The Physics of Particle Detectors

Lecture Notes

SS 2012

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Physics of Particle Detection

Every effect of particles or radiation can be used as a working principle for a particle detector.

Claus Grupen

Precise knowledge of the processes leading to signals in particle detectors is necessary.

The detectors are nowadays working close to the limits of theoretically achievable measurement accuracy – even in large systems.

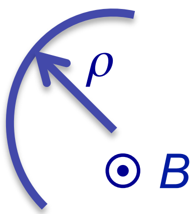
Werner Riegler

Detection and identification of particles

- Detection = particle counting (is there a particle?)
- Identification = measurement of mass and charge of the particle
  (most elementary particle have Ze=±1)

How:

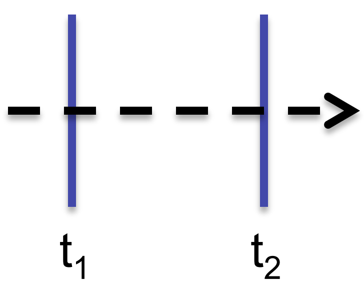
- charged particles are deflected by B fields such that:



$$\rho = \frac{p}{ZeB} \propto \frac{p}{Z} = \frac{\gamma \boxed{m_0 \beta c}}{Z}$$

p = particle momentum
m_0 = rest mass
βc = particle velocity

- particle velocity measured with time-of-flight method



$$\beta \propto \frac{1}{\Delta t}$$

Detection and identification of particles

- Detection = particle counting (is there a particle?)
- Identification = measurement of mass and charge of the particle
  (most elementary particle have Ze=±1)

How:

- kinetic energy determined via a calorimetric measurement

$$E^{kin} = (\gamma - 1) m_0 c^2 \qquad \gamma = \frac{1}{\sqrt{1+\beta^2}}$$

- for Z=1 the mass is extracted from E^{kin} and p
- to determine Z (particle charge) a Z-sensitive variable is e.g. the ionization energy loss

$$\frac{dE}{dx} \propto \frac{Z^2}{\beta^2} \ln\left(a\beta^2\gamma^2\right)$$

a = material-dependent constant

Interaction of particles and γ-radiation with matter

Different type of interactions for charged and neutral particles

Difference "scale" of processes for electromagnetic and strong interactions

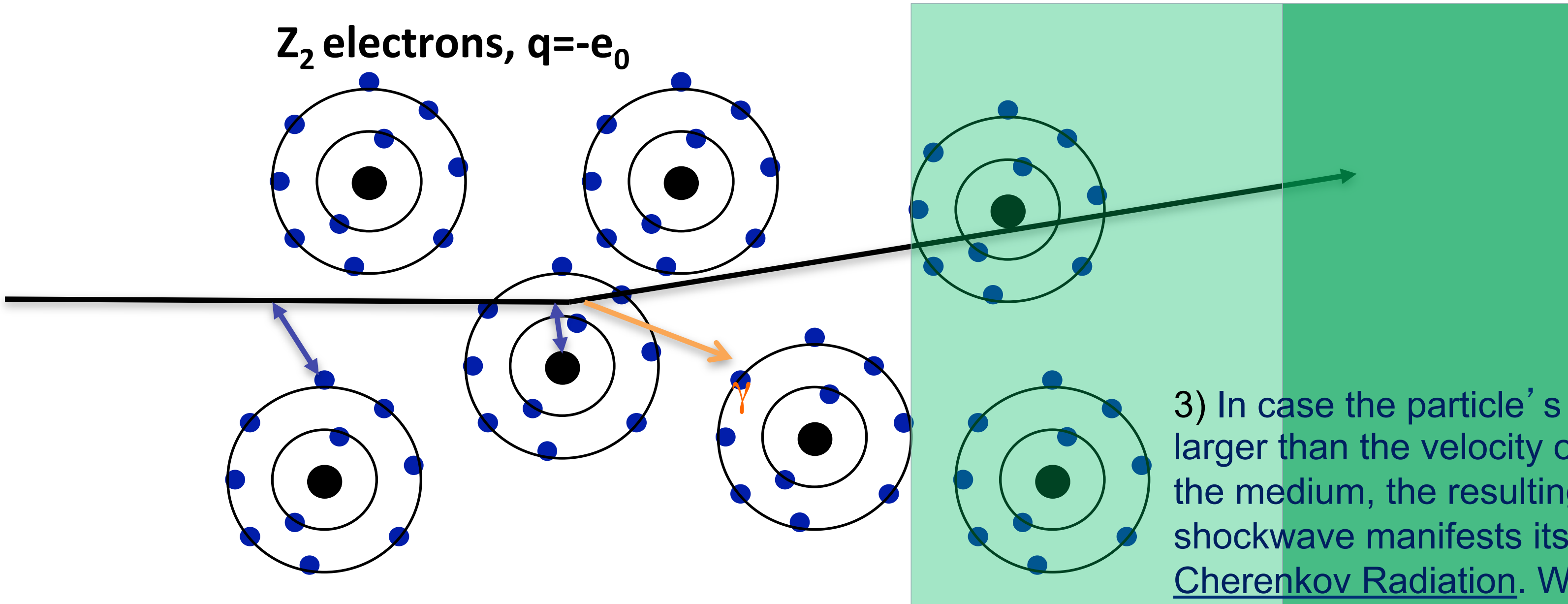
- Detection of charged particles (Ionization, Bremsstrahlung, Cherenkov …)
- Detection of γ-rays (Photo/Compton effect, pair production)
- Detection of neutrons (strong interaction)
- Detection of neutrinos (weak interaction)

Mind: a phenomenological treatment is given, no emphasis on derivation of the formulas, but on the meaning and implication for detector design.

Interaction of charged particles

Three type of electromagnetic interactions:

1. Ionization (of the atoms of the traversed material)
2. Emission of Cherenkov light
3. Emission of transition radiation

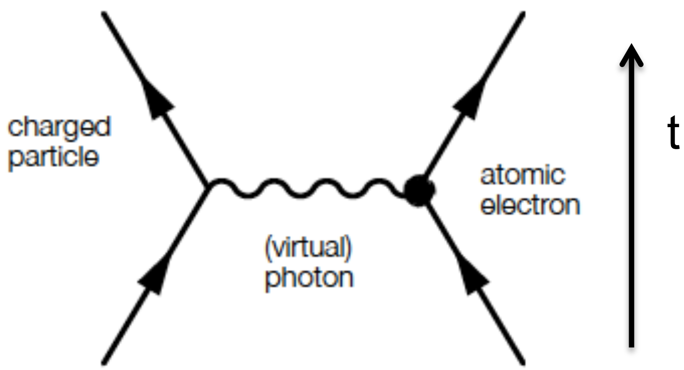


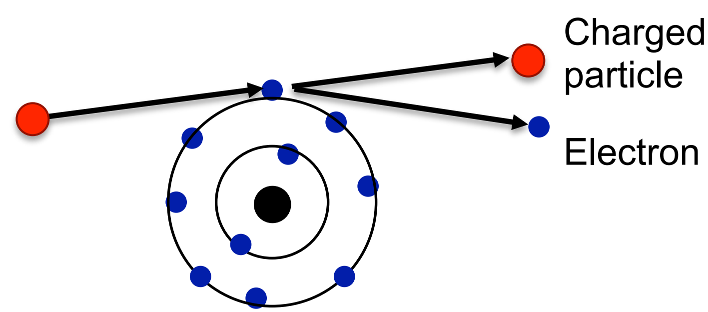
1) Interaction with the atomic electrons. The incoming particle loses energy and the atoms are <u>excited</u> or <u>ionized</u>

2) Interaction with the atomic nucleus. The particle is deflected (scattered) causing <u>multiple scattering</u> of the particle in the material. During this scattering a <u>Bremsstrahlung</u> photon can be emitted.

3) In case the particle's velocity is larger than the velocity of light in the medium, the resulting EM shockwave manifests itself as <u>Cherenkov Radiation</u>. When the particle crosses the boundary between two media, there is a probability of the order of 1% to produced and X ray photon, called <u>Transition radiation</u>.

Energy loss by ionization – dE/dx





First calculate for $Mc^2 \gg m_e c^2$:

Energy loss for heavy charged particle [dE/dx for electrons more complex]

The trajectory of the charged particle is unchanged after scattering

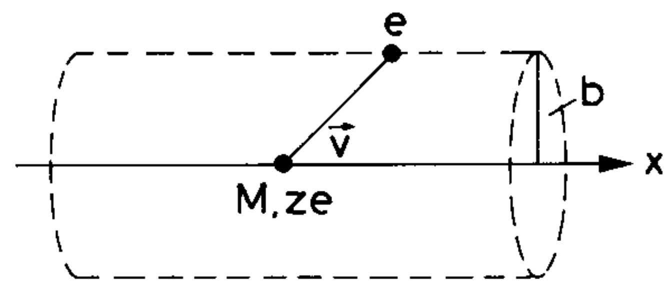
$$\frac{dE}{dx} \propto \frac{Z^2}{\beta^2} \ln\left(a\beta^2\gamma^2\right)$$

a = material-dependent constant

Bohr's calculation of dE/dx

Particle with charge Ze and velocity v moves through a medium with electron density n.

Electrons considered free and initially at rest



The momentum transferred to the electron is:

$$\Delta p_\perp = \int F_\perp dt = \int F_\perp \frac{dt}{dx} dx = \int F_\perp \frac{dx}{v}$$

Symmetry!

$$\Delta p_\parallel \ : \quad \text{averages to zero}$$

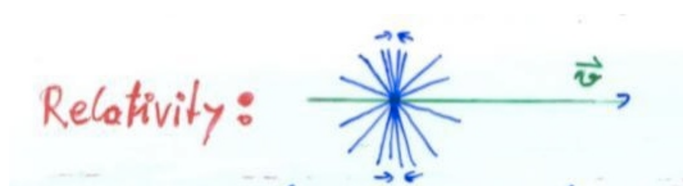
Gauss law: *The electric flux through any closed surface is proportional to the enclosed electric charge*

$$F_\perp = eE_\perp$$

$$\Delta p_\perp = e \int E_\perp \frac{dx}{v} \quad \leftarrow \quad \int E_\perp (2\pi b)\, dx = 4\pi(ze) \rightarrow \int E_\perp dx = \frac{2ze}{b}$$

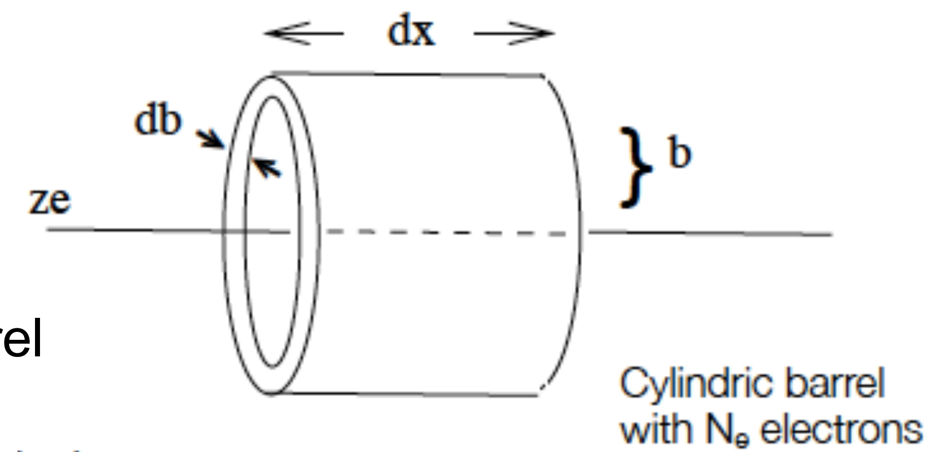
$$\downarrow$$

$$= \frac{2ze^2}{bv}$$



Bohr's calculation of dE/dx

$$\boxed{\Delta E(b) = \frac{\Delta p^2}{2m_e}}$$ with $$\boxed{\Delta p_\perp = \frac{2ze^2}{bv}}$$



Energy transfer to a single electron

For n electrons distributed on a barrel

n = $N_e \cdot (2\pi b) \cdot$ db dx

Energy loss per path length dx for distance between b and b+db in medium with electron density N_e:

$$-dE(b) = \frac{\Delta p^2}{2m_e} \cdot 2\pi N_e b\ db\ dx = \frac{(2ze^2)^2}{2m_e(bv)^2} \cdot 2\pi N_e b\ db\ dx = \frac{4\pi N_e z^2 e^4}{m_e v^2} \cdot \frac{db}{b} dx$$

(-) Energy loss !

$$-\frac{dE}{dx} = \frac{4\pi N_e z^2 e^4}{m_e v^2} \cdot \int_{b_{\min}}^{b_{\max}} \frac{db}{b} = \frac{4\pi N_e z^2 r_e^2 m_e c^2}{\beta^2} \ln \frac{b_{\max}}{b_{\min}} \qquad (r_e m_e c^2 = e^2)$$

Diverges for b ➛ 0; integration only for relevant range [b_{min}, b_{max}]

Bohr's calculation of dE/dx

Stopping power: $$-\frac{dE}{dx} = \frac{4\pi N_e z^2 r_e^2 m_e c^2}{\beta^2} \ln\frac{b_{\max}}{b_{\min}}$$

Determination of the relevant range [b_{min}, b_{max}]:

b_{min} : for head-on collisions in which the kinetic energy transferred is maximum

$$\rightarrow E_{\max}(b_{\min}) = \frac{(2ze^2)^2}{2m_e v^2 b_{\min}^2}$$

$$\leftarrow E_{\max} = \frac{1}{2}\gamma^2 m_e (2v)^2 = 2m_e c^2 \beta^2 \gamma^2$$

(calculated from conservation of momentum)

$$b_{\min} = \frac{ze^2}{\gamma m_e v^2}$$

b_{max}: particle still moves faster than the e in the atomic orbit (~ electron at rest).

electrons are bound to atoms with an average orbital frequency $\langle \upsilon_e \rangle$

the interaction time has to be smaller or equal to $1/\langle \upsilon_e \rangle$

$$b_{\max} = \frac{\gamma v}{\langle \upsilon_e \rangle}$$

or distance at which the kinetic energy transferred is minimum E_{min}= I (mean ionization potential)

In this interval the stopping power does not diverge: Bohr classical formula

$$\boxed{-\frac{dE}{dx} = \frac{4\pi N_e z^2 r_e^2 m_e c^2}{\beta^2} \ln\frac{\gamma^2 m v^3}{ze^2 \langle \upsilon_e \rangle} = \frac{4\pi N_e z^2 r_e^2 m_e c^2}{\beta^2} \ln\left(\frac{2m_e c^2 \beta^2 \gamma^2}{I}\right)}$$

Behte-Bloch equation

Quantum mechanic calculation of Bohr stopping power

Valid for heavy charged particles ($m_{incident}$>>m_e), e.g. proton, k, π, μ

$$-\left\langle \frac{dE}{dx} \right\rangle = \boxed{2\pi N_a r_e^2 m_e c^2} \rho \frac{Z}{A} \frac{z^2}{\beta^2} \left[\ln\left(\frac{2m_e c^2 \beta^2 \gamma^2}{I^2} W_{\max}\right) - 2\beta^2 - \delta(\beta\gamma) - \frac{C}{Z} \right]$$

=0.1535 MeV cm²/g

$$\frac{dE}{dx} \propto \frac{Z^2}{\beta^2} \ln\left(a\beta^2\gamma^2\right)$$

Fundamental constants

r_e=classical radius of electron
m_e=mass of electron
N_a=Avogadro’s number
c =speed of light

Absorber medium

I = mean ionization potential
Z = atomic number of absorber
A = atomic weight of absorber
ρ = density of absorber
δ = density correction
C = shell correction

Incident particle

z = charge of incident particle
β = v/c of incident particle
γ = $(1-\beta^2)^{-1/2}$
W_{max}= max. energy transfer in one collision

Note: the classical dE/dx formula contains many features of the QM version: $(z/\beta)^2$, & ln[]

$$\frac{-dE}{dx} = \frac{4\pi N_e z^2 r_e^2 m_e c^2}{\beta^2} \ln \frac{b_{\max}}{b_{\min}}$$

Exercise:

- Calculate the stopping power of 5 MeV **α** -particles in air.

