COMPLETE EXACT $O(\alpha)$ CORRECTIONS TO $e^+e^- \to 4f$

- $e^+e^- \to \mu^- \bar{\nu}_\mu u \bar{d}$

- Oberna1 :

analytic expression	correct
software packages	working
numerical tests	none
physical structure	missing

- Present status :

physical structure	2 options
numerical tests	95% of the diagrams OK
	large cancellations
	numerical instabilities

OVERVIEW

diagrams generation OK

diagrams simplification
organization of the amplitude $\Big\}$ 2 options

preparation of the modules for the numerical part OK

propagators, fermion lines, loop integrals, amplitudes

1 special case is missing

Montecarlo integration : sampling of the phase space

real integration

Main philosophy:

better to sum small numbers than to calculate large differences

Diagram simplification / Organization of the amplitude

Obernai: in evidence the 453 box integrals

$$\mathcal{M} = \sum_{i=1}^{453} I_i \qquad I_i \left\{ \begin{array}{l} \text{scalar integral} \\ \text{tensor coefficients} \\ \text{coupling} \\ \text{propagators} \\ \text{spinors} \end{array} \right.$$

BAD:

1) large gauge cancellation among different terms
2) redundant number of fermion chains $(\bar{u} \not{k} v)$

<u>option 1:</u> in evidence the fermion chains

$$\mathcal{M} = \sum_{i=1}^{N} c_i \left(\bar{u} \not{k}_{i_1} v \;\; \bar{u} \not{k}_{i_2} v \;\; \bar{u} \not{k}_{i_3} v \right)_i$$

- contraction of Lorentz indices, Fierz identities
- simplification of $\int d^4k \dfrac{k \cdot p}{\cdots \left[(k+p)^2 - m^2 \right] \cdots}$
- 6-pt integrals → combination of 6 5-pt integrals
- covariant decomposition of tensor integrals
- eliminate redundant fermion lines

- large cancellations <u>naturally</u> <u>within</u> each c_i
- fewer sources of numerical errors
- N is not known a priori
- size of each term in the sum closer to the final result
  10 MB for … 2.6 MB for

Option 2 :

$$\mathcal{M} = \left(\sum_{i=1}^{76} c_i \; t^{(i)}_{\alpha\beta\gamma} \right) J^{\alpha}_{e^+e^-} \, J^{\beta}_{\mu\nu} \, J^{\gamma}_{ud}$$

- break the fermion lines
- covariant decomposition of tensor integrals
- 76 tensors $\qquad g^{\alpha\beta} p_i^{\gamma} \,,\; p_i^{\alpha} p_j^{\beta} p_k^{\gamma}$
- reduce $t^{(i)}_{\alpha\beta\gamma}$ to a basis of tensors
- c_i gauge parameter independent each one separately

Option 1 WELL tested, presently used

Option 2 tested only for numerical equivalence to opt.1

FUTURE : Opt. 2 c_i are the best candidates to operate significant analytical simplifications

NUMERICAL EVALUATION

main cross

scalar products
fermionic lines
propagators
selfenergies, vertices
boxes, 5-pt integrals
amplitude Born, virtual
counterterms
soft bremsstrahlung

$$\left.\begin{array}{l} \\ \\ \\ \\ \\ \\ \\ \end{array}\right\} \quad \begin{array}{l} |\mathcal{M}_{Born}|^2, \\ |\mathcal{M}_{Born}|^2 \delta_{SB}, \\ 2\,\mathrm{Re}\left(\mathcal{M}_{Born}\mathcal{M}^*_{virt}\right) \end{array}$$

analytical IR finiteness OK

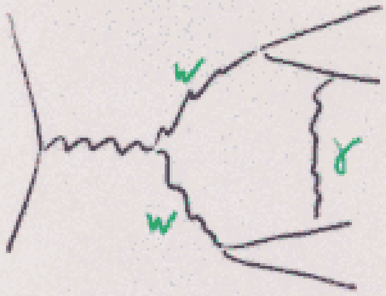
boxes with complex masses

- cancellation of IR divergences

$$\underbrace{|\mathcal{M}_{Born}|^2 \delta_{SB}}_{\uparrow \text{ complex } M_W} \qquad 2\,\mathrm{Re}\left(\underbrace{\mathcal{M}_{Born}}_{\uparrow \text{ complex } M_W,}\ \underbrace{\mathcal{M}^*_{virt}}_{\uparrow \text{ real } M_W}\right)$$

- a real m_W spoils the cancellation: large effects
- a complex M_W is needed to better describe the resonance and to EXACTLY CANCEL the IR divergence

Single and double resonant 5pt integrals



$$E_0 = \sum_{i=0}^{4} e_i \, \Delta(i)$$

- e_i evaluