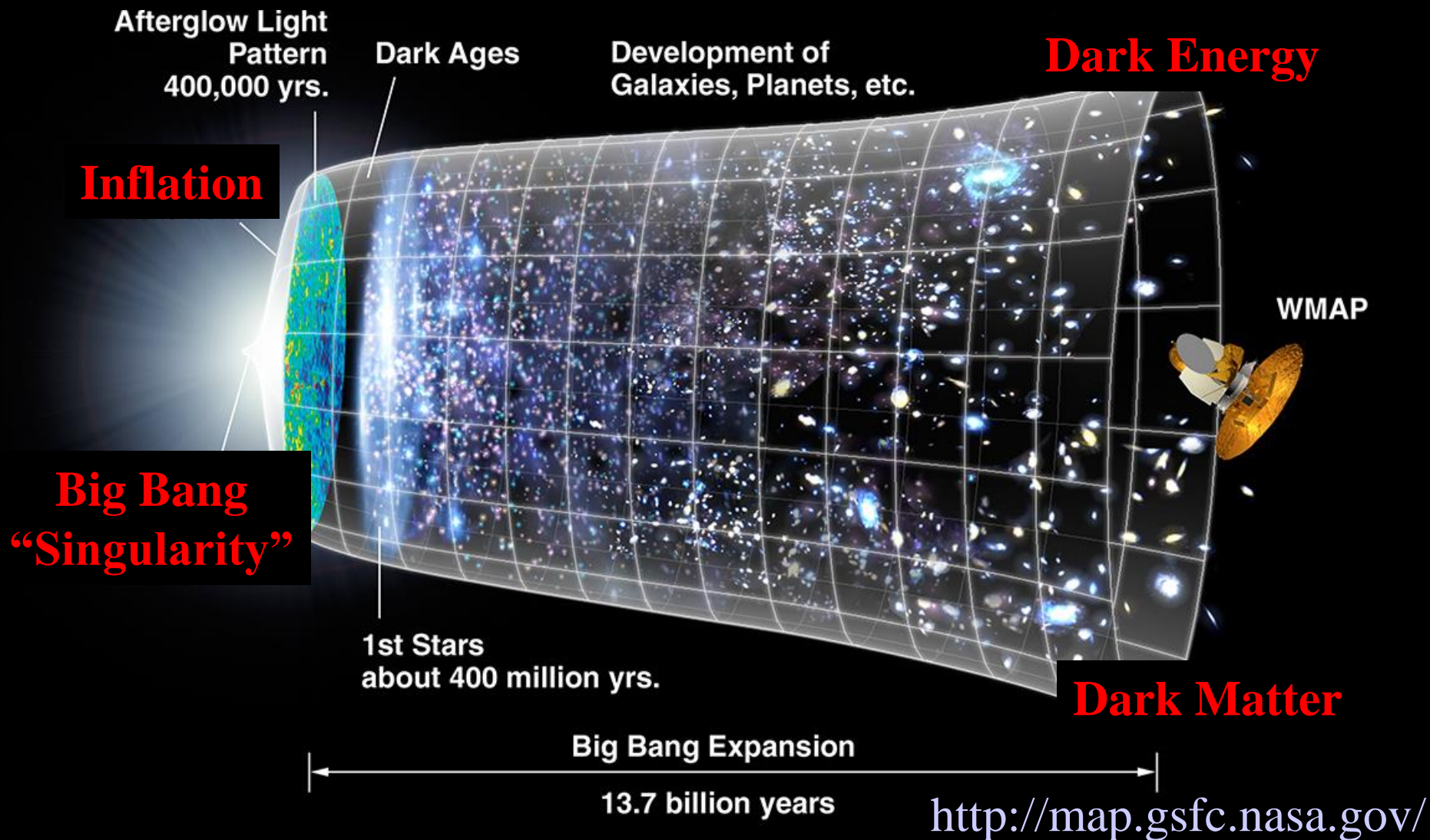
Massive gravity and cosmology

Shinji Mukohyama
Yukawa Institute for Theoretical Physics
Kyoto University

Based on collaborations with

Katsuki Aoki, Antonio DeFelice, Garrett Goon, Emir Gumrukcuoglu, Lavinia Heisenberg, Kurt Hinterbichler, Sachiko Kuroyanagi, David Langlois, Chunshan Lin, Charles Mazuet, Ryo Namba, Atsushi Naruko, Michele Oliosi, Takahiro Tanaka, Norihiro Tanahashi, Mark Trodden, Jean-Philippe Uzan, Mikhail Volkov

Why alternative gravity theories?



Three conditions for good alternative theories of gravity (my personal viewpoint)

1. Theoretically consistent
e.g. no ghost instability
2. Experimentally viable
solar system / table top experiments
3. Predictable
e.g. protected by symmetry

Some examples

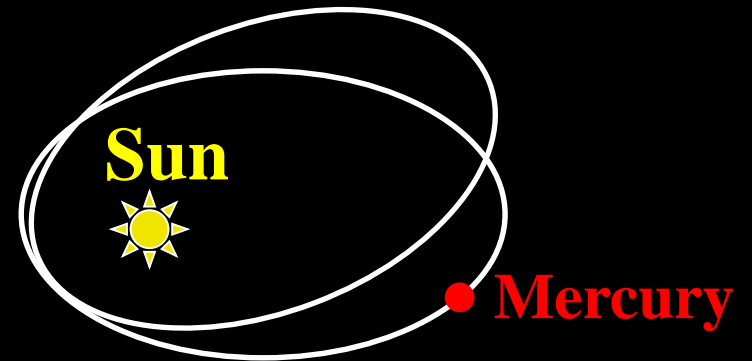
- I. Ghost condensation
IR modification of gravity
motivation: dark energy/matter
- II. Nonlinear massive gravity
IR modification of gravity
motivation: “Can graviton have mass?”
- III. Horava-Lifshitz gravity
UV modification of gravity
motivation: quantum gravity
- IV. Superstring theory
UV modification of gravity
motivation: quantum gravity, unified theory

