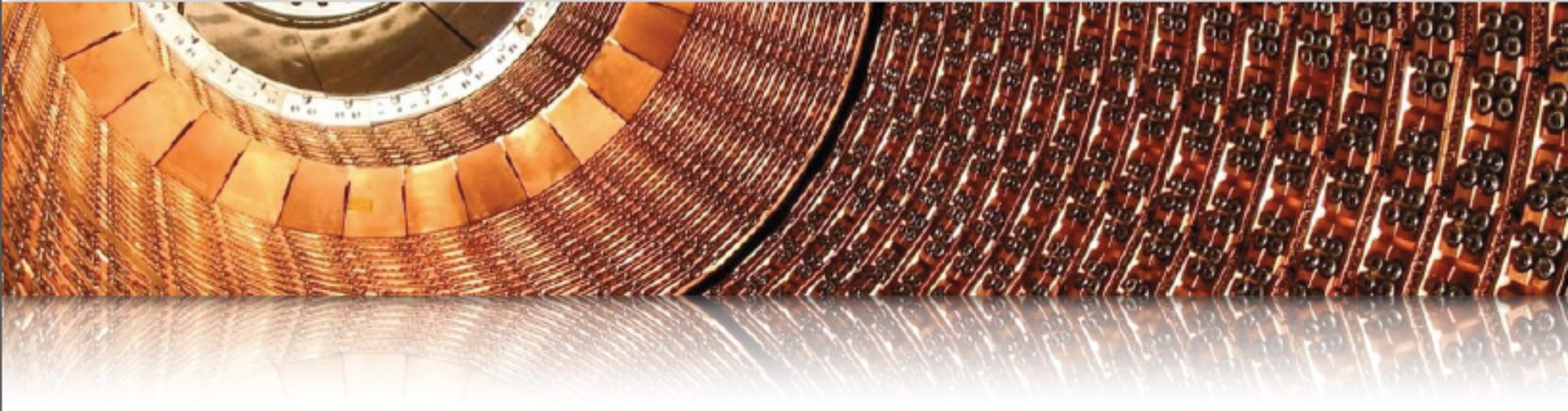


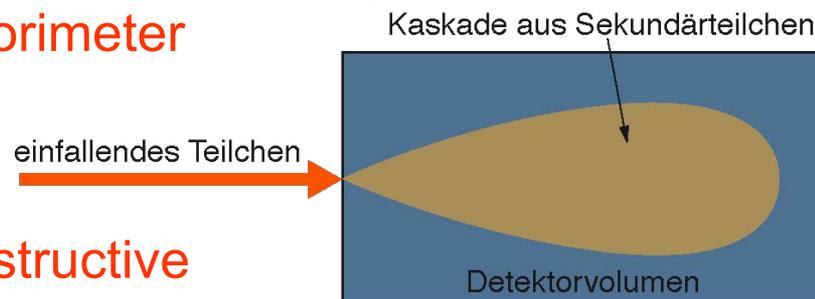
Calorimeters

Energy measurement



Calorimeter

- In nuclear and particle physics calorimetry refers to the detection of particles, and measurements of their properties, through total absorption in a block of matter, the **calorimeter**
- Common feature of all calorimeters is that the measurement process is **destructive**
 - Unlike, for example, wire chambers that measure particles by tracking in a magnetic field, the particles are no longer available for inspection once the calorimeter is done with them.
 - The only exception concerns **muons**. The fact that muons can penetrate a substantial amount of matter is an important mean for muon identification.
- In the absorption, almost all particle's energy is eventually converted to **heat**, hence the term calorimeter

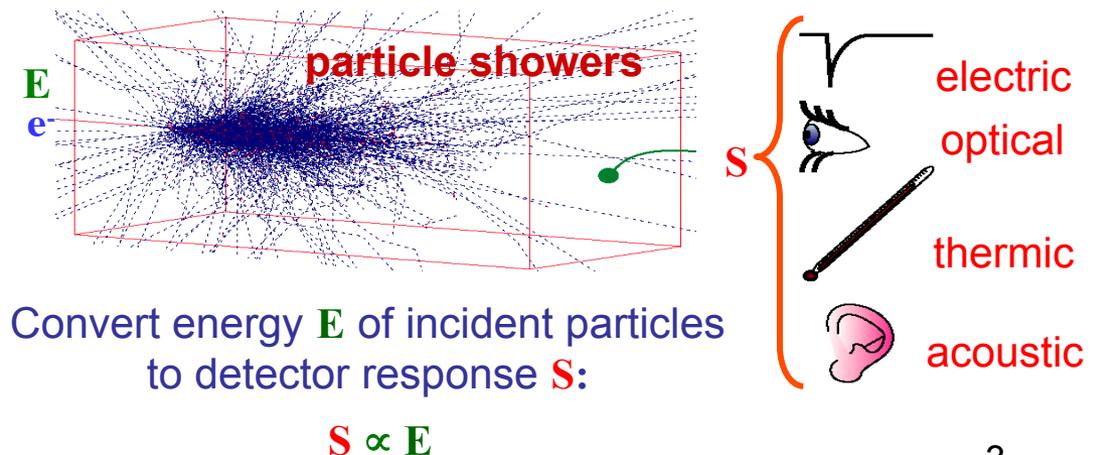


Calorimetry in particle physics

- Calorimetry is a widespread technique in particle physics:
 - instrumented targets
 - neutrino experiments
 - proton decay / cosmic ray detectors
 - shower counters
 - 4π detectors for collider experiments

- Calorimetry makes use of various detection mechanisms:

- Scintillation
- Cherenkov radiation
- Ionization
- Cryogenic phenomena



Why calorimetry?

- Measure *charged + neutral* particles
- Performance of calorimeters *improves with energy* and is \sim constant over 4π (Magn. Spectr. anisotropy due to B field)

Calorimeter: $\frac{\sigma_E}{E} \sim \frac{1}{\sqrt{E}}$
[see below]

e.g. ATLAS:

$$\frac{\sigma_E}{E} \approx \frac{0.1}{\sqrt{E}}$$

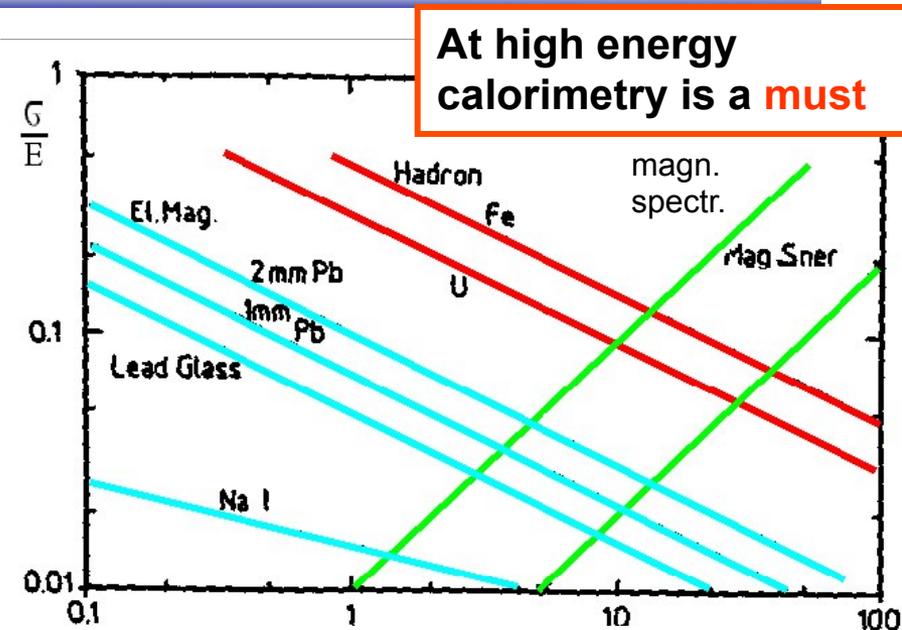
i.e. $\sigma_E/E = 1\% @ 100 \text{ GeV}$

Gas detector: $\frac{\sigma_p}{p} \sim p$
[see above]

e.g. ATLAS:

$$\frac{\sigma_p}{p} \approx 5 \cdot 10^{-4} \cdot p_t$$

i.e. $\sigma_p/p = 5\% @ 100 \text{ GeV}$



- Obtain information *fast* (<100ns feasible)
→ recognize and select interesting events in real time (*trigger*)

Electromagnetic Calorimeters

Electromagnetic shower

Dominant processes at high energies ($E > \text{few MeV}$) :

Photons : Pair production

$$\sigma_{\text{pair}} \approx \frac{7}{9} \left(4\alpha r_e^2 Z^2 \ln \frac{183}{Z^{1/3}} \right)$$

$$= \frac{7}{9} \frac{A}{N_A X_0} \quad [X_0: \text{radiation length}]$$

[in cm or g/cm²]

Absorption coefficient:

$$\mu = n\sigma = \rho \frac{N_A}{A} \cdot \sigma_{\text{pair}} = \frac{7}{9} \frac{\rho}{X_0}$$

$X_0 = \text{radiation length in [g/cm}^2\text{]}$

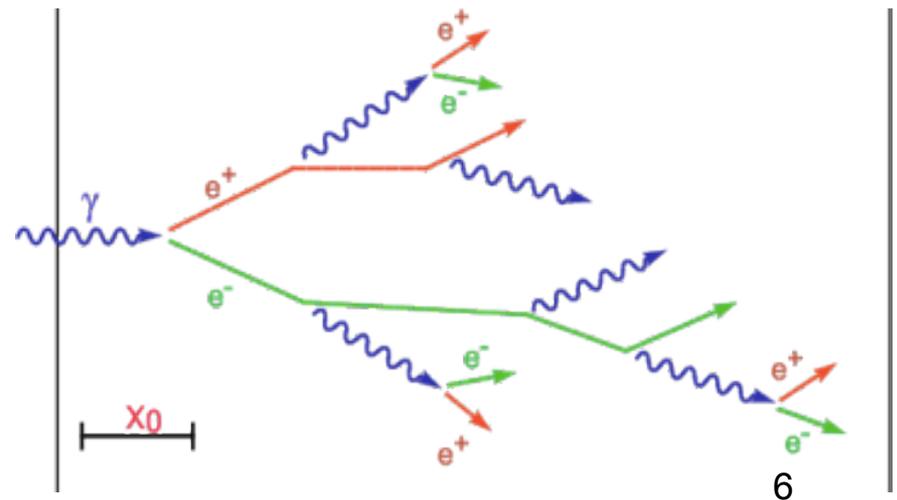
$$X_0 = \frac{A}{4\alpha N_A Z^2 r_e^2 \ln \frac{183}{Z^{1/3}}}$$

Electrons : Bremsstrahlung

$$\frac{dE}{dx} = 4\alpha N_A \frac{Z^2}{A} r_e^2 \cdot E \ln \frac{183}{Z^{1/3}} = \frac{E}{X_0}$$

$$\rightarrow E = E_0 e^{-x/X_0}$$

After passage of one X_0 electron has only (1/e)th of its primary energy ...
[i.e. 37%]

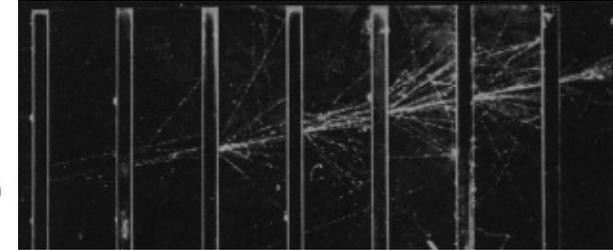


Analytic shower model

Simplified model [Heitler]: shower development governed by X_0

e^- loses $[1 - 1/e] = 63\%$ of energy in 1 X_0 (Brems.)

the *mean free path* of a γ is $9/7 X_0$ (pair prod.)



Lead absorbers in cloud chamber

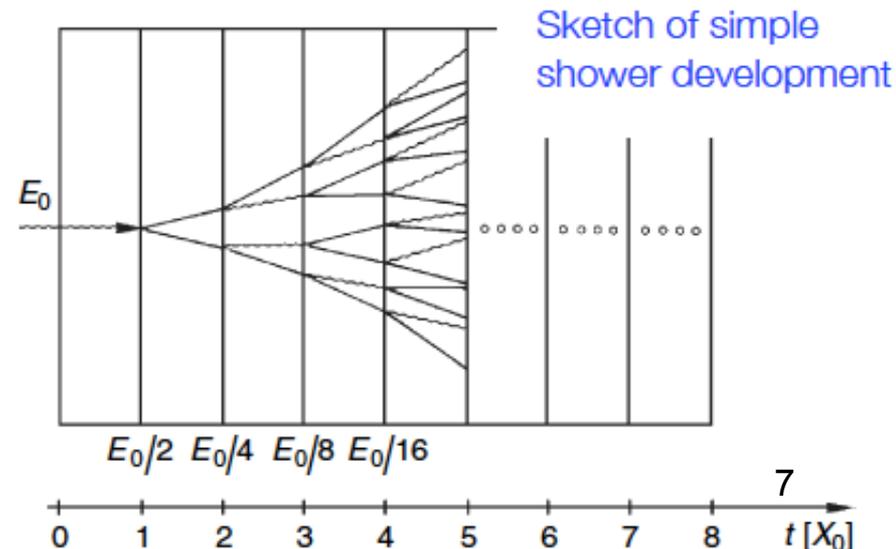
Assume:

$E > E_c$: no energy loss by ionization/excitation

$E < E_c$: energy loss **only** via ionization/excitation

Simple shower model:

- 2^t particles after $t [X_0]$
- each with energy $E/2^t$
- Stops if $E < \text{critical energy } \epsilon_c$
- Number of particles $N = E/\epsilon_c$
- Maximum at $t_{\text{max}} \propto \ln(E_0/E_c)$



Analytic shower mode

Simple shower model quite powerful → characterized shower by:

- Number of particles in shower
- Location of shower maximum
- Transverse shower distribution
- Longitudinal shower distribution

$$N_{\max} = 2^{t_{\max}} = \frac{E_0}{E_c}$$

$$t_{\max} \propto \ln(E_0/E_c)$$

$$L \sim \ln \frac{E}{E_c}$$

Longitudinal shower distribution increases only logarithmically with the primary energy of the incident particle, i.e. calorimeters can be compact

Some numbers: $E_c \approx 10 \text{ MeV}$, $E_0 = 1 \text{ GeV}$ → $t_{\max} = \ln 100 \approx 4.5$; $N_{\max} = 100$
 $E_0 = 100 \text{ GeV}$ → $t_{\max} = \ln 10000 \approx 9.2$; $N_{\max} = 10000$

	Szint.	LAr	Fe	Pb	W
$X_0(\text{cm})$	34	14	1.76	0.56	0.35

→ 100 GeV electron contained in 16 cm Fe or 5 cm Pb

Longitudinal development of EM shower

Longitudinal profile

Parametrization:
[Longo 1975]

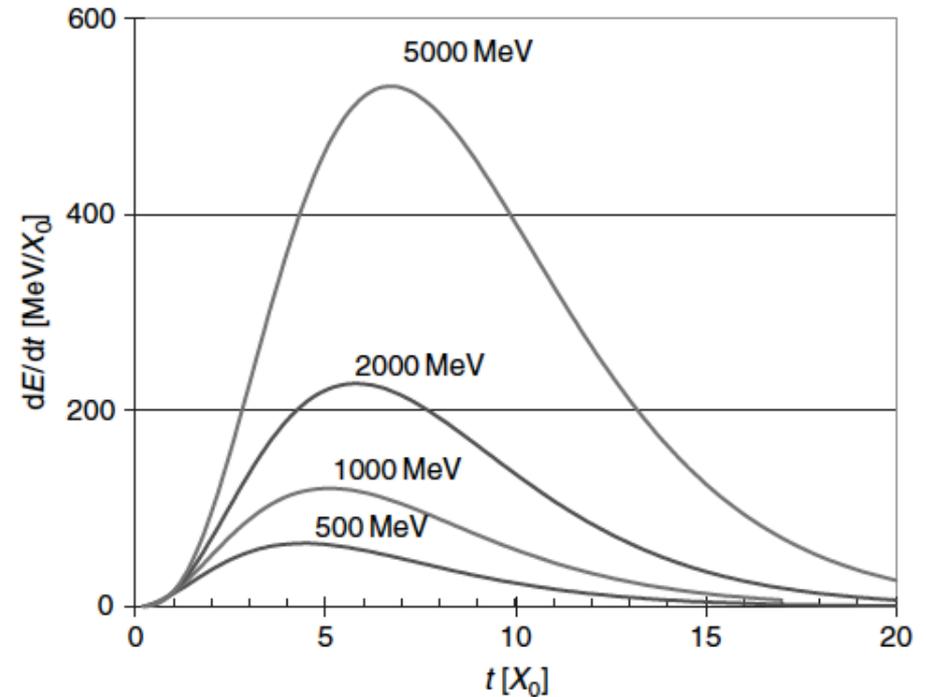
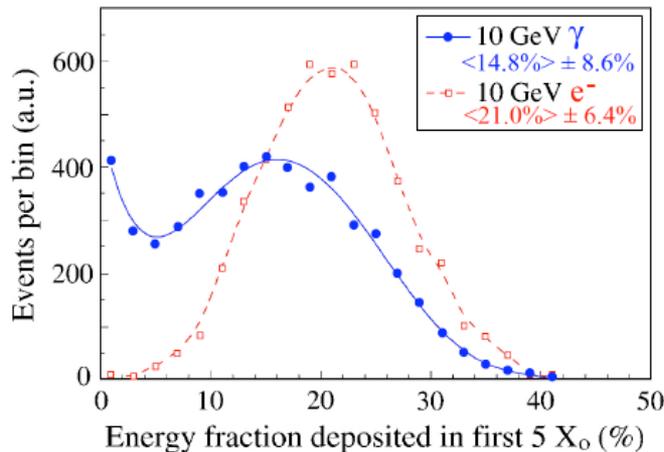
$$\frac{dE}{dt} = E_0 t^\alpha e^{-\beta t}$$

α, β : free parameters

t^α : at small depth number of secondaries increases ...

$e^{-\beta t}$: at larger depth absorption dominates ...

Numbers for $E = 2$ GeV (approximate):
 $\alpha = 2, \beta = 0.5, t_{\max} = \alpha/\beta$



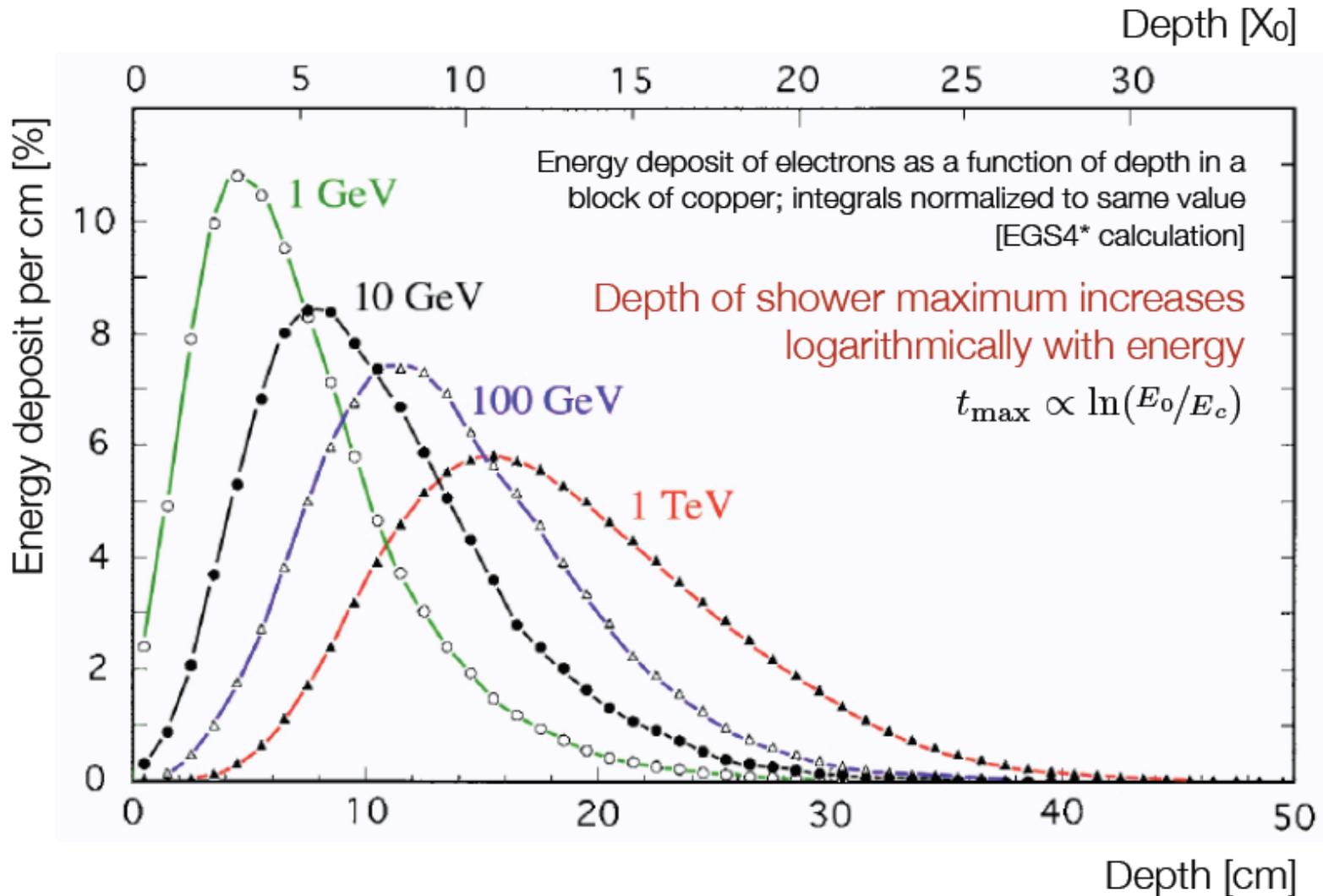
important *differences between* showers induced by e, γ

$$t_{\max} = \frac{\alpha - 1}{\beta} = \ln \left(\frac{E_0}{E_c} \right) + C_{e\gamma}$$

with:

- $C_{e\gamma} = -0.5$ [γ -induced]
- $C_{e\gamma} = -1.0$ [e -induced]

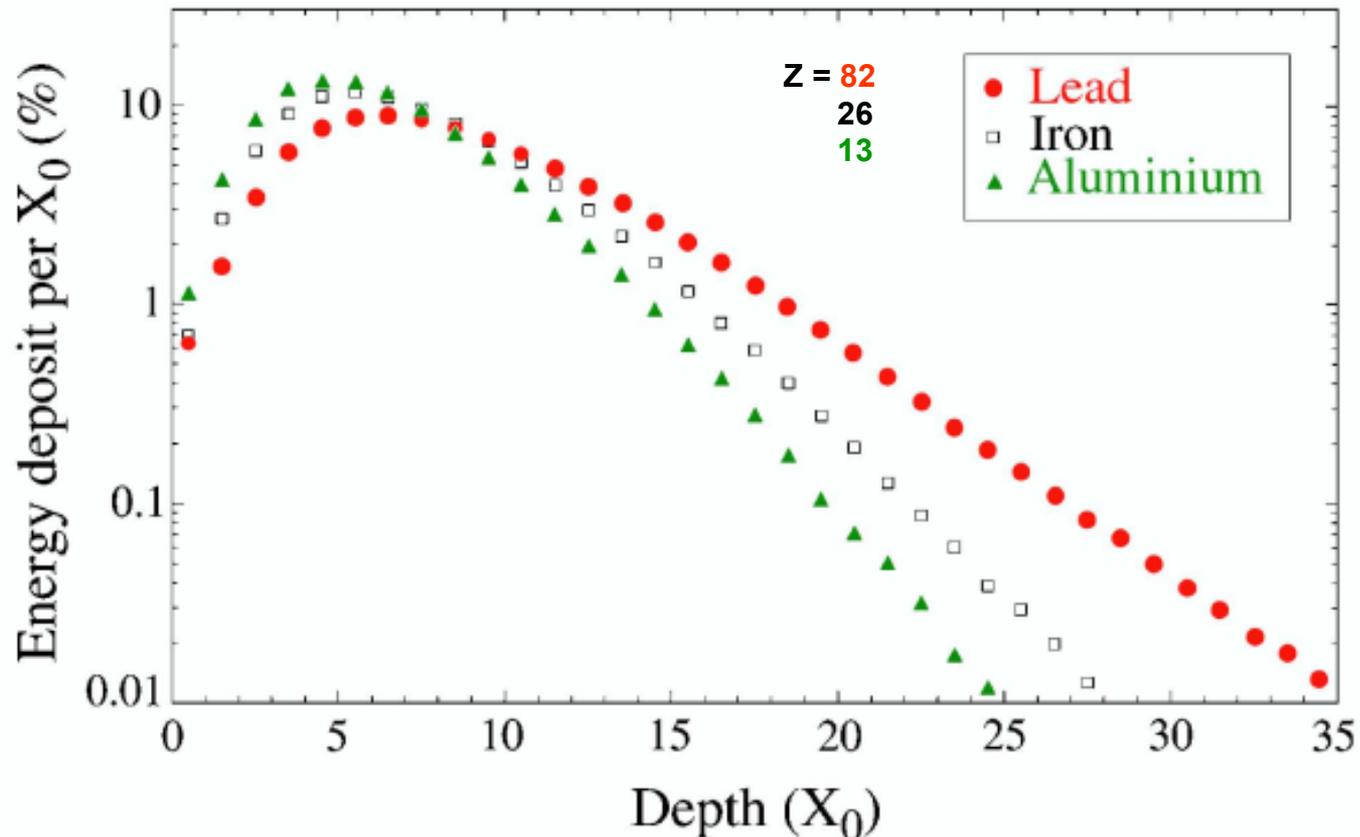
Longitudinal development of EM shower



Longitudinal development of EM shower

Shower decay:

after the shower maximum the shower decays slowly through ionization and Compton scattering → NOT proportional to X_0



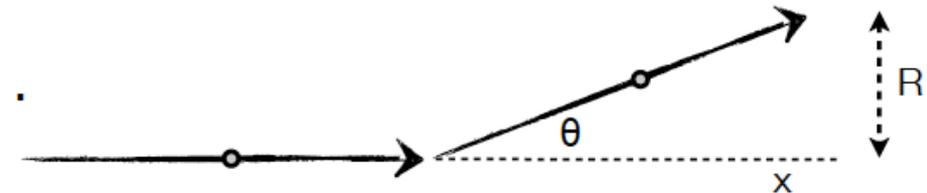
Lateral development of EM shower

Opening angle:

1) bremsstrahlung and pair production

$$\langle \theta^2 \rangle \approx (m/E)^2 = 1/\gamma^2$$

Small contribution as $m_e/E_c = 0.05$



Lateral extension: $R = x \cdot \tan \theta \approx x \cdot \theta$, if θ small ...

2) multiple coulomb scattering
[Mollier theory]

$$\langle \theta \rangle = \frac{21.2 \text{ MeV}}{E_e} \sqrt{\frac{x}{X_0}} \quad [\beta = 1, c = 1, z = 1] \quad E_s = \sqrt{\frac{4\pi}{\alpha}} (m_e c^2) = 21.2 \text{ MeV} \quad [\text{Scale Energy}]$$

Lateral spread:

Main contribution from low energy electrons as $\langle \theta \rangle \sim 1/E_e$, i.e. for electrons with $E = E_c$

Assuming the approximate range of electrons to be X_0 yields $\langle \theta \rangle \approx 21 \text{ MeV}/E_e \rightarrow$
lateral extension: $R = \langle \theta \rangle X_0$

Mollier radius: $R_M = \frac{E_s}{E_c} X_0 \approx \frac{21 \text{ MeV}}{E_c} X_0$

Lateral development of EM shower

Transverse profile

Parametrization:

$$\frac{dE}{dr} = \alpha e^{-r/R_M} + \beta e^{-r/\lambda_{\min}}$$

α, β : free parameters

R_M : Molière radius

λ_{\min} : range of low energetic photons ...