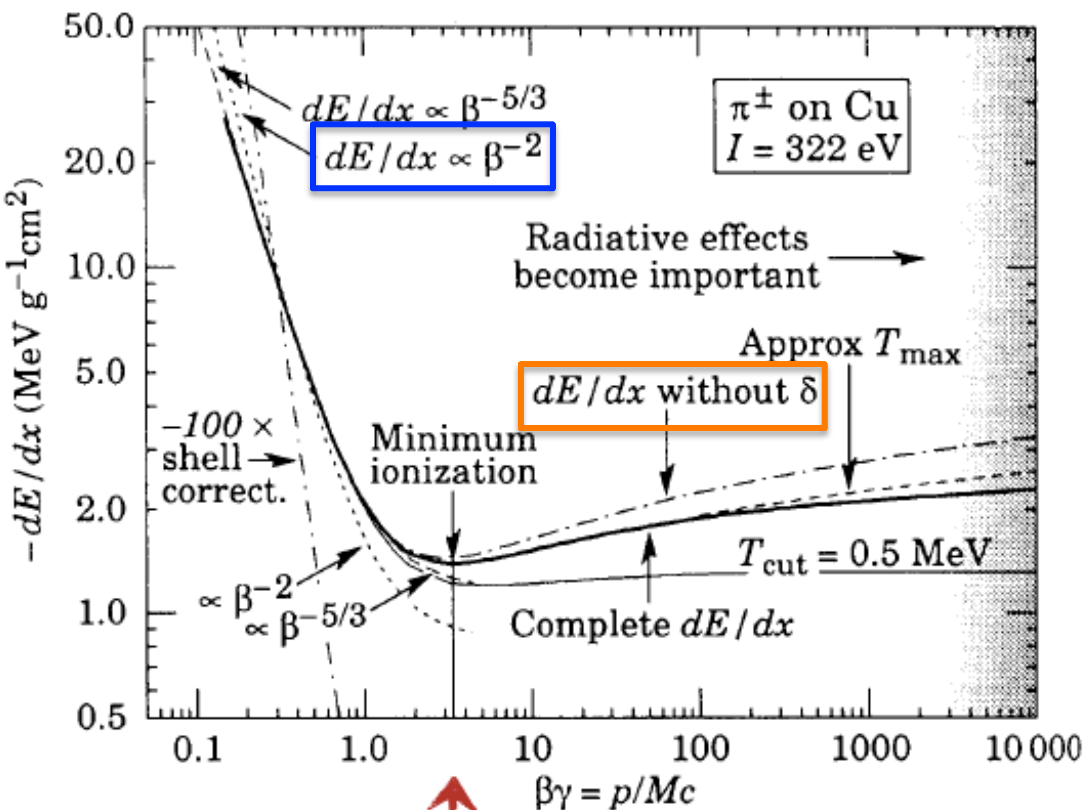
$$-\left\langle \frac{dE}{dx} \right\rangle = 4\pi N_a r_e^2 m_e c^2 \rho \frac{Z}{A} \frac{z^2}{\beta^2} \left[\ln(\frac{W_{\max}}{I}) - \beta^2 \right]$$

$$W_{\max} = 2m_e c^2 \beta^2 \gamma^2$$
$$= 2 \cdot 0.511 \cdot (0.05177)^2$$
$$= 2.739 \cdot 10^{-3} MeV$$

$$I = 12Z + 7, Z < 13 \quad \text{[eV]}$$
$$I = 9.76Z + 5.58Z^{-0.19}, Z \geq 13$$

$$-\left\langle \frac{dE}{\rho dx} \right\rangle = 0.3054 \frac{1}{2} \frac{2^2}{(0.0517)^2} \left[\ln(\frac{W_{\max}}{I}) - (0.0517)^2 \right]$$

Understanding Bethe-Bloch



$\beta\gamma = 3\text{-}4$

$$\frac{dE}{dx} \propto \frac{Z^2}{\beta^2} \ln\left(a\beta^2\gamma^2\right)$$

Minimum ionizing particles (MIP): $\boxed{\beta\gamma = 3\text{-}4}$

dE/dx falls $\boxed{\sim \beta^{-2}}$; kinematic factor
[precise dependence: ~ $\beta^{-5/3}$]

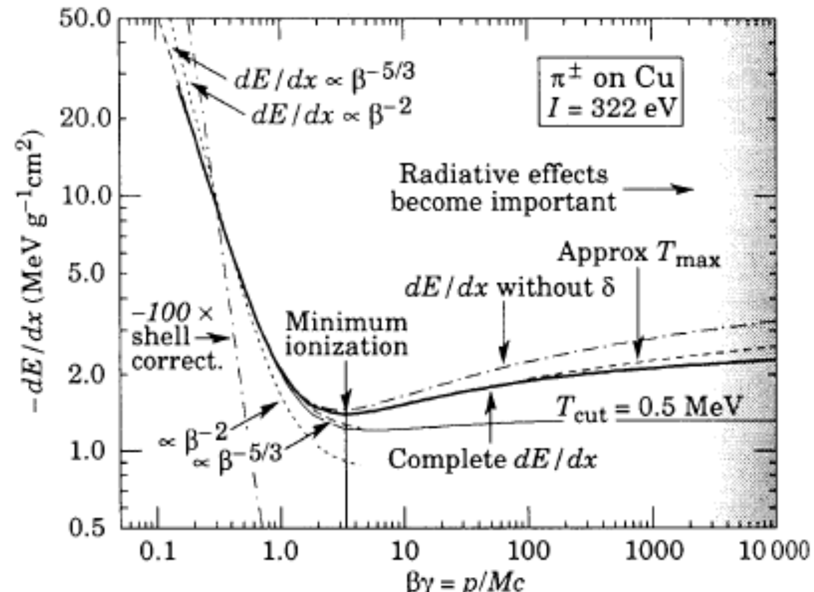
dE/dx rises $\boxed{\sim \ln(\beta\gamma)^2}$; relativistic rise
[rel. extension of transversal E-field]

Saturation at large ($\beta\gamma$) due to density effect (correction δ)
[polarization of medium]

Units: MeV g^{-1} cm^2

MIP looses ~ 13 MeV/cm
[density of copper: 8.94 g/cm^3]

Understanding Bethe-Bloch



$1/\beta^2$-dependence:

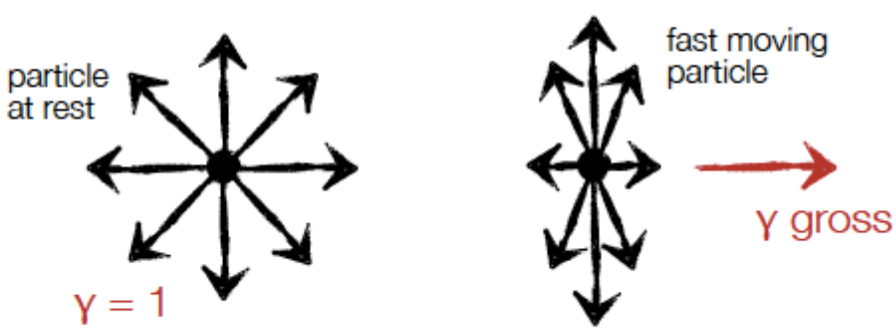
Remember:

$$\Delta p_\perp = \int F_\perp dt = \int F_\perp \frac{dx}{v}$$

i.e. slower particles feel electric force of atomic electron for longer time ...

Relativistic rise for $\beta\gamma > 4$:

High energy particle: transversal electric field increases due to Lorentz transform; $E_y \to \gamma E_y$. Thus interaction cross section increases ...



Corrections:

low energy : shell corrections

high energy : density corrections

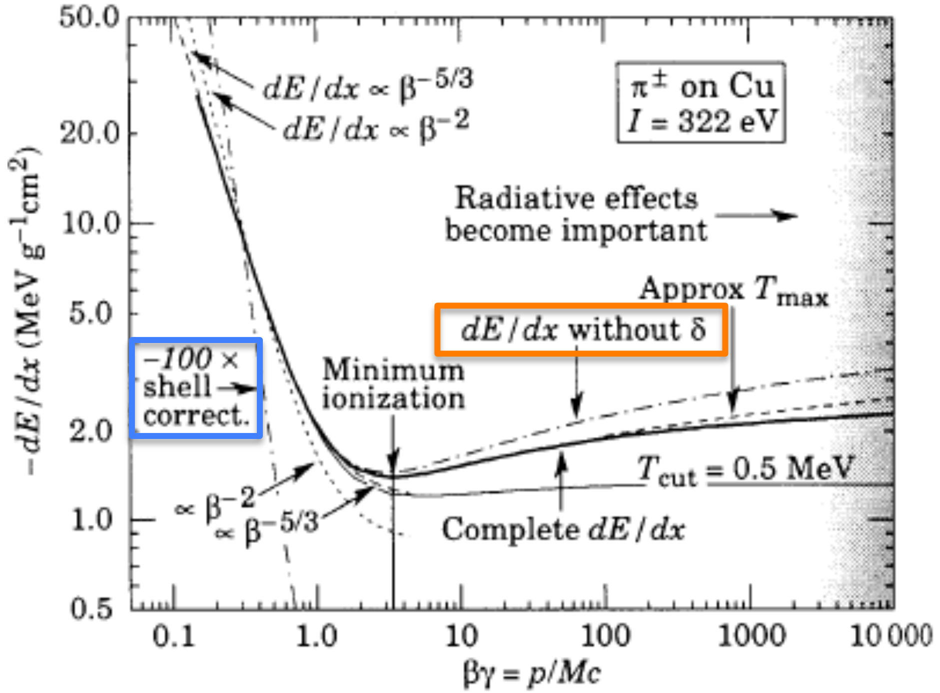
Understanding Bethe-Bloch

$$-\left\langle \frac{dE}{dx} \right\rangle = 2\pi N_a r_e^2 m_e c^2 \rho \frac{Z}{A}\frac{z^2}{\beta^2}\left[\ln\left(\frac{2m_e c^2 \beta^2 \gamma^2}{I^2} W_{\max}\right) - 2\beta^2 - \boxed{\delta(\beta\gamma)} - \boxed{\frac{C}{Z}}\right]$$

Density correction
[saturation at high energy]
Density dependent polarization effect ...

Shielding of electrical field far from particle path; effectively cuts of the long range contribution ...
More relevant at high γ

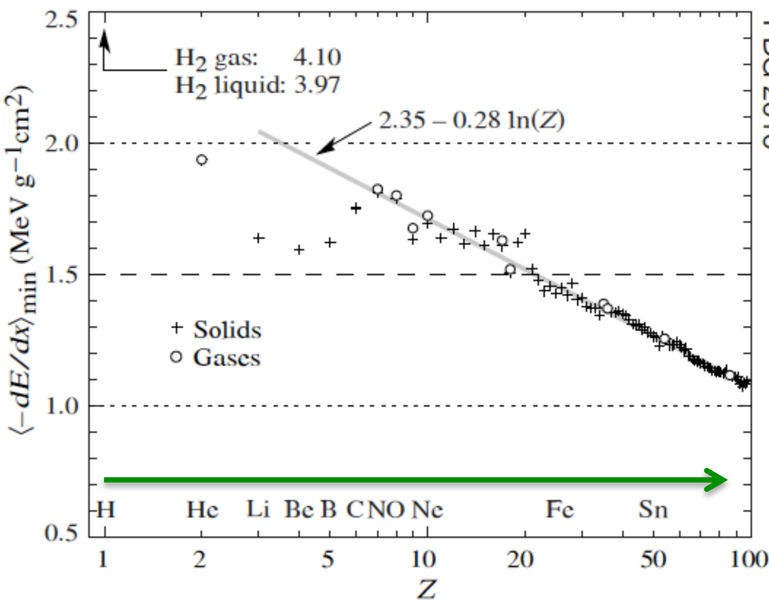
Shell correction [small effect]
For small velocity assumption that electron is at rest breaks down, Capture process is possible

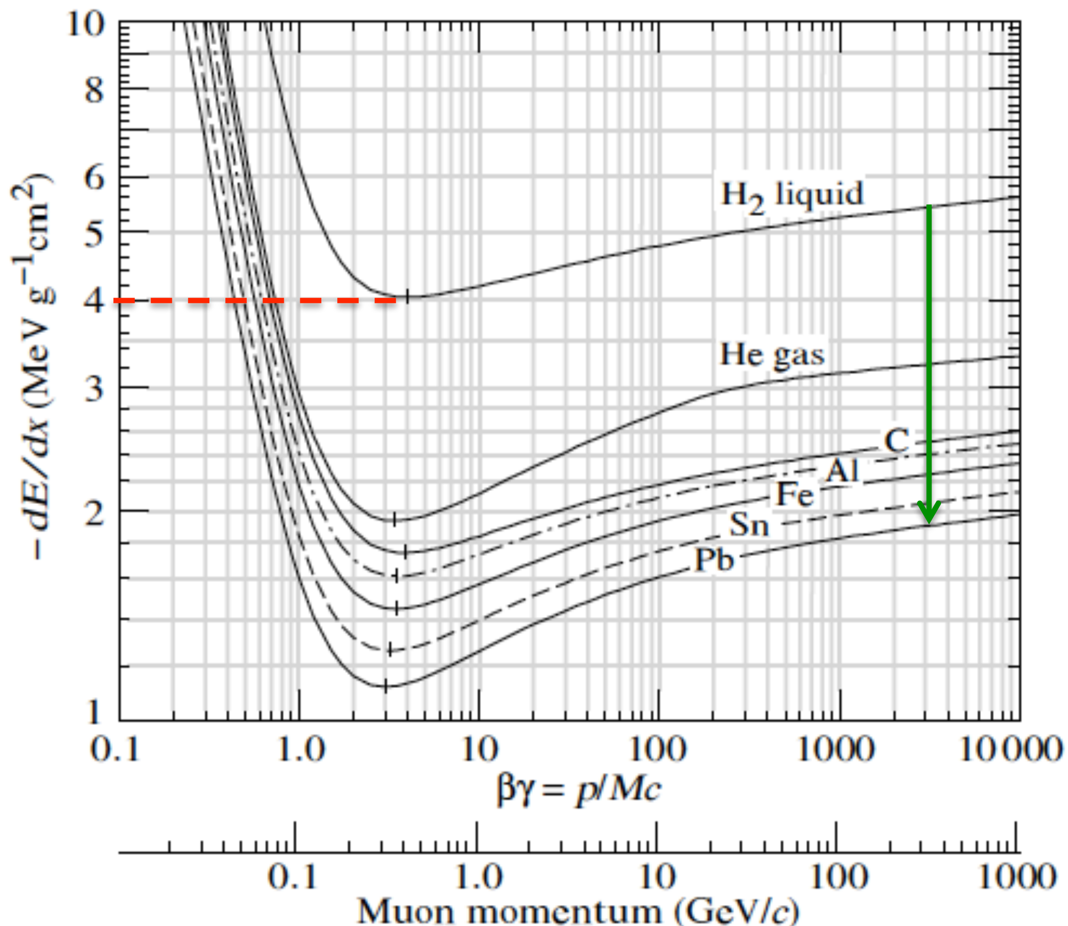


Dependence of A and Z

$$-\left\langle \frac{dE}{dx} \right\rangle = 2\pi N_a r_e^2 m_e c^2 \rho \frac{Z}{A} \frac{z^2}{\beta^2} \left[\ln\left(\frac{2 m_e c^2 \beta^2 \gamma^2}{I^2} W_{max}\right) - 2\beta^2 - \delta(\beta\gamma) - \frac{C}{Z} \right]$$

Minimum ionization:
ca. 1 - 2 MeV/g cm^{-2}
[H_2: 4 MeV/g cm^{-2}]





Some numbers

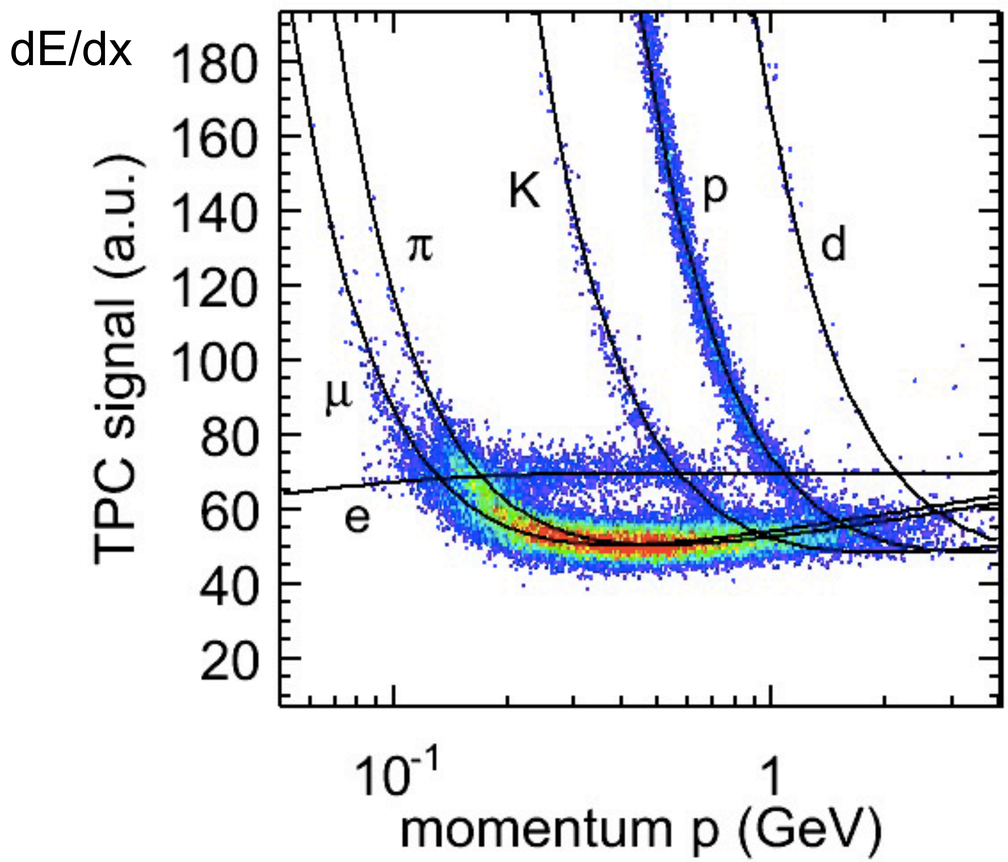
Minimum ionization:
ca. 1 - 2 MeV/g cm^{-2}

i.e. for a material with $\rho = 1\ g/cm^3$ ➔ $dE/dx = 1\text{-}2\ MeV/cm$

Example :
Iron: Thickness = 100 cm; $\rho = 7.87\ g/cm^3$
$dE \approx 1.4\ MeV\ g^{-1}\ cm^2 * 100\ cm * 7.87 g/cm^3 = 1102\ MeV$