A motivation for IR modification

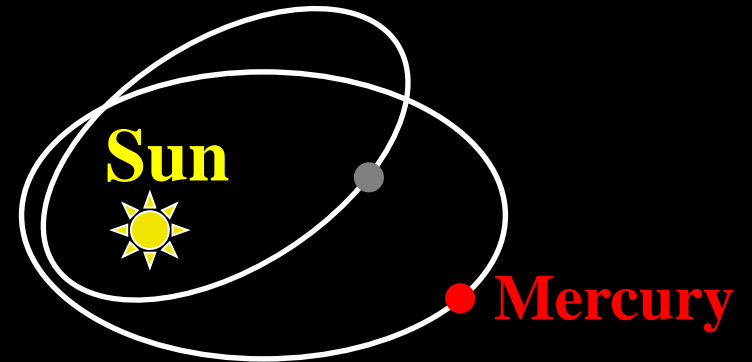
- Gravity at long distances
Flattening galaxy rotation curves
extra gravity
Dimming supernovae
accelerating universe
- Usual explanation: new forms of matter (DARK MATTER) and energy (DARK ENERGY).

Dark component in the solar system?

Precession of perihelion
observed in 1800's...



which people tried to
explain with a “dark
planet”, Vulcan,



But the right answer wasn't “dark planet”, it was
“change gravity” from Newton to GR.

Can we change gravity in IR?

➤ Change Theory?

Massive gravity

Fierz-Pauli 1939

DGP model

Dvali-Gabadadze-Porrati 2000

➤ Change State?

Higgs phase of gravity

The simplest: Ghost condensation

Arkani-Hamed, Cheng, Luty and Mukohyama, JHEP 0405:074,2004.

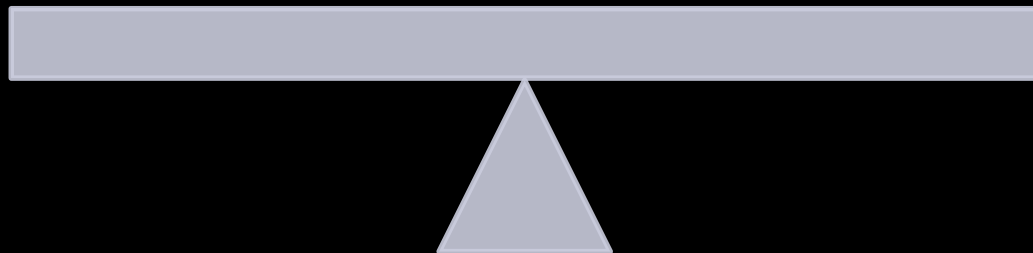
Massive gravity: history

Simple question: Can graviton have mass?

May lead to acceleration without dark energy

Yes?

No?



Massive gravity: history

Simple question: Can graviton have mass?

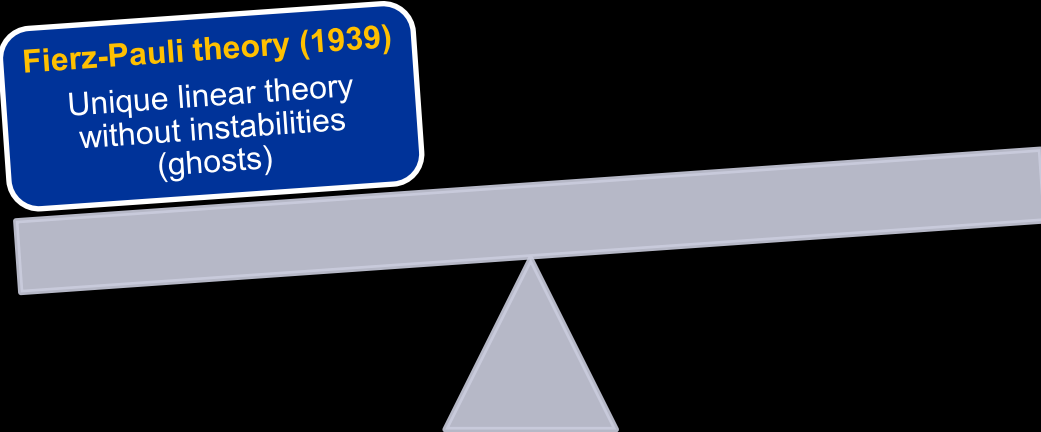
May lead to acceleration without dark energy

Yes?

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Fierz-Pauli theory (1939)

Unique linear theory
without instabilities
(ghosts)

A grey seesaw is positioned horizontally, tilted upwards on the right side. A blue box with a white border is placed on the left side of the seesaw, containing text about the Fierz-Pauli theory. The seesaw is supported by a grey triangular fulcrum in the center.

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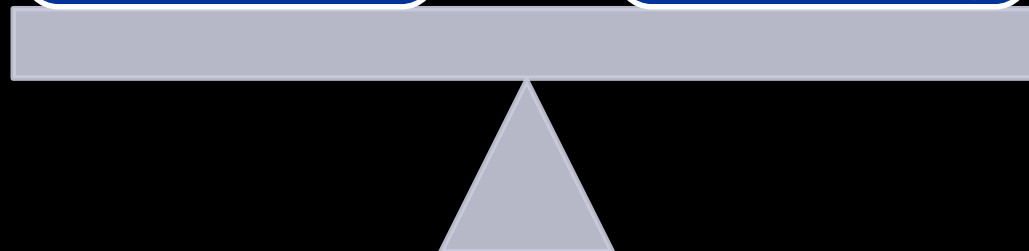
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van Dam-Veltman-
Zhakharov discontinuity
(1970)