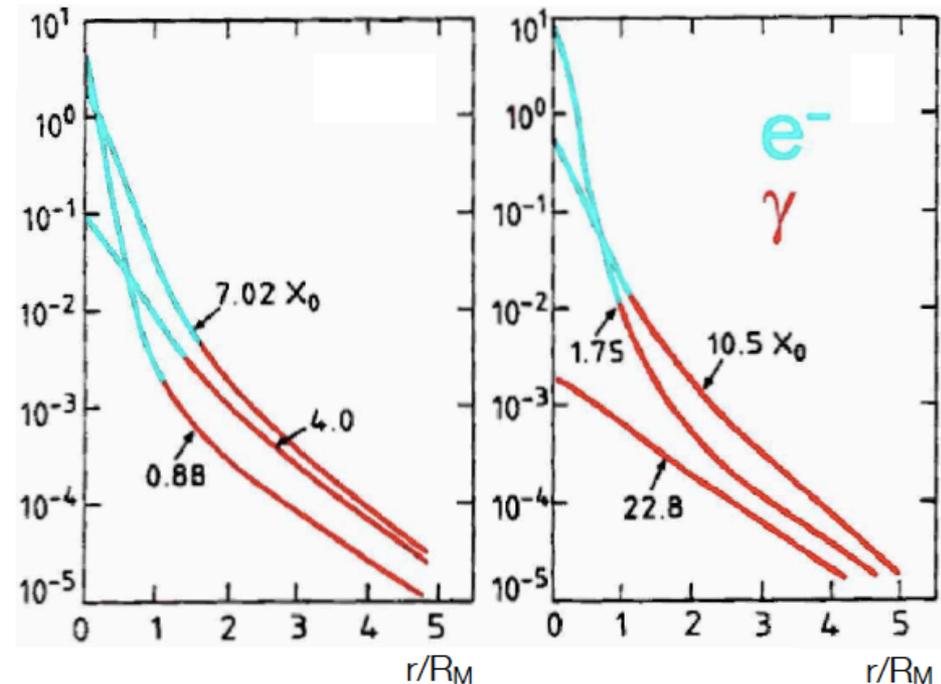
Inner part: coulomb scattering ...

Electrons and positrons move away from shower axis due to multiple scattering ...

Outer part: low energy photons ...

Photons (and electrons) produced in isotropic processes (Compton scattering, photo-electric effect) move away from shower axis; predominant beyond shower maximum, particularly in high-Z absorber media...

energy deposit
[arbitrary unites]

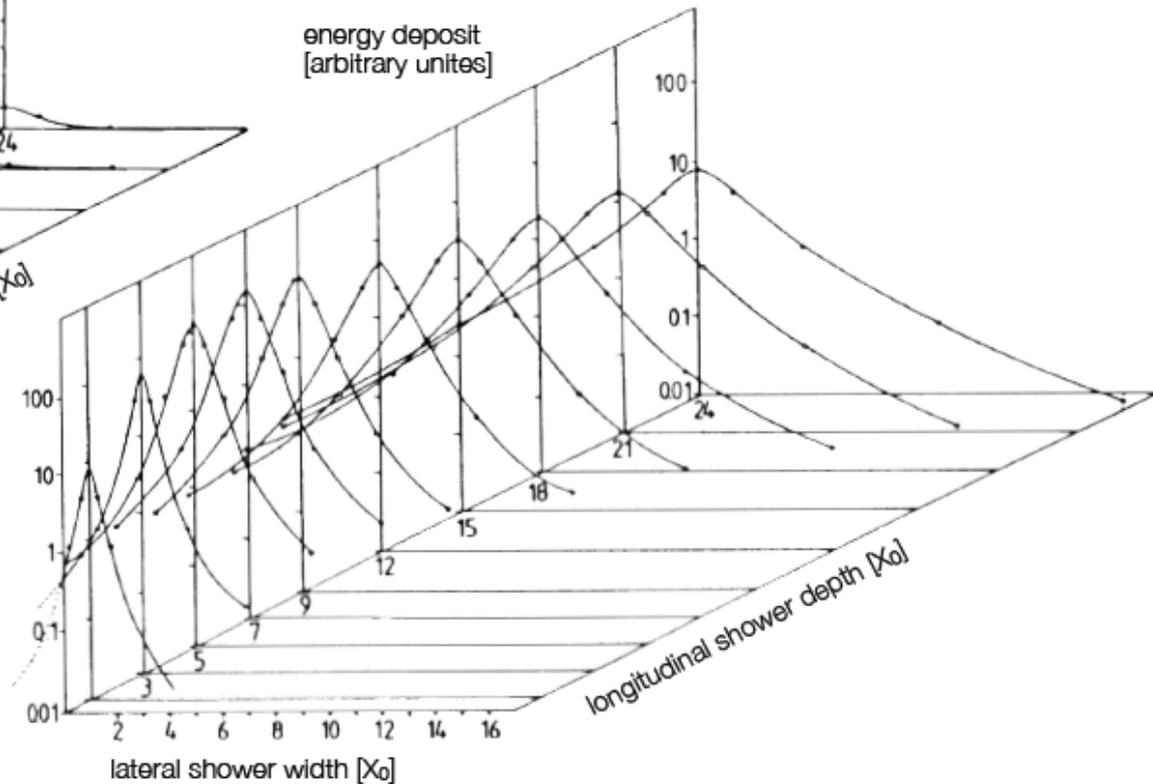
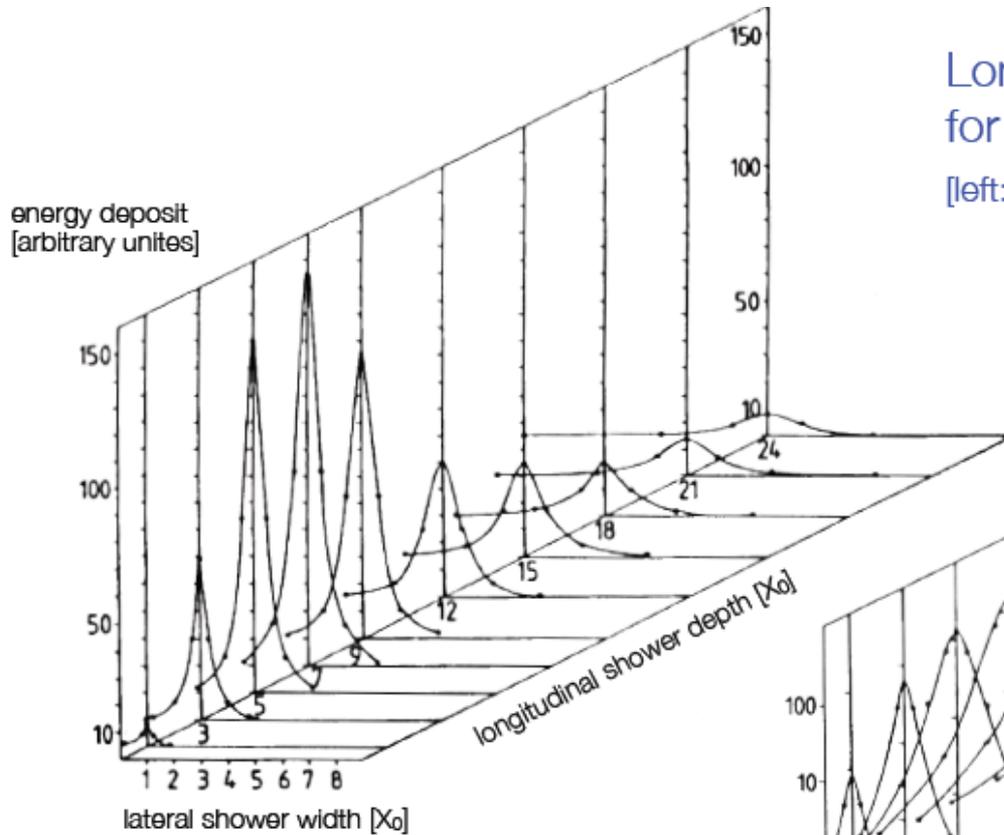


The shower gets wider at larger depth

3D shower development

Longitudinal and transversal shower profile for a 6 GeV electron in lead absorber ...

[left: linear scale; right: logarithmic scale]



Useful back of the envelop calculations

Radiation length:

$$X_0 = \frac{180A}{Z^2} \frac{\text{g}}{\text{cm}^2}$$

Critical energy:

[Attention: Definition of Rossi used]

$$E_c = \frac{550 \text{ MeV}}{Z}$$

Shower maximum:

$$t_{\max} = \ln \frac{E}{E_c} - \begin{cases} 1.0 & e^- \text{ induced shower} \\ 0.5 & \gamma \text{ induced shower} \end{cases}$$

Longitudinal
energy containment:

$$L(95\%) = t_{\max} + 0.08Z + 9.6 [X_0]$$

Transverse
Energy containment:

$$R(90\%) = R_M$$
$$R(95\%) = 2R_M$$

Problem:

Calculate how much Pb, Fe or Cu is needed to stop a 10 GeV electron.

Pb : $Z=82$, $A=207$, $\rho=11.34 \text{ g/cm}^3$

Fe : $Z=26$, $A=56$, $\rho=7.87 \text{ g/cm}^3$

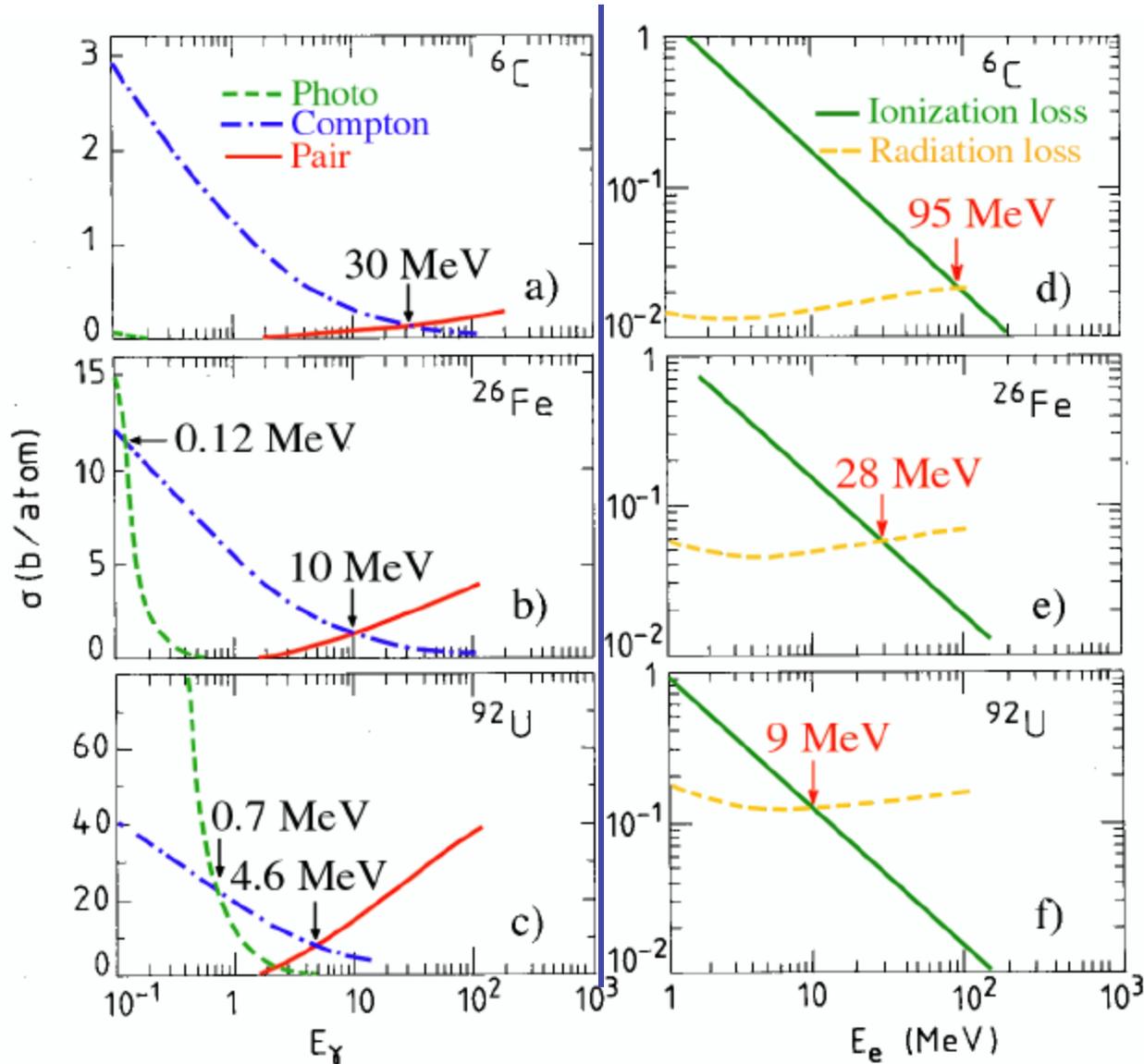
Cu : $Z=29$, $A=63$, $\rho=8.92 \text{ g/cm}^3$

Material dependence

Increasing Z

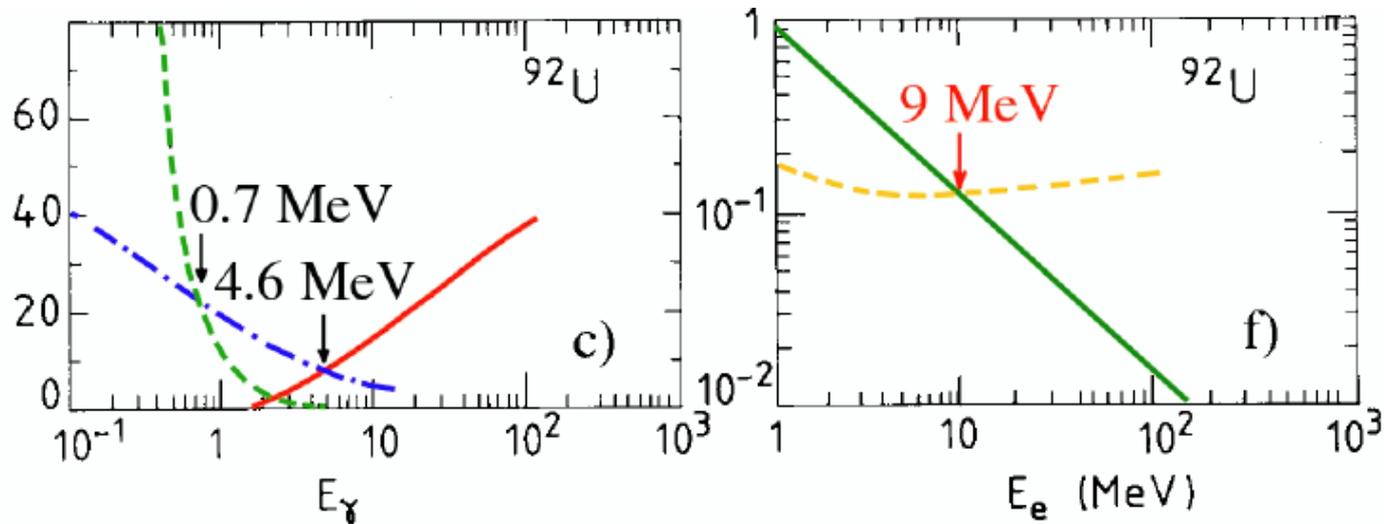


Gammas



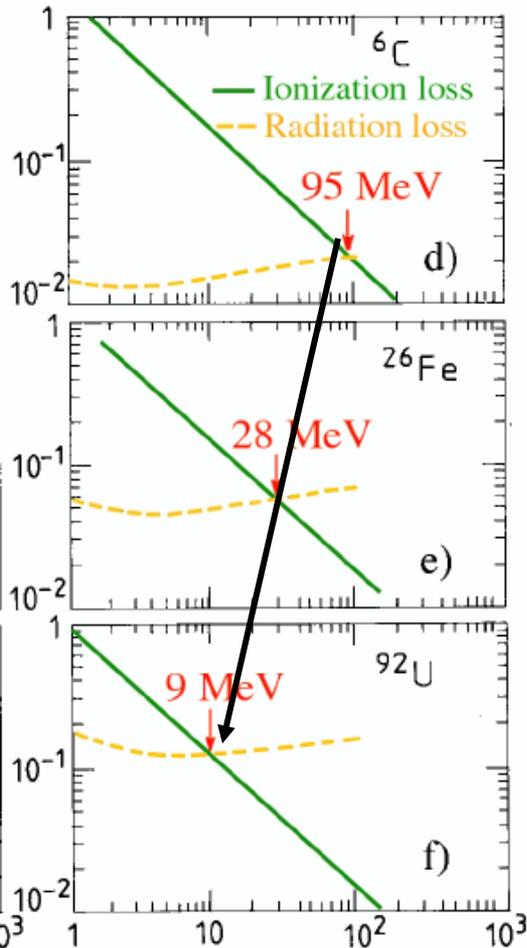
Electrons

Interpretation / comments



Energy scale:
even though calorimeters are intended to
measure GeV, TeV energy deposits,
their performance is determined by what happens
at the MeV - keV - eV level

Electrons



Increasing Z

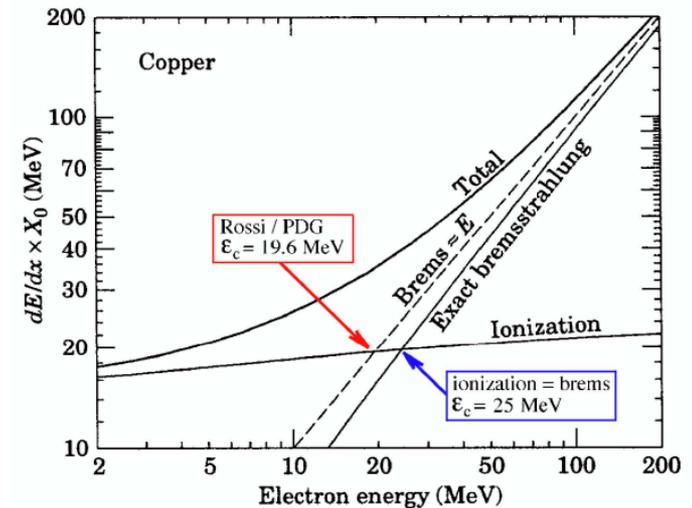
Electrons lose energy by: *ionization* *radiation*

Critical energy ϵ_c :

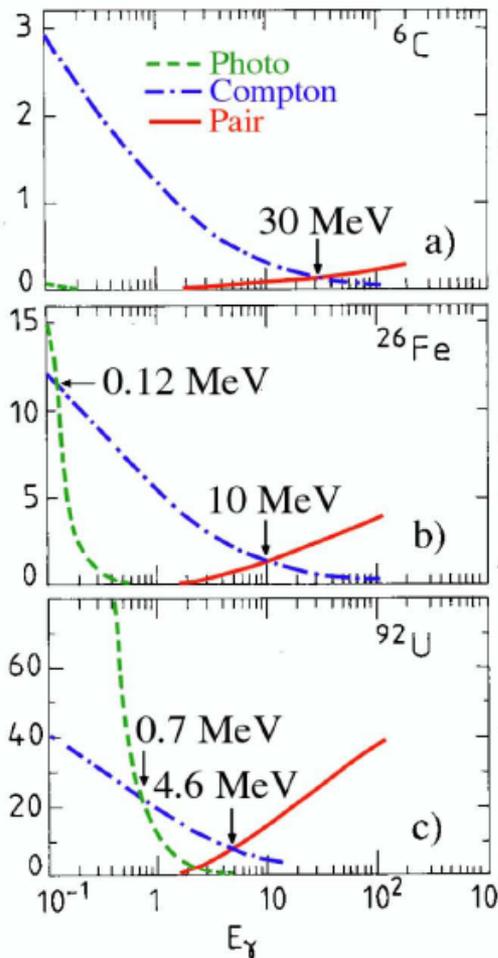
$$\frac{dE}{dx} (\text{ion}) = \frac{dE}{dx} (\text{rad})$$

$$\epsilon_c \propto 1/Z \quad \text{PDG: } \epsilon_c = 610 \text{ MeV}/(Z + 1.24)$$

In high Z materials
particle multiplication
at lower energies



Photons



Increasing Z

• *Photons* interact by:

1) Photoelectric effect

$$\sigma \propto Z^5, E^{-3}$$

2) Compton scattering

$$\sigma \propto Z, E^{-1}$$

3) Conversion into e^+e^-

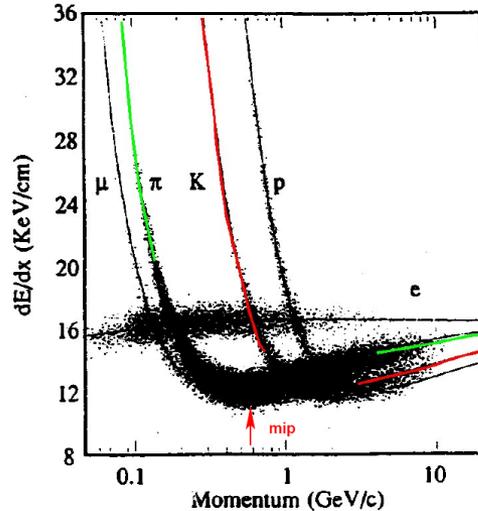
σ increases with E , Z , asymptotic at ~ 1 GeV

Differences between high-Z/low-Z materials:

- Energy at which *radiation* becomes dominant
- Energy at which *photoelectric effect* becomes dominant
- Energy at which e^+e^- *pair production* becomes dominant

What about the muons?

Heavy particles: $M \gg m_e$
 → Bethe-Bloch



Minimum Ionizing Particle:
 $dE/dx = \text{minimum}$

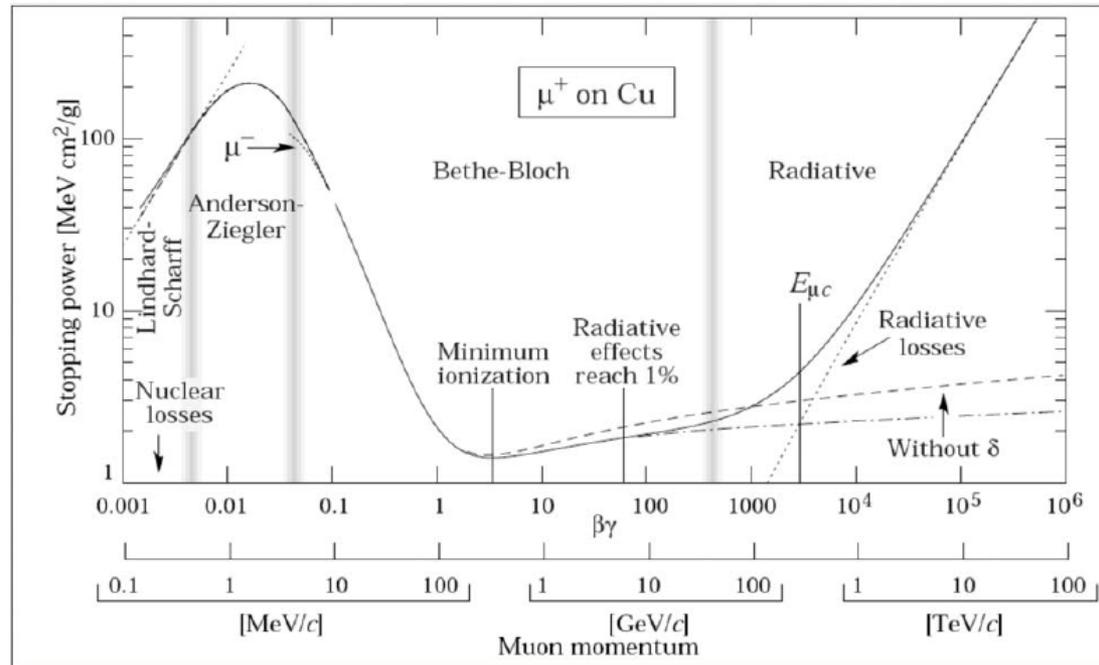
$$E_c^\mu = E_c^e \left(\frac{m_\mu}{m_e} \right)^2 \approx 4 \cdot 10^4 E_c^e$$

$E_c(e^-)$ in Cu = 20 MeV

$E_c(\mu)$ in Cu = 1 TeV

$Z_{Cu} = 29$

Muon energy losses mainly via ionization → “no shower”



dE/dx: some typical values

Typically $dE/dx = 1-2 \text{ MeV / g cm}^2 \times \rho \text{ [g/cm}^3]$

Iron $\rho=7.87 \text{ g/cm}^3$: $dE/dx = 11 \text{ MeV / cm} = 1.1 \text{ GeV / m}$

Silicon $300 \text{ }\mu\text{m}$: $dE/dx = 115 \text{ keV (MPV} = 82\text{keV)} (\sim 4 \text{ MeV / cm})$

Gas: $dE/dx = \text{few keV / cm}$

Ionization energy: $\sim Z \times 10 \text{ eV}$

$300 \text{ }\mu\text{m Silicon: } 30'000 \text{ e/h pairs} \quad (\sim 10^6 \text{ e/h pairs /cm})$

Small band gap, 3.6 eV/pair

Still a small charge: depletion

Gas: $\text{few } 10 \text{ electron ion pairs/cm}$

Need gas amplification

To be compared to typical pre-amplifier electronic noise equivalent: 1000 e

dE/dx fluctuations

Distance between interactions: exponential distribution

- $P(d) \sim \exp(-d / \lambda)$ with $\lambda = A / N_A \sigma \rho$

Number of collisions in given thickness: Poisson distribution

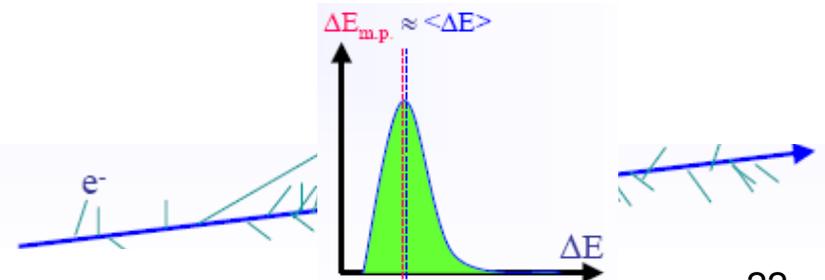
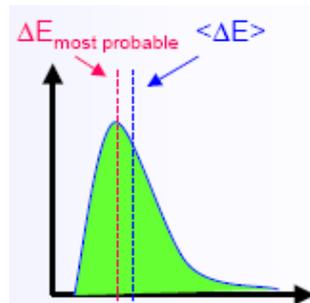
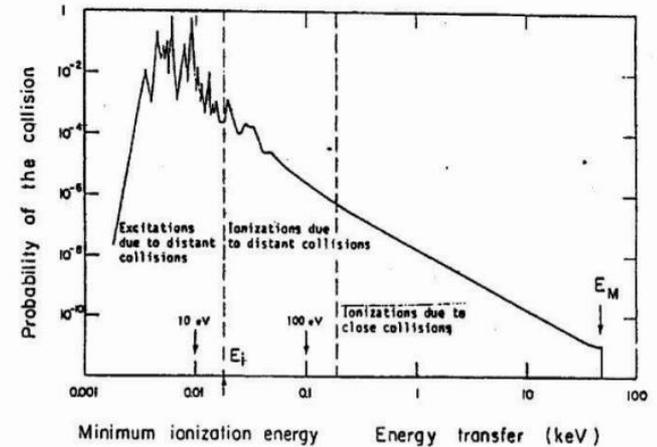
- Can fluctuate to zero \rightarrow inefficiencies

Energy loss distribution in each collision \rightarrow

- Large values possible (δ electrons)

$P(dE/dx)$ is a **Landau distribution**

- Asymmetric (tail to high dE/dx)
- Mean \neq most probable value
- Approaches Gaussian for thick layers



Muons are not MIP

The effects of radiation are clearly visible in calorimeters, especially for high-energy muons in high-Z absorber material

like Pb (Z=82)

$E_c(e^-) = 6 \text{ MeV}$

$E_c(\mu) = 250 \text{ GeV}$

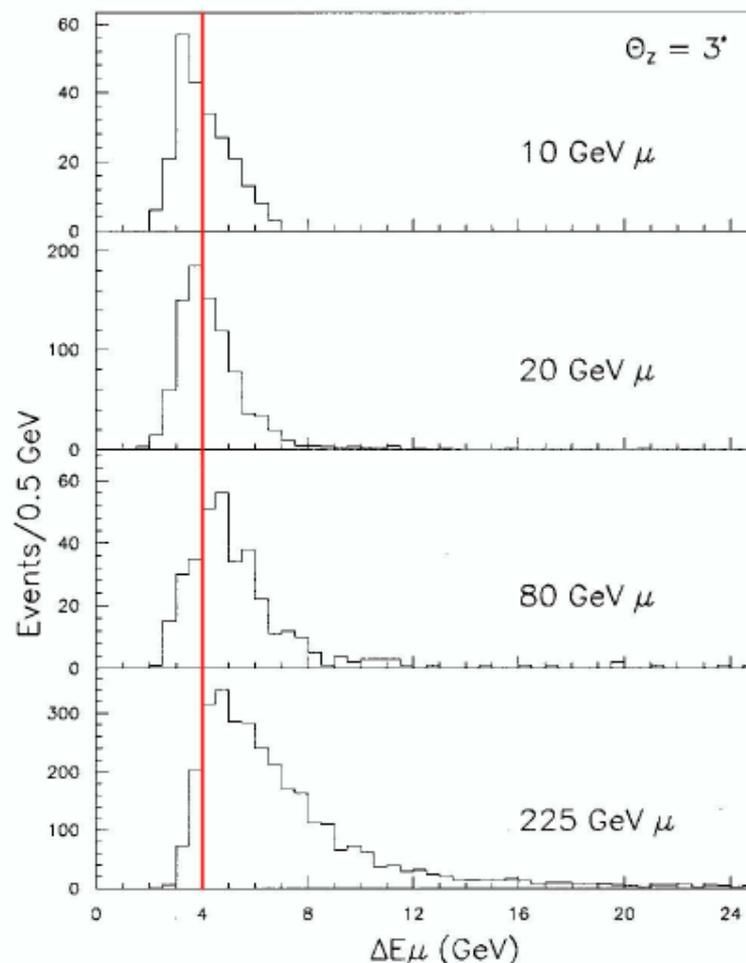


FIG. 2.19. Signal distributions for muons of 10, 20, 80 and 225 GeV traversing the $9.5\lambda_{\text{int}}$ deep SPACAL detector at $\theta_z = 3^\circ$. From [Aco 92c].