→ A 1 GeV Muon can traverse 1m of Iron

dE/dx for particle identification



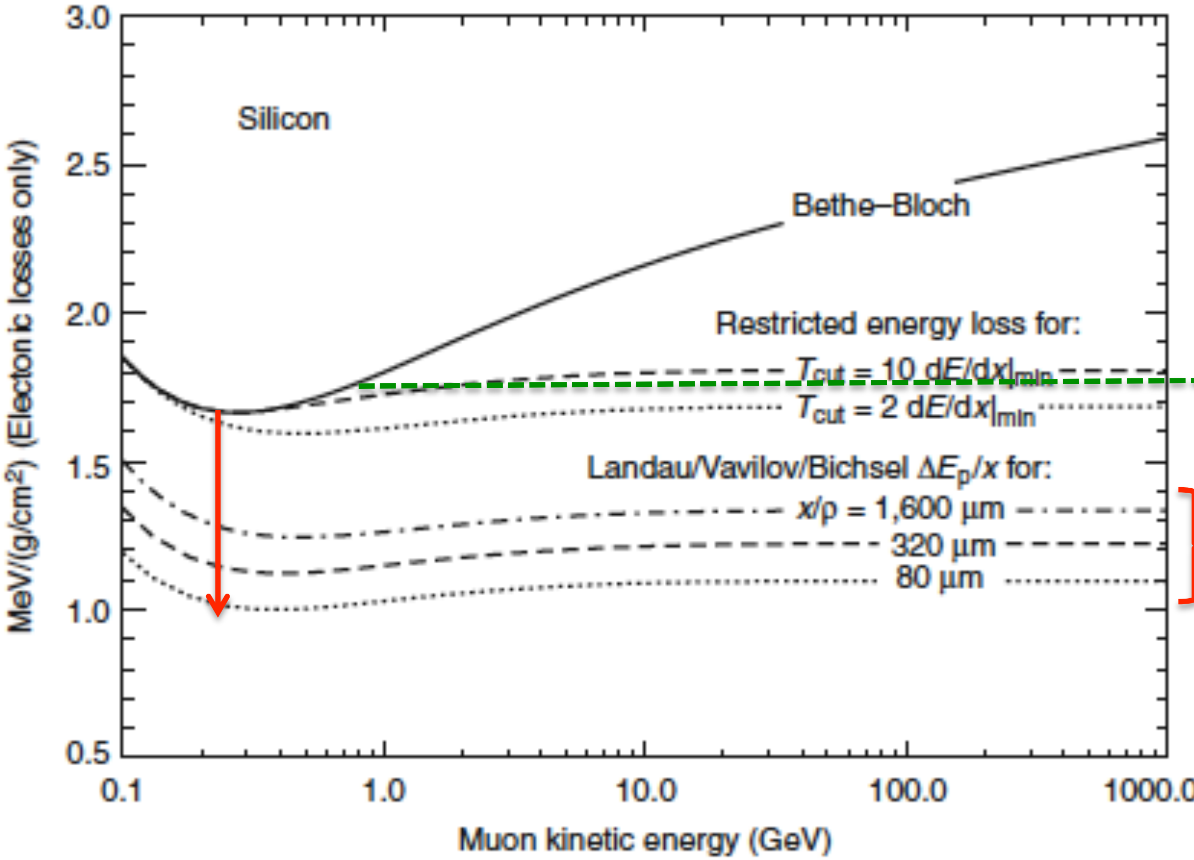
The energy loss as a function of momentum p=mcβγ is dependent on the particle mass

By measuring the particle momentum (deflection in a magnetic field) and the energy loss one gets the mass of the particle, i.e. particle ID

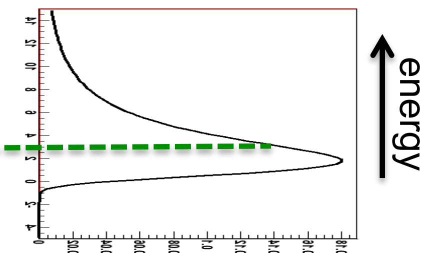
(at least in a certain energy region)

Dependence on absorber thickness

- The Bethe-Bloch equation describes the mean energy loss
- When a charged particle passes the layer of material with thickness x , the energy distribution of the δ-electrons and the fluctuations of their number (n_δ) cause fluctuations of the energy losses ΔE



The energy loss ΔE in a layer of material is distributed according to the Landau function:

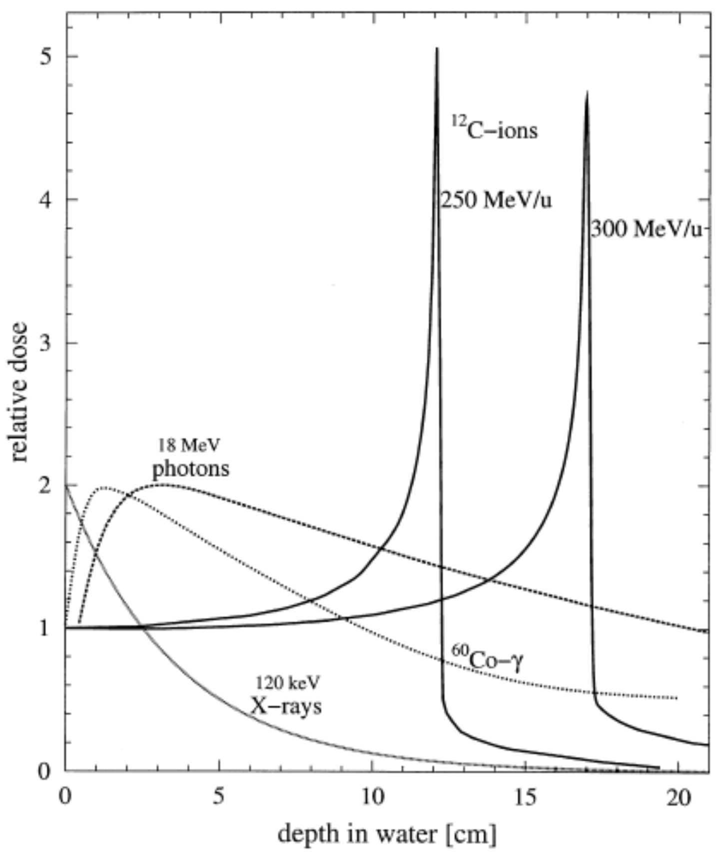


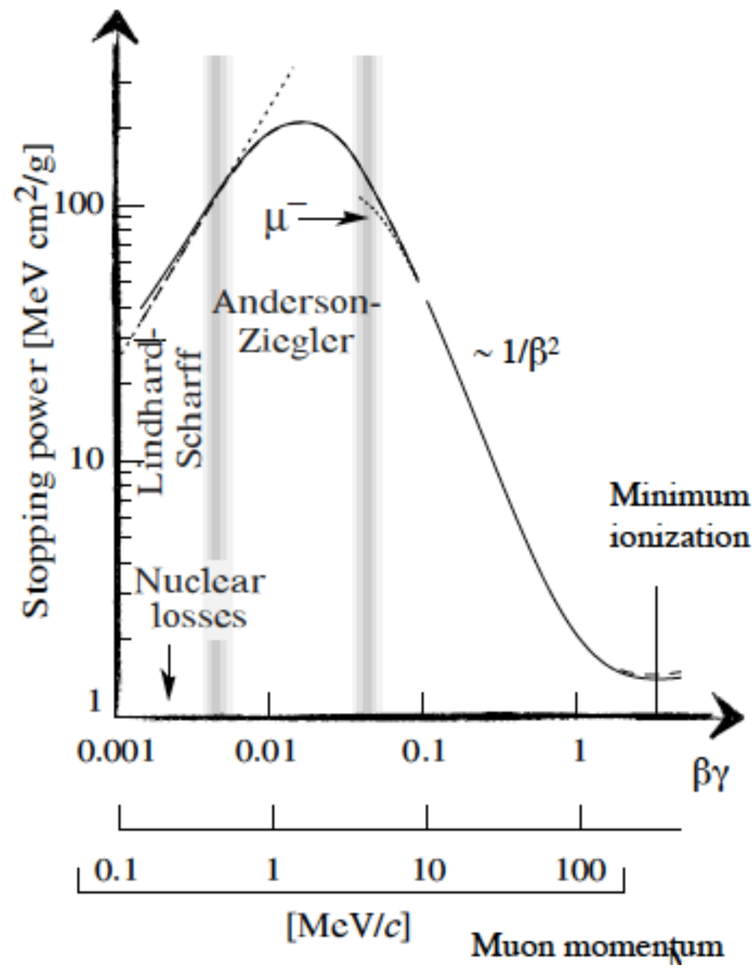
For a realistic thin silicon detector $n_\delta \lesssim$ 1-10, fluctuations do not follow the Landau distribution

Energy loss at small momenta

- energy loss increases at small βγ
- particles deposit most of their energy at the end of their track

➔ Bragg peak

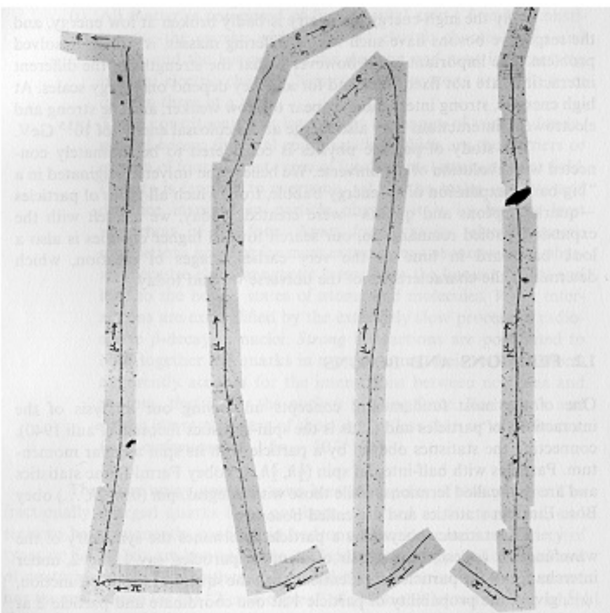




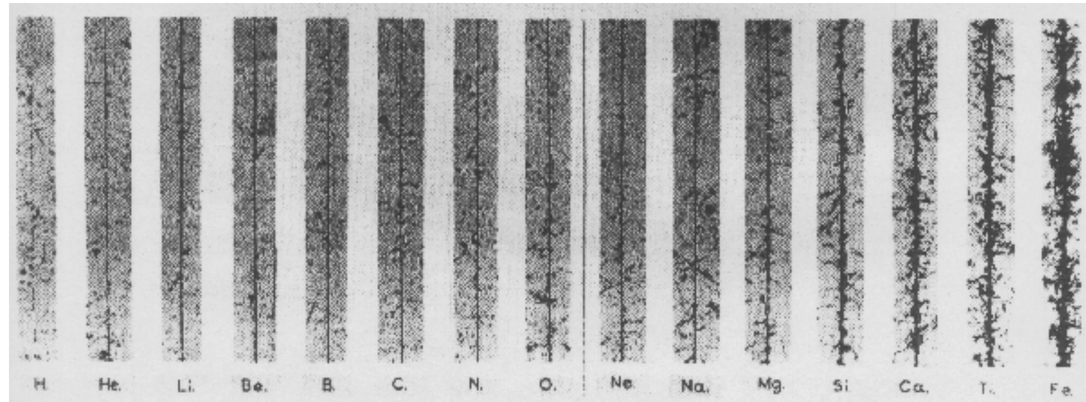
➔ Important effect for tumor therapy

Energy loss at small momenta

Small energy loss
→ Fast Particle

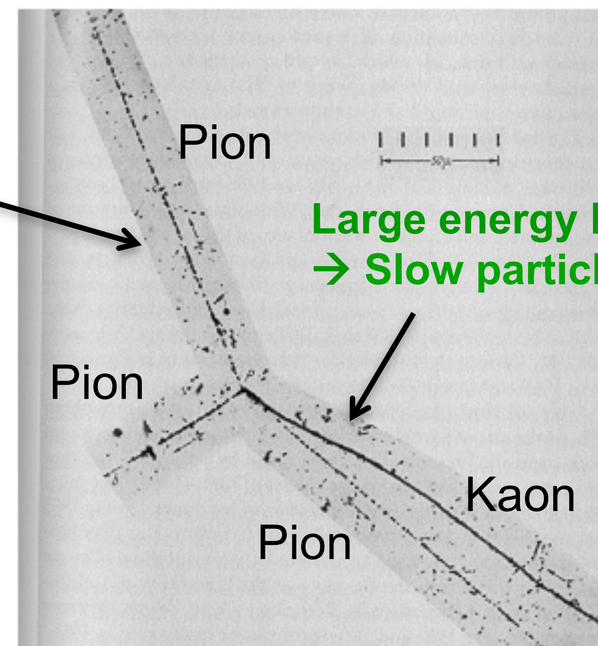


Discovery of muon and pion



Cosmis rays: dE/dx α Z^2

Small energy loss
→ Fast particle



Large energy loss
→ Slow particle

Mean particle range

Integrate over energy loss

from the total energy T to zero

$$R(T) = \int_0^T \left[-\frac{dE}{dx} \right]^{-1} dE$$

More often use empirical formula

(➔ see exercise)

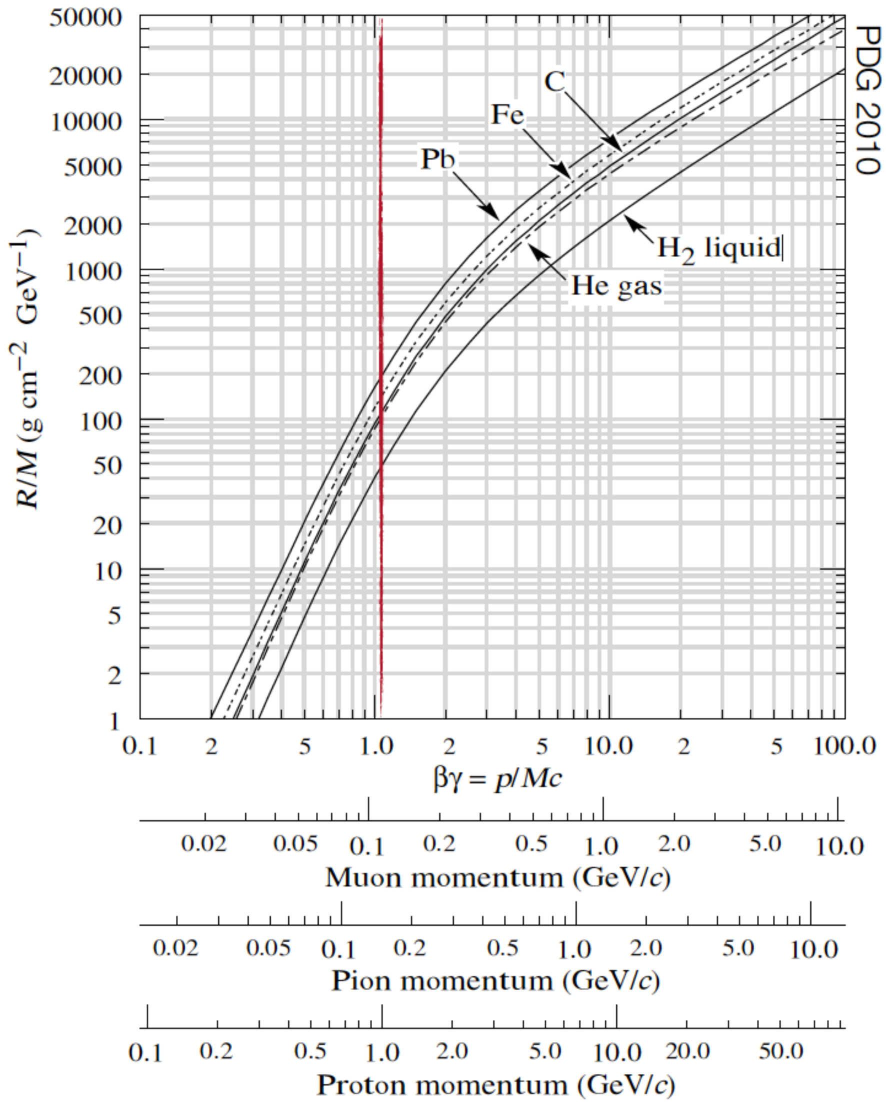
Example:

Proton with p = 1 GeV

Target: lead with ρ = 11.34 g/cm^3

R/M = 200 g cm^{-2} GeV^{-1}

➙ R = 200/11.34/1 cm ~ 20 cm



Exercise:

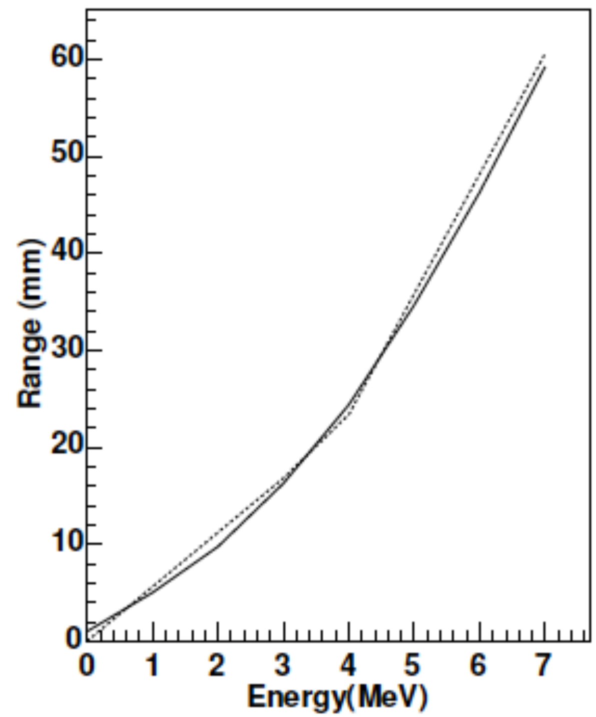
- Compute the range of 4 MeV α -particles in air and tissue.