**Massless limit \neq
General Relativity**



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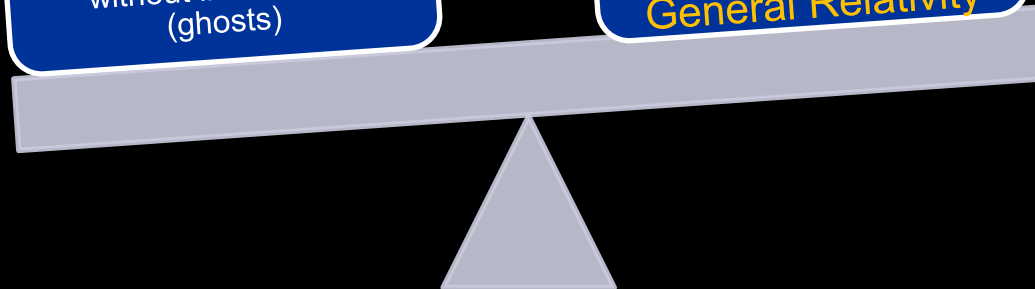
Yes?

No?

Vainshtein mechanism
(1972)
Nonlinearity \rightarrow Massless
limit = General Relativity

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6th d.o.f. @ Nonlinear level
 \rightarrow Instability (ghost)

van Dam-Veltman-
Zhakharov discontinuity
(1970)
Massless limit \neq
General Relativity

Nonlinear massive gravity

de Rham, Gabadadze 2010

de Rham, Gabadadze & Tolley 2010

- First example of fully nonlinear massive gravity without BD ghost since 1972!
- Purely classical (but technically natural)
- Properties of 5 d.o.f. depend on background

- **4 scalar fields ϕ^a ($a=0,1,2,3$)**

- **Poincare symmetry in the field space:**

$$\phi^a \rightarrow \phi^a + c^a, \quad \phi^a \rightarrow \Lambda_b^a \phi^b$$



$$f_{\mu\nu} \equiv \eta_{ab} \partial_\mu \phi^a \partial_\nu \phi^b$$

fiducial metric

Pullback of
Minkowski metric in field space
to spacetime

Systematic resummation

de Rham, Gabadadze & Tolley 2010

$$I_{mass}[g_{\mu\nu}, f_{\mu\nu}] = M_{Pl}^2 m_g^2 \int d^4x \sqrt{-g} (\mathcal{L}_2 + \alpha_3 \mathcal{L}_3 + \alpha_4 \mathcal{L}_4)$$

$$f_{\mu\nu} \equiv \eta_{ab} \partial_\mu \phi^a \partial_\nu \phi^b$$

$$\mathcal{K}_\nu^\mu = \delta_\nu^\mu - \left(\sqrt{g^{-1} f} \right)_\nu^\mu$$

$$\mathcal{L}_2 = \frac{1}{2} ([\mathcal{K}]^2 - [\mathcal{K}^2])$$

$$\mathcal{L}_3 = \frac{1}{6} ([\mathcal{K}]^3 - 3 [\mathcal{K}] [\mathcal{K}^2] + 2 [\mathcal{K}^3]) \quad [\mathcal{A}] \equiv Tr \mathcal{A}$$

$$\mathcal{L}_4 = \frac{1}{24} ([\mathcal{K}]^4 - 6 [\mathcal{K}]^2 [\mathcal{K}^2] + 3 [\mathcal{K}^2]^2 + 8 [\mathcal{K}] [\mathcal{K}^3] - 6 [\mathcal{K}^4])$$

No helicity-0 ghost, i.e. no BD ghost, in decoupling limit

$$\mathcal{K}_{\mu\nu} = \partial_\mu \partial_\nu \pi \quad \longrightarrow \quad \mathcal{L}_{2,3,4} = (\text{total derivative})$$

No BD ghost away from decoupling limit (Hassan&Rosen)

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No?

de Rham-Gabadadze-Tolley (2010)

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Massless limit \neq General Relativity

No FLRW universe?

D'Amico, de Rham, Dubovsky, Gabadadze, Pirtshalava, Tolley (2011)

- Flat FLRW ansatz in “Unitary gauge”

$$g_{\mu\nu} dx^\mu dx^\nu = -N^2(t) dt^2 + a^2(t) (dx^2 + dy^2 + dz^2)$$

$$\phi^a = x^a \quad \longrightarrow \quad f_{\mu\nu} = \eta_{\mu\nu}$$

- Bianchi “identity” $\rightarrow a(t) = \text{const.}$

$$\text{c.f.} \quad \nabla^\mu \left(\frac{2}{\sqrt{-g}} \frac{\delta I}{\delta g^{\mu\nu}} \right) = \frac{1}{\sqrt{-g}} \frac{\delta I_g}{\delta \phi^a} \partial_\nu \phi^a$$

\rightarrow no non-trivial flat FLRW cosmology

- “Our conclusions on the absence of the homogeneous and isotropic solutions do not change if we allow for a more general maximally symmetric 3-space”
- c.f. Inhomogeneous solutions do exist.

[Koyama, Niz, Tasinato 2011; Chamseddine & Volkov 2011]

Massive gravity: history

Simple question: Can graviton have mass?

May lead to acceleration without dark energy

Yes?

No?

Consistent Theory
found in 2010 but

No Viable Cosmology?

de Rham, Gabadadze, Tolley (2010)
First example of nonlinear massive gravity without BD ghost since 1971

de Rham, Gabadadze, Tolley (2010)
Nonlinearity \rightarrow Massless limit = General Relativity

Fierz-Pauli theory (1939)
Unique linear theory without instabilities (ghosts)

D'Amico, Denicol, Esposito, Luciani (2010)
No exact solution for FRW (homogeneous isotropic) universe!