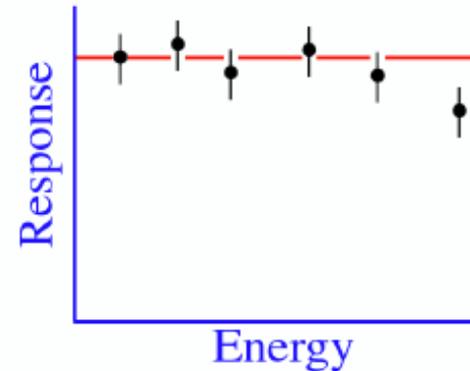
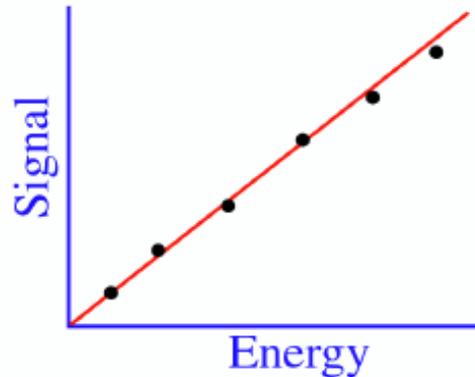
Measurement of showers

- To make a statement about the energy of a particle:
 1. relationship between measured signal and deposited energy
 - **Detector response → Linearity**
 - The average calorimeter signal vs. the energy of the particle
 - Homogenous and sampling calorimeters
 - Compensation (for hadronic showers)
 2. precision with which the unknown energy can be measured
 - **Detector resolution → Fluctuations**
 - Event to event variations of the signal
 - Resolution
 - What limits the accuracy at different energies?

Response and linearity

“**response** = average signal per unit of deposited energy”
e.g. # photoelectrons/GeV, picoCoulombs/MeV, etc

A **linear** calorimeter has a **constant response**



In general

Electromagnetic calorimeters are linear

→ All energy deposited through ionization/excitation of absorber

Hadronic calorimeters are not ... (later)

Sources of non-linearity

- Instrumental effects
 - Saturation of gas detectors, scintillators, photo-detectors, electronics
- Response varies with something that varies with energy
- Examples:
 - Deposited energy “counts” differently, depending on depth
 - And depth increases with energy
- Leakage (increases with energy)

Example of non-linearity

Signal linearity for electromagnetic showers

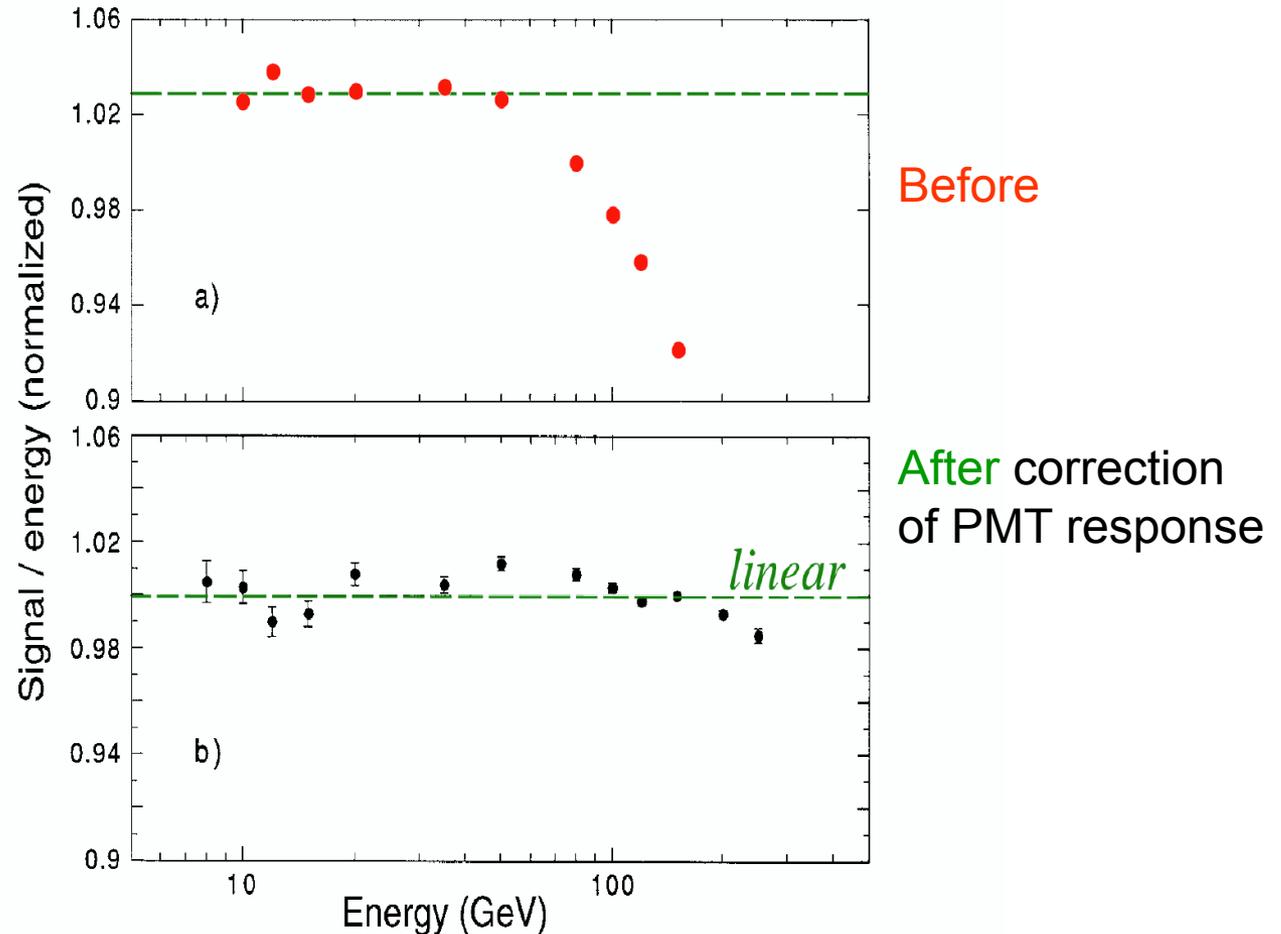


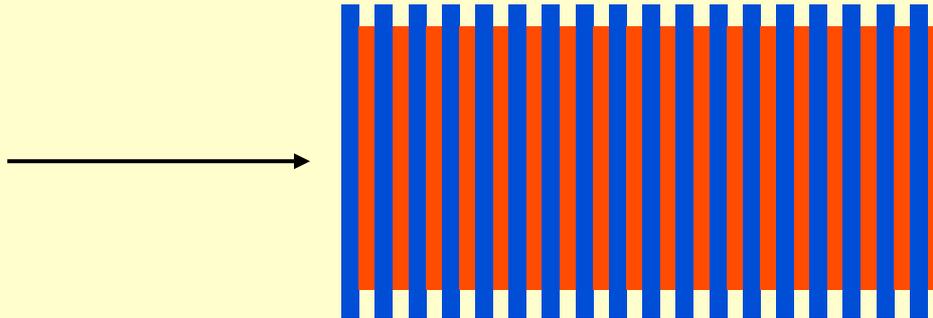
FIG. 3.1. The em calorimeter response as a function of energy, measured with the QFCAL calorimeter, before (a) and after (b) precautions were taken against PMT saturation effects. Data from [Akc 97].

Calorimeter types

There are two general classes of calorimeter:

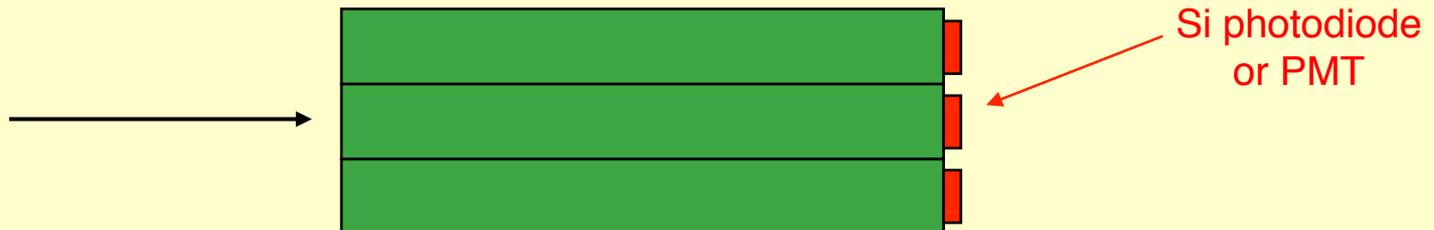
Sampling calorimeters:

Layers of passive absorber (such as Pb, or Cu) alternate with active detector layers such as Si, scintillator or liquid argon



Homogeneous calorimeters:

A single medium serves as both absorber and detector, eg: liquified Xe or Kr, dense crystal scintillators (BGO, PbWO_4 ), lead loaded glass.



Homogenous calorimeters

One block of material serves as **absorber and active medium** at the same time
Scintillating crystals with high density and high Z

Advantages:

see all charged particles in the shower → best statistical precision
same response from everywhere → good linearity

Disadvantages:

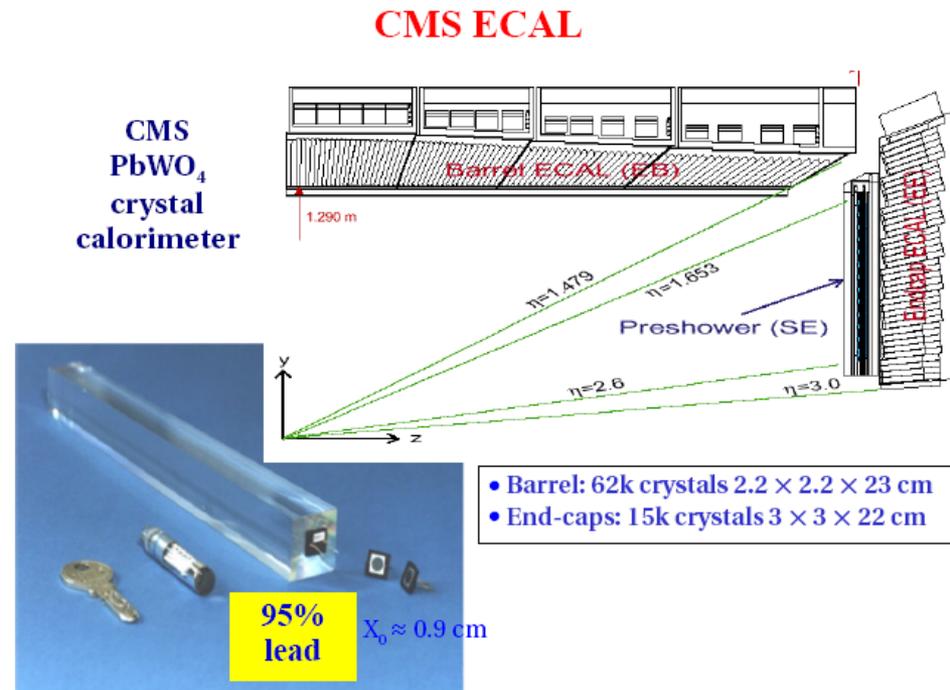
cost and limited segmentation

Examples:

B factories: small photon energies

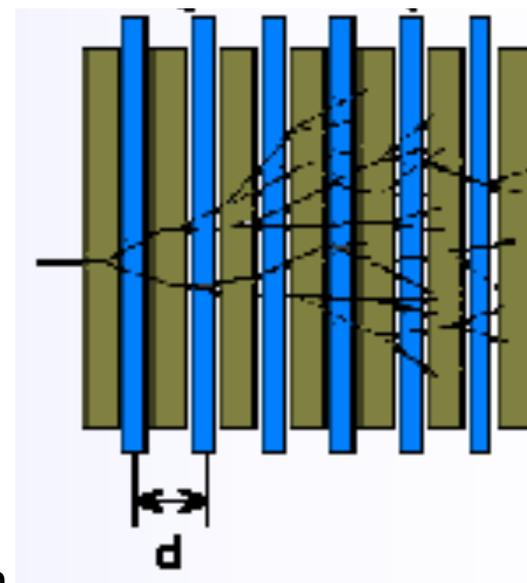
CMS ECAL:

optimized for $H \rightarrow \gamma\gamma$



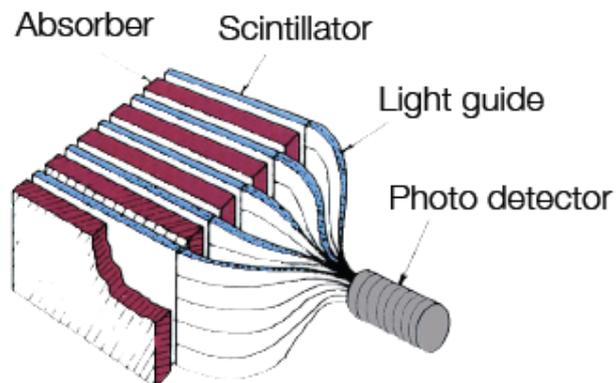
Sampling calorimeters

- Use different media
 - High density absorber
 - Interleaved with active readout devices
 - Most commonly used: sandwich structures →
 - But also: embedded fibres,
- Sampling fraction
 - $f_{\text{sampl}} = E_{\text{visible}} / E_{\text{total deposited}}$
- Advantages:
 - Cost, transverse and longitudinal segmentation
- Disadvantages:
 - Only part of shower seen, less precise
- Examples:
 - ATLAS ECAL
 - All HCALs (I know of)

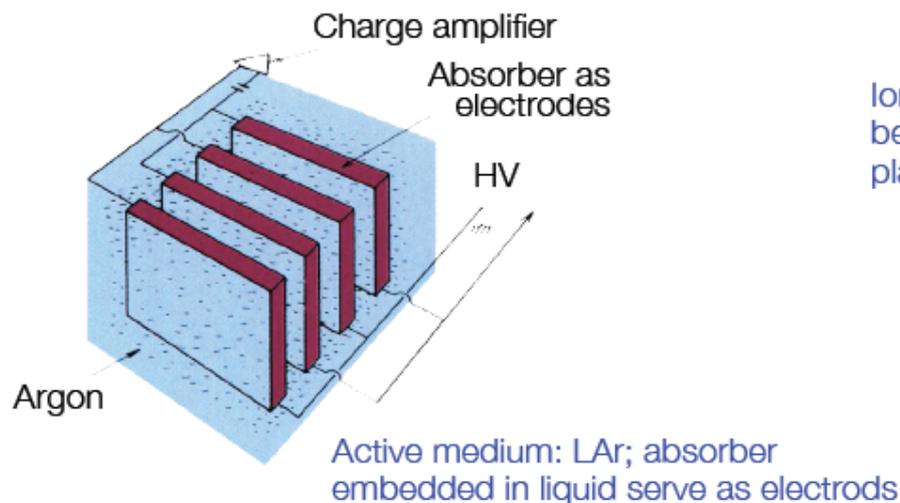
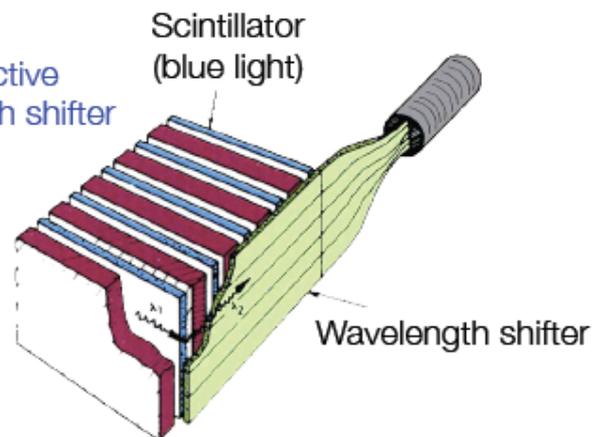


Sampling calorimeters

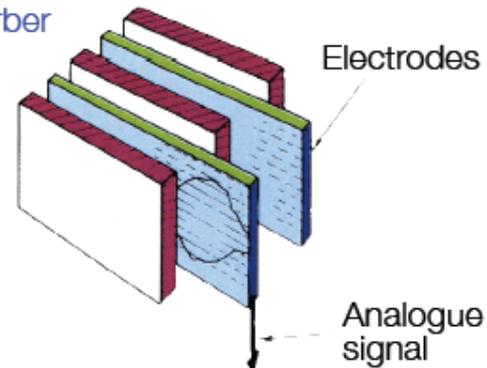
Scintillators as active layer;
signal readout via photo multipliers



Scintillators as active layer; wave length shifter to convert light

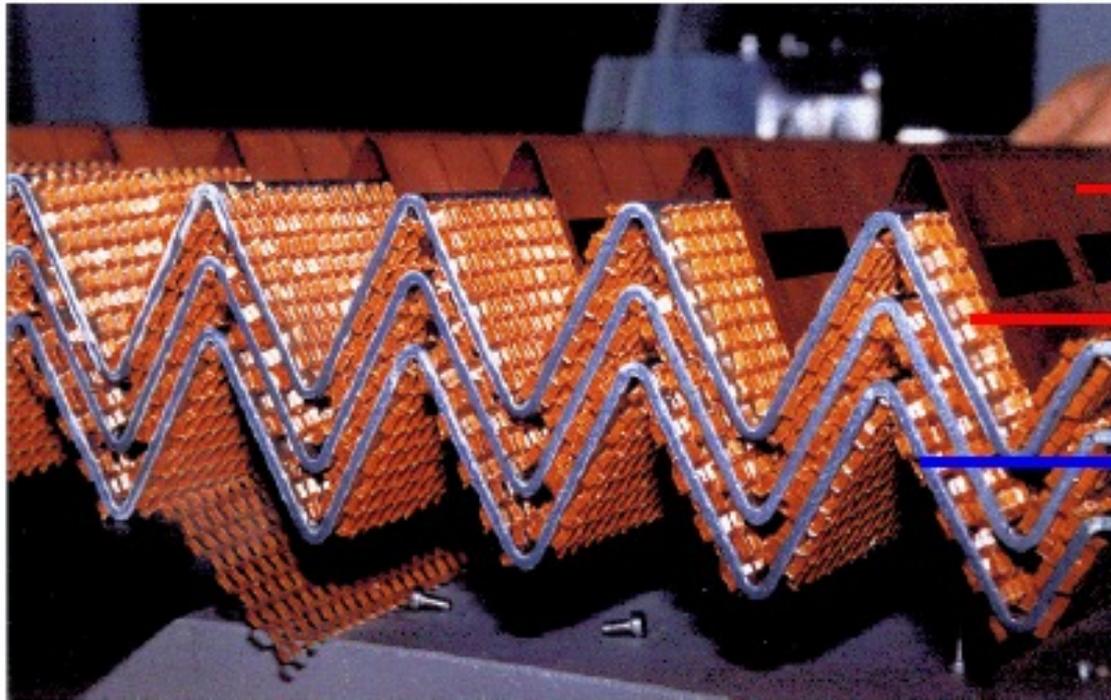


Ionization chambers
between absorber
plates



Possible setups

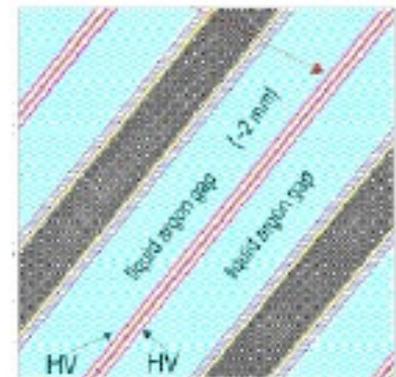
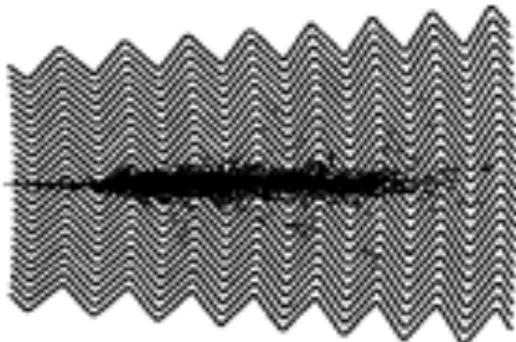
ATLAS LAr ECAL



Cu electrodes at +HV

Spacers define LAr gap
 2×2 mm

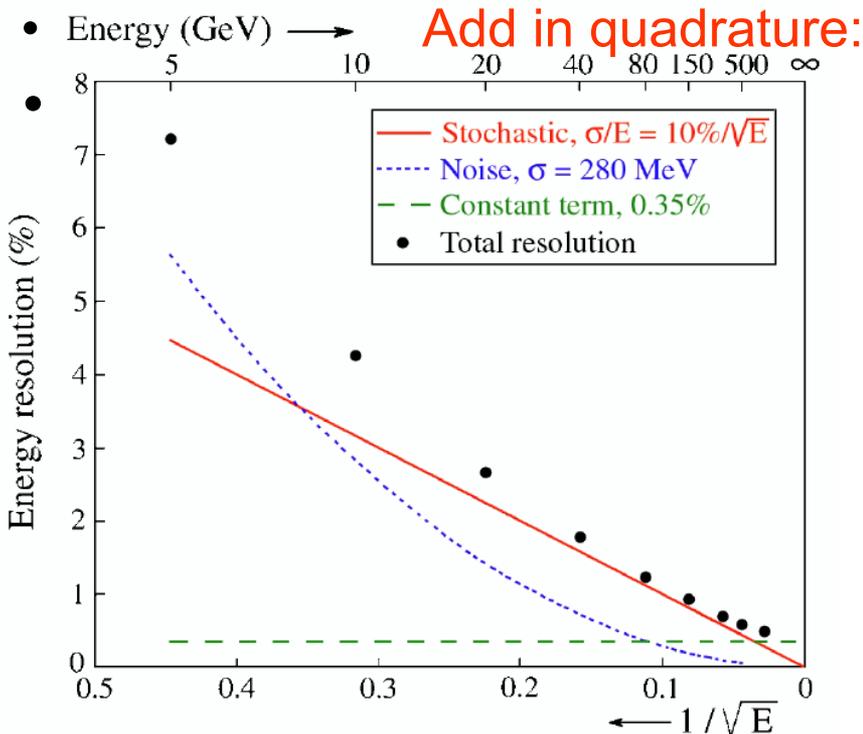
2 mm Pb absorber
clad in stainless steel.



Fluctuations

Different effects have different energy dependence

- quantum, sampling fluctuations $\sigma/E \sim E^{-1/2}$
- shower leakage $\sigma/E \sim E^{-1/4}$
- electronic noise $\sigma/E \sim E^{-1}$
- structural non-uniformities $\sigma/E = \text{constant}$



$$\sigma_{\text{tot}}^2 = \sigma_1^2 + \sigma_2^2 + \sigma_3^2 + \sigma_4^2 + \dots$$

← example: ATLAS EM calorimeter

Energy resolution

Ideally, if all shower particles counted:

$$E \sim N, \quad \sigma \sim \sqrt{N} \sim \sqrt{E}$$

In practice:

absolute $\sigma = a \sqrt{E} \oplus b E \oplus c$

relative $\sigma / E = a / \sqrt{E} \oplus b \oplus c / E$

a: stochastic term

intrinsic statistical shower fluctuations

sampling fluctuations

signal quantum fluctuations (e.g. photo-electron statistics)

b: constant term

inhomogeneities (hardware or calibration)

imperfections in calorimeter construction (dimensional variations, etc.)

non-linearity of readout electronics

fluctuations in longitudinal energy containment (leakage can also be $\sim E^{-1/4}$)

fluctuations in energy lost in dead material before or within the calorimeter

c: noise term

readout electronic noise

Radio-activity, pile-up fluctuations

Intrinsic Energy Resolution of EM calorimeters

Homogeneous calorimeters:

signal = sum of all E deposited by charged particles with $E > E_{\text{threshold}}$

If W is the mean energy required to produce a 'signal quantum' (eg an electron-ion pair in a noble liquid or a 'visible' photon in a crystal) \rightarrow mean number of 'quanta' produced is

$$\langle n \rangle = E / W$$

The intrinsic energy resolution is given by the fluctuations on n.

$$\sigma_E / E = 1/\sqrt{n} = 1/\sqrt{(E/W)}$$

i.e. in a semiconductor crystals (Ge, Ge(Li), Si(Li))

$W = 2.9$ eV (to produce e-hole pair)

$\rightarrow 1$ MeV $\gamma = 350000$ electrons $\rightarrow 1/\sqrt{n} = 0.17\%$ stochastic term

Silicon detectors	:	$W \approx 3.6$ eV
Gas detectors	:	$W \approx 30$ eV
Plastic scintillator	:	$W \approx 100$ eV

In addition, fluctuations on n are reduced by correlation in the production of consecutive e-hole pairs: the Fano factor F

$$\sigma_E / E = \sqrt{(FW/E)}$$

For GeLi γ detector $F \sim 0.1 \rightarrow$ stochastic term $\sim 1.7\%/\sqrt{E[\text{GeV}]}$

Resolution of crystal EM calorimeters

Study the example of CMS: PbWO_4 crystals r/o via APD:

Fano factor $F \sim 2$ for the crystal/APD combination
in crystals $F \sim 1$ + fluctuations in the avalanche multiplication process of APD
(‘excess noise factor’)

PbWO_4 is a relatively weak scintillator. In CMS, ~ 4500 photo-electrons/1 GeV
(with QE $\sim 80\%$ for APD)

Thus, expected stochastic term:

$$a_{pe} = \sqrt{F/N_{pe}} = \sqrt{2/4500} = 2.1\%$$

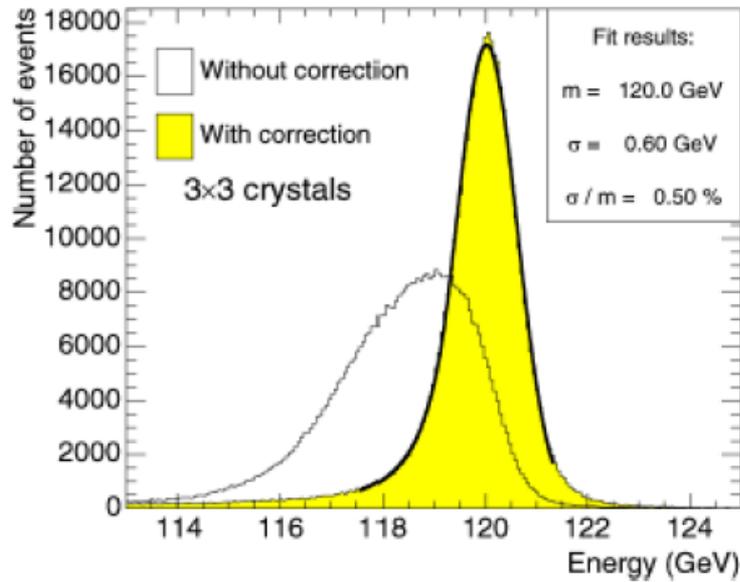
Including effect of lateral leakage from limited clusters of crystals (to minimise electronic noise and pile up) one has to add

$$a_{leak} = 1.5\% (\Sigma(5 \times 5)) \quad \text{and} \quad a_{leak} = 2\% (\Sigma(3 \times 3))$$

Thus for the $\Sigma(3 \times 3)$ case one expects $a = a_{pe} \oplus a_{leak} = 2.9\%$

→ compared with the measured value: $a_{meas} = 3.4\%$

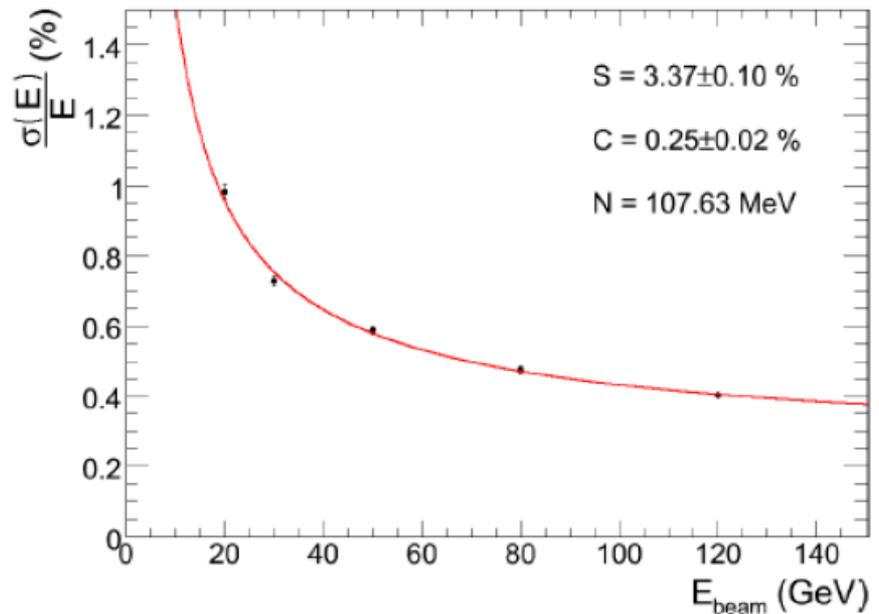
Example: CMS ECAL resolution



Correction for radial loss

The sampling term is 3 times smaller than ATLAS; other terms are similar

$$\left(\frac{\sigma}{E}\right)^2 = \underbrace{\left(\frac{3.37\%}{\sqrt{E}}\right)^2}_{\text{stoch.}} + \underbrace{\left(\frac{0.107}{E}\right)^2}_{\text{noise}} + \underbrace{(0.25\%)^2}_{\text{const.}}$$



Resolution of sampling calorimeters

Main contribution: sampling fluctuations, from variations in the number of **charged** particles crossing the active layers.

Increases linearly with incident energy and with the fineness of the sampling.

Thus:

$$n_{ch} \propto E/t \quad (t \text{ is the thickness of each absorber layer})$$

For statistically independent sampling the sampling contribution to the stochastic term is:

$$\sigma_{samp}/E \propto 1/\sqrt{n_{ch}} \propto \sqrt{t/E}$$

Thus the resolution improves as t is decreased.

For EM order 100 samplings required to approach the resolution of typical homogeneous devices → impractical.

Typically:

$$\sigma_{samp}/E \sim 10\%/\sqrt{E}$$

Dependence on sampling

Measure energy resolution of a sampling calorimeter for different absorber thicknesses

t_{abs} : absorber thickness in X_0

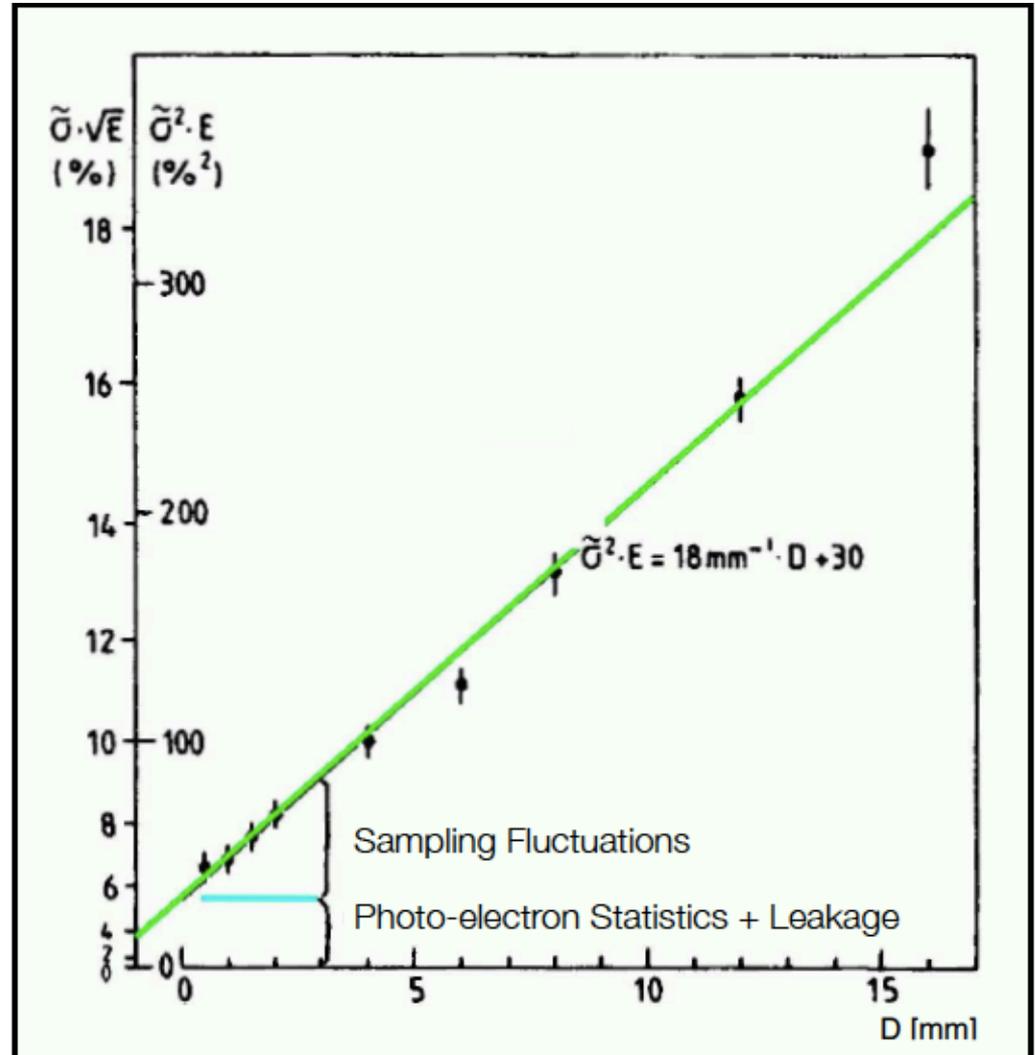
D : absorber thickness in mm

Sampling contribution:

$$\frac{\sigma_E}{E} = 3.2\% \sqrt{\frac{E_c [\text{MeV}] \cdot t_{\text{abs}}}{F \cdot E [\text{GeV}]}}$$

Choose: E_c small (large Z)

t_{abs} small (fine sampling)



EM calorimeters: energy resolution

Homogeneous calorimeters: all the energy is deposited in an active medium.
Absorber \equiv active medium \longrightarrow All e^+e^- over threshold produce a signal

Excellent energy resolution

Compare processes with different energy threshold

Scintillating crystals

$$E_s \cong \beta E_{\text{gap}} \sim \text{eV}$$

$$\approx 10^2 \div 10^4 \gamma / \text{MeV}$$

$$\sigma / E \sim (1 \div 3)\% / \sqrt{E(\text{GeV})}$$

Cherenkov radiators

$$\beta > \frac{1}{n} \rightarrow E_s \sim 0.7 \text{MeV}$$

$$\approx 10 \div 30 \gamma / \text{MeV}$$

$$\sigma / E \sim (10 \div 5)\% / \sqrt{E(\text{GeV})}$$



Lowest possible limit

Homogeneous vs Sampling

Technology (Experiment)	Depth	Energy resolution	Date
NaI(Tl) (Crystal Ball)	$20X_0$	$2.7\%/E^{1/4}$	1983
Bi ₄ Ge ₃ O ₁₂ (BGO) (L3)	$22X_0$	$2\%/ \sqrt{E} \oplus 0.7\%$	1993
CsI (KTeV)	$27X_0$	$2\%/ \sqrt{E} \oplus 0.45\%$	1996
CsI(Tl) (BaBar)	$16-18X_0$	$2.3\%/E^{1/4} \oplus 1.4\%$	1999
CsI(Tl) (BELLE)	$16X_0$	1.7% for $E_\gamma > 3.5$ GeV	1998
PbWO ₄ (PWO) (CMS)	$25X_0$	$3\%/ \sqrt{E} \oplus 0.5\% \oplus 0.2/E$	1997
Lead glass (OPAL)	$20.5X_0$	$5\%/ \sqrt{E}$	1990
Liquid Kr (NA48)	$27X_0$	$3.2\%/ \sqrt{E} \oplus 0.42\% \oplus 0.09/E$	1998
Scintillator/depleted U (ZEUS)	$20-30X_0$	$18\%/ \sqrt{E}$	1988
Scintillator/Pb (CDF)	$18X_0$	$13.5\%/ \sqrt{E}$	1988
Scintillator fiber/Pb spaghetti (KLOE)	$15X_0$	$5.7\%/ \sqrt{E} \oplus 0.6\%$	1995
Liquid Ar/Pb (NA31)	$27X_0$	$7.5\%/ \sqrt{E} \oplus 0.5\% \oplus 0.1/E$	1988
Liquid Ar/Pb (SLD)	$21X_0$	$8\%/ \sqrt{E}$	1993
Liquid Ar/Pb (H1)	$20-30X_0$	$12\%/ \sqrt{E} \oplus 1\%$	1998
Liquid Ar/depl. U (DØ)	$20.5X_0$	$16\%/ \sqrt{E} \oplus 0.3\% \oplus 0.3/E$	1993
Liquid Ar/Pb accordion (ATLAS)	$25X_0$	$10\%/ \sqrt{E} \oplus 0.4\% \oplus 0.3/E$	1996

Homogeneous

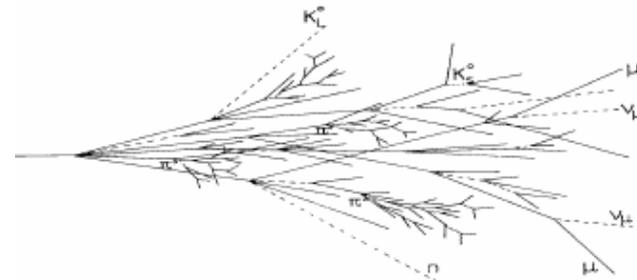
Sampling

* E in GeV

Hadronic calorimeters

Hadron showers

- Extra complication: **The strong interaction** with detector material
- Importance of calorimetric measurement
 - Charged hadrons: complementary to track measurement
 - Neutral hadrons: the only way to measure their energy
- In nuclear collisions numbers of secondary particles are produced
 - Partially undergo secondary, tertiary **nuclear reactions** → formation of hadronic cascade
 - Electromagnetically decaying particles (π, η) initiate EM showers
 - Part of the energy is absorbed as nuclear binding energy or target recoil (**Invisible energy**)
- Similar to EM showers, but much more complex
 - need simulation tools (MC)
- Different scale: hadronic interaction length



Hadronic interactions

1st stage: the hard collision

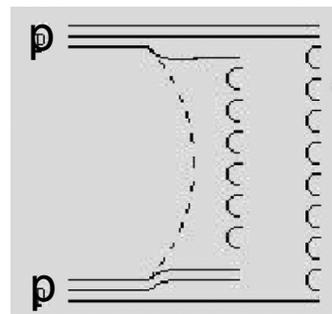
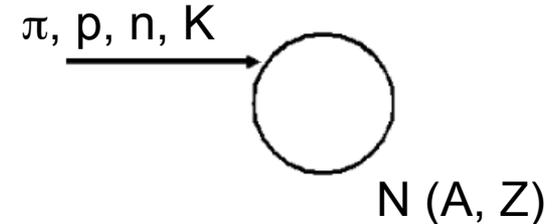
before first interaction:

- pions travel 25-50% longer than protons (~2/3 smaller in size)
- a pion loses ~100-300 MeV by ionization (Z dependent)

- particle multiplication
(one example: string model)

average energy needed to produce a pion 0.7 (1.3) GeV in Cu (Pb)

Particle nucleus collision according to cross-sections



q-qbar pairs

Nucleon is split in quark di-quark
Strings are formed
String hadronisation (adding qqbar pair)
fragmentation of damaged nucleus

- Multiplicity scales with E and particle type
- $\sim 1/3 \pi^0 \rightarrow \gamma\gamma$ produced in charge exchange processes:
 $\pi^+p \rightarrow \pi^0n$ / $\pi^-n \rightarrow \pi^0p$
- Leading particle effect: depends on incident hadron type
e.g fewer π^0 from protons, baryon number conservation

Hadronic interactions

2nd stage: spallation

– Intra-nuclear cascade

Fast hadron traversing the nucleus frees protons and neutrons in number proportional to their numerical presence in the nucleus.

Some of these n and p can escape the nucleus

For $^{208}_{82}\text{Pb}$ ~1.5 more cascade n than p

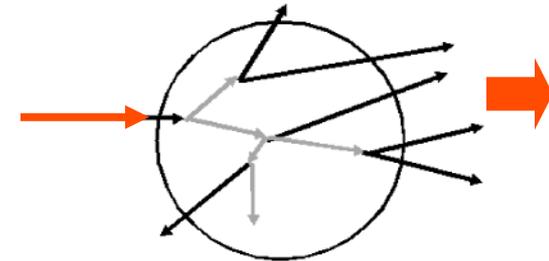
– The nucleons involved in the cascade transfer energy to the nucleus which is left in an excited state

– Nuclear de-excitation

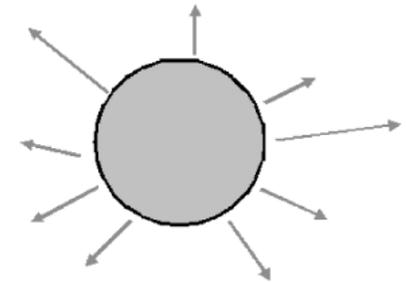
- Evaporation of soft (~10 MeV) nucleons and α
- + fission for some materials

The number of nucleons released depends on the binding E (7.9 MeV in Pb, 8.8 MeV in Fe)

Mainly neutrons released by evaporation → protons are trapped by the Coulomb barrier (12 MeV in Pb, only 5 MeV in Fe)



dominating momentum component along incoming particle direction

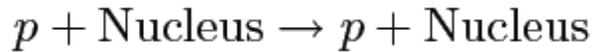


isotropic process

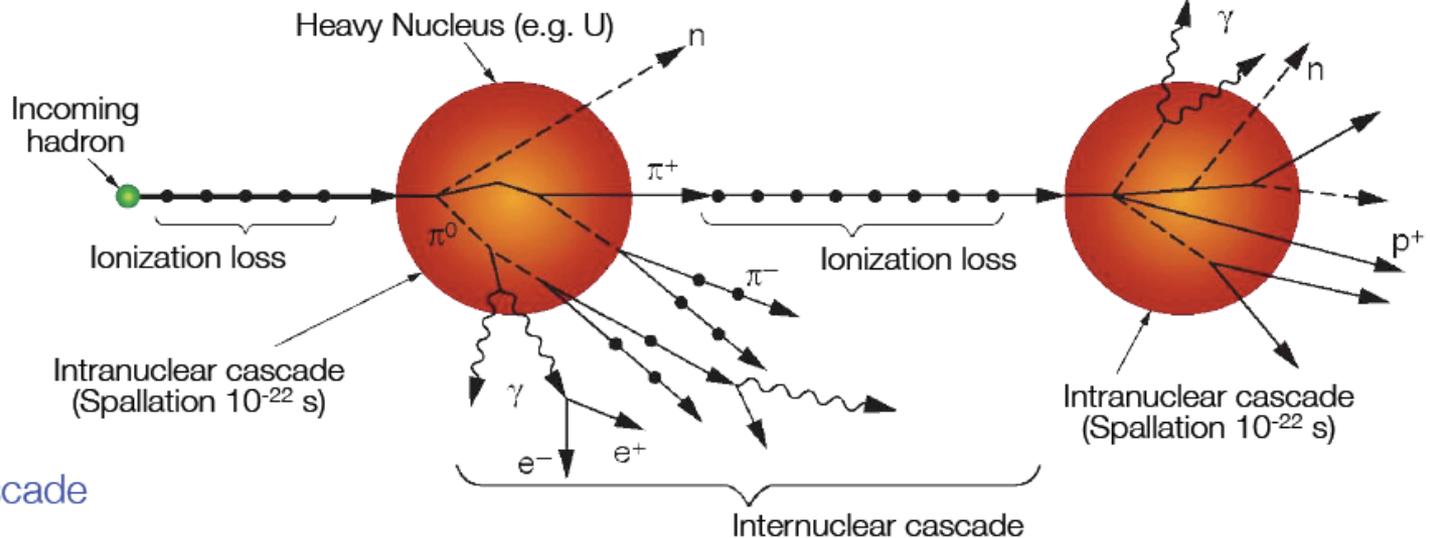
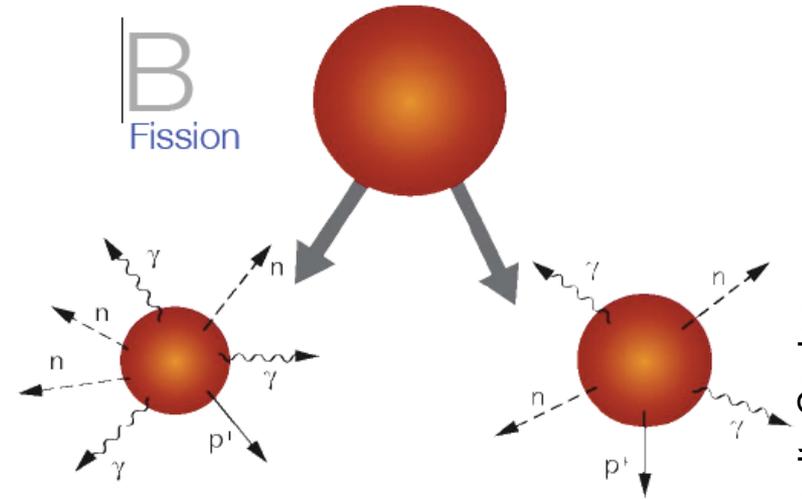
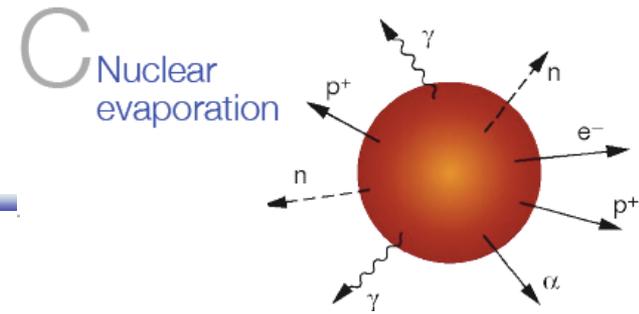
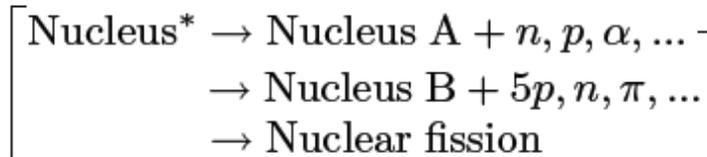
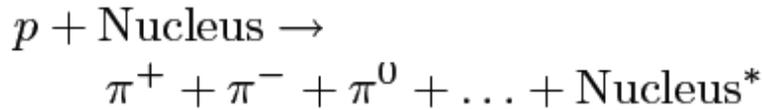
Hadronic showers

Hadronic interaction:

Elastic:



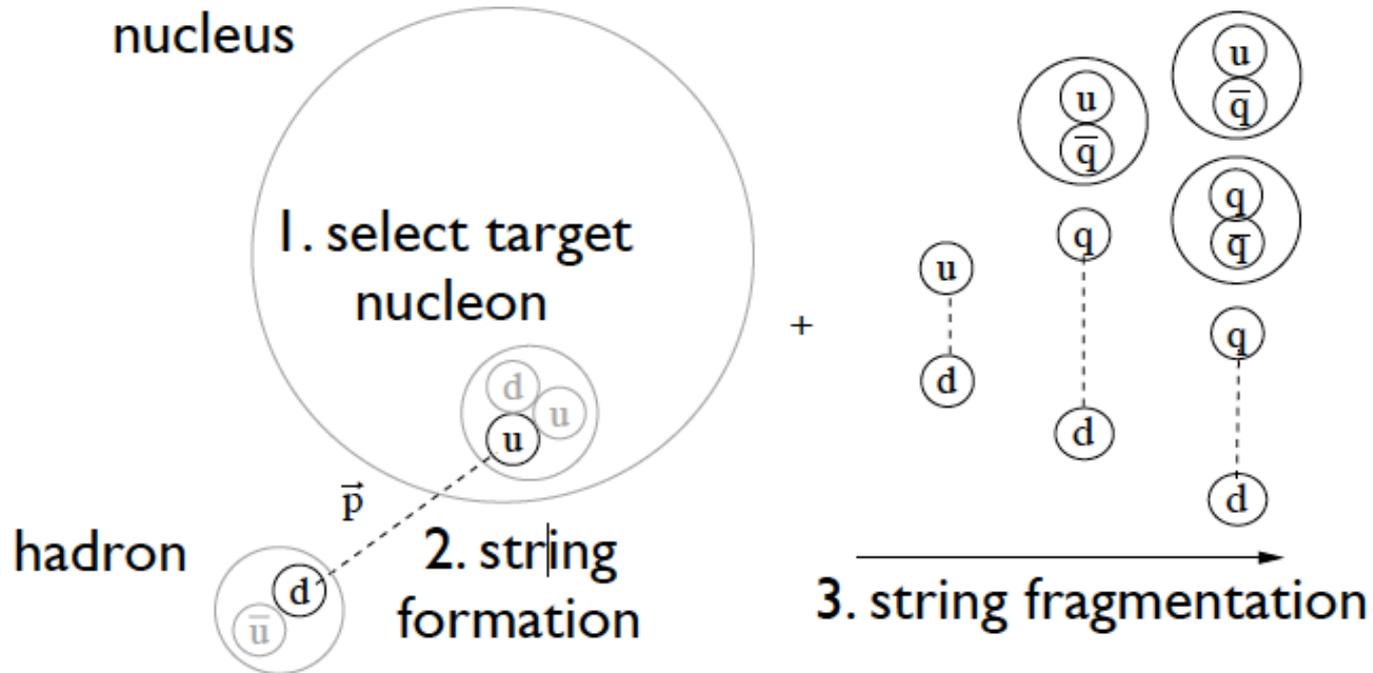
Inelastic:



Courtesy of H. C. Schoultz Coulon

“naïve” model (simulation programs)

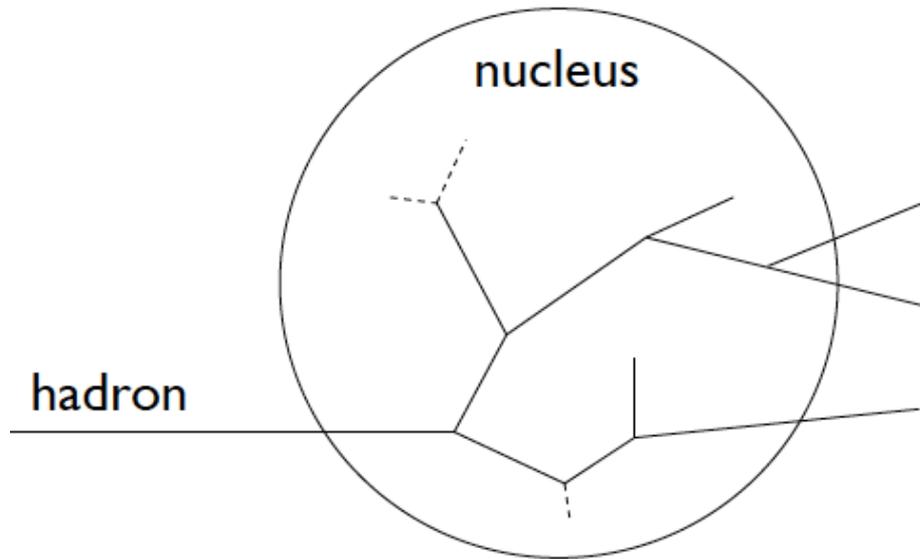
Interaction of hadrons with $E > 10$ GeV described by string models



- projectile interacts with single nucleon (p,n)
- a string is formed between quarks from interacting nucleons
- the string fragmentation generates hadrons

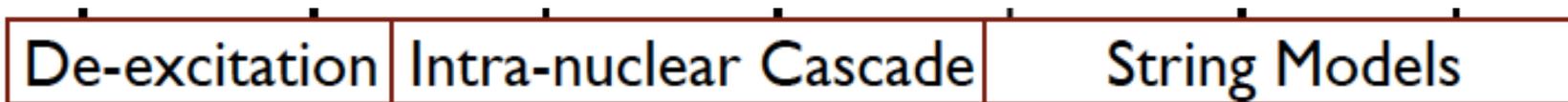
“naïve” model (simulation programs)

Interaction of hadrons with $10 \text{ MeV} < E < 10 \text{ GeV}$ via intra-nuclear cascades



- $\lambda_{\text{deBroglie}} \leq d \text{ nucleon}$
- nucleus = Fermi gas (all nucleons included)
- Pauli exclusion: allow only secondaries above Fermi energy

For $E < 10 \text{ MeV}$ only relevant are fission, photon emission, evaporation, ...