Several empirical and semi-empirical formulae have been proposed to compute range of α-particles in air. For example, (56),

$$R_\alpha^{air}[mm] = \begin{cases} e^{1.61\sqrt{E_\alpha}} & \text{for } E_\alpha < 4\,MeV \\ (0.05E_\alpha + 2.85)E_\alpha^{3/2} & \text{for } 4\,MeV \le E_\alpha \le 15\,MeV \end{cases} \tag{2.4.23}$$

and

$$R_\alpha^{air}[cm] = \begin{cases} 0.56E_\alpha & \text{for } E_\alpha < 4\,MeV \\ 1.24E_\alpha - 2.62 & \text{for } 4\,MeV \le E_\alpha < 8\,MeV \end{cases}$$



Scaling the range to other materials

$$R_\alpha^x = 3.37 \times 10^{-4} R_\alpha^{air} \frac{\sqrt{A_x}}{\rho_x}.$$

Energy loss for electrons

Bethe-Bloch formula needs modification

Incident and target electron have same mass m_e

Scattering of identical, undistinguishable particles

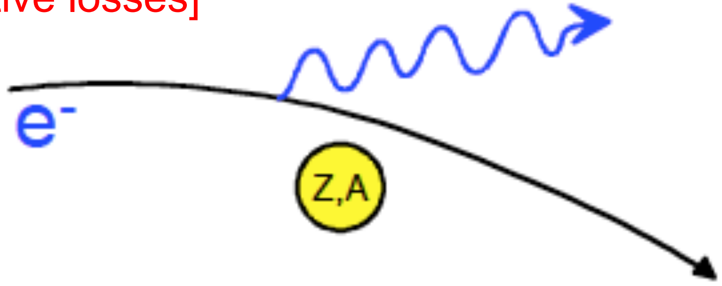
$$-\left\langle \frac{dE}{dx} \right\rangle_{Ionization} \propto \ln(E)$$

Dominating process for E_e > 10-30 MeV is not anymore ionization but

Bremsstrahlung: photon emission by an electron accelerated in Coulomb field of nucleus

[radiative losses]

$$-\left\langle \frac{dE}{dx} \right\rangle_{Brems} \propto \frac{E}{m^2}$$



energy loss proportional to $1/m^2$ → main relevance for electrons (or ultra-relativistic muons)

Bremsstrahlung

$$-\left\langle \frac{dE}{dx} \right\rangle_{Brems} = \frac{E}{X_0}$$

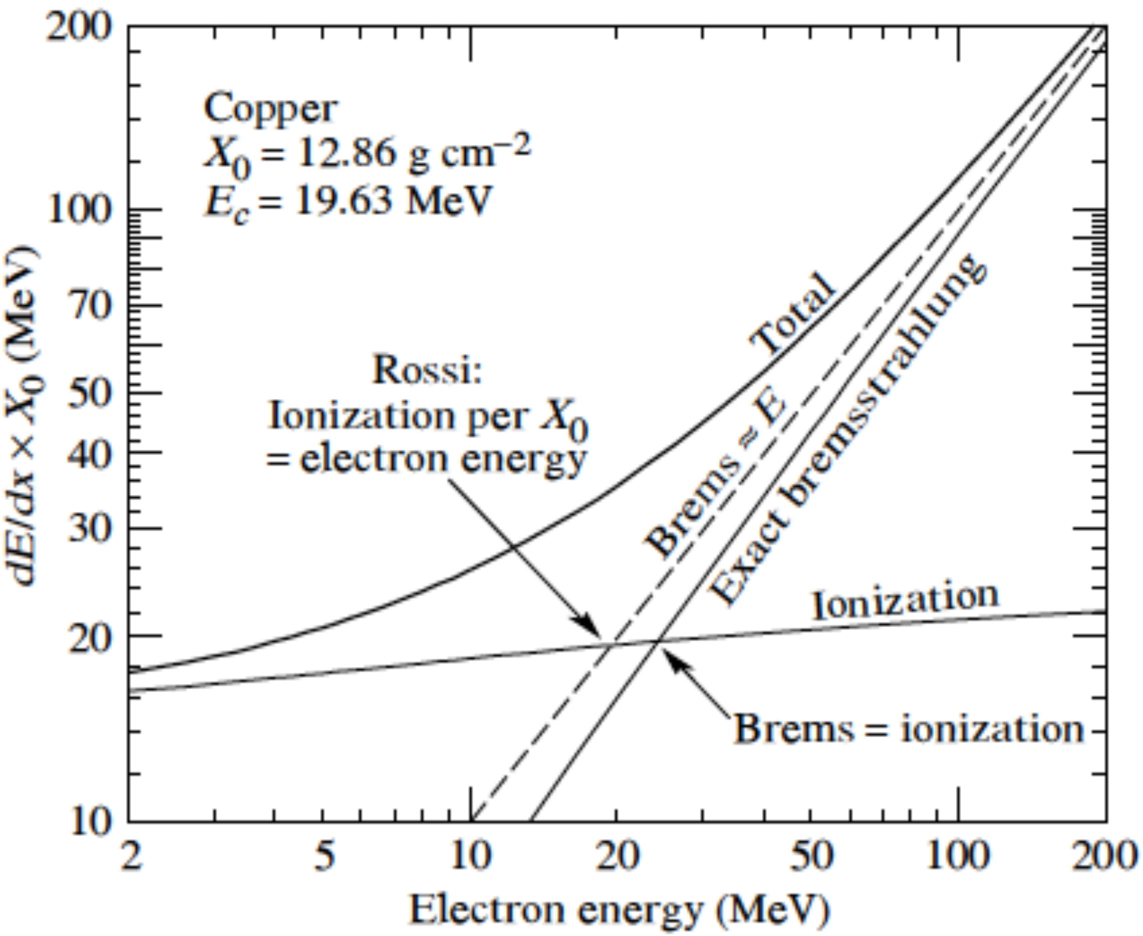
X_0 = radiation length in [g/cm^2]

$$\boxed{X_0 = \frac{A}{4\alpha N_A Z^2 r_e^2 \ln \frac{183}{Z^{1/3}}}}$$

After passage of one X_0 electron has lost all but $(1/e)^{th}$ of its energy (63%)

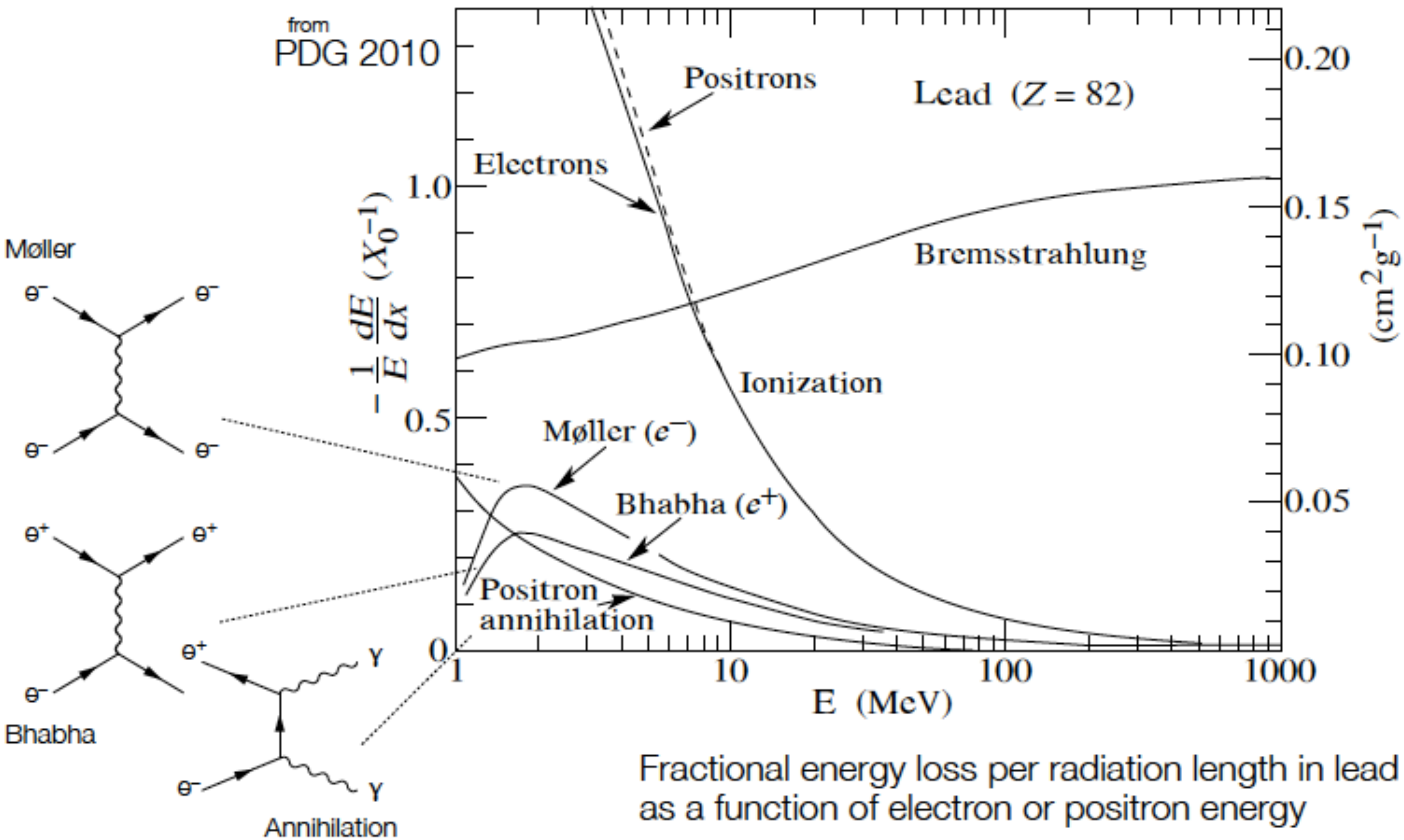
E_c = critical energy

$$\boxed{\left.\frac{dE}{dx}(E_c)\right|_{Brems} = \left.\frac{dE}{dx}(E_c)\right|_{Ion}}$$



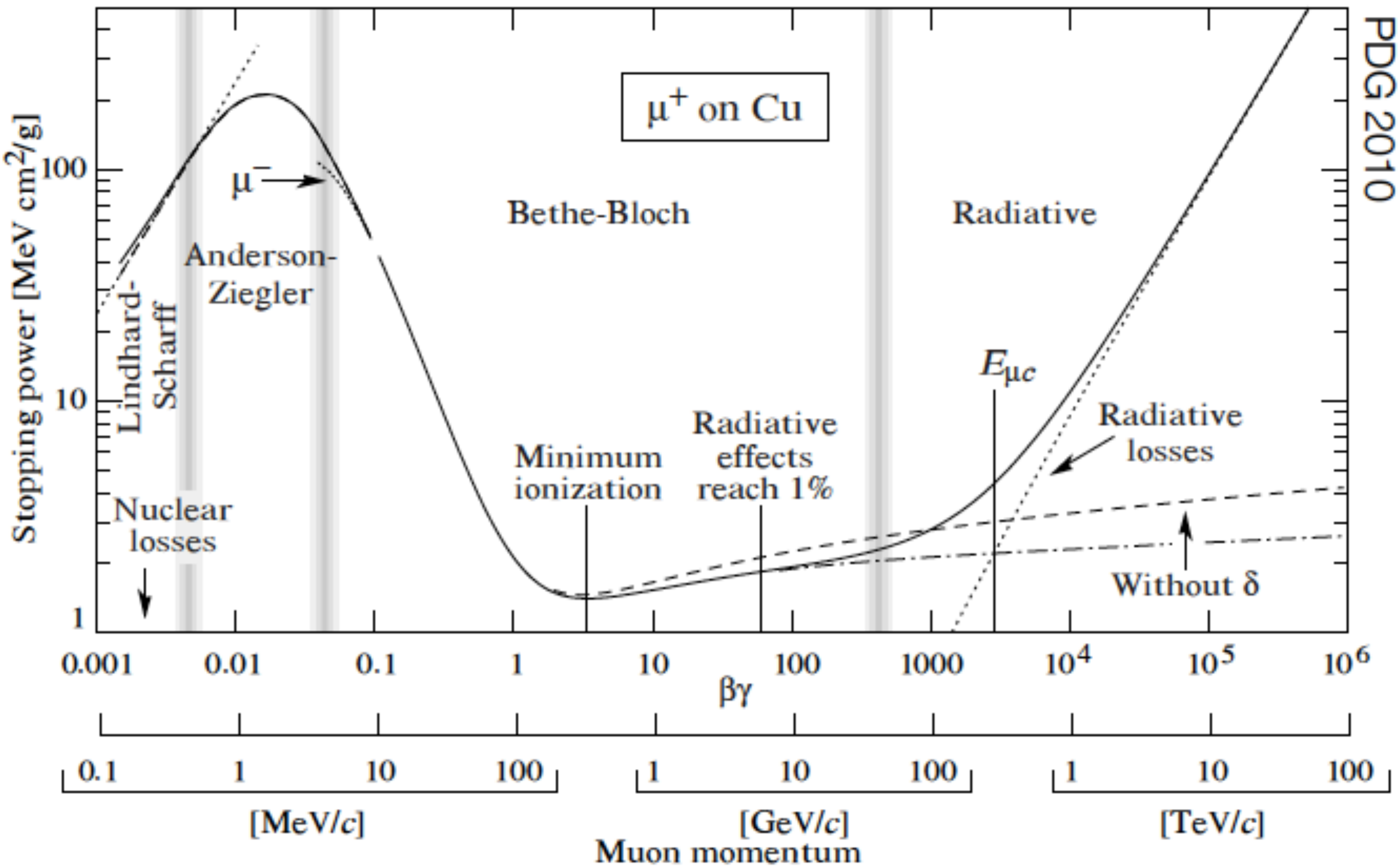
Total energy loss for electrons

from PDG 2010



Fractional energy loss per radiation length in lead as a function of electron or positron energy

Energy loss summary

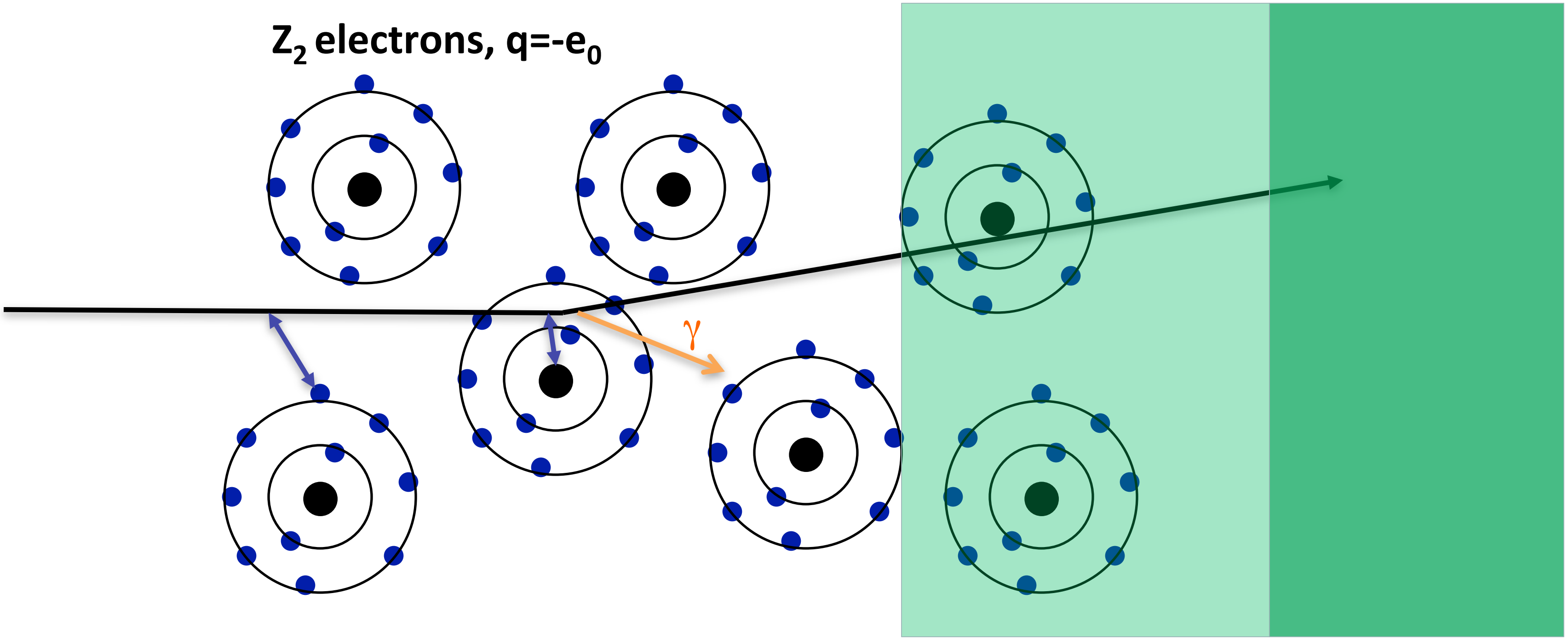


Second part

Interaction of charged particles

Three type of electromagnetic interactions:

1. Ionization (of the atoms of the traversed material) ✔
2. Emission of Cherenkov light
3. Emission of transition radiation



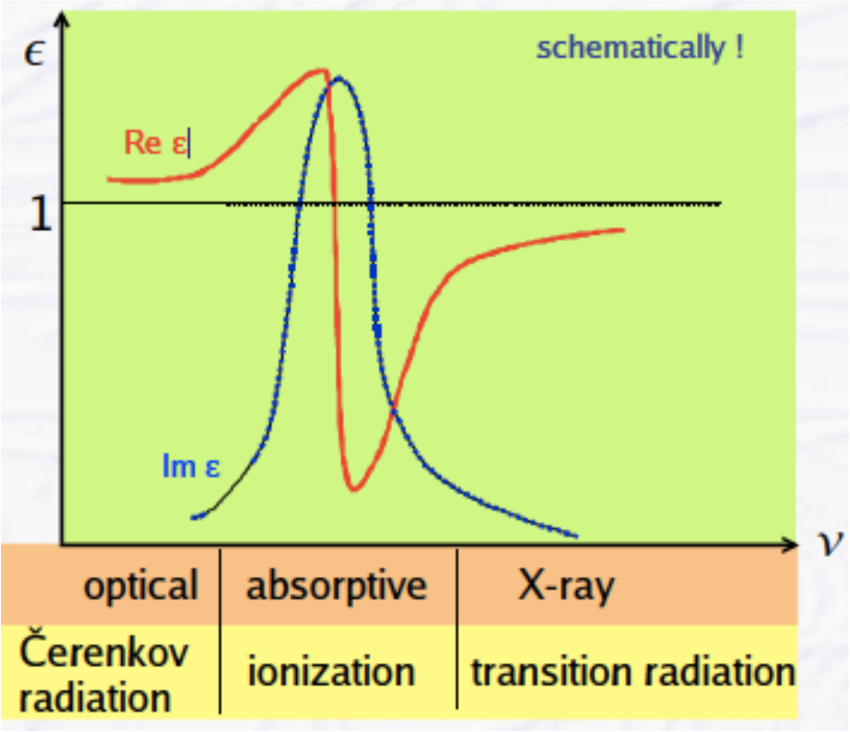
Energy Loss by Photon Emission

Ionization is one way of energy loss
emission of photons is another...

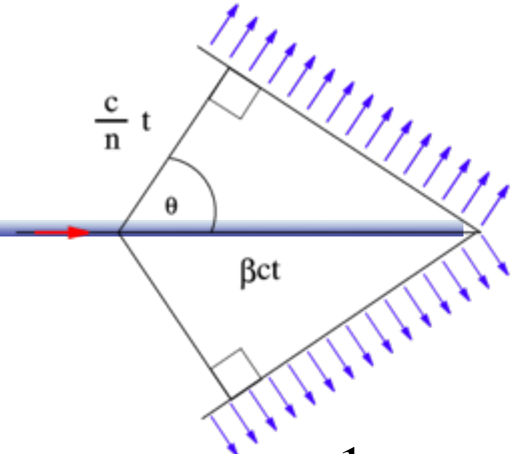
Optical behavior of medium is characterized by the (complex) dielectric constant **ε**

Re √**ε** **=** n Refractive index

Im **ε** **=** k Absorption parameter



Cherenkov radiation



Velocity of the particle: v
Velocity of light in a medium of refractive index n: c/n

Threshold condition for Cherenkov light emission: $v_{th} \geq \frac{c}{n} \Rightarrow \beta_{th} \geq \frac{1}{n}$

$$-\left\langle \frac{dE}{dx} \right\rangle_{Cherenkov} \propto z^2 \sin^2 \theta_c \qquad \cos\theta_c = \frac{1}{n\beta}$$

for water $\theta_c^{max} = 42^\circ$
for neon at 1 atm θ_c^{max} =11mrad

Energy loss by Cherenkov radiation very small w.r.t. ionization (< 1%)

Typically O(1-2 keV / cm) or O(200-1000) visible photons / cm

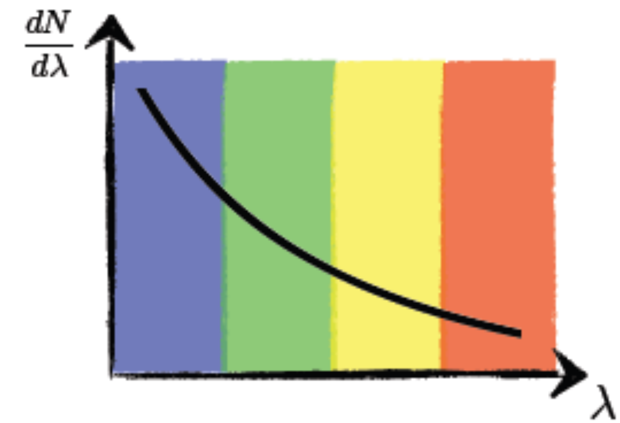
Visible photons:
E = 1 - 5 eV; λ = 300 - 600 nm

Cherenkov radiation

In a Cherenkov detector the produced photons are measured

Number of emitted photons per unit of length:

- wavelength dependence ~ $1/\lambda^2$



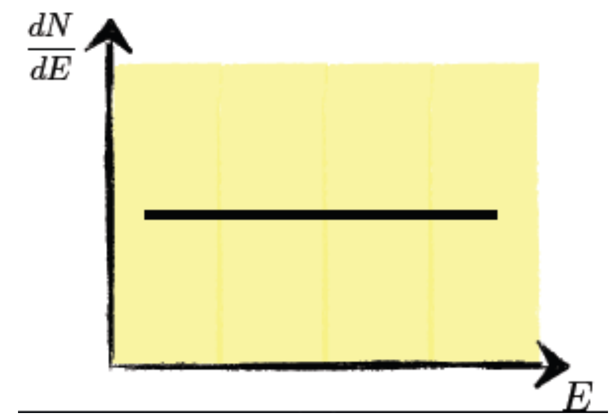
$$\frac{d^2N}{d\lambda dx} = \frac{2\pi\alpha z^2}{\lambda^2}\left(1 - \frac{1}{\beta^2 n^2(\lambda)}\right) = \frac{2\pi\alpha z^2}{\lambda^2}\sin^2\theta_C$$

Integrate over sensitivity range:
[for typical Photomultiplier]

$$\frac{dN}{dx} = \int_{350\text{ nm}}^{550\text{ nm}} d\lambda \frac{d^2N}{d\lambda dx}$$

$$= 475\, z^2 \sin^2\theta_C \text{ photons/cm}$$

- energy dependence ~ constant



$$\frac{d^2N}{dEdx} = \frac{z^2\alpha}{\hbar c}\left(1 - \frac{1}{\beta^2 n^2(\lambda)}\right) = \frac{z^2\alpha}{\hbar c}\sin^2\theta_C$$

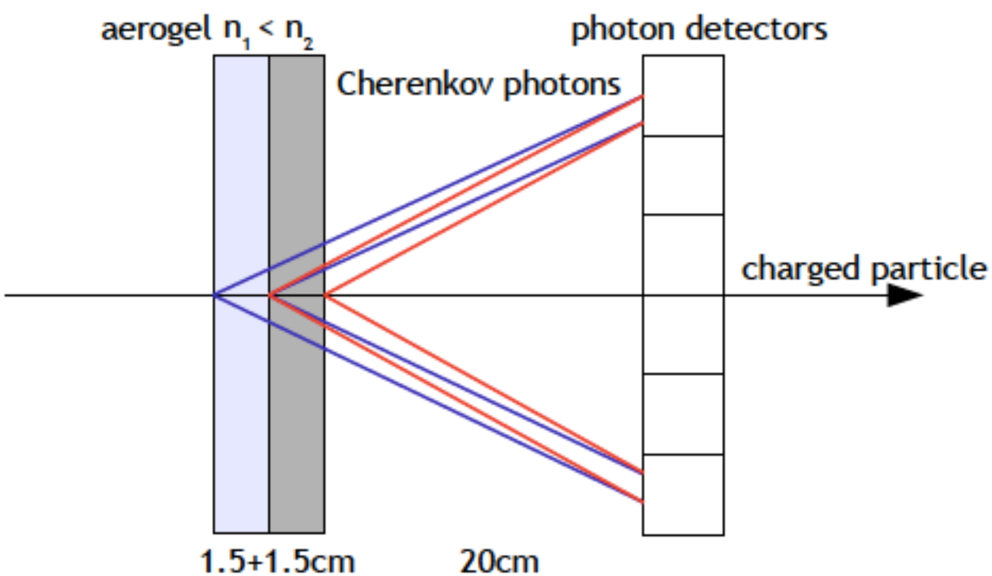
$$\approx \text{const}$$

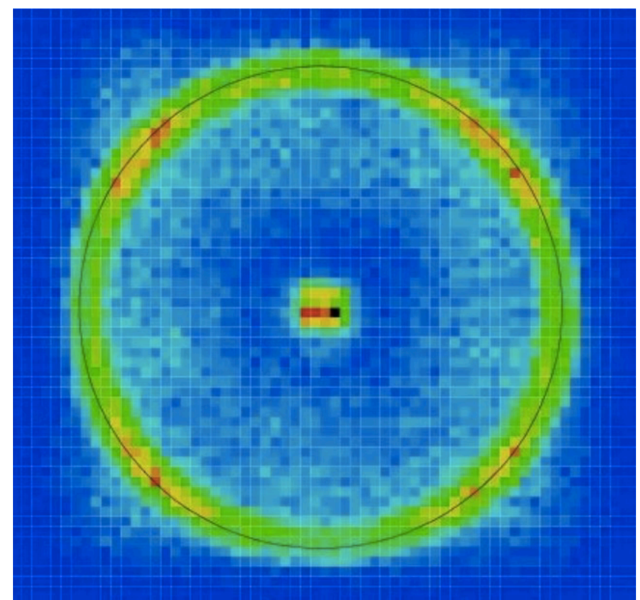
$$\frac{d^2N}{dEdx} = 370 \sin^2\theta_C \text{ eV}^{-1}\text{ cm}^{-1}$$

Detection of Cherenkov radiation

Parameters of Typical Radiator

Medium	n	β_{thr}	θ_{max} [β=1]	N_{ph} [eV^{-1} cm^{-1}]
Luft	1.000283	0.9997	1.36	0.208
Isobutan	1.00127	0.9987	2.89	0.941
Wasser	1.33	0.752	41.2	160.8
Quartz	1.46	0.685	46.7	196.4

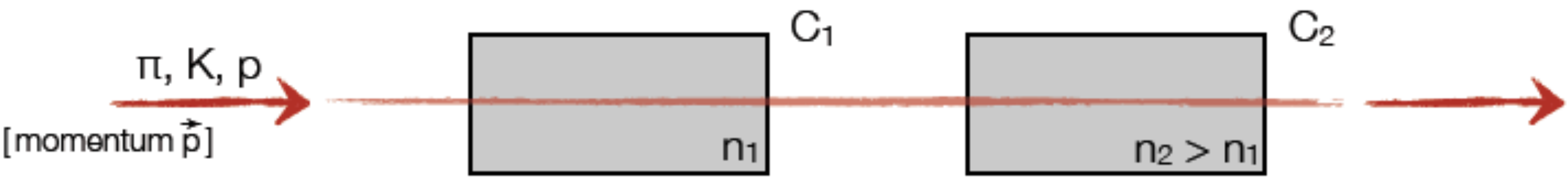




Cherenkov radiation - application

Threshold detection:

Observation of Cherenkov radiation → $\beta > \beta_{thr}$



Choose n_1, n_2 in such a way that for:

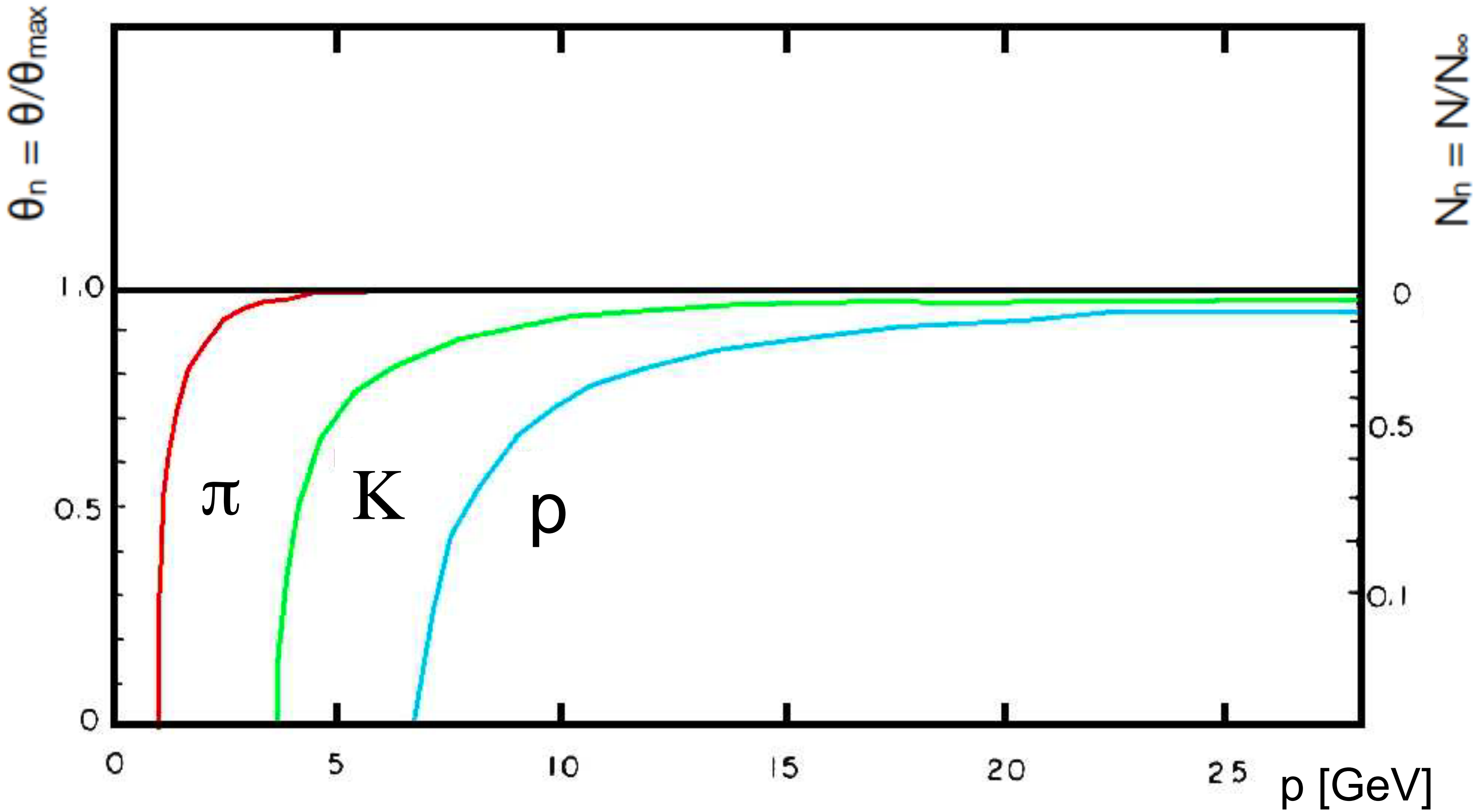
n_2 : $\beta_\pi, \beta_K > 1/n_2$ and $\beta_p < 1/n_2$

n_1 : $\beta_\pi > 1/n_1$ and $\beta_K, \beta_p < 1/n_1$

Note:
e always visible in Cherenkov counters

Light in C_1 and C_2	→	identified pion
Light in C_2 and not in C_1	→	identified kaon
Light neither in C_1 and C_2	→	identified proton

Cherenkov Radiation – Momentum Dependence



Cherenkov angle θ and number of photons N grow with β

Asymptotic value for β=1: $\cos \theta_{max} = 1/n$; $N_\infty = x \cdot 370$ / cm $(1-1/n^2)$

Exercise

Compute the threshold energies an electron and a proton must possess in light water to emit Cherenkov radiation.

$N_{water} = 1.3$ $\beta_{thresh} = 1/1.3 = 0.77$

$E_{thresh} = E_0(\gamma_{thresh} - 1)$

An electron moving in water emits Cherenkov radiation in a cone making an angle of 40^o with electron's direction of motion.

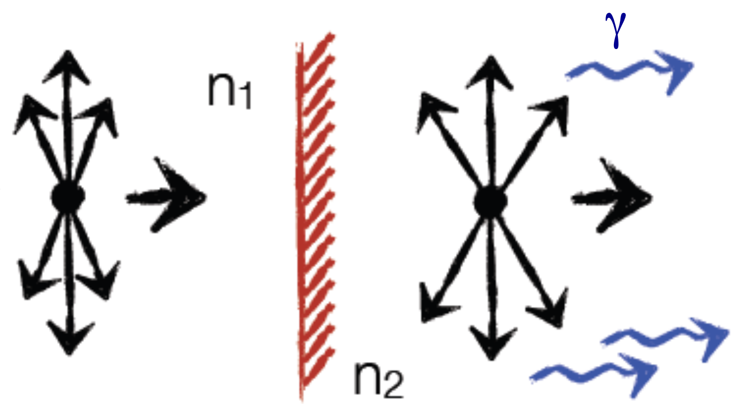
Compute the number of photons emitted per centimeter by the electron.

$$\frac{dN}{dx} = 475 \sin^2 \Theta \ [photon / cm]$$

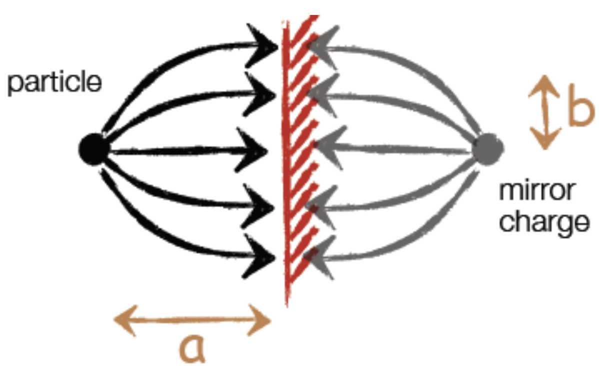
Transition Radiation

Transition radiation occurs if a relativist particle (large γ) passes the boundary between two media with different refraction indices ($n_1 \neq n_2$)
[predicted by Ginzburg and Frank 1946; experimental confirmation 70ies]

Effect can be explained by
re-arrangement of electric field:



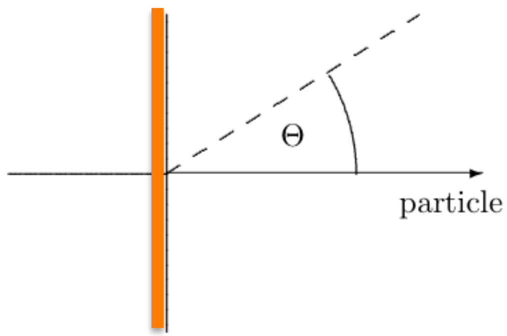
A charged particle approaching a boundary created a magnetic dipole with its mirror charge



The time-dependent dipole field causes the emission of electromagnetic radiation

Energy radiated from a single boundary: $$S = \frac{1}{3}\alpha z^2 \gamma \hbar \omega_P \quad (\hbar\omega_P \approx 20eV)$$

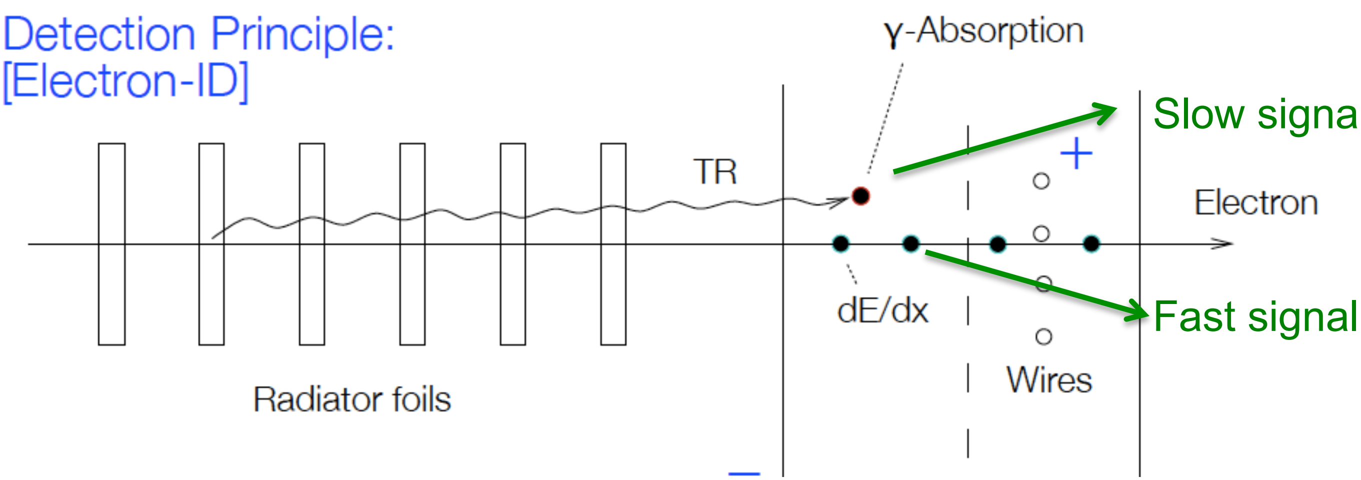
Transition Radiation



- Typical emission angle: $\Theta = 1/\gamma$
- Energy of radiated photons: $\sim \gamma$
- Number of radiated photons: αz^2
- Effective threshold: $\gamma > 1000$

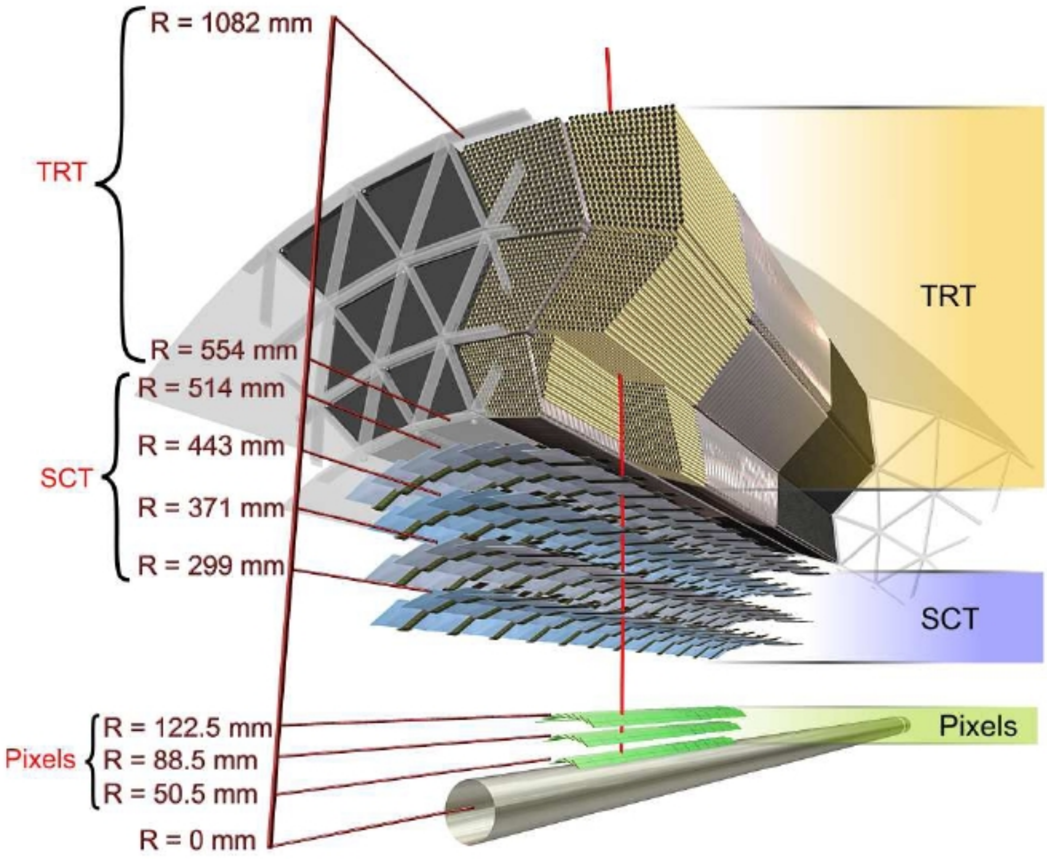
➔ Use stacked assemblies of low Z material with many transitions + a detector with high Z gas

Detection Principle:
[Electron-ID]



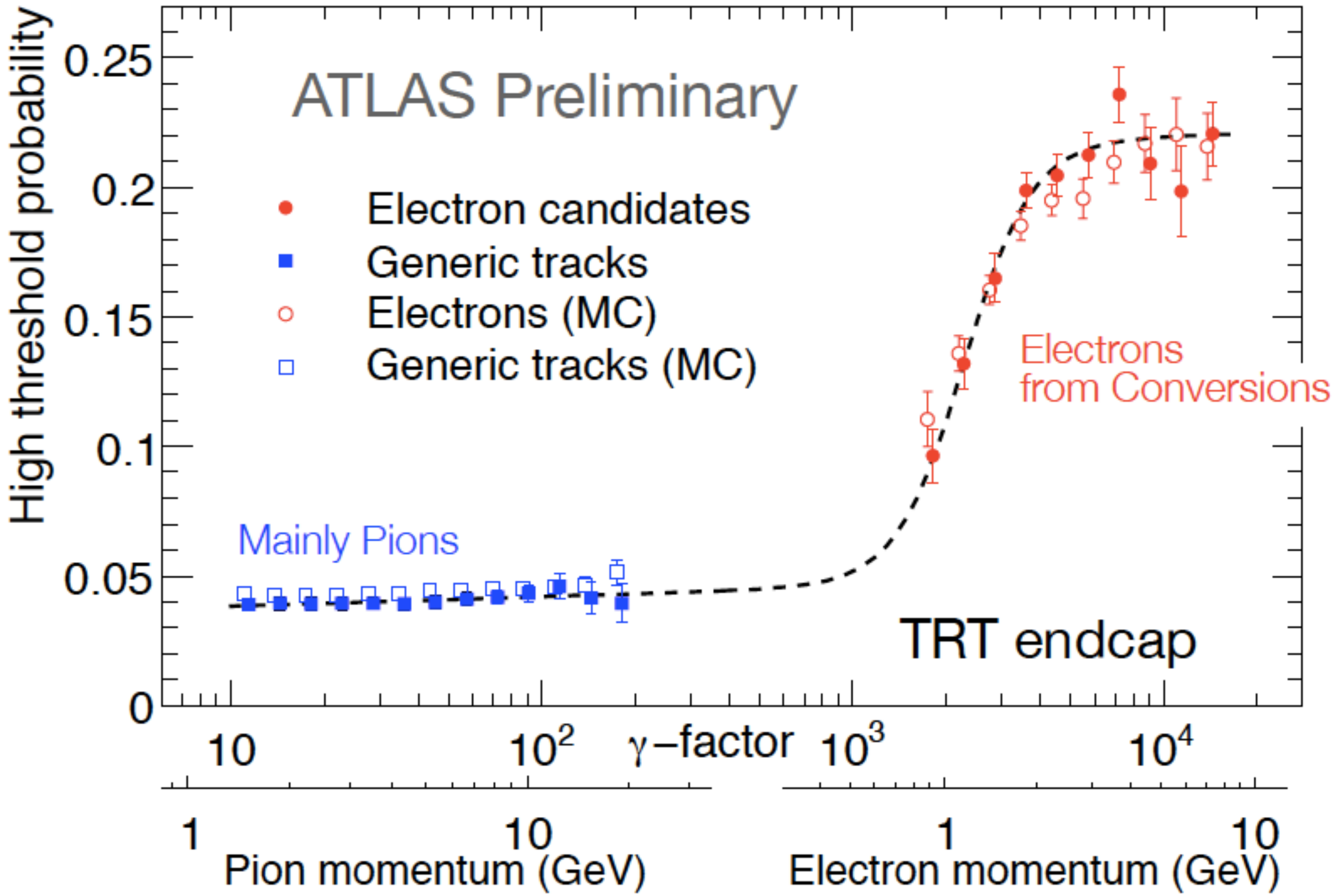
Note: Only X-ray (E>20keV) photons can traverse the many radiators without being absorbed

Transition radiation detectors



- straw tubes with xenon-based gas mixture
- 4 mm in diameter, equipped with a 30 μm diameter gold-plated W-Re wire

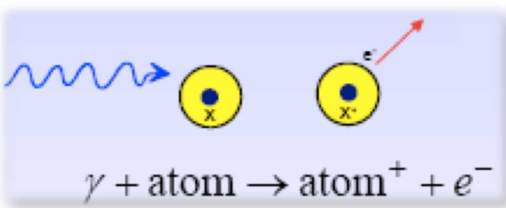
Transition radiation detector (ATLAS)



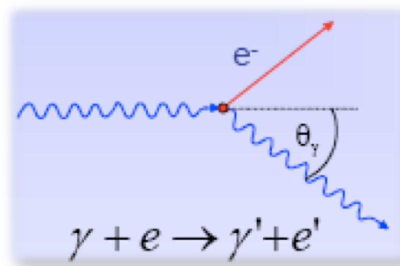
Interactions of photons with matter

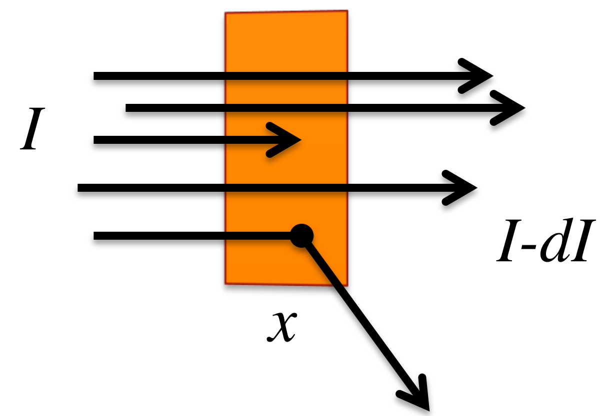
Characteristic for interactions of photons with matter:
A photon is removed from the beam after one single interaction either because of total absorption or scattering

1) Photoelectric Effect

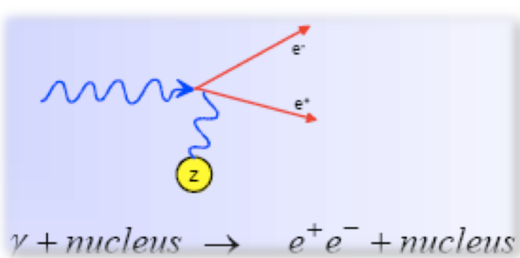


2) Compton Scattering





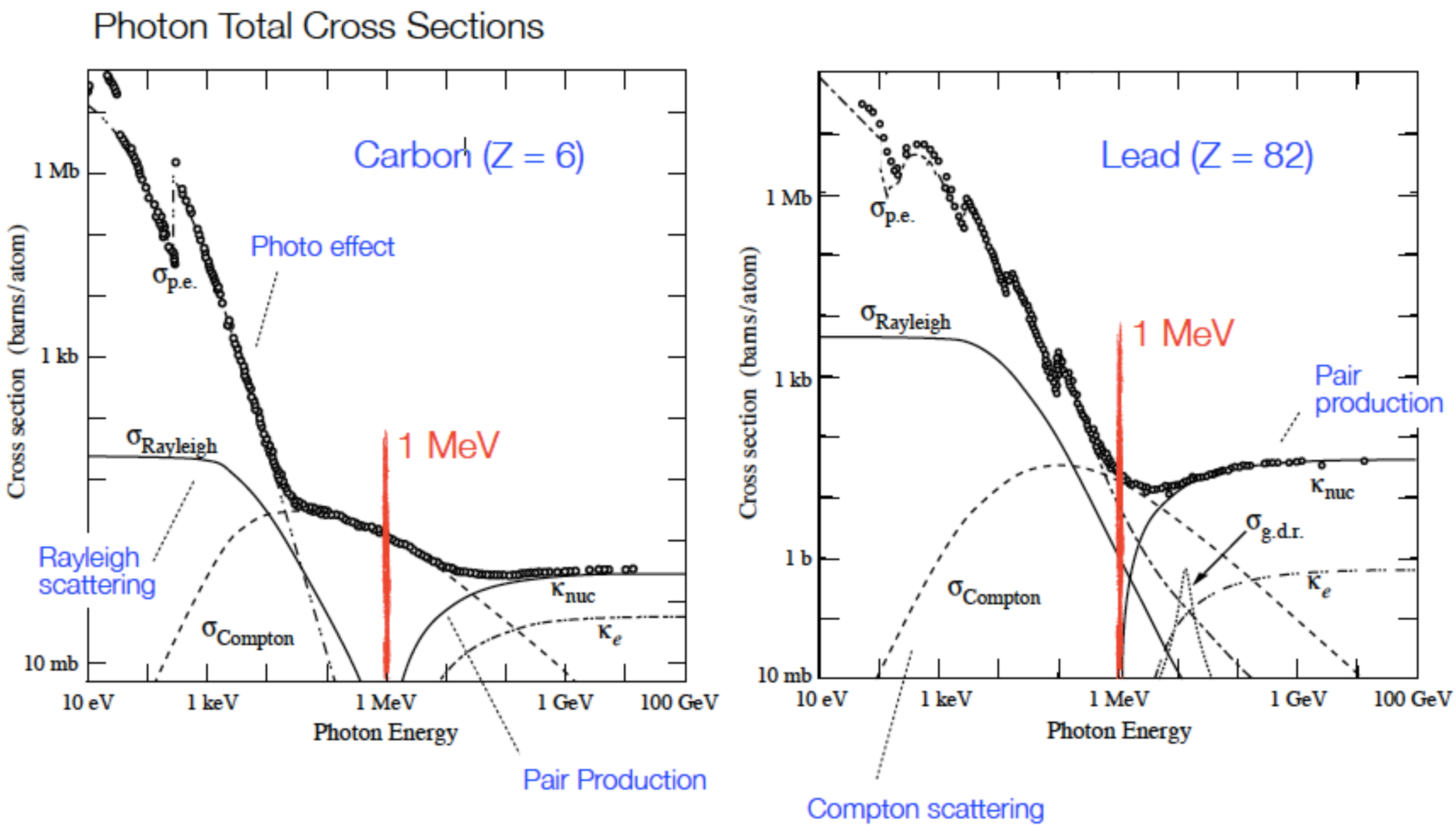
3) Pair Production



$$I(x) = I_0 e^{-\mu x}, \;\; \mu = \frac{N}{A}\sum_{i=1}^{3}\sigma_i$$

$\lambda = 1/\mu$ Mean free path

Interactions of photons with matter



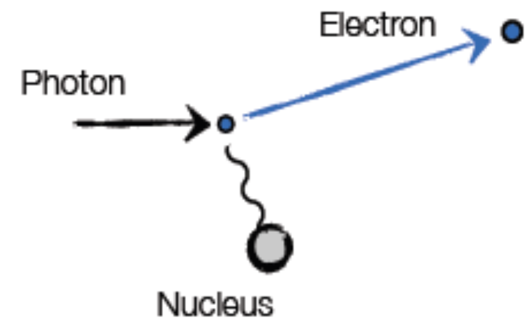
Photoelectric effect

From energy conservation:

$$E_e = E_\gamma - E_N = h\upsilon - I_b$$

I_b = Nucleus binding energy
introduces strong Z dependence

Naive picture

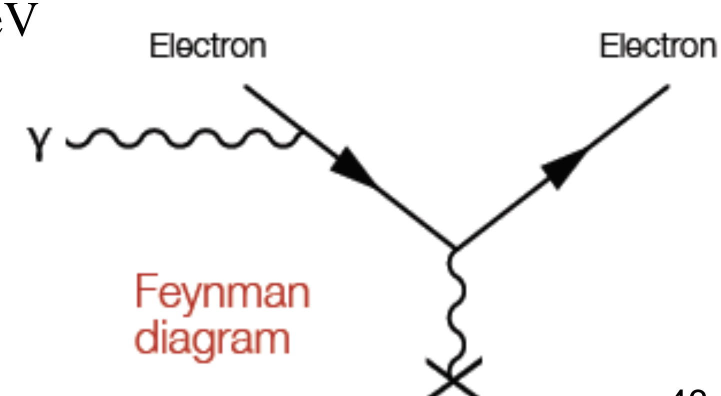


Cross-section largest for $E_\gamma \approx$ K-shell energy
Strongest E dependence for $I_0 < E_\gamma < m_e c^2$

$\gamma + \text{atom} \rightarrow e^- + \text{atom}^+$

$$\sigma_{ph} = \alpha \pi a_B Z^5 (I_0 / E_\gamma)^{7/2}$$

$$a_B = 0.53 \ \text{Å}$$
$$I_0 = 13.6 \ \text{eV}$$



E-dependence softer for $E_\gamma > m_e c^2$

$$\sigma_{ph} = 2\pi r_e^2 \alpha^4 Z^5 (mc)^2 / E_\gamma$$

Exercise

Calculate the wavelength below which it would be impossible for photons to ionize hydrogen atoms. The first ionization potential for hydrogen is $E_\gamma \geq 13.6$ eV .

$$\lambda = \frac{hc}{E_\gamma} = \frac{2\pi\hbar c}{E_\gamma}$$

$$\lambda_{\max} \leq \frac{2\pi\hbar c}{E_\gamma}$$

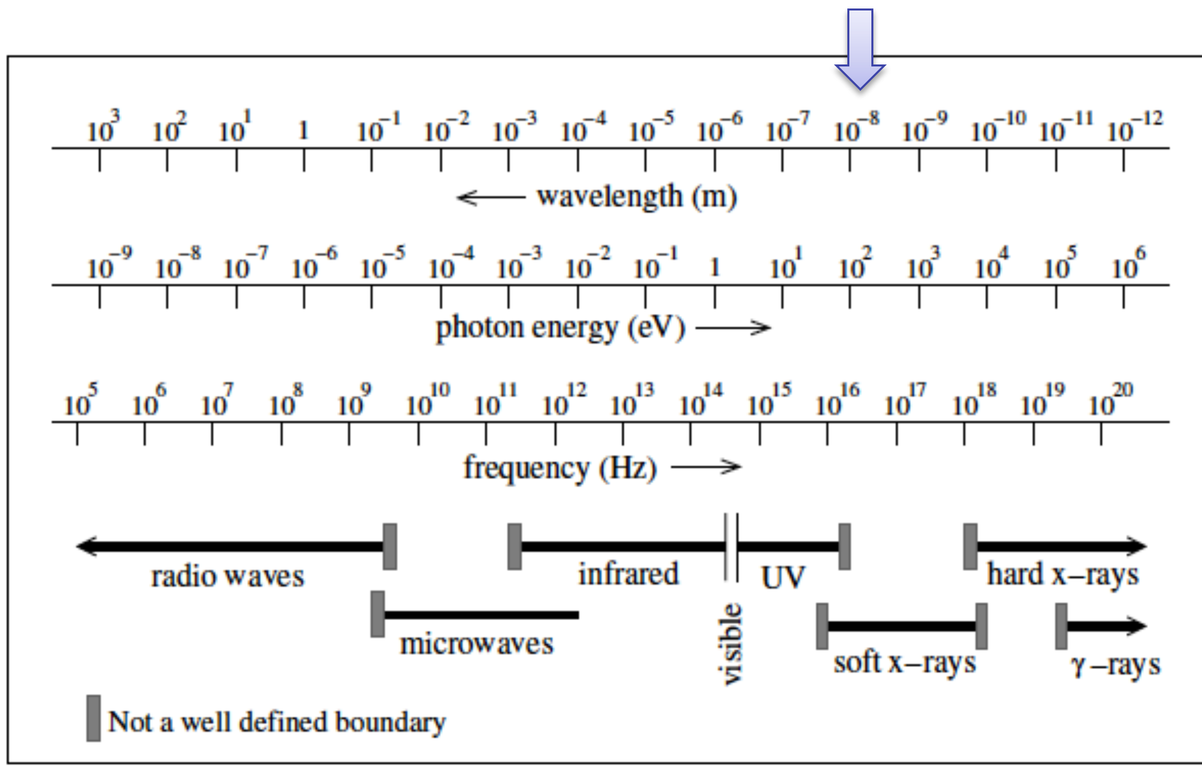


Figure 1.6.1: Electromagnetic spectrum.

Compton scattering

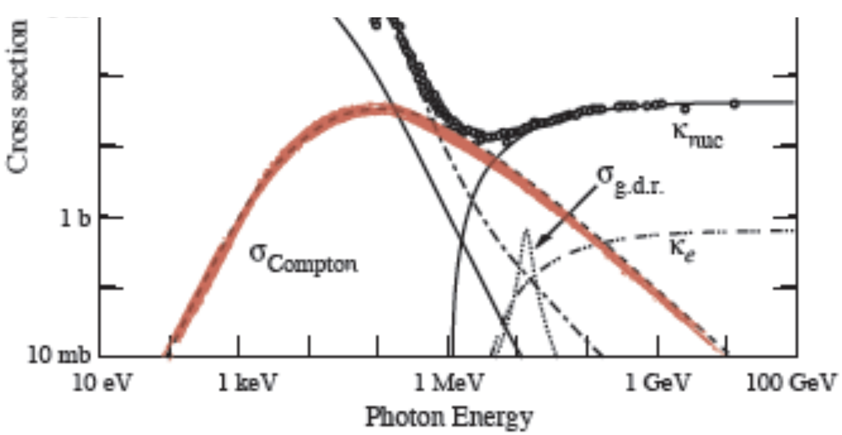
Best known electromagnetic process (Klein–Nishina formula)

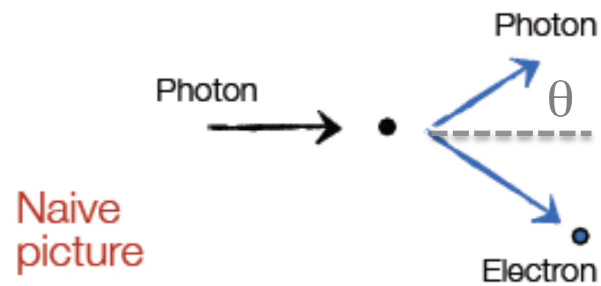
for $E_\lambda << m_e c^2$ $\boxed{\sigma_c \propto \sigma_{Th}(1-2\varepsilon)}$

Thompson cross-section:
$\sigma_{Th} = 8\pi/3\ r_e^2 = 0.66$ barn

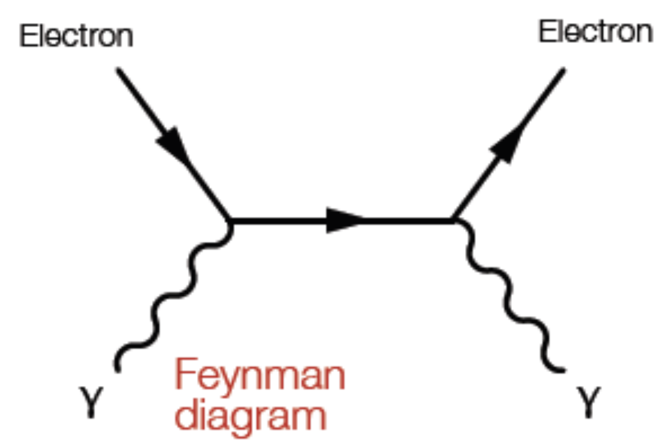
$$\varepsilon = \frac{E_\lambda}{m_e c^2}$$

for $E_\lambda >> m_e c^2$ $\boxed{\sigma_c \propto \frac{\ln \varepsilon}{\varepsilon} Z}$





$\gamma + e^- \rightarrow (\gamma)' + (e^-)'$



Compton scattering

From E and p conservation get the energy of the scattered photon

$$E_\gamma' = \frac{E_\gamma}{1+\varepsilon(1-\cos\theta)} \qquad \varepsilon = \frac{E_\lambda}{m_e c^2}$$

Kinetic energy of the outgoing electron:

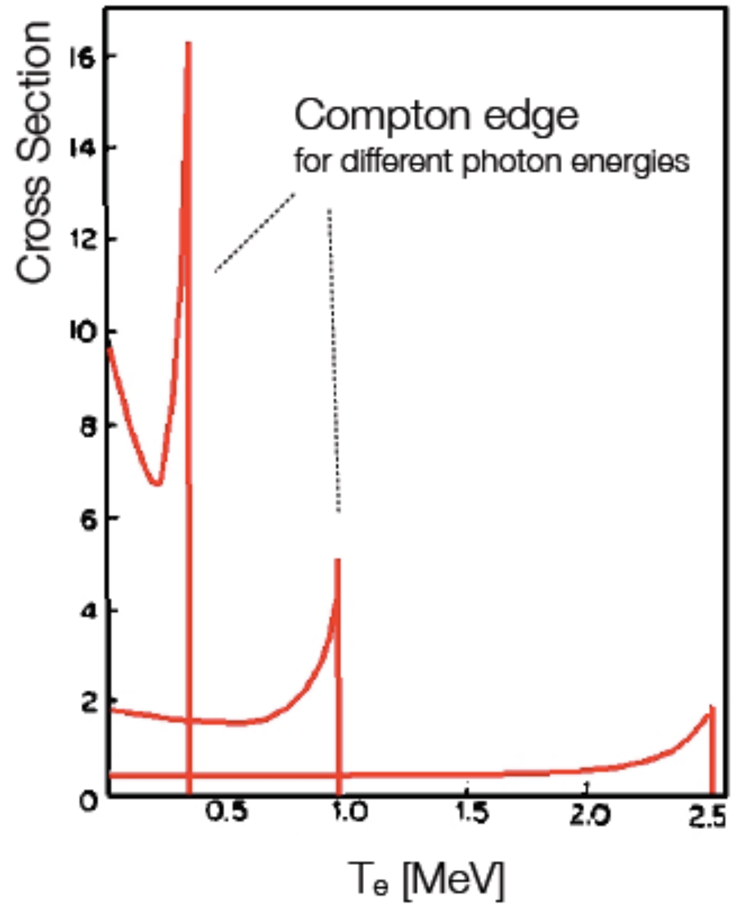
$$T_e = E_\gamma - E_\gamma' = E_\gamma \frac{\varepsilon(1-\cos\theta)}{1+\varepsilon(1-\cos\theta)}$$

Max. electron recoil energy for θ = π:

$$\boxed{T_{\max} = E_\gamma \frac{2\varepsilon}{1+2\varepsilon}}$$

Transfer of complete γ-energy via Compton scattering not possible:

$$\Delta E = E_\gamma - T_{\max} = E_\gamma \frac{1}{1+2\varepsilon}$$



Important for single photon detection; if photon is not completely absorbed a minimal amount of energy is missing (Compton rejection in PET)

Exercise

A photon incident on an atom scatters off at an angle of 55^o with an energy of 150 keV. Determine the initial energy of the photon and the energy of the scattered electron.

$$E_\gamma' = \frac{E_\gamma}{1+\varepsilon(1-\cos\theta)} \qquad \varepsilon = \frac{E_\gamma}{m_e c^2} \qquad \lambda = \frac{hc}{E_\gamma} = \frac{2\pi\hbar c}{E_\gamma}$$

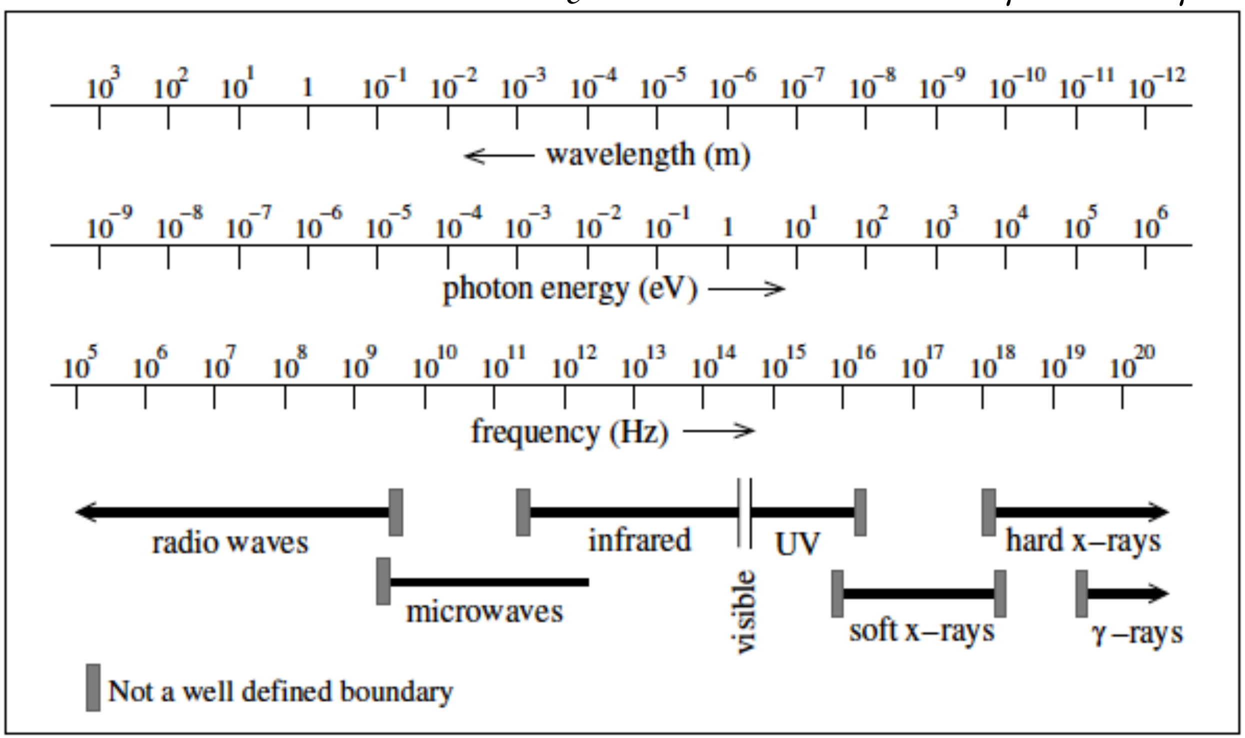
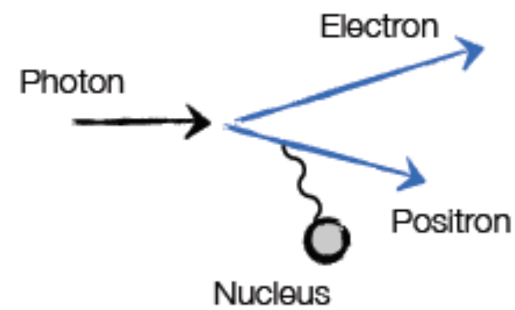


Figure 1.6.1: Electromagnetic spectrum.

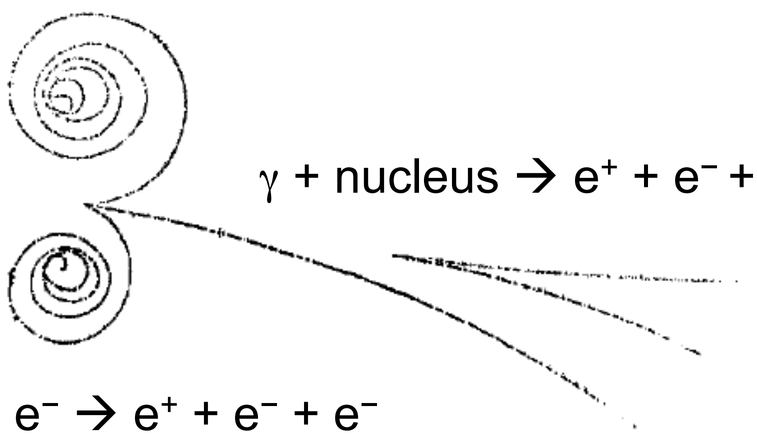
Pair production

Minimum energy required for this process
2 m_e + Energy transferred to the nucleus

$$E_\gamma \geq 2m_e c^2 + \frac{2m_e c^2}{m_{Nucleus}}$$

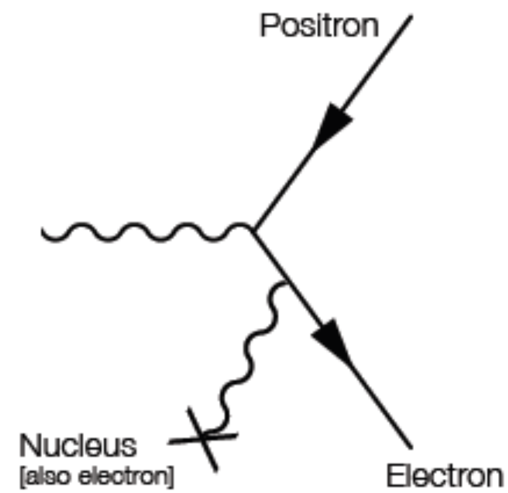


γ + atom → e^+ + e^-



γ + nucleus → e^+ + e^- + nucleus'

γ + e^- → e^+ + e^- + e^-



Pair production

for $E_\lambda >> m_e c^2$

$$\sigma_{\text{pair}} = 4\alpha r_e^2 Z^2 \left(\frac{7}{9} \ln \frac{183}{Z^{1/3}} - \frac{1}{54} \right) \quad [\mathrm{cm}^2/\mathrm{atom}]$$

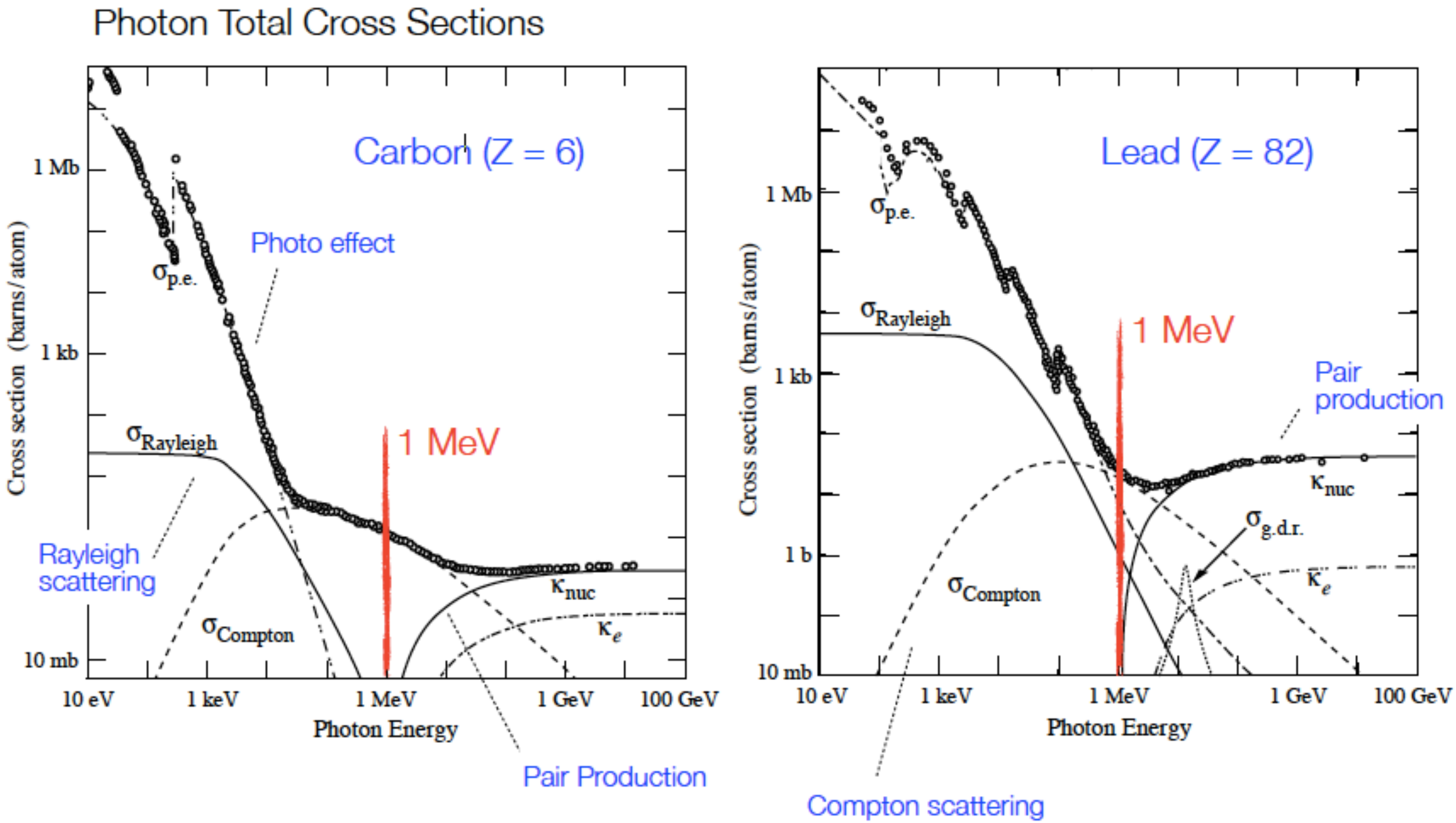
Using as for Bremsstrahlung the radiation length

$$X_0 = \frac{A}{4\alpha N_A Z^2 r_e^2 \ln \frac{183}{Z^{1/3}}}$$

$$\boxed{\sigma_{pair} = \frac{7}{9} \frac{N_A}{A} \cdot \frac{1}{X_0}}$$

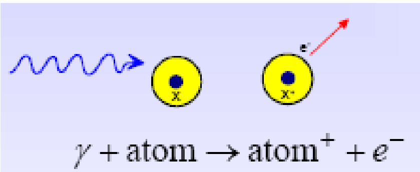
	ρ [g/cm³]	X_0 [cm]
H_2 [fl.]	0.071	865
C	2.27	18.8
Fe	7.87	1.76
Pb	11.35	0.56
Luft	$1.2 \cdot 10^{-3}$	$30 \cdot 10^{3}$

Interactions of photons with matter

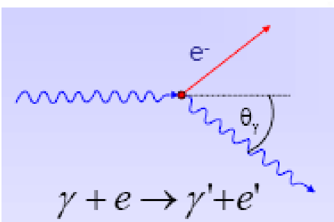


Electromagnetic interactions

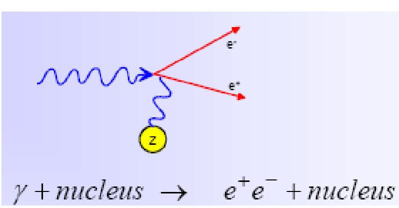
Gammas



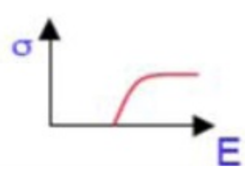
- Photoelectric effect



- Compton effect

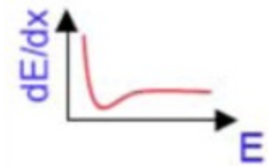


- Pair production

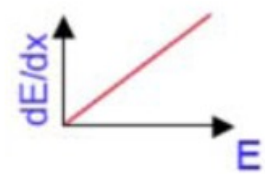


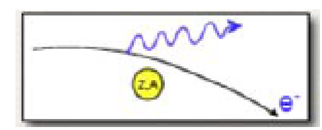
Electrons

- Ionisation



- Bremsstrahlung

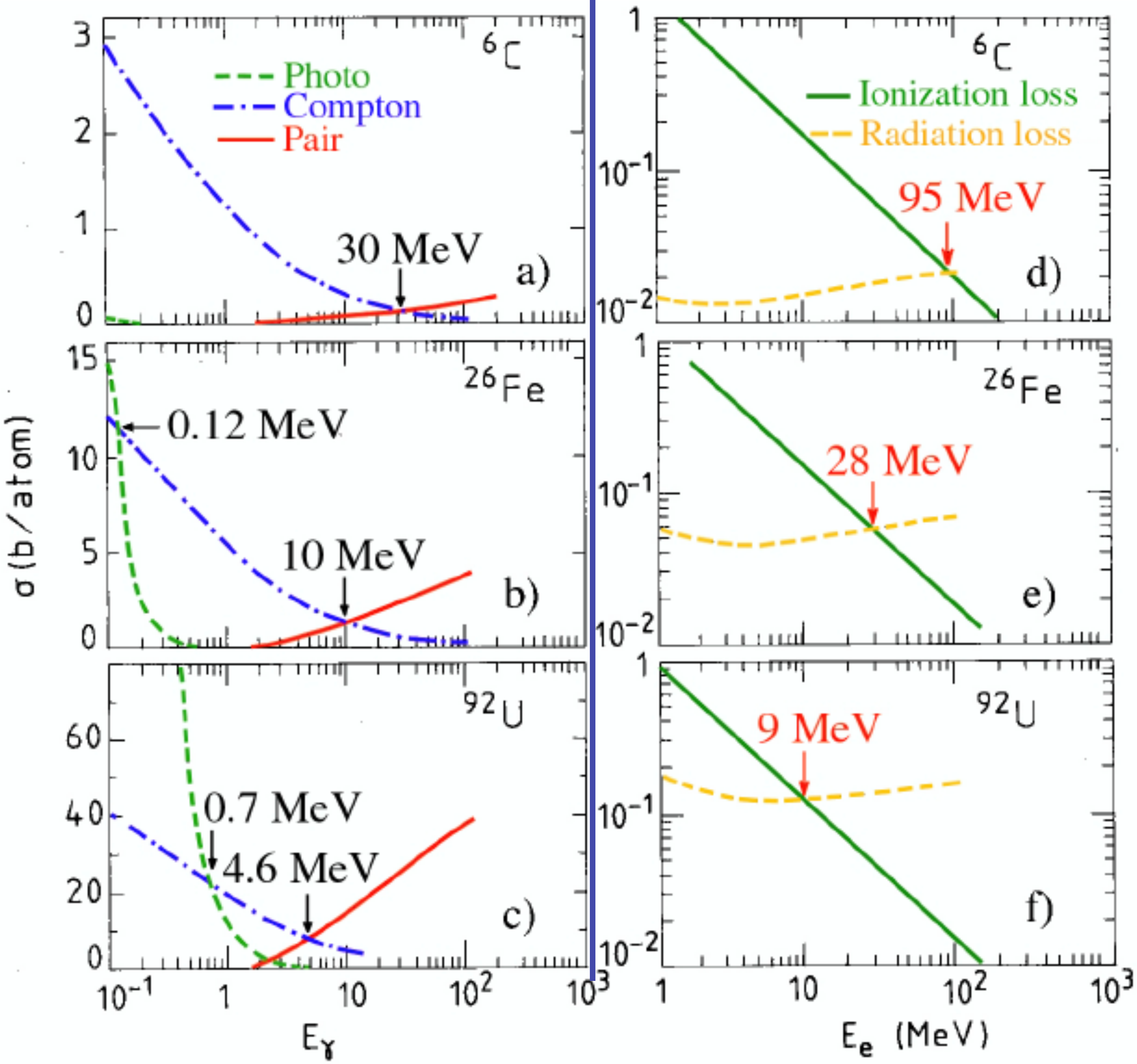




Material dependence

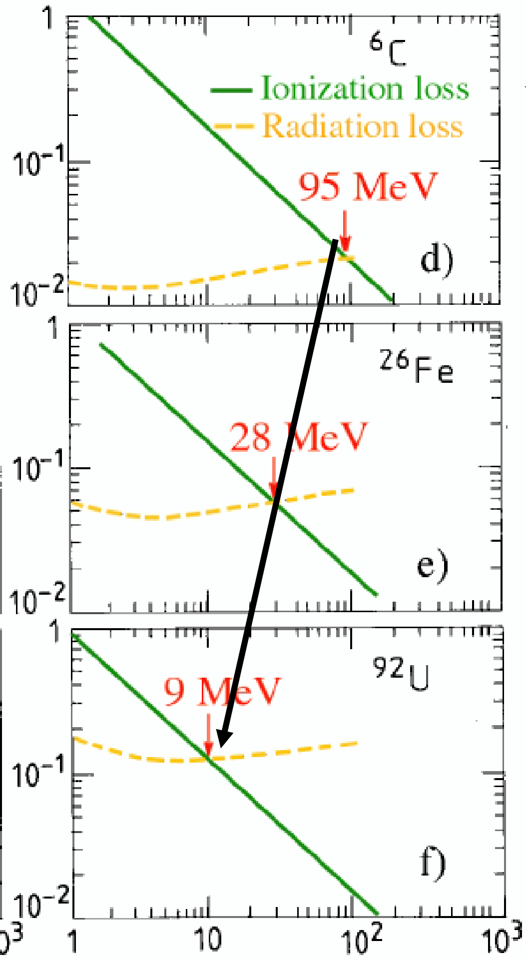
Increasing Z

Gammas



Electrons

Electrons



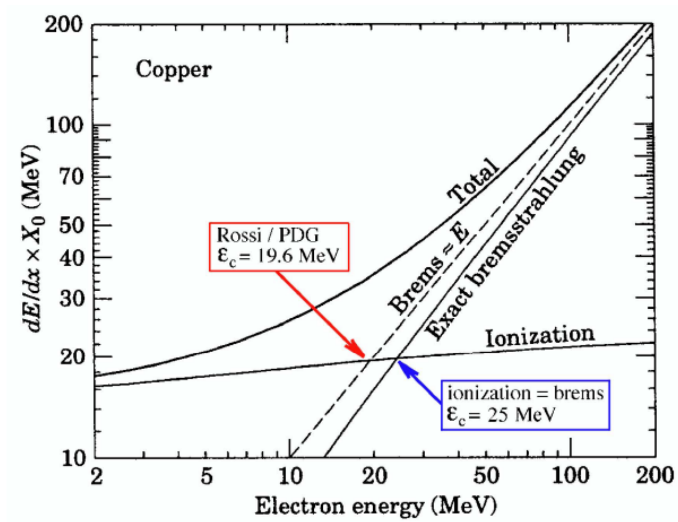
Increasing Z

Electrons lose energy by: *ionization* *radiation*

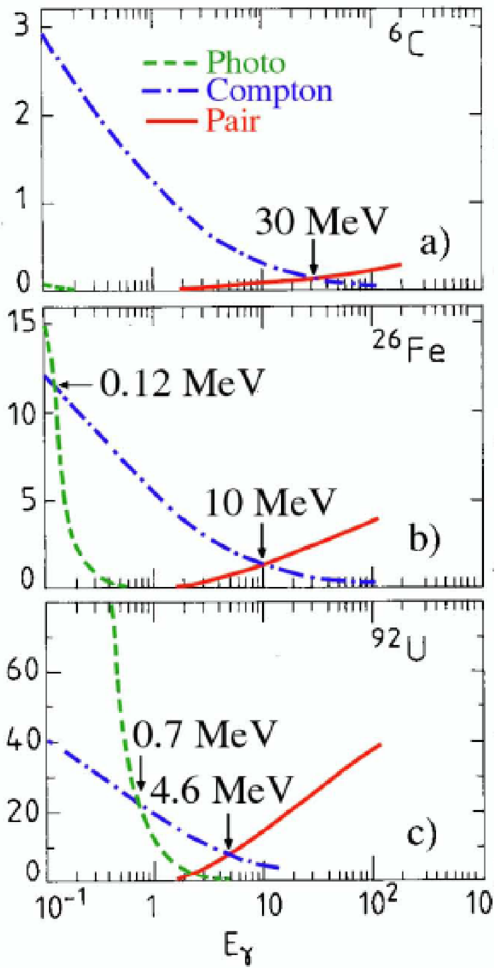
Critical energy ϵ_c: $\frac{dE}{dx}\text{(ion)} = \frac{dE}{dx}\text{(rad)}$

$\epsilon_c \propto 1/Z$ PDG: $\epsilon_c = 610\ \text{MeV}/(Z + 1.24)$

In high Z materials particle multiplication at lower energies



Photons



Increasing Z

- *Photons* interact by:

1) *Photoelectric effect* $\sigma \propto Z^5,\ E^{-3}$
2) *Compton scattering* $\sigma \propto Z,\ E^{-1}$
3) *Conversion into* e^+e^- σ increases with E, Z, asymptotic at ~ 1 GeV

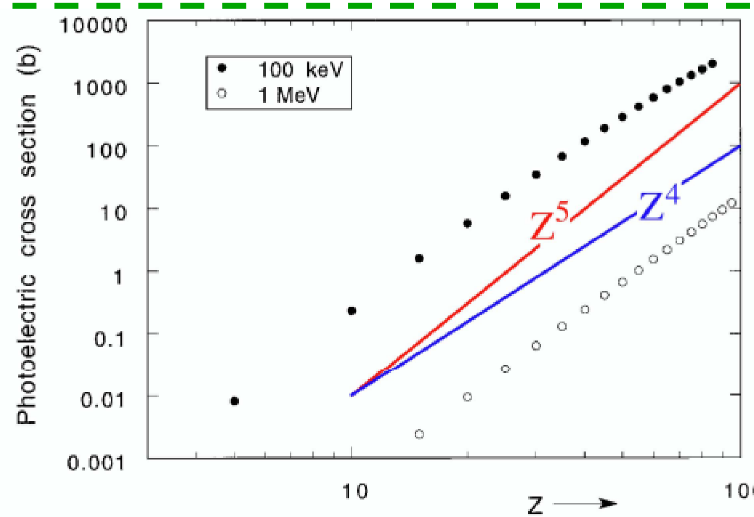


FIG. 2.3. Cross section for the photoelectric effect as a function of the Z value of the absorber. Data for 100 keV and 1 MeV γs.

References and credits

It is hard to keep track of the original source of material contained in a lecture, my apologies to those who originally created the plots and graphs collected here and are not properly quoted.

This lecture is largely based on:

K.Kleinknecht: Detectors for Particle Radiation, Cambridge
C.Grupen: Particle Detectors, Cambridge
C Grupen and I. Buvat, Handbook of Particle Detection and Imaging, Springer
W.R.Leo: Techniques for Nuclear and Particle Physics Experiments, Springer

Many of the nice animations, graphs and ideas are taken from:
The Physics of Particle Detectors by Prof. H.-C. Schultz-Coulon
http://www.kip.uni-heidelberg.de/~coulon/Lectures/Detectors/