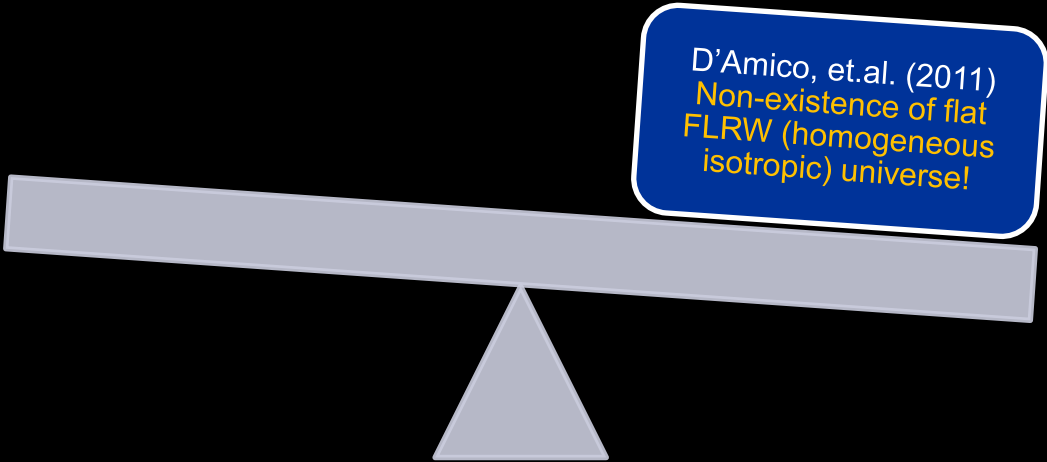
Souvaine, Deser, Esposito (1972)
6th d.o.f. @ Nonlinear level \rightarrow Instability (ghost)

Vincenti, Vasiliev, Zhakharov discontinuity (1970)
Massless limit \neq General Relativity

Cosmological solutions in nonlinear massive gravity

Good?

Bad?



D'Amico, et.al. (2011)
Non-existence of flat
FLRW (homogeneous
isotropic) universe!

Open FLRW solutions

Gumrukcuoglu, Lin, Mukohyama, arXiv: 1109.3845 [hep-th]

- $f_{\mu\nu}$ spontaneously breaks diffeo.
- Both $g_{\mu\nu}$ and $f_{\mu\nu}$ must respect FLRW symmetry
- Need FLRW coordinates of Minkowski $f_{\mu\nu}$

- No closed FLRW chart

$$\phi^0 = f(t)\sqrt{1 + |K|(x^2 + y^2 + z^2)},$$

$$\phi^1 = \sqrt{|K|}f(t)x,$$

$$\phi^2 = \sqrt{|K|}f(t)y,$$

$$\phi^3 = \sqrt{|K|}f(t)z.$$

- Open FLRW ansatz

$$f_{\mu\nu}dx^\mu dx^\nu = -(\dot{f}(t))^2 dt^2 + |K| (f(t))^2 \Omega_{ij}(x^k) dx^i dx^j$$

$$g_{\mu\nu}dx^\mu dx^\nu = -N(t)^2 dt^2 + a(t)^2 \Omega_{ij} dx^i dx^j,$$

$$\Omega_{ij} dx^i dx^j = dx^2 + dy^2 + dz^2 - \frac{|K|(x dx + y dy + z dz)^2}{1 + |K|(x^2 + y^2 + z^2)},$$

Open FLRW solutions

Gumrukcuoglu, Lin, Mukohyama, arXiv: 1109.3845 [hep-th]

- EOM for ϕ^a ($a=0,1,2,3$)

$$(\dot{a} - \sqrt{|K|}N) \left[\left(3 - \frac{2\sqrt{|K|}f}{a} \right) + \alpha_3 \left(3 - \frac{\sqrt{|K|}f}{a} \right) \left(1 - \frac{\sqrt{|K|}f}{a} \right) + \alpha_4 \left(1 - \frac{\sqrt{|K|}f}{a} \right)^2 \right] = 0$$

- The first sol $\dot{a} = \sqrt{|K|}N$ implies $g_{\mu\nu}$ is Minkowski

→ we consider other solutions

$$f = \frac{a}{\sqrt{|K|}} X_{\pm}, \quad X_{\pm} \equiv \frac{1 + 2\alpha_3 + \alpha_4 \pm \sqrt{1 + \alpha_3 + \alpha_3^2 - \alpha_4}}{\alpha_3 + \alpha_4}$$

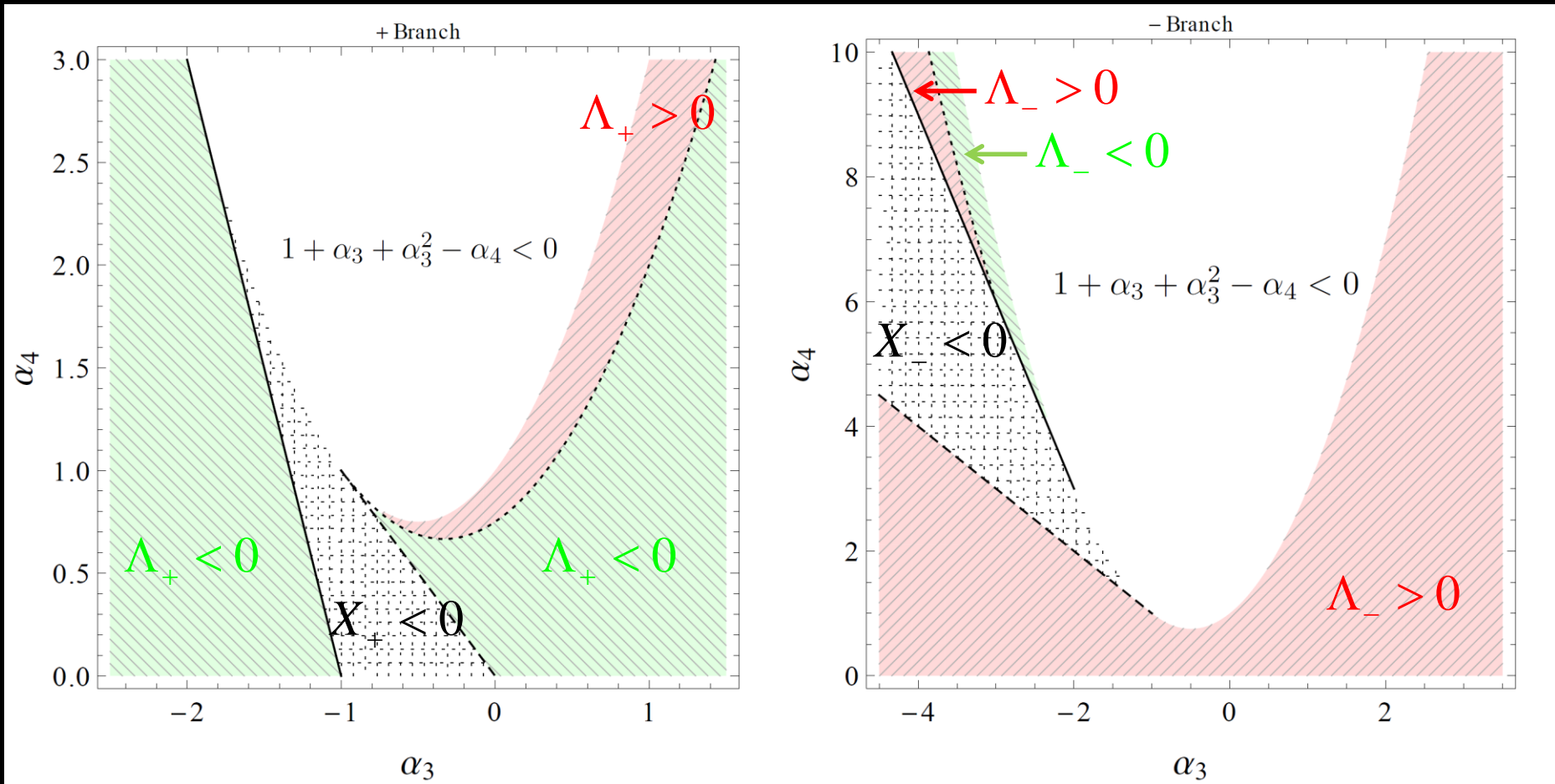
- Latter solutions do not exist if $K=0$

- Metric EOM → self-acceleration

$$3H^2 + \frac{3K}{a^2} = \Lambda_{\pm} + \frac{1}{M_{Pl}^2} \rho$$

$$\Lambda_{\pm} \equiv -\frac{m_g^2}{(\alpha_3 + \alpha_4)^2} \left[(1 + \alpha_3) (2 + \alpha_3 + 2\alpha_3^2 - 3\alpha_4) \pm 2 (1 + \alpha_3 + \alpha_3^2 - \alpha_4)^{3/2} \right]$$

Self-acceleration



$$f = \frac{a}{\sqrt{|K|}} X_{\pm}, \quad X_{\pm} \equiv \frac{1 + 2\alpha_3 + \alpha_4 \pm \sqrt{1 + \alpha_3 + \alpha_3^2 - \alpha_4}}{\alpha_3 + \alpha_4}$$

Cosmological solutions in nonlinear massive gravity

Good?

Bad?

Open universes with self-acceleration
GLM (2011a)

D'Amico, et.al. (2011)
Non-existence of flat FLRW (homogeneous isotropic) universe!

GLM = Gumrukcuoglu-Lin-Mukohyama

Cosmological solutions in nonlinear massive gravity

Good?

Bad?

More general fiducial metric $f_{\mu\nu}$
closed/flat/open FLRW universes allowed
GLM (2011b)

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Summary of Introduction + α

- Nonlinear massive gravity
free from BD ghost
- FLRW background
No closed/flat universe
Open universes with self-acceleration!
- More general fiducial metric $f_{\mu\nu}$
closed/flat/open FLRW universes allowed
Friedmann eq does not depend on $f_{\mu\nu}$
- Cosmological linear perturbations
Scalar/vector sectors \rightarrow same as in GR
Tensor sector \rightarrow time-dependent mass

Nonlinear instability

DeFelice, Gumrukcuoglu, Mukohyama, arXiv: 1206.2080 [hep-th]

- de Sitter or FLRW fiducial metric
- Pure gravity + bare cc \rightarrow FLRW sol = de Sitter
- Bianchi I universe with axisymmetry + linear perturbation (without decoupling limit)
- Small anisotropy expansion of Bianchi I + linear perturbation
 \rightarrow nonlinear perturbation around flat FLRW
- **Odd-sector:**
1 healthy mode + 1 healthy or ghosty mode
- **Even-sector:**
2 healthy modes + 1 ghosty mode
- This is not BD ghost nor Higuchi ghost.

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More general fiducial metric $f_{\mu\nu}$
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NEW
Nonlinear instability of FLRW solutions
DGM (2012)

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Non-existence of flat FLRW (homogeneous isotropic) universe!

New class of cosmological solution

Gumrukcuoglu, Lin, Mukohyama, arXiv: 1206.2723 [hep-th]
+ De Felice, arXiv: 1303.4154 [hep-th]

- Healthy regions with (relatively) large anisotropy
- Are there attractors in healthy region?
- Classification of fixed points
- Local stability analysis
- Global stability analysis

At attractors, physical metric is isotropic but fiducial metric is anisotropic.

→ Anisotropic FLRW universe!