1 MeV 10 MeV 100 MeV 1 GeV 10 GeV 100 GeV 1 TeV \Rightarrow 48

Hadronic shower

Hadronic interaction:

Cross Section:

at high energies
also diffractive contribution

$$\sigma_{\text{tot}} = \sigma_{\text{el}} + \sigma_{\text{inel}}$$

For substantial energies
 σ_{inel} dominates:

$$\sigma_{\text{el}} \approx 10 \text{ mb}$$

$$\sigma_{\text{inel}} \propto A^{2/3} \text{ [geometrical cross section]}$$

$$\therefore \sigma_{\text{tot}} = \sigma_{\text{tot}}(pA) \approx \sigma_{\text{tot}}(pp) \cdot A^{2/3}$$

[σ_{tot} slightly grows with \sqrt{s}]

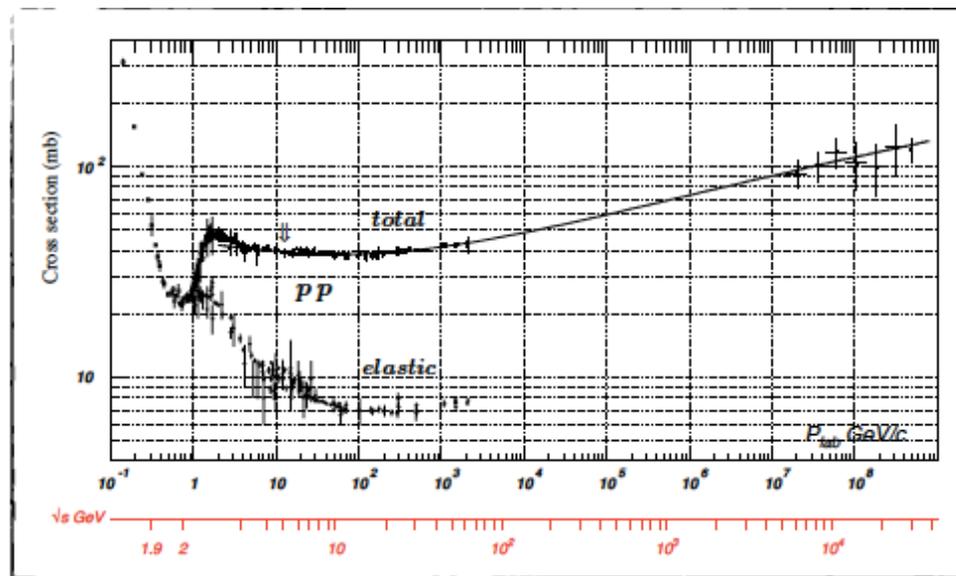
Hadronic interaction length:

$$\lambda_{\text{int}} = \frac{1}{\sigma_{\text{tot}} \cdot n} = \frac{A}{\sigma_{pp} A^{2/3} \cdot N_A \rho} \sim A^{1/3} \quad \text{[for } \sqrt{s} \approx 1 - 100 \text{ GeV]}$$

$$\approx 35 \text{ g/cm}^2 \cdot A^{1/3}$$

which yields:

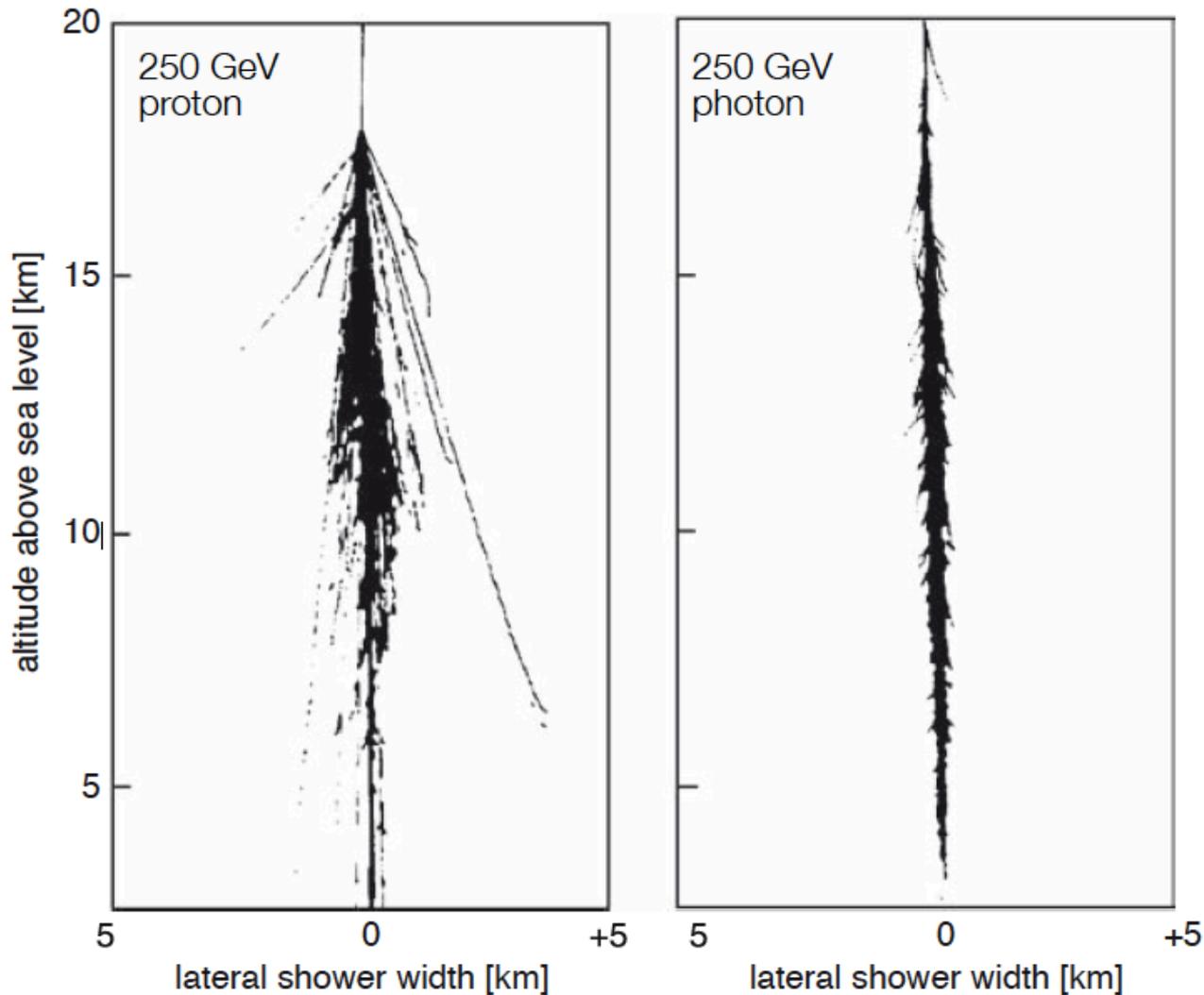
$$N(x) = N_0 \exp(-x/\lambda_{\text{int}})$$



Total proton-proton cross section
[similar for p+n in 1-100 GeV range]

Interaction length characterizes both,
longitudinal and transverse profile of
hadronic showers ...

Comparison hadronic vs EM showers



Simulated air showers

Comparison hadronic vs EM showers

Hadronic vs. electromagnetic interaction length:

$$\left. \begin{aligned} X_0 &\sim \frac{A}{Z^2} \\ \lambda_{\text{int}} &\sim A^{1/3} \end{aligned} \right\} \rightarrow \frac{\lambda_{\text{int}}}{X_0} \sim A^{4/3}$$

$$\lambda_{\text{int}} \gg X_0$$

[$\lambda_{\text{int}}/X_0 > 30$ possible; see below]

Typical
Longitudinal size: $6 \dots 9 \lambda_{\text{int}}$ [EM: $15-20 X_0$]
[95% containment]

Typical
Transverse size: one λ_{int} [EM: $2 R_M$; compact]
[95% containment]

Hadronic calorimeter need more depth than electromagnetic calorimeter ...

Some numerical values for materials typical used in hadron calorimeters

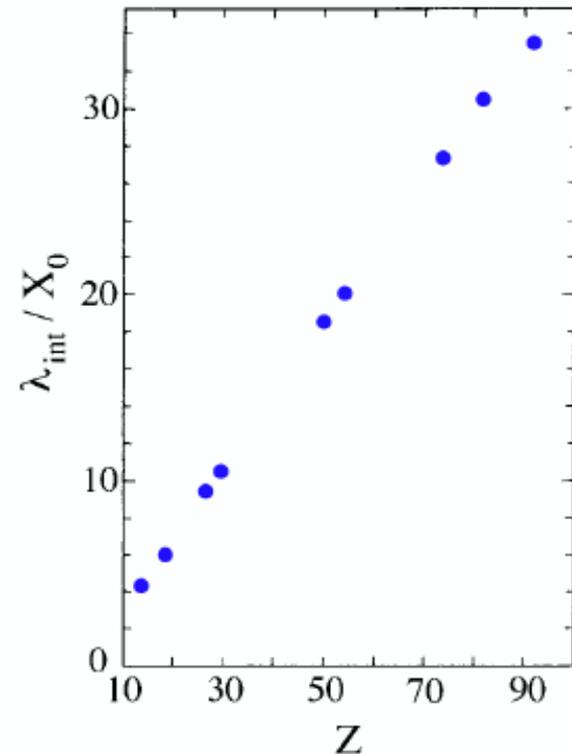
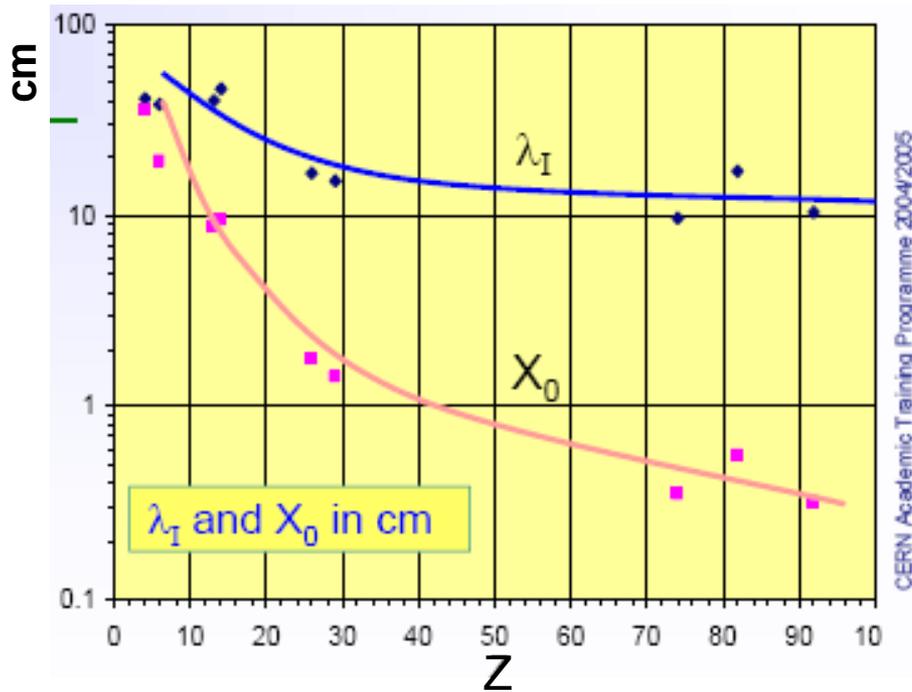
	λ_{int} [cm]	X_0 [cm]
Szint.	79.4	42.2
LAr	83.7	14.0
Fe	16.8	1.76
Pb	17.1	0.56
U	10.5	0.32
C	38.1	18.8

Material dependence

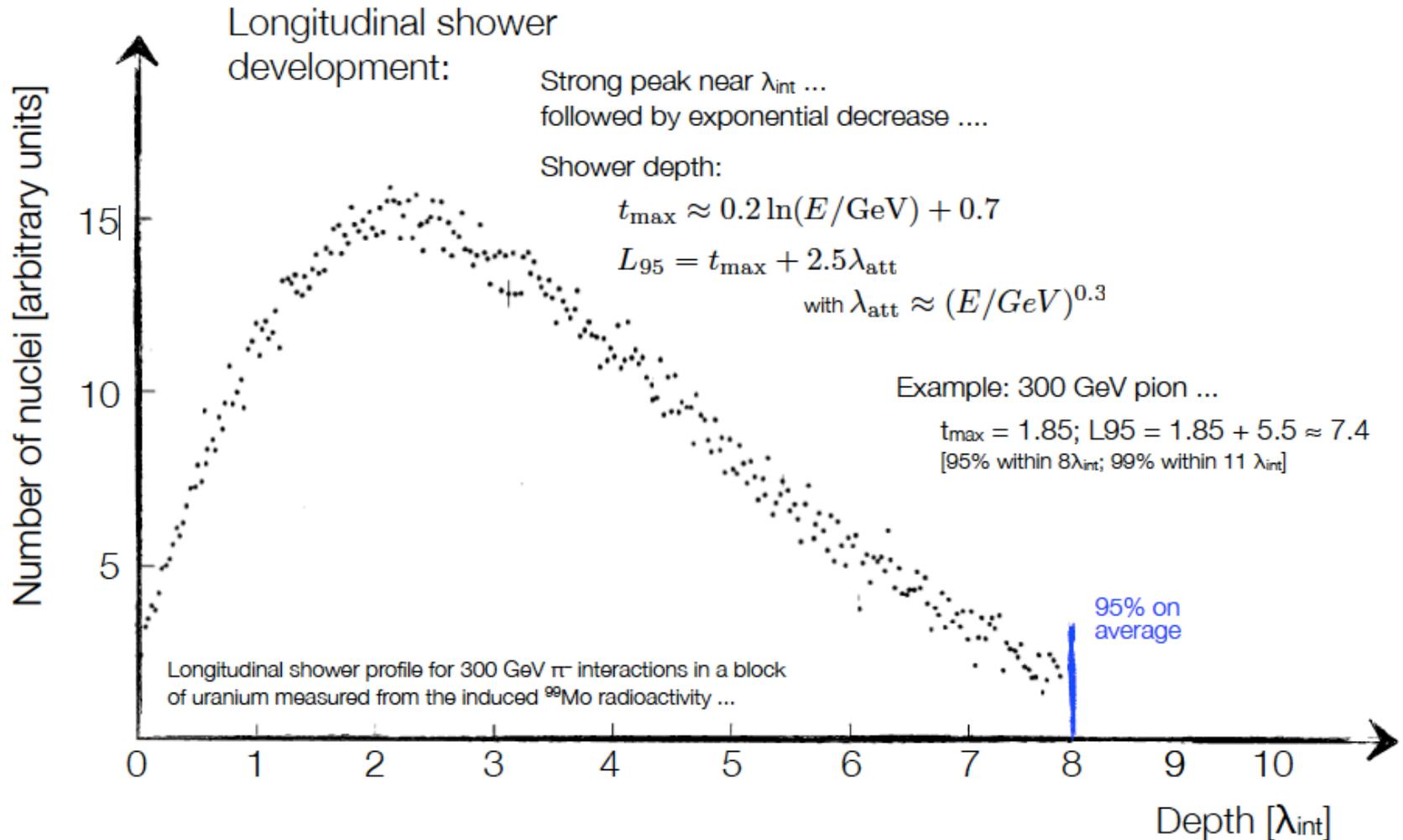
λ_{int} : mean free path between nuclear collisions

$$\lambda_{\text{int}} (\text{g cm}^{-2}) \propto A^{1/3}$$

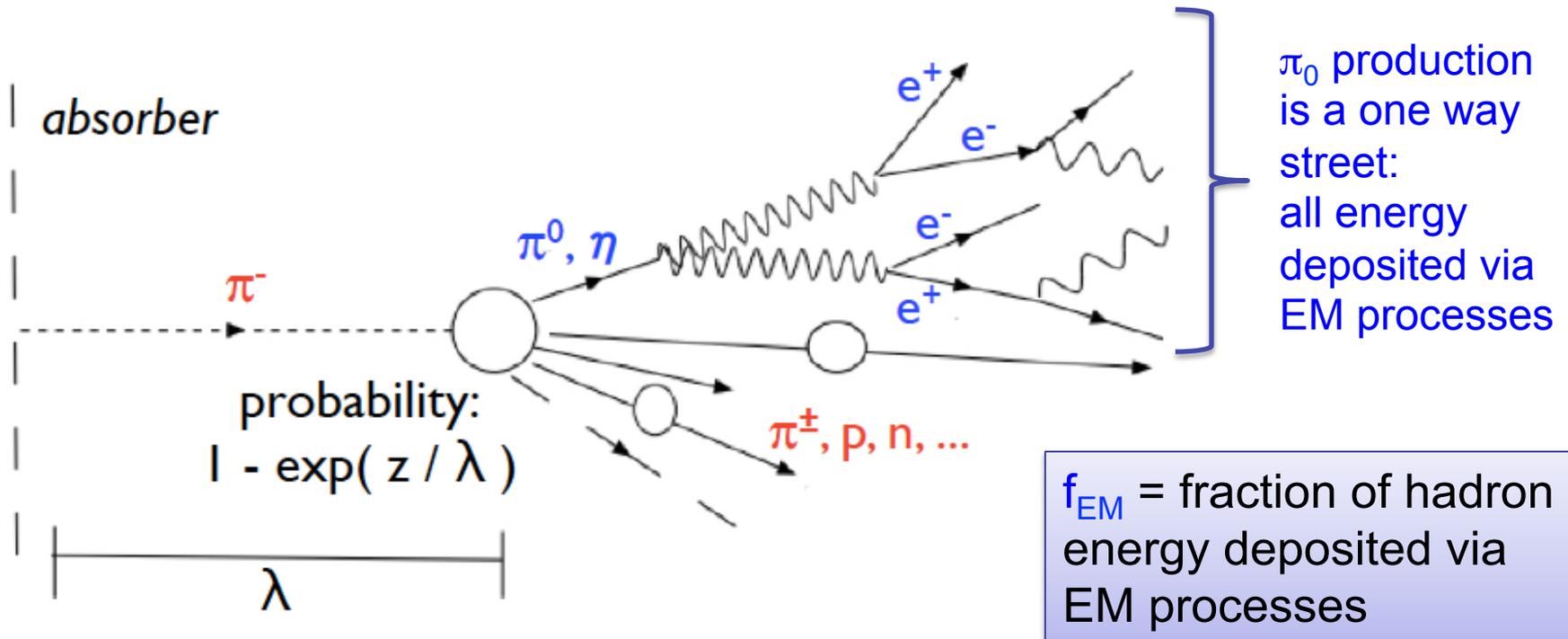
Hadron showers are much longer than EM ones – how much, depends on Z



Longitudinal development



Hadronic showers



- Electromagnetic** → ionization, excitation (e^\pm)
 → photo effect, scattering (γ)
- Hadronic** → ionization (π^\pm, ρ)
 → invisible energy (binding, recoil)

Electromagnetic fraction

$f_{\text{em}} \rightarrow 1$ (high energy limit)

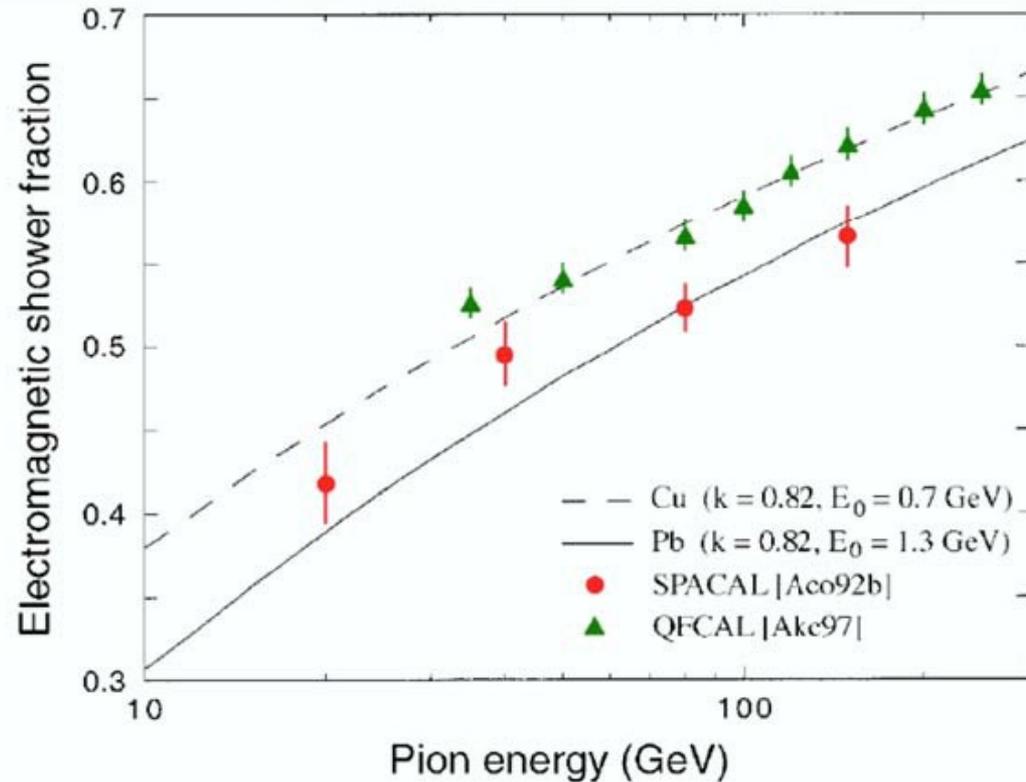
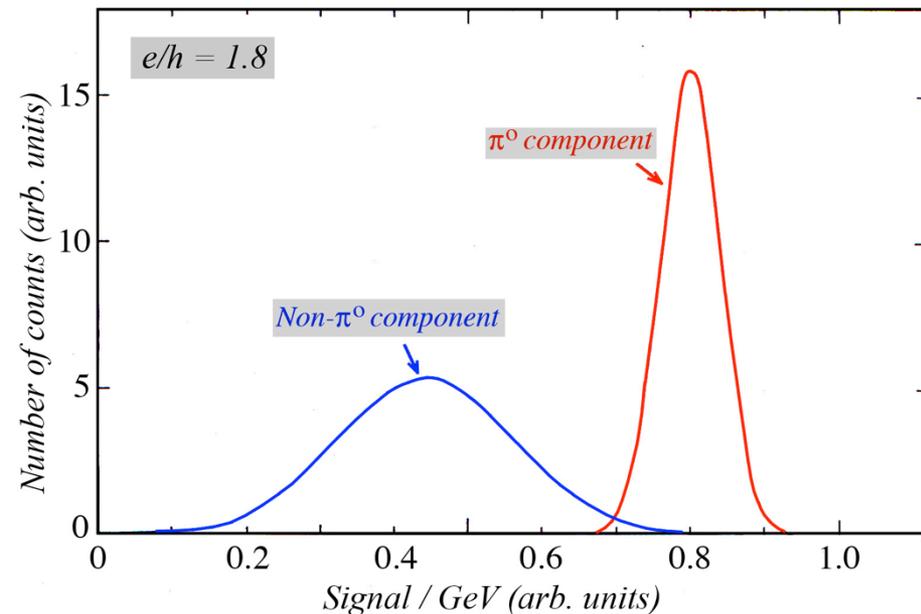


FIG. 2.22. Comparison between the experimental results on the em fraction of pion-induced showers in the (copper-based) QFCAL and (lead-based) SPACAL detectors. Data from [Akc 97] and [Aco 92b].

EM fraction in hadron showers

The origin of the non-compensation problems



Charge conversion of $\pi^{+/-}$ produces **electromagnetic component** of hadronic shower (π^0)

e = response to the EM shower component

h = response to the non-EM component

Response to a pion initiated shower:

$$\pi = f_{em} e + (1 - f_{em}) h$$

Comparing pion and electron showers:

$$\frac{e}{\pi} = \frac{e}{f_{em} e + (1 - f_{em}) h} = \frac{e}{h} \cdot \frac{1}{1 + f_{em} (e/h - 1)}$$

Calorimeters can be:

- Overcompensating $e/h < 1$
- Undercompensating $e/h > 1$
- Compensating $e/h = 1$

e/h and e/π

e/h: not directly measurable → give the degree of non-compensation

e/π: ratio of response between electron-induced and pion-induced shower

$$\left(\frac{e}{\pi}\right) = \frac{e}{f_{em} e + (1-f_{em}) h} = \left(\frac{e}{h}\right) \cdot \frac{1}{1 + f_{em} (e/h - 1)}$$

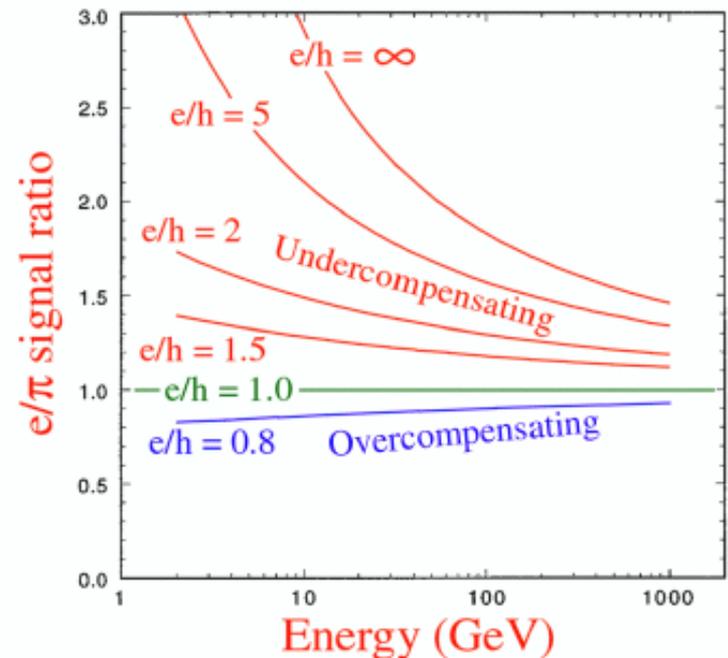
e/h is energy independent

e/π depends on E via $f_{em}(E)$ → non-linearity

Approaches to achieve compensation:

e/h → 1 right choice of materials or

f_{em} → 1 (high energy limit)



Hadron non-linearity and e/h

Non-linearity determined by e/h value of the calorimeter

Measurement of non-linearity is one of the methods to determine e/h

Assuming linearity for EM showers, $e(E_1)=e(E_2)$:

$$\frac{\pi(E_1)}{\pi(E_2)} = \frac{f_{em}(E_1) + [1 - f_{em}(E_1)] \cdot e/h}{f_{em}(E_2) + [1 - f_{em}(E_2)] \cdot e/h}$$

For $e/h=1 \rightarrow$

$$\frac{\pi(E_1)}{\pi(E_2)} = 1$$

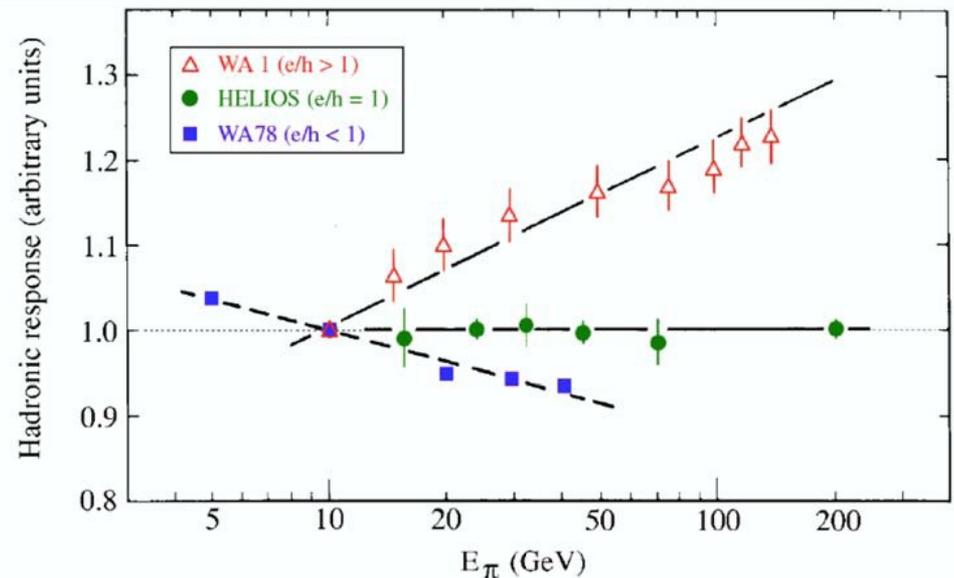


FIG. 3.14. The response to pions as a function of energy for three calorimeters with different e/h values: the WA1 calorimeter ($e/h > 1$, [Abr 81]), the HELIOS calorimeter ($e/h \approx 1$, [Ake 87]) and the WA78 calorimeter ($e/h < 1$, [Dev 86, Cat 87]). All data are normalized to the results for 10 GeV.

e/h ratio

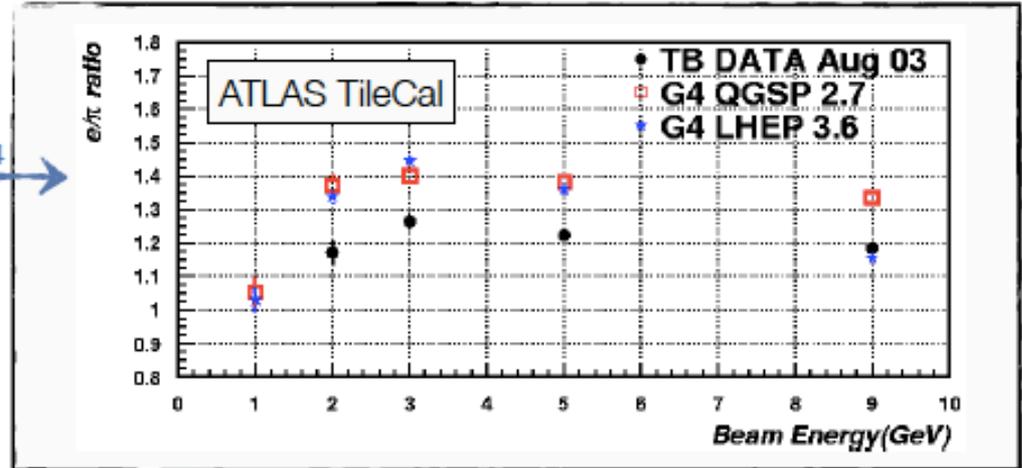
Response of calorimeters very different to electromagnetic (e) and hadronic (h) energy deposits

Usually higher weight for electromagnetic component
i.e. $e/h > 1$

$e/h \neq 1$ leads to non-uniform energy response
due to fluctuations in f_{em}

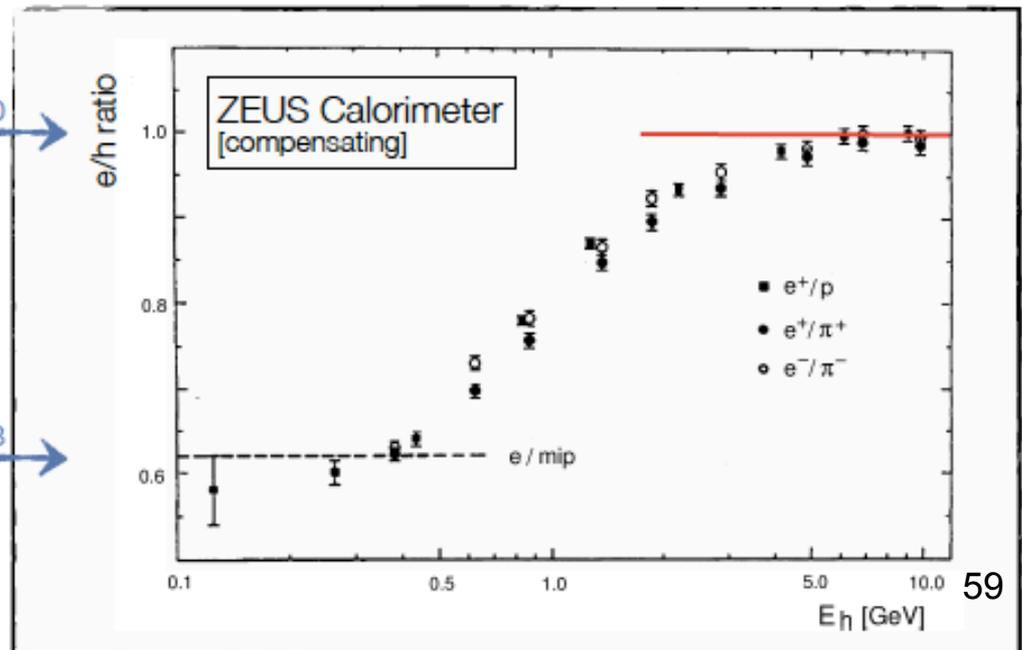
Compensation important!
 $e/h = 1$ [ZEUS calorimeter]

$e/h = 1.4$ →



$e/h = 1.0$ →

$e/h = 0.8$ →



Hadronic response (I)

- Energy deposition mechanisms relevant for the absorption of the non-EM shower energy:
- **Ionization by charged pions** f_{rel} (Relativistic shower component).
- **spallation protons** f_p (non-relativistic shower component).
- **Kinetic energy carried by evaporation neutrons** f_n
- The energy used to release protons and neutrons from calorimeter nuclei, and the kinetic energy carried by recoil nuclei do not lead to a calorimeter signal. This is the **invisible fraction** f_{inv} of the non-em shower energy

The total hadron response can be expressed as:

$$h = f_{rel} \cdot rel + f_p \cdot p + f_n \cdot n + f_{inv} \cdot inv$$

Normalizing to mip and ignoring (for now) the invisible component

$$f_{rel} + f_p + f_n + f_{inv} = 1$$

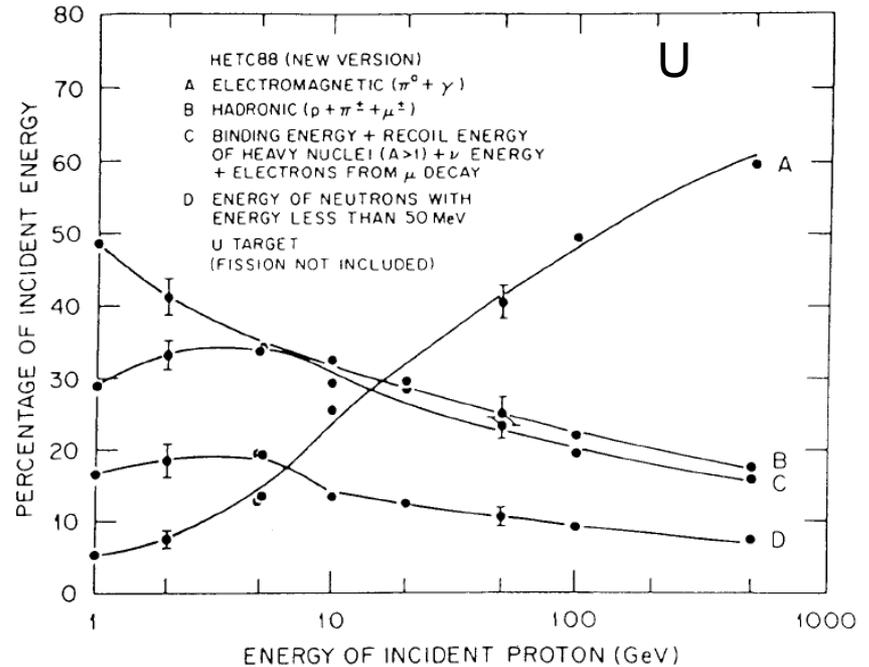
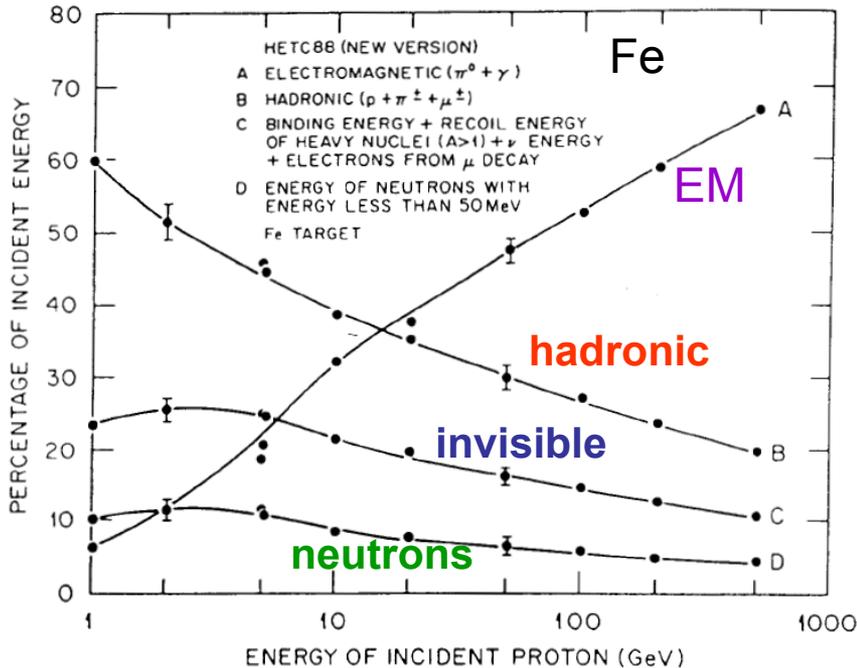
$$\frac{e}{h} = \frac{e/mip}{f_{rel} \cdot rel/mip + f_p \cdot p/mip + f_n \cdot n/mip}$$

The e/h value can be determined once we know the calorimeter response to the three components of the non-em shower

Hadronic shower: energy fractions

$$E_p = f_{em} e + (1 - f_{em}) h$$

$$h = f_{rel} \cdot rel + f_p \cdot p + f_n \cdot n + f_{inv} \cdot inv$$



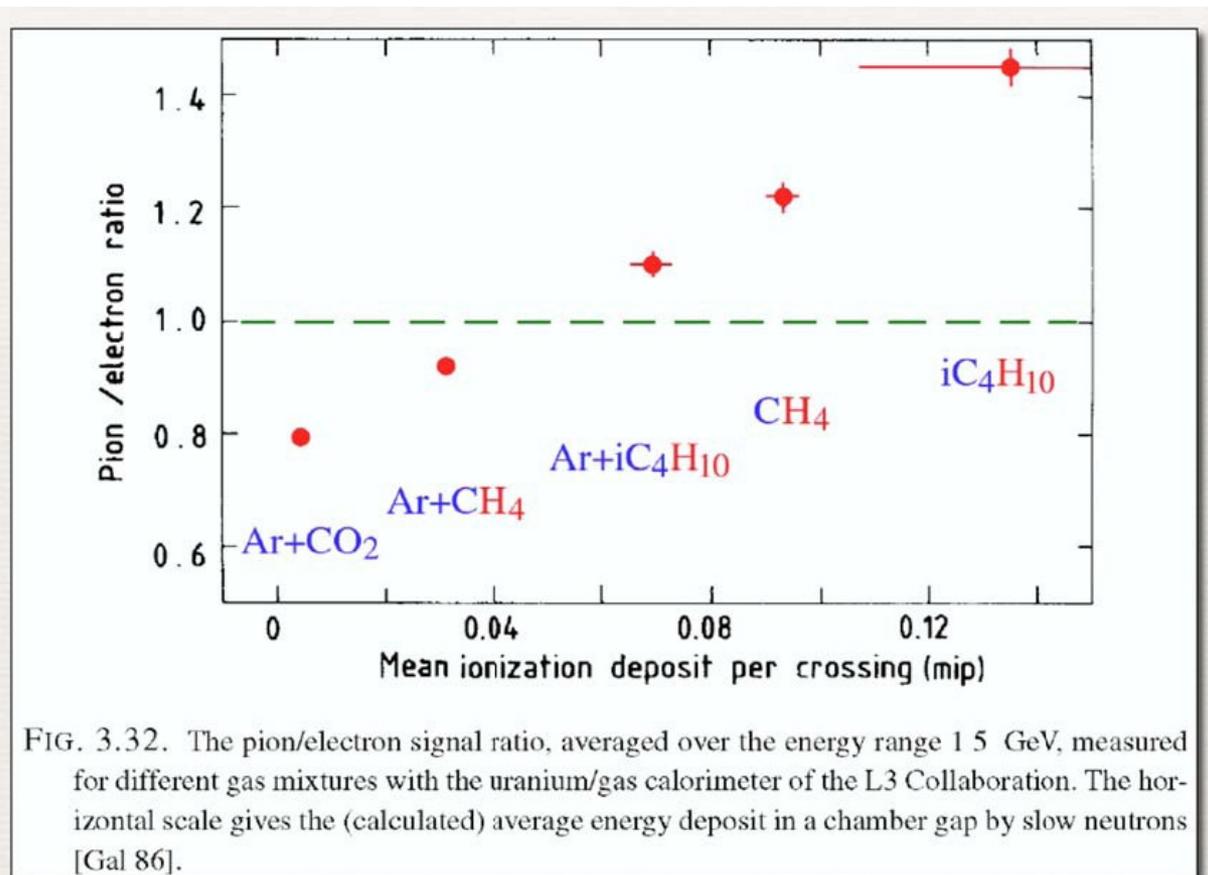
Compensation by tuning neutron response

Compensation with hydrogenous active detector

Elastic scattering of soft neutrons on protons

High energy transfer

Outgoing soft protons have high specific energy loss

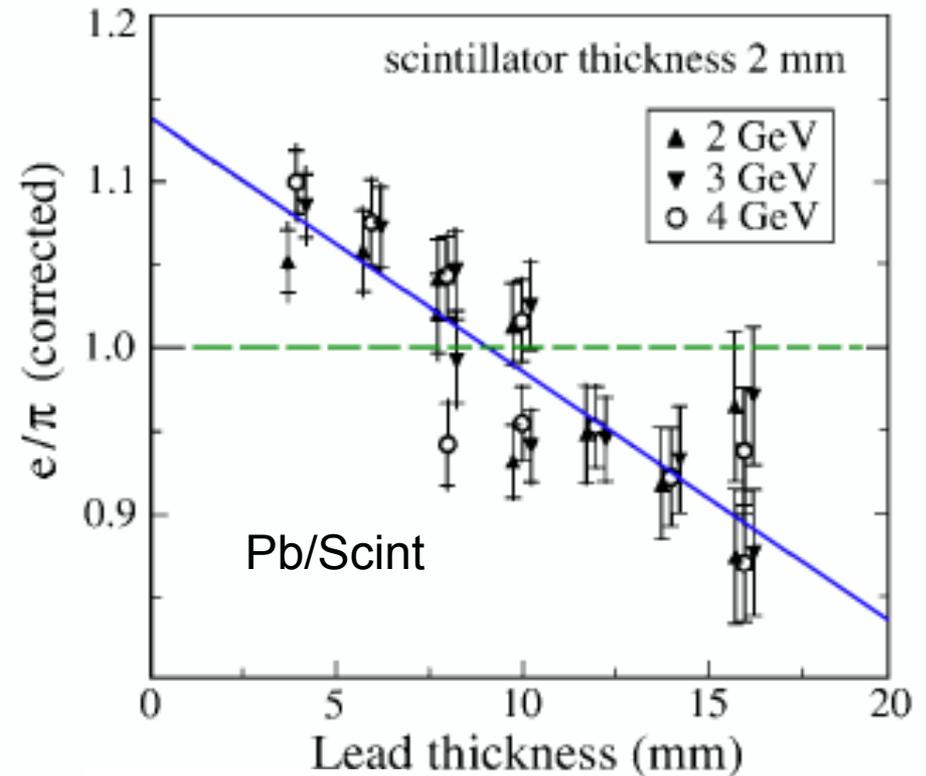
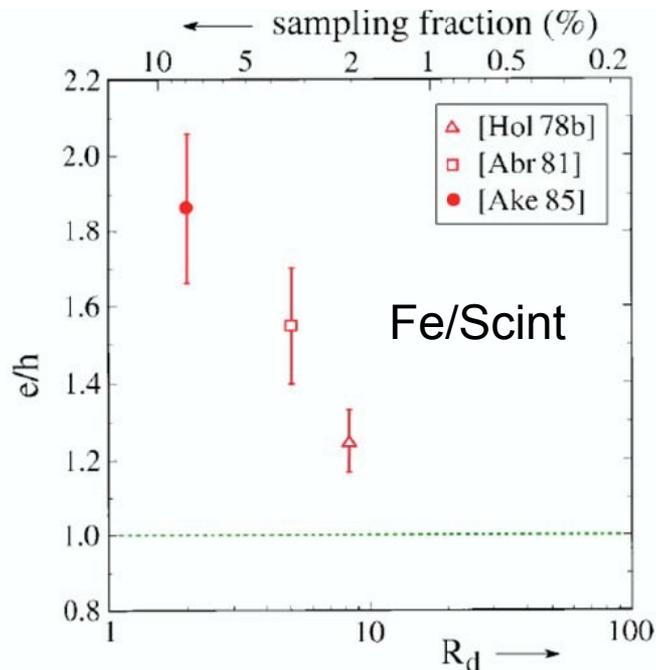


Compensation by tuning neutron response

Compensation adjusting the sampling frequency

Works best with Pb and U

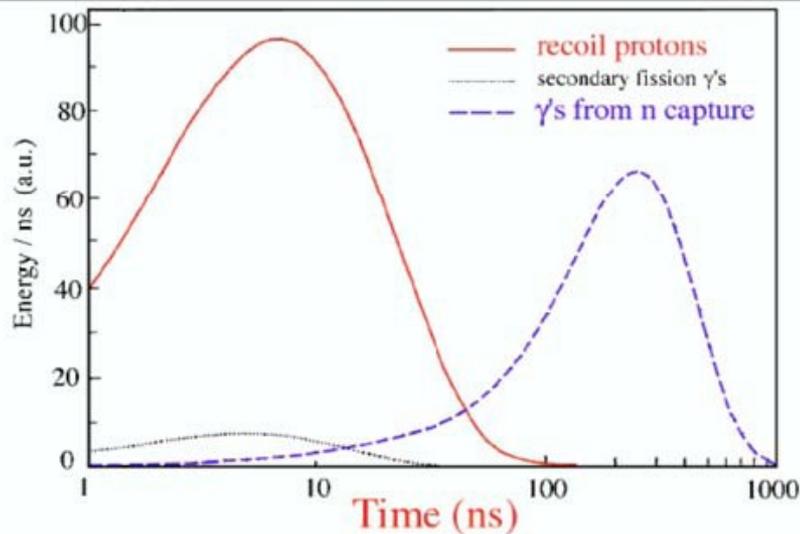
In principle also possible with Fe, but only few n generated



the ratio 4:1 gives compensation for Pb/Scint

in Fe/Scint need ratio > 10:1 → deterioration of longitudinal segmentation

Energy released by slow neutrons



Large fraction of neutron energy captured and released after >100 ns

FIG. 3.22. Time structure of various contributions from neutron-induced processes to the hadronic signals of the ZEUS uranium/plastic-scintillator calorimeter [Bru 88].

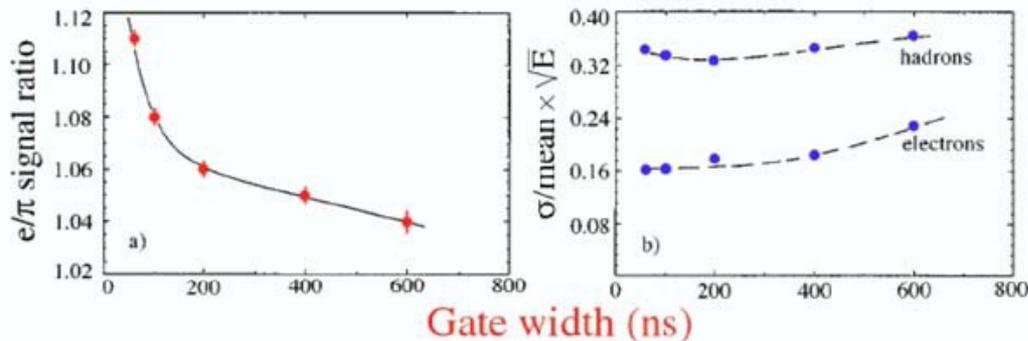


FIG. 3.23. The ratio of the average ZEUS calorimeter signals from 5 GeV/c electrons and pions (a) and the energy resolutions for detecting these particles (b), as a function of the charge integration time [Kru 92].

Long integration time:
 - collect more hadron E
 → closer to compensation
 - integrate additional noise
 → worse resolution

Sampling fluctuations in EM and hadronic showers

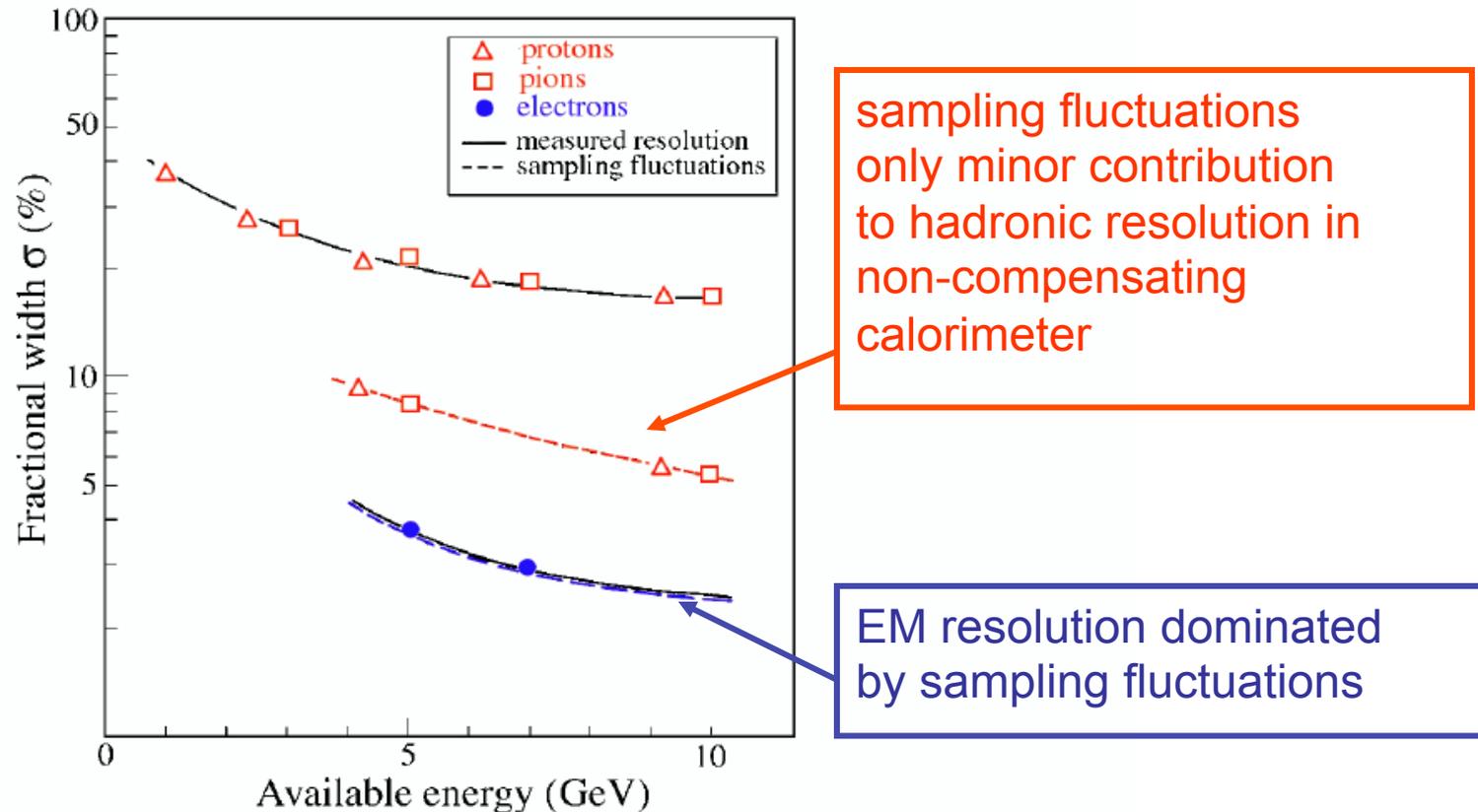


FIG. 4.15. The energy resolution and the contribution from sampling fluctuations to this resolution measured for electrons and hadrons, in a calorimeter consisting of 1.5 mm thick iron plates separated by 2 mm gaps filled with liquid argon. From [Fab 77].