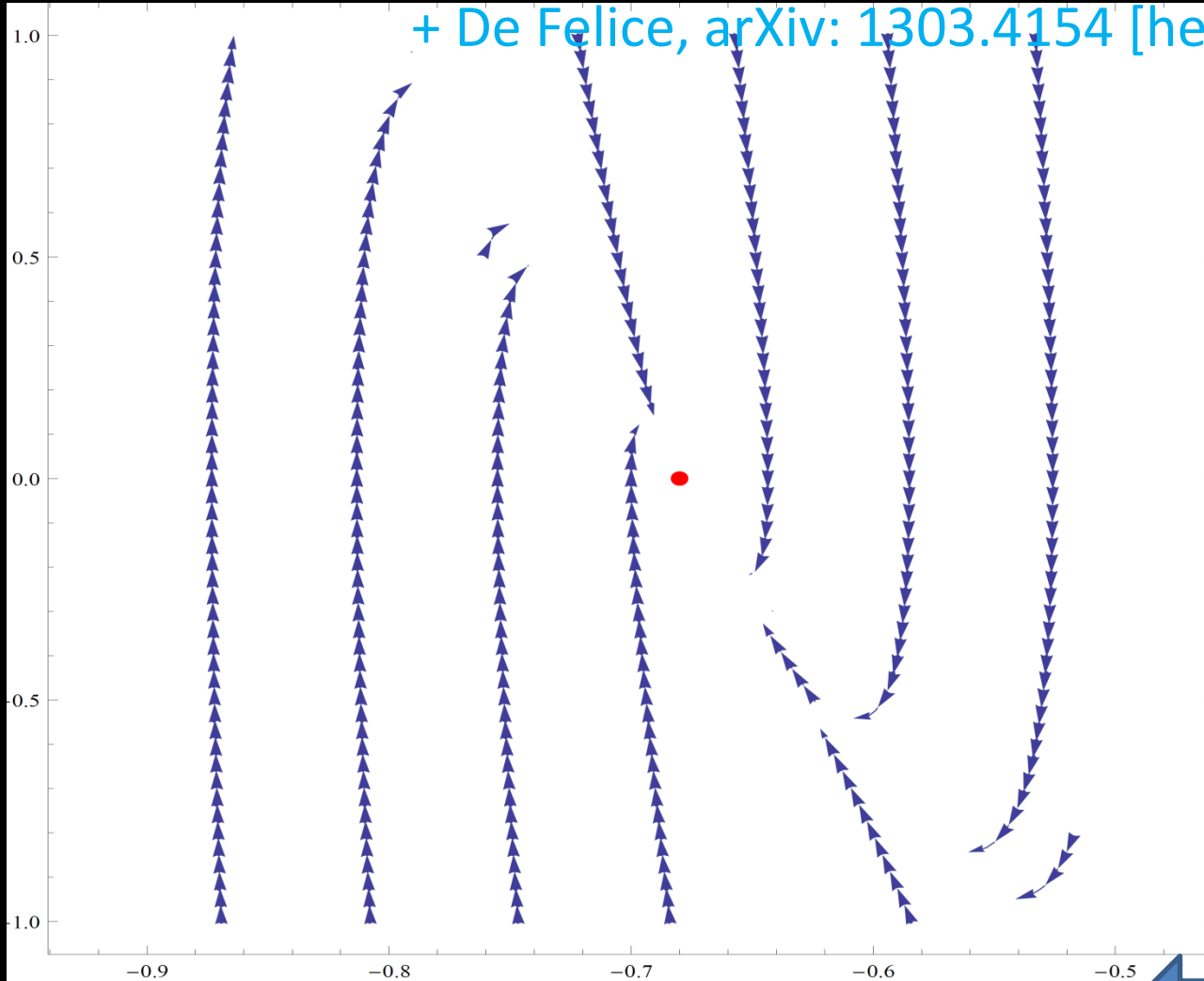

statistical anisotropy expected
(suppressed by small m_g^2)

New class of cosmological solution

Gumrukcuoglu, Lin, Mukohyama, arXiv: 1206.2723 [hep-th]

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Anisotropy
in
Expansion



Anisotropy in fiducial metric



Cosmological solutions in nonlinear massive gravity

Good?

Bad?

NEW Class of Solutions
Anisotropic FLRW
universe
GLM (2012)

More general fiducial
metric $f_{\mu\nu}$
closed/flat/open FLRW
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GLM = Gumrukcuoglu-Lin-Mukohyama
DGM = DeFelice-Gumrukcuoglu-Mukohyama

New backgrounds or Extended theories

- New nonlinear instability [DeFelice, Gumrukcuoglu, Mukohyama 2012]
→ (i) new backgrounds, or (ii) extended theories
- (i) Anisotropic FLRW (Gumrukcuoglu, Lin, Mukohyama 2012): physical metric is isotropic but fiducial metric is anisotropic
- (ii) Extended quasidilaton (De Felice&Mukohyama 2013), Bimetric theory (Hassan, Rosen 2011; DeFelice, Nakamura, Tanaka 2013; DeFelice, Gumrukcuoglu, Mukohyama, Tanahashi, Tanaka 2014), Rotation-invariant theory (Rubakov 2004; Dubovsky 2004; Blas, Comelli, Pilo 2009; Comelli, Nesti, Pilo 2012; Langlois, Mukohyama, Namba, Naruko 2014), Composite metric (de Rham, Heisenberg, Ribeiro 2014; Gumrukcuoglu, Heisenberg, Mukohyama 2014, 2015), New quasidilaton (Mukohyama 2014; De Felice, Gumrukcuoglu, Heisenberg, Mukohyama, Tanahashi 2016), Chameleonic bigravity (De Felice, Mukohyama, Uzan 2017), ...
- They provide stable cosmology.