Fluctuations in hadronic showers

- Some types of fluctuations as in EM showers, **plus:**
- 1) Fluctuations in **visible energy**
(ultimate limit of hadronic energy resolution)
- 2) Fluctuations in the **EM shower fraction**, f_{em}
 - **Dominating effect** in most hadron calorimeters ($e/h > 1$)
 - Fluctuations are **asymmetric** in pion showers (one-way street)
 - Differences between **p, π** induced showersNo leading π^0 in proton showers (barion # conservation)

$$E_p = f_{em} e + (1 - f_{em}) h$$
$$h = f_{rel} \cdot rel + f_p \cdot p + f_n \cdot n + f_{inv} \cdot inv$$

1) Fluctuations in visible energy

Fluctuations in losses due to nuclear binding energy

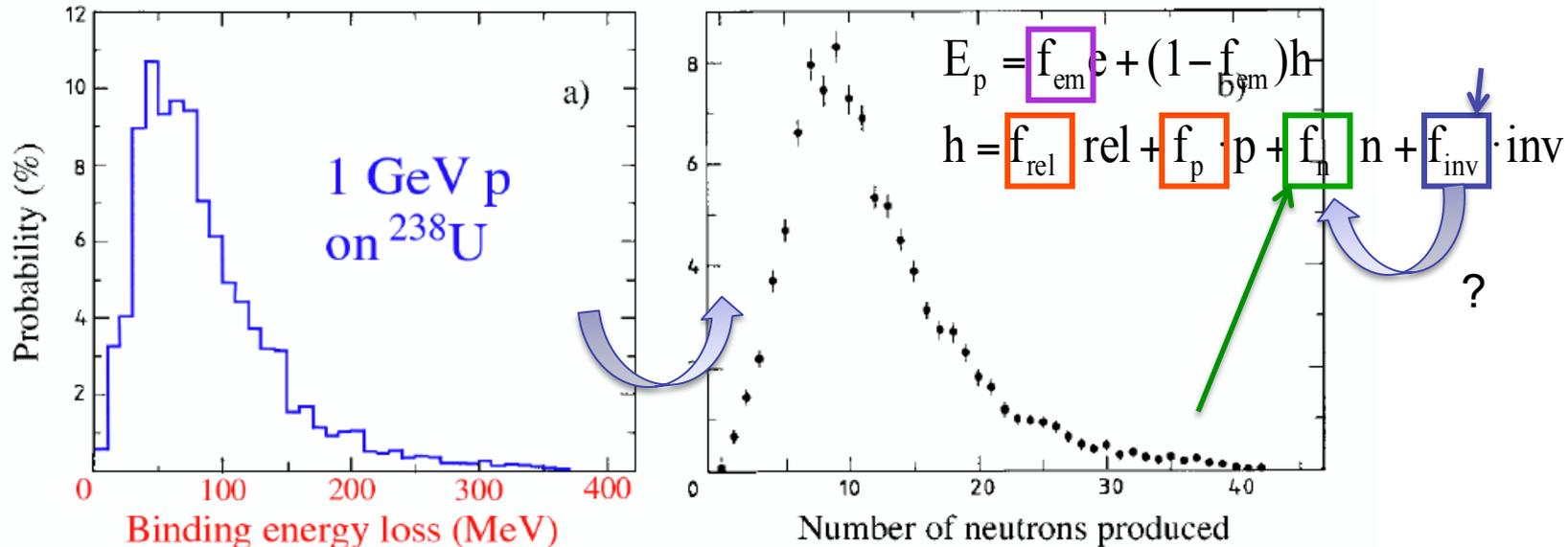


FIG. 4.43. The nuclear binding energy lost in spallation reactions induced by 1 GeV protons on ^{238}U nuclei (a), and the number of neutrons produced in such reactions (b). From [Wig 87].

- Estimate of the **fluctuations of nuclear binding energy** loss in high-Z materials $\sim 15\%$
- Note the strong **correlation** between the distribution of the binding energy loss and the distribution of the **number of neutrons produced in the spallation** reactions
- There may be also a strong **correlation** between the **kinetic energy** carried by these **neutrons** and the nuclear binding energy loss

2) Fluctuations in the EM shower fraction

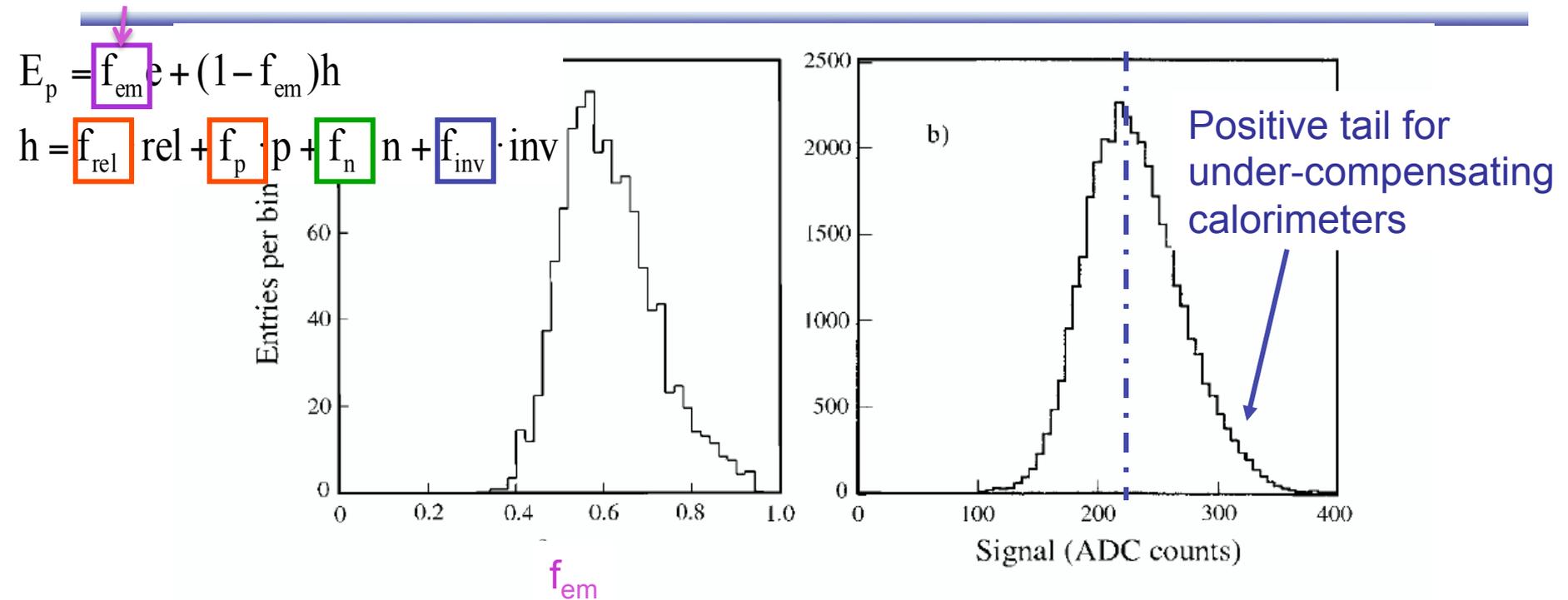


FIG. 4.44. The distribution of the fraction of the energy of 150 GeV π^- showers contained in the em shower core, as measured with the SPACAL detector (a) [Aco 92b] and the signal distribution for 300 GeV π^- showers in the CMS Quartz-Fiber calorimeter (b) [Akc 98].

Pion showers: Due to the **irreversibility** of the production of π_0 s and because of the **leading particle effect**, there is an **asymmetry** in the probability that an anomalously large fraction of the energy goes into the EM shower component

Differences in p / π induced showers

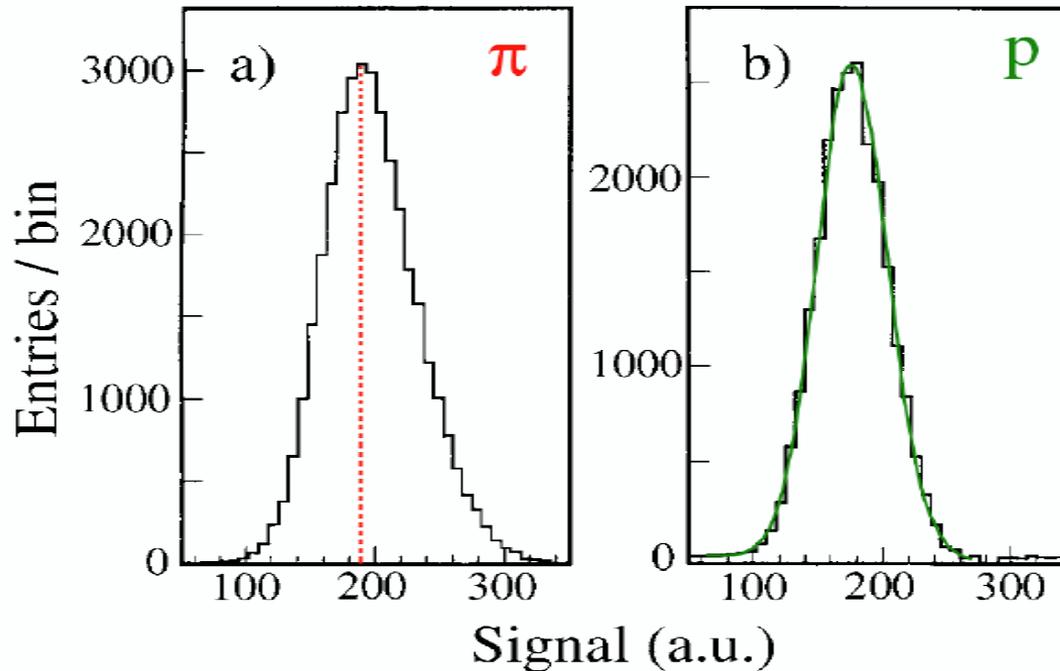


FIG. 4.49. Signal distributions for 300 GeV pions (a) and protons (b) detected with a quartz-fiber calorimeter. The curve represents the result of a Gaussian fit to the proton distribution [Akc 98].

$\langle fem \rangle$ is **smaller** in proton-induced showers than in pion induced ones: **barion number conservation** prohibits the production of leading π_0 s and thus reduces the EM component respect to pion-induced showers

Energy resolution of hadron showers

Hadronic energy resolution of non-compensating calorimeters does not scale with $1/\sqrt{E}$

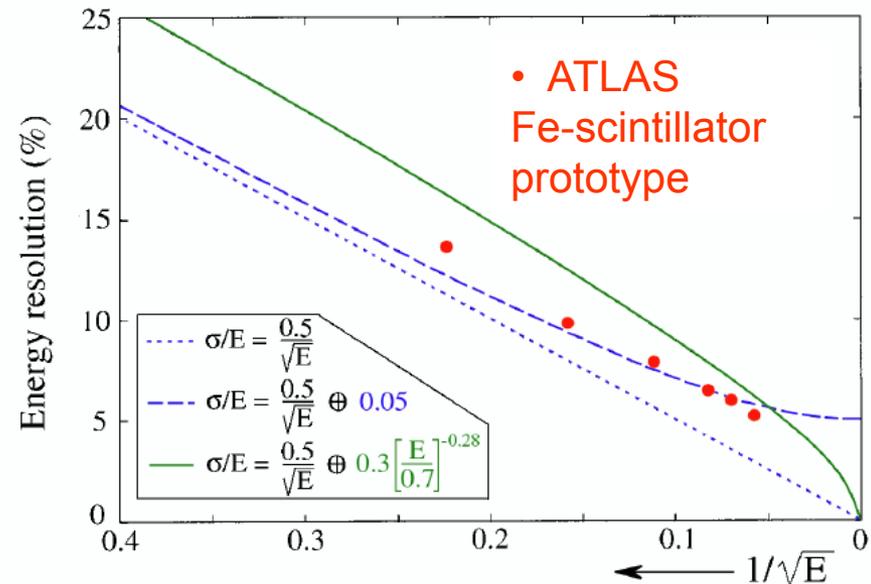
→ $\sigma / E = a / \sqrt{E} \oplus b$ does not describe the data

Effects of non-compensation on σ/E is are better described by an energy dependent term:

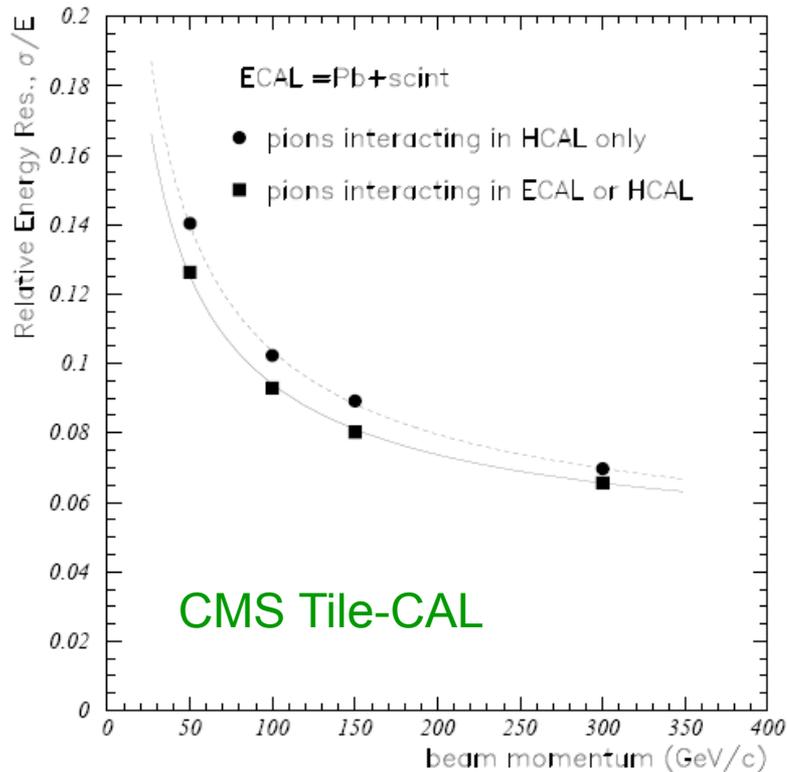
$$\sigma / E = a / \sqrt{E} \oplus b (E/E_0)^{L-1}$$

In practice a good approximation is:

$$\sigma / E = a / \sqrt{E} + b$$



Examples: HCAL E resolution

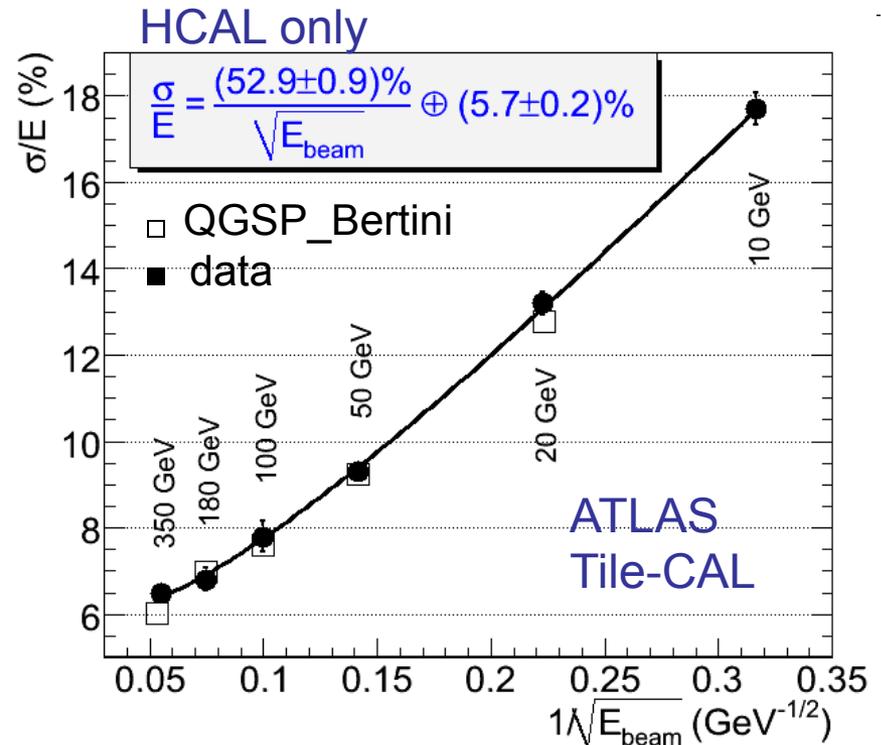


HCAL only

$$\sigma/E = (93.8 \pm 0.9)\%/\sqrt{E} \oplus (4.4 \pm 0.1)\%$$

ECAL+HCAL

$$\sigma/E = (82.6 \pm 0.6)\%/\sqrt{E} \oplus (4.5 \pm 0.1)\%$$



Improved resolution using full calorimetric system (ECAL+HCAL)

ATLAS LAr + Tile for pions: $\frac{\sigma(E)}{E} = \frac{42\%}{\sqrt{E}} \oplus 2\%$

A realistic calorimetric system

Typical Calorimeter: two components ...

Electromagnetic (EM) +
Hadronic section (Had) ...

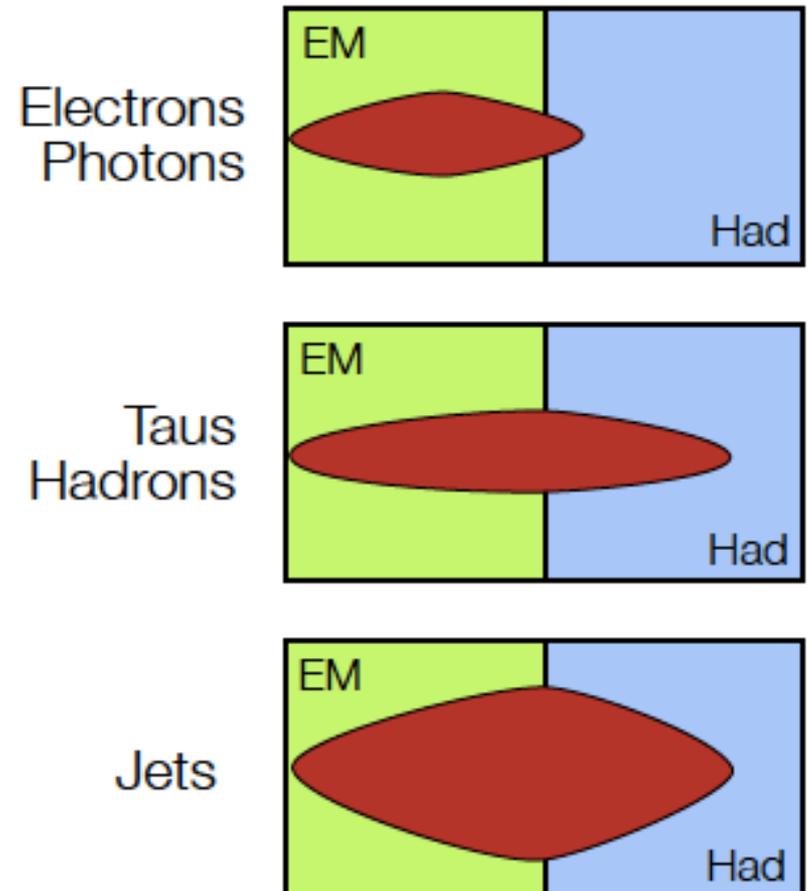
Different setups chosen for
optimal energy resolution ...

But:

Hadronic energy measured in
both parts of calorimeter ...

Needs careful consideration of
different response ...

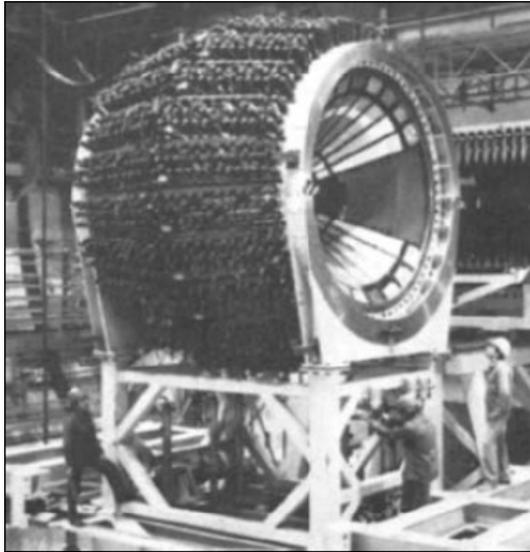
Schematic of a
typical HEP calorimeter



What is really needed in terms of E res.?

- 1) Hadron energy resolution can be improved with weighting algorithms
 - what is the limit?
- 2) HEP experiments measure jets, not single hadrons (?)
 - How does the jet energy resolution relate to the hadron res.?
- 3) Jet energy resolution depends on whole detector and only partially on HCAL performance (→ Particle Flow Algorithms)
 - What is the true hadron energy resolution required?
- 4) What is the ultimate jet energy resolution achievable?
 - Dual readout (DREAM) vs Particle Flow

Importance of jet energy resolution



UA2 (CERN SPS) Discovery of W and Z from their leptonic decay

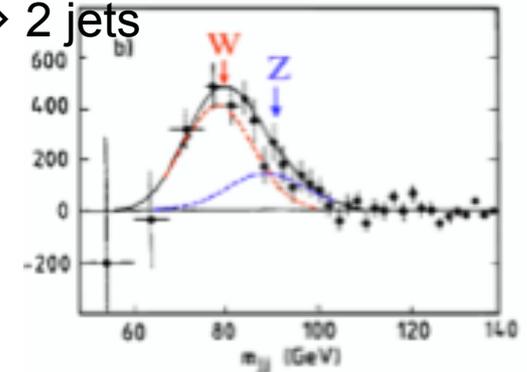
1981

Search for $W^\pm \rightarrow qq$ and $Z \rightarrow qq \Rightarrow 2$ jets

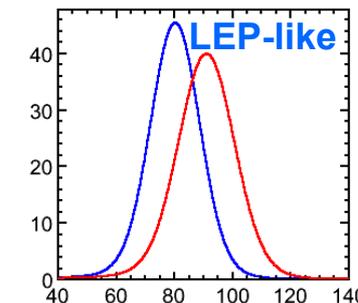
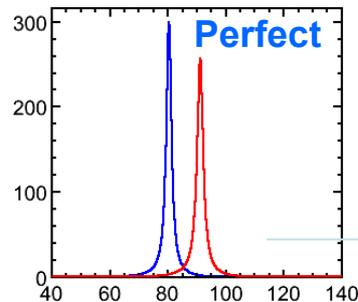
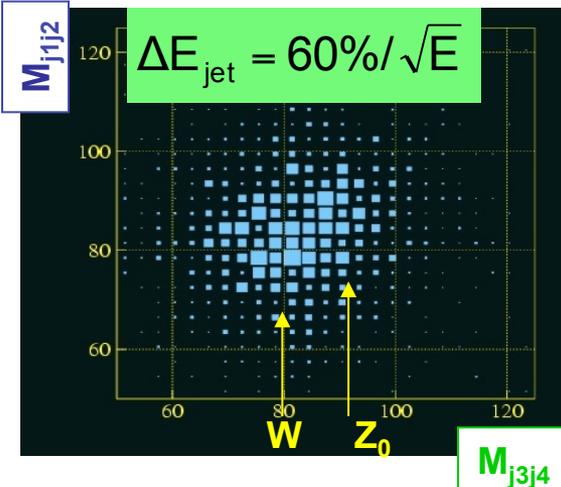
Calorimeter performance:

ECAL: $\sigma_E/E = 15\%/\sqrt{E}$

HCAL: $\sigma_E/E = 80\%/\sqrt{E}$



LEP-like detector



What is the best W/Z separation?

$$W/Z \text{ sep} = (m_Z - m_W) / \sigma_m$$

$$\Delta m = 10.8 \text{ GeV} / 2.5 \text{ GeV} \sim 4.3\sigma$$

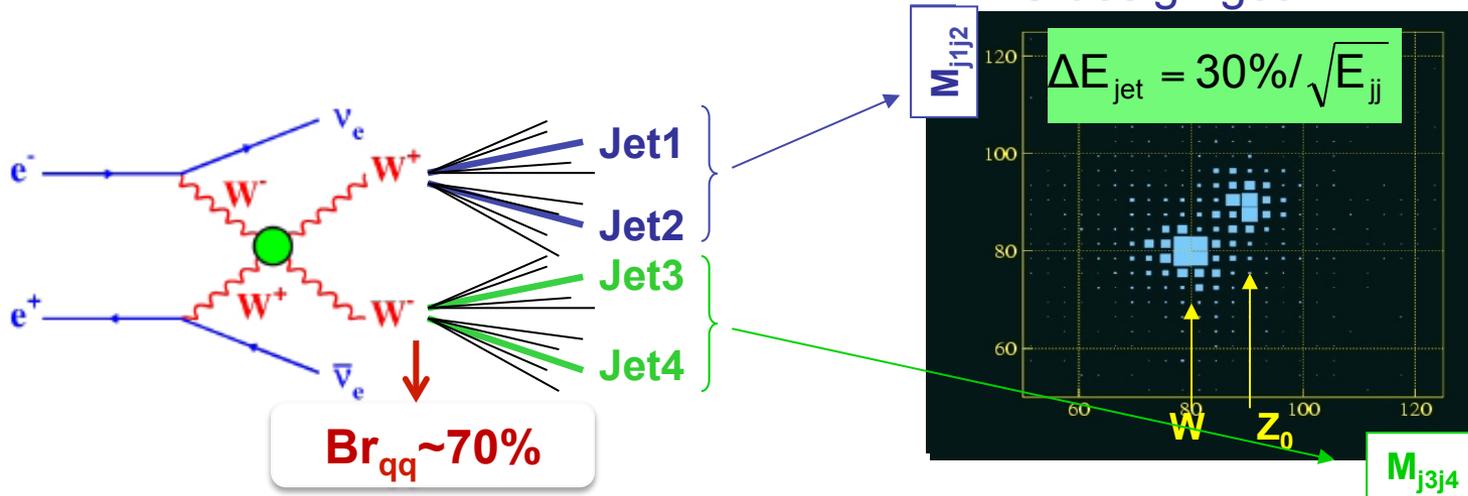
in practice reduced due to Brei-Wigner tail

Required for 2-Gaussians identification

→ separation of means $> 2\sigma$

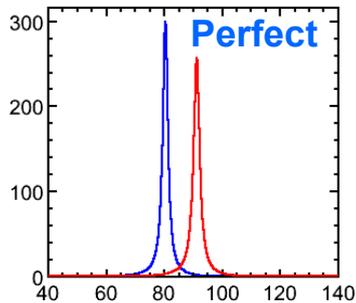
LC physics = Jet physics

precision physics → lepton machine (ILC: $e^+ e^-$ @ 0.5-1 TeV, CLIC: @ 1-3 TeV)
ILC design goal σ_r

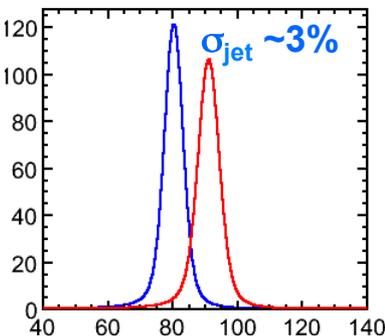


→ Require jet energy resolution improvement by a factor of 2

→ Worse jet energy resolution ($60\%/\sqrt{E}$) is equivalent to a loss of $\sim 40\%$ lumi



Note due to Breit-Wigner tails **best possible** separation is 96 %

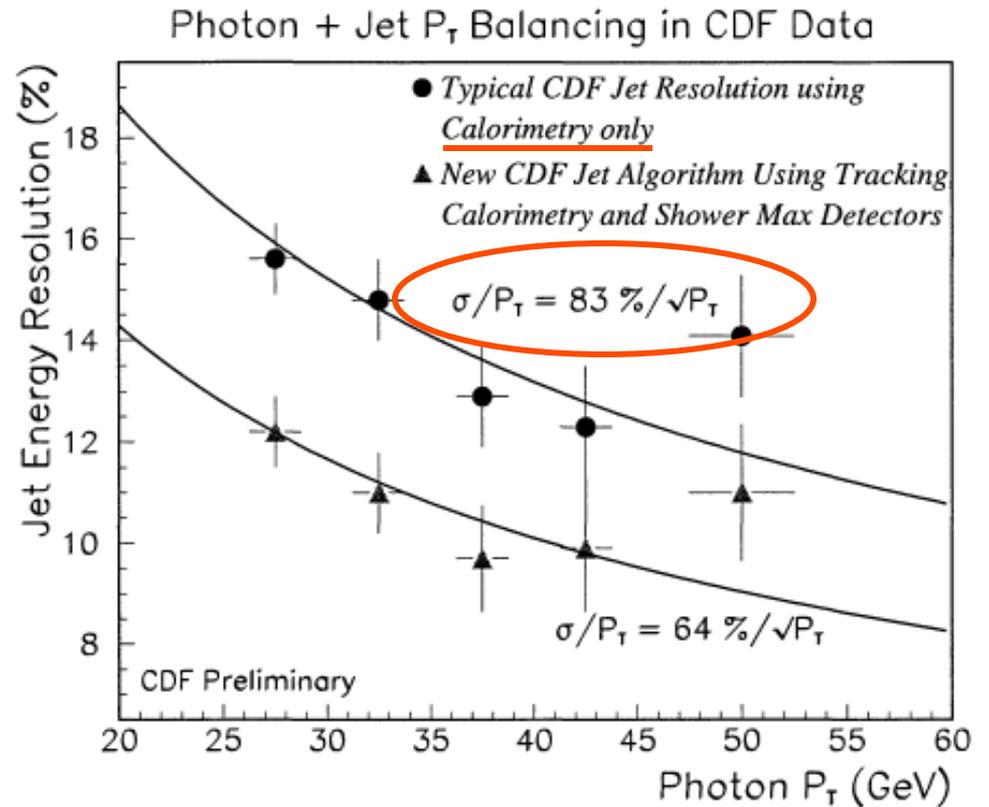


build a detector with excellent jet energy resolution

reasonable choice for LC jet energy resolution:
minimal goal $\sigma_E/E < 3.5\%$

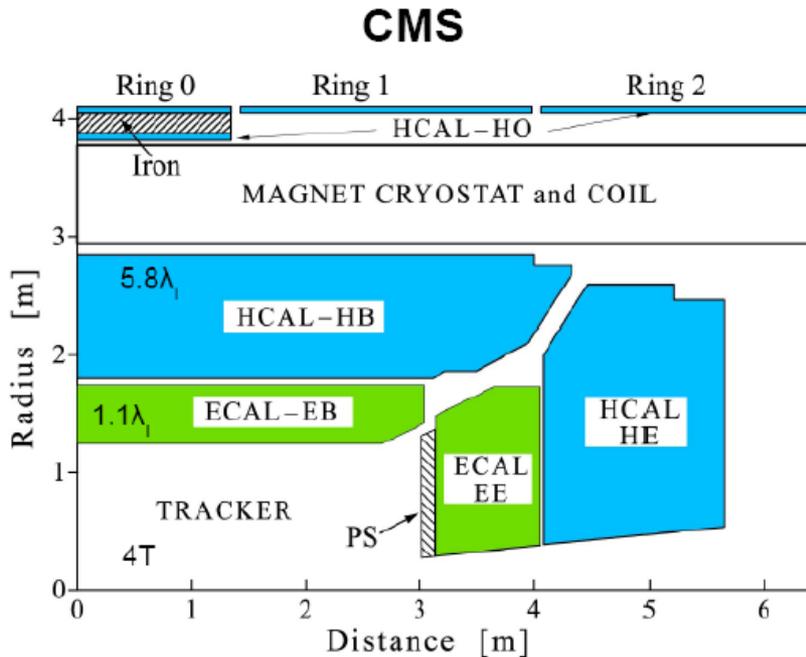
Jets at CDF

	Central	Plug
EM thickness	19 X_0 , 1 λ	21 X_0 , 1 λ
sample(Pb)	0.6 X_0	0.8 X_0
sample(scint.)	5 mm	4.5 mm
wavelength sh.	sheet	fiber
resolution	$\frac{13.5\%}{\sqrt{E_T}} \oplus 2\%$	$\frac{14.5\%}{\sqrt{E}} \oplus 1\%$
HAD thickness	4.5 λ	7 λ
sample(Fe)	25-50 mm	50 mm
sample(scint.)	10 mm	6 mm
wavelength sh.	finger	fiber
resolution	$\frac{50\%}{\sqrt{E_T}} \oplus 3\%$	$\frac{70\%}{\sqrt{E}} \oplus 4\%$



Jet energy performance in calorimeter worse than hadron performance !!

Examples: jet energy resolution

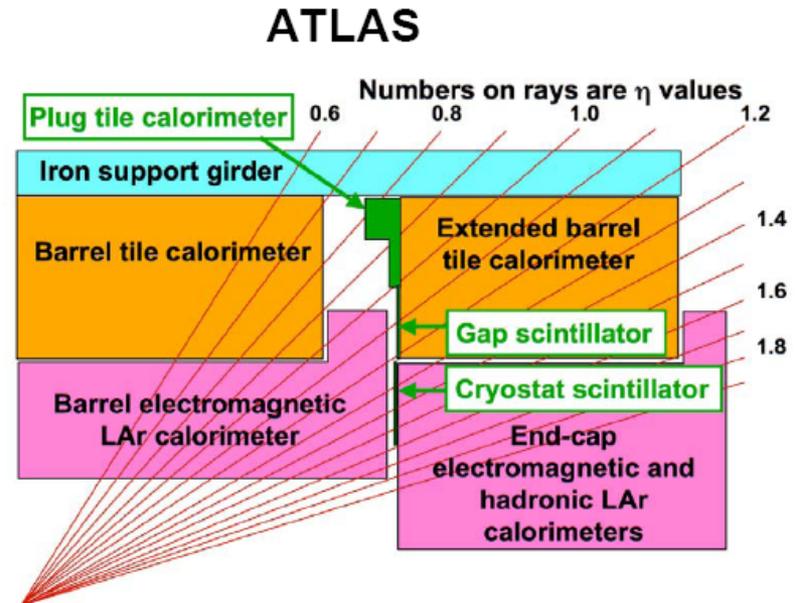


5 cm brass / 3.7 cm scint.
Embedded fibres, HPD readout

Expected jet resolution:

$$\frac{\sigma}{E} = \frac{125\%}{\sqrt{E}} \oplus \frac{5.6 \text{ GeV}}{E} \oplus 3.3\%$$

Stochastic term for hadrons was ~93% and 42% respectively



14 mm iron / 3 mm scint.
sci. fibres, read out by phototubes

Jet resolution with weighting:

$$\frac{\sigma}{E} = \frac{60\%}{\sqrt{E}} \oplus 3\%$$

FUTURE CALORIMETRY

Energy resolution: the next generation

Concentrate on improvement of jet energy resolution

to match the requirement of the new physics expected in the next 30-50 years:

→ Attack fluctuations

Hadronic calorimeter largest fluctuations (if not compensating)

Two approaches:

- minimize the influence of the calorimeter

→ measure jets using the combination of all detectors

Particle Flow

- measure the shower hadronic shower components in each event & weight

→ directly access the source of fluctuations

Dual (Triple) Readout

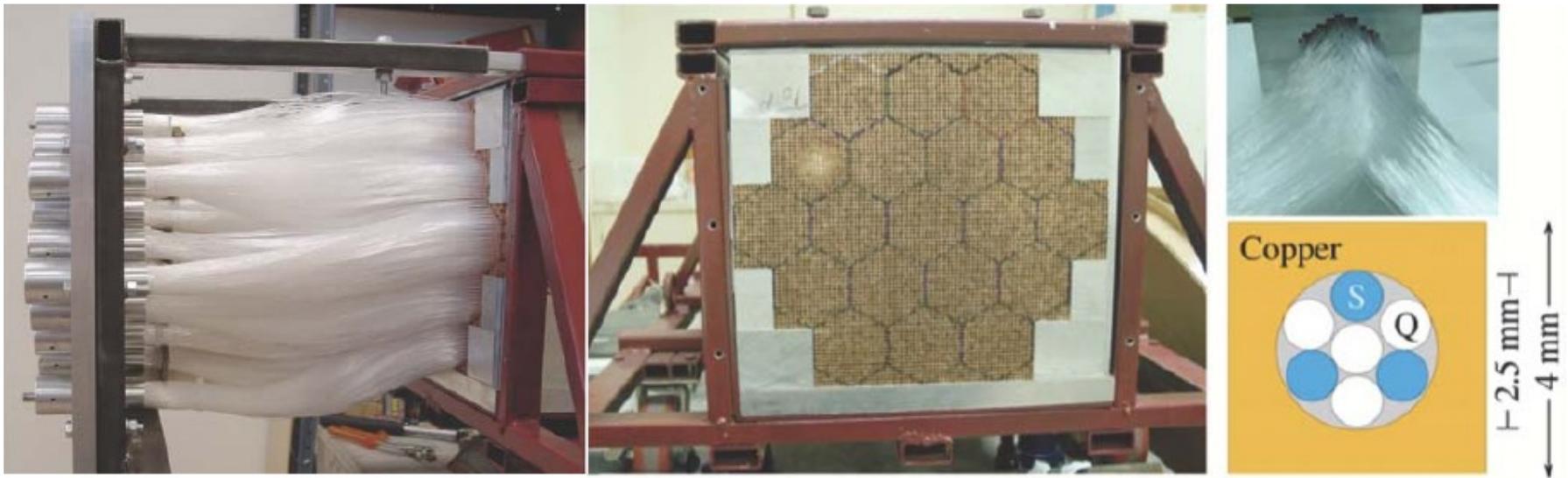
Dual Readout Calorimetry

the DREAM Collaboration

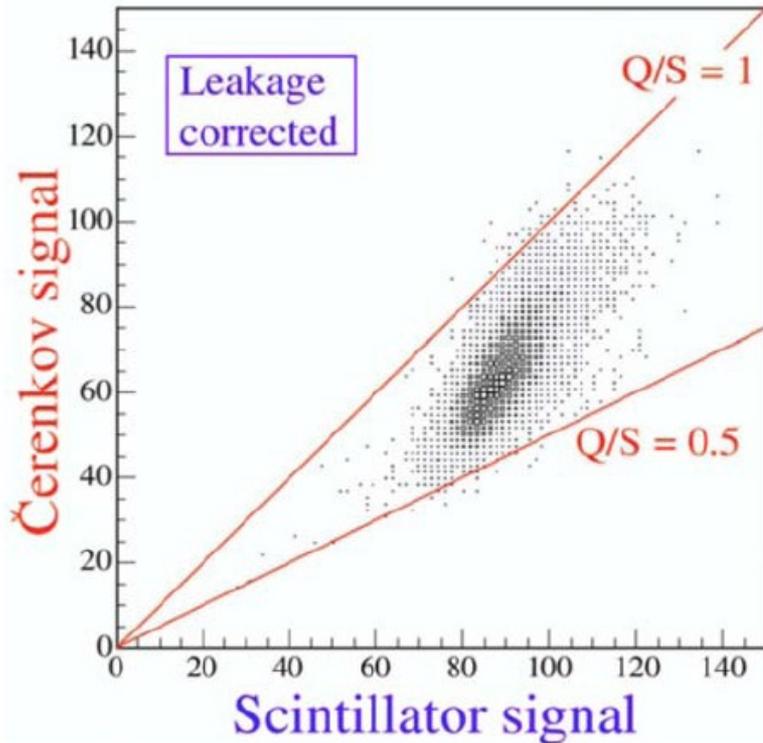
- Measure f_{EM} cell-by-cell by comparing Cherenkov and dE/dx signals
- Densely packed SPACAL calorimeter with interleaved **Quartz** (Cherenkov) and **Scintillating** Fibers
- Production of Cherenkov light only by em particles (f_{EM})
from CMS-HF ($e/h=5$) ~80% of non-em energy deposited by non-relativistic particles
- 2 m long rods ($10 \lambda_{int}$) with no longitudinal segmentation

What is the dream? Measure jets as accurately as electrons, i.e.

$$\sigma_E/E \sim 15\%/\sqrt{E}$$



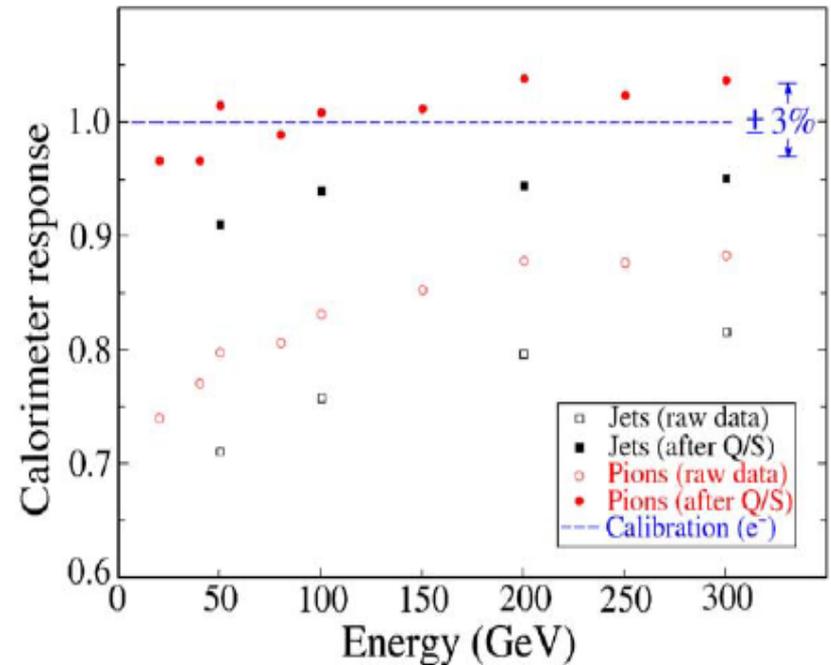
Determination of f_{EM}



$Q/S < 1 \rightarrow$ ~25% of the scintillator signal from pion showers is caused by non-relativistic particles, typically protons from spallation or elastic neutron scattering

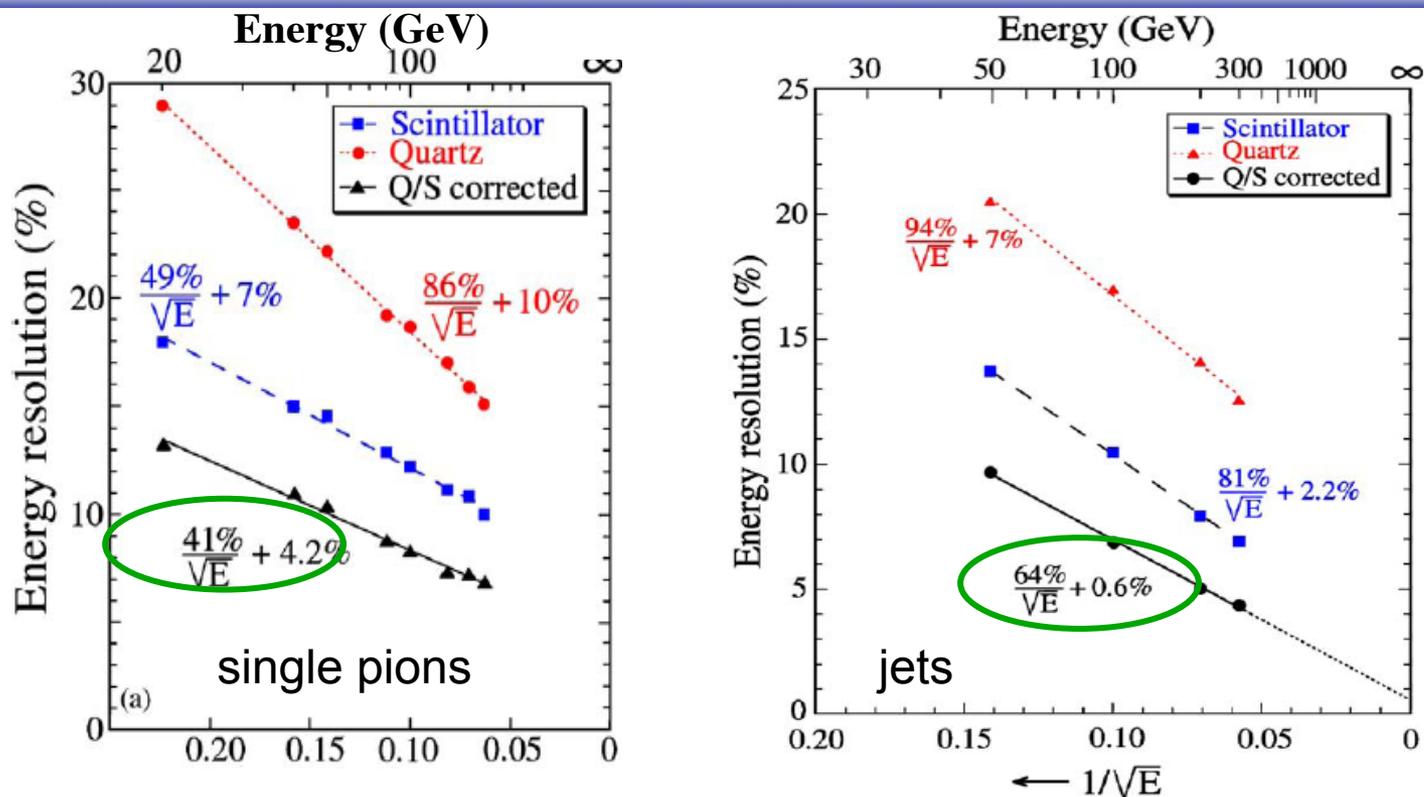
\rightarrow Extract f_{EM} from the Q/S ratio

$$\frac{Q}{S} = \frac{R_Q}{R_S} = \frac{f_{em} + 0.20(1 - f_{em})}{f_{em} + 0.77(1 - f_{em})}$$



Recovered linearity of response to pions and “jets”

Energy resolution



Significant improvement in energy resolution especially for jets

Next challenges:

- 1) re-gain partial longitudinal segmentation (ECAL/HCAL) → Dual readout of BGO crystals exploiting the fast Cherenkov response
- 2) add Triple readout → measure the neutron component with hydrogenous materials

Particle Flow

- **Particle flow** is a concept to improve the jet energy resolution of a HEP detector based on:

- proper **detector** design (high granular calorimeter!!!)

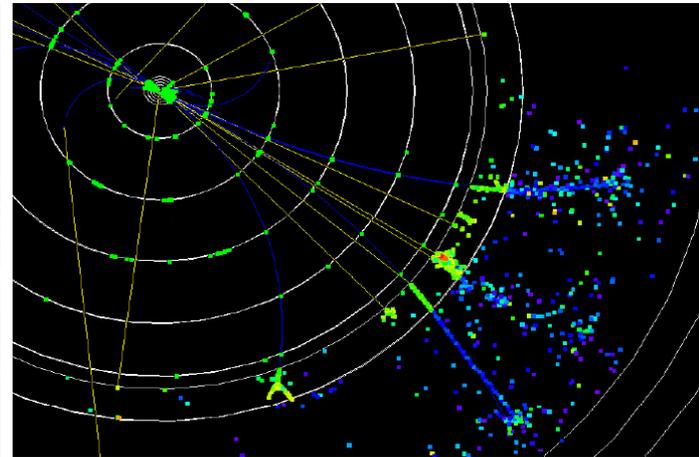
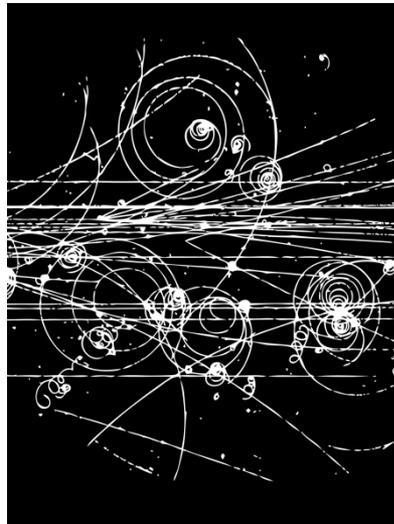
- + sophisticated reconstruction **software**

- PFlow techniques have been shown to improve jet E resolution in existing detectors, but the full benefit can only be seen on the future generation of PFlow designed detectors

Requires the design of

- a highly granular calorimeter, $O(1\text{cm}^2)$ cells
- dedicated electronics, $O(20\text{M})$ channels)
- high level of integration

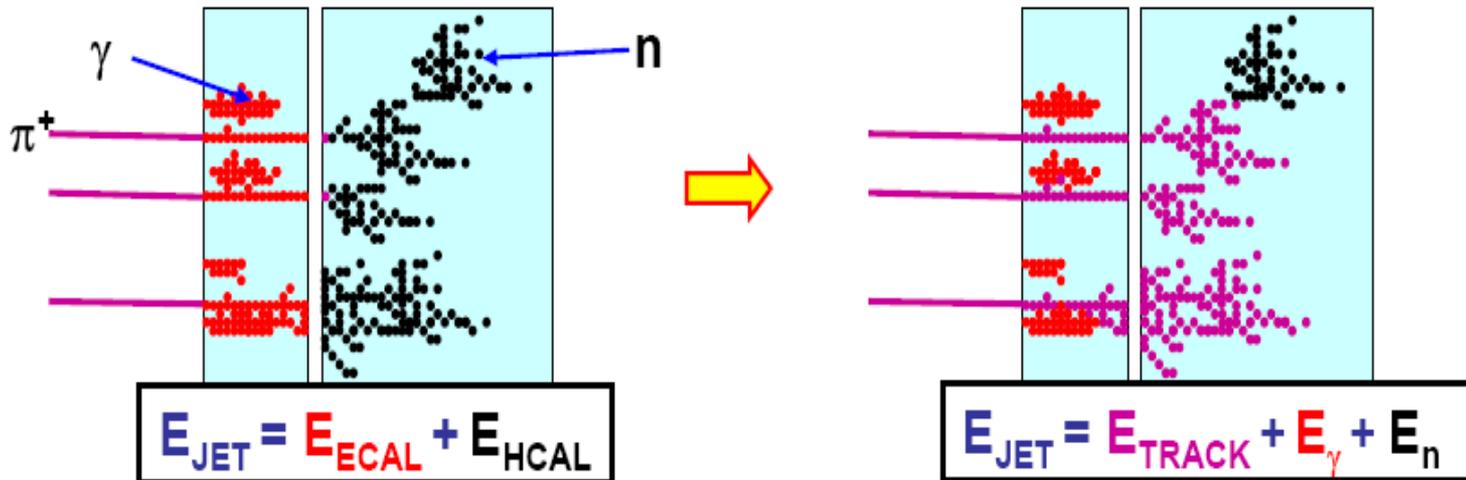
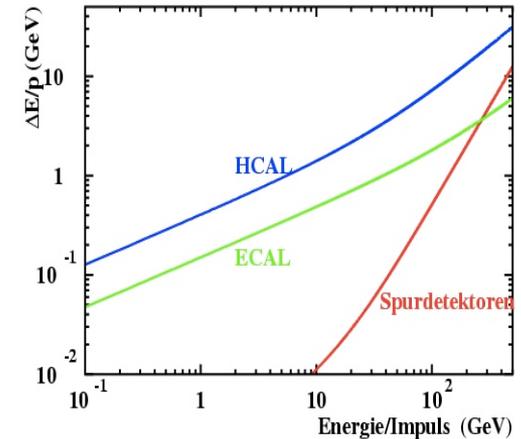
Doesn't it remind you of much more common pictures?



Full event reconstruction with a particle flow algorithm

Particle Flow paradigm

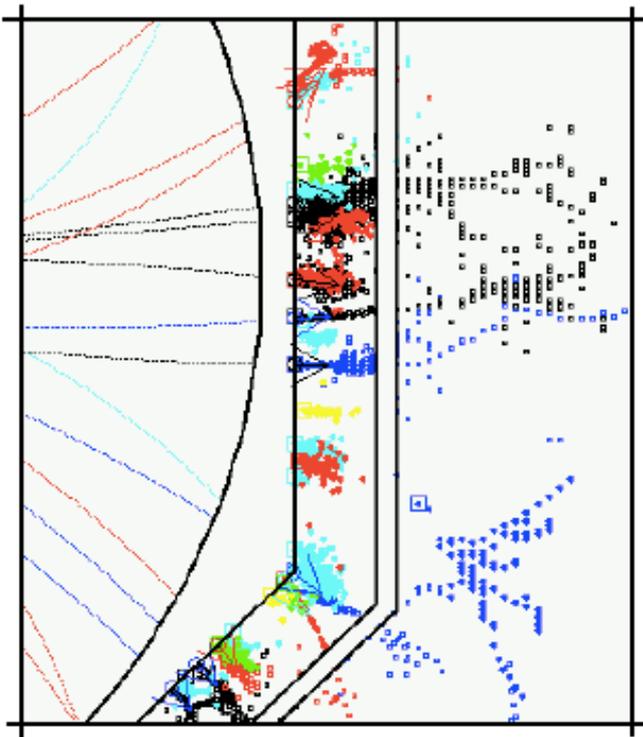
- reconstruct **every** particle in the event
- up to ~100 GeV **Tracker** is superior to calorimeter →
- use tracker to reconstruct e^\pm, μ^\pm, h^\pm (<65%> of E_{jet})
- use **ECAL** for γ reconstruction (<25%>)
- (**ECAL+**) **HCAL** for h^0 reconstruction (<10%>)
- HCAL E resolution still dominates E_{jet} resolution
- But much improved resolution (only 10% of E_{jet} in HCAL)



**PFLOW calorimetry = Highly granular detectors (CALICE)
+ Sophisticated reconstruction software**

Particle Flow

Component	Detector	Fraction	Part. resolution	Jet Energy Res.
Charged (X^\pm)	Tracker	60%	$10^{-4} E_x$	negligible
Photons (γ)	ECAL	30%	$0.1/\sqrt{E_\gamma}$	$.06/\sqrt{E_{\text{jet}}}$
Neutral Hadrons (h)	E/HCAL	10%	$0.5/\sqrt{E_{\text{had}}}$	$.16/\sqrt{E_{\text{jet}}}$



Particle Flow (PFA):

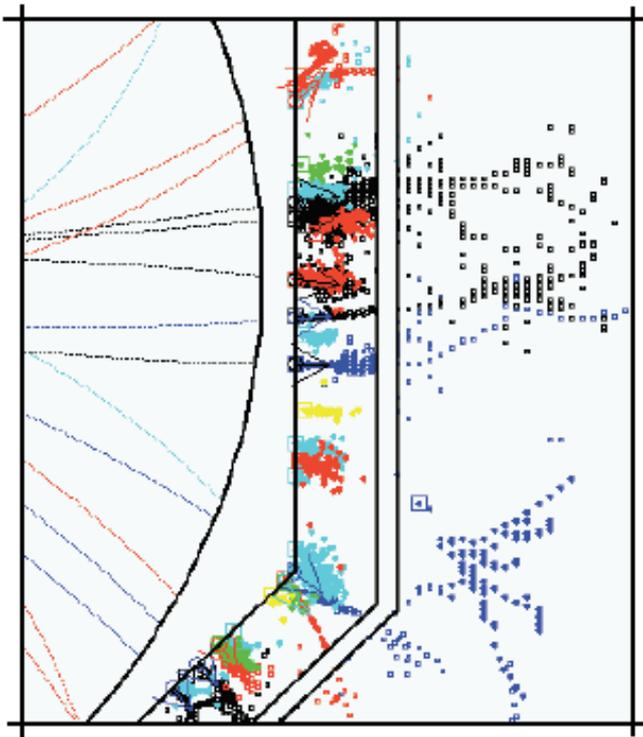
Choose detector best suited for particular particle type ...

i.e.: use tracks and distinguish 'charged' from 'neutral' energy to avoid double counting

distinguish electromagnetic and hadronic energy deposits for software compensation

Particle flow

Component	Detector	Fraction	Part. resolution	Jet Energy Res.
Charged (X^\pm)	Tracker	60%	$10^{-4} E_x$	negligible
Photons (γ)	ECAL	30%	$0.1/\sqrt{E_\gamma}$	$.06/\sqrt{E_{\text{jet}}}$
Neutral Hadrons (h)	E/HCAL	10%	$0.5/\sqrt{E_{\text{had}}}$	$.16/\sqrt{E_{\text{jet}}}$



PFA – Energy Resolution:

$$\sigma_{\text{jet}}^2 = \sigma_X^2 + \sigma_\gamma^2 + \sigma_{\text{had}}^2$$

$$.17/\sqrt{E}$$

$$+ \sigma_{\text{confusion}}^2 + \dots$$

$$< .25/\sqrt{E}$$

?

Granularity more important
than energy resolution !?