Massive gravitons as dark matter and gravitational waves

Katsuki Aoki^{1,*} and Shinji Mukohyama^{2,3,†}

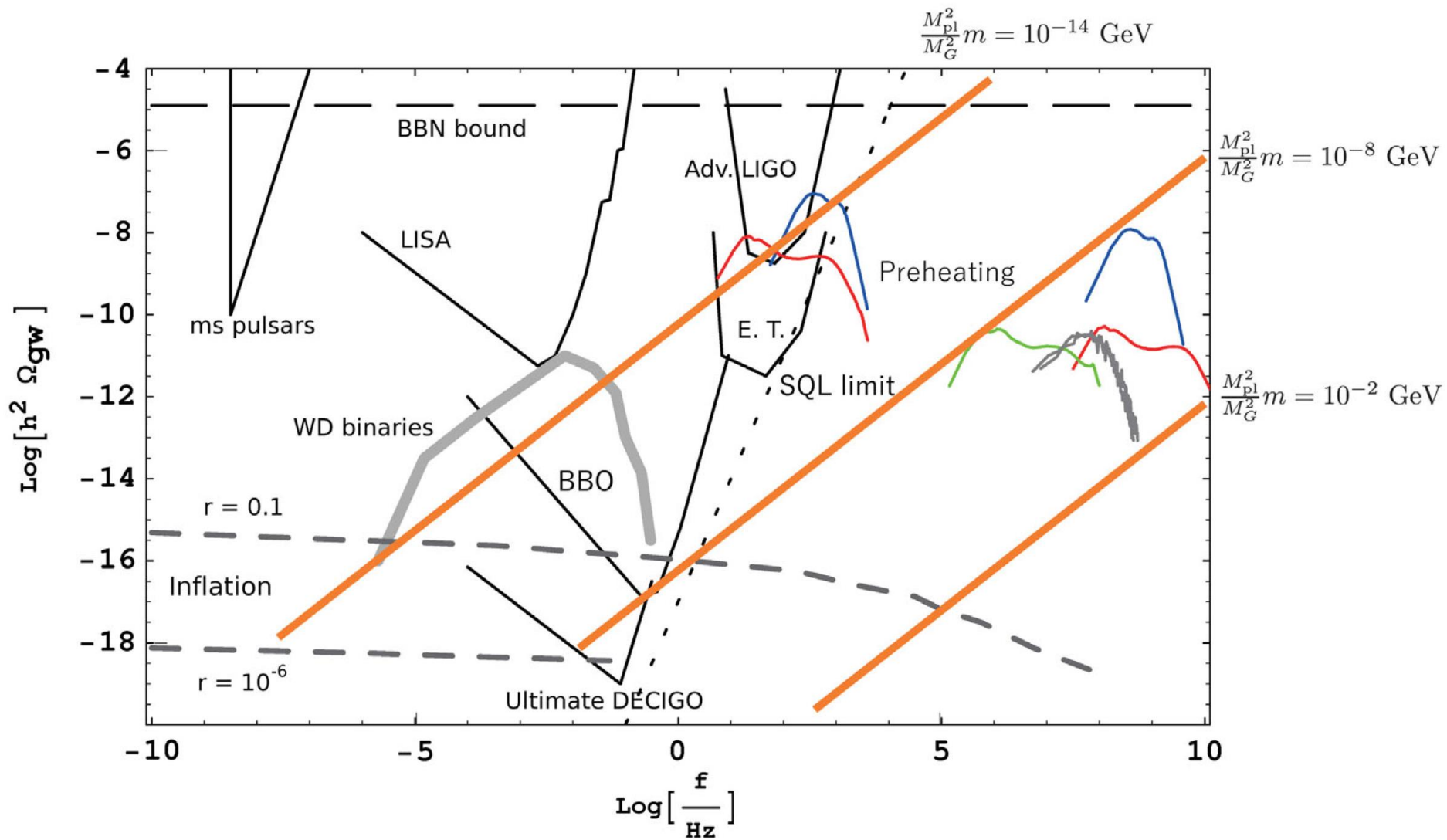
¹*Department of Physics, Waseda University, Shinjuku, Tokyo 169-8555, Japan*

²*Center for Gravitational Physics, Yukawa Institute for Theoretical Physics,
Kyoto University, 606-8502 Kyoto, Japan*

³*Kavli Institute for the Physics and Mathematics of the Universe (WPI), UTIAS,
The University of Tokyo, Kashiwa, Chiba 277-8583, Japan*

(Received 2 May 2016; published 1 July 2016)

We consider the possibility that the massive graviton is a viable candidate for dark matter in the context of bimetric gravity. We first derive the energy-momentum tensor of the massive graviton and show that it indeed behaves as that of dark matter fluid. We then discuss a production mechanism and the present abundance of massive gravitons as dark matter. Since the metric to which ordinary matter fields couple is a linear combination of the two mass eigenstates of bigravity, production of massive gravitons, i.e., the dark matter particles, is inevitably accompanied by generation of massless gravitons, i.e., the gravitational waves. Therefore, in this scenario some information about dark matter in our Universe is encoded in gravitational waves. For instance, if LIGO detects gravitational waves generated by the preheating after inflation, then the massive graviton with the mass of ~ 0.01 GeV is a candidate for dark matter.



Cosmological solutions in nonlinear massive gravity

Good?

Bad?

Extended theories:
Extended quasidilaton,
biometric theory, rotation-
invariant theory,
composite metric, ...

More general fiducial
metric $f_{\mu\nu}$
closed/flat/open FLRW
universes allowed
GLM (2011b)

**Open universes with self-
acceleration**
GLM (2011a)

NEW
Nonlinear instability of
FLRW solutions
DGM (2012)

D'Amico, et.al. (2011)
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More recent development

Minimal Theory of Massive Gravity

De Felice & Mukohyama, arXiv: 1506.01594

1512.04008

- 2 physical dof only = massive gravitational waves
- exactly same FLRW background as in dRGT
- no BD ghost, no Higuchi ghost, no nonlinear ghost

Three steps to the Minimal Theory

1. Fix local Lorentz to realize ADM vielbein in dRGT
2. Switch to Hamiltonian
3. Add 2 additional constraints

(It is easy to go back to Lagrangian after 3.)

Gravitational wave signal from massive gravity

A Emir Gümrükçüoğlu¹, Sachiko Kuroyanagi², Chunshan Lin¹,
Shinji Mukohyama¹ and Norihiro Tanahashi³

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² Research Center for the Early Universe (RESCEU), Graduate School of Science, The University of Tokyo, Tokyo 113-0033, Japan

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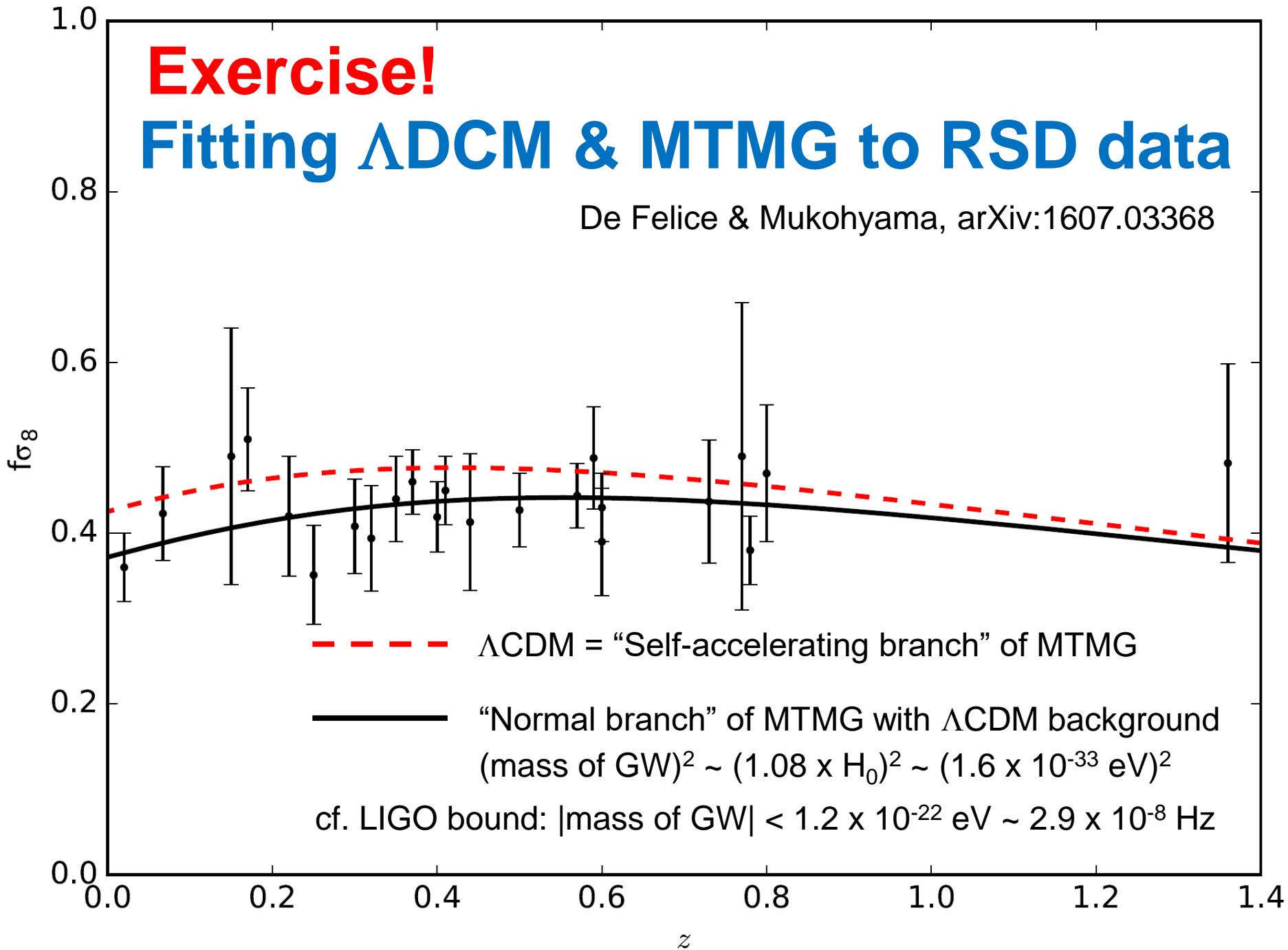
Abstract

We discuss the detectability of gravitational waves with a time-dependent mass contribution, by means of the stochastic gravitational wave observations. Such a mass term typically arises in the cosmological solutions of massive gravity theories. We conduct the analysis based on a general quadratic action, and thus the results apply universally to any massive gravity theories in which the modification of general relativity appears primarily in the tensor modes. The primary manifestation of the modification in the gravitational wave spectrum is a sharp peak. The position and height of the peak carry information on the present value of the mass term, as well as the duration of the inflationary stage. We also discuss the detectability of such a gravitational wave signal using the future-planned gravitational wave observatories.

Exercise!

Fitting Λ CDM & MTMG to RSD data

De Felice & Mukohyama, arXiv:1607.03368



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Minimal Theory of Massive Gravity
DeFelice&Mukohyama (2015)

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DGM = DeFelice-Gumrukcuoglu-Mukohyama

DGHM = DeFelice-Gumrukcuoglu-Heisenberg-Mukohyama

Summary

- Nonlinear massive gravity
free from BD ghost
- FLRW background
No closed/flat universe
Open universes with self-acceleration!
- More general fiducial metric $f_{\mu\nu}$
closed/flat/open FLRW universes allowed
Friedmann eq does not depend on $f_{\mu\nu}$
- Cosmological linear perturbations
Scalar/vector sectors \rightarrow same as in GR
Tensor sector \rightarrow time-dependent mass
- All homogeneous and isotropic FLRW solutions in the original dRGT theory have infinitely strong coupling and ghost instability
- Stable cosmology realized in (i) new class of cosmological solution or (ii) extended theories
- Minimal theory of massive gravity with 2dof provides a nonlinear completion of dRGT self-accelerating cosmology

Why alternative gravity theories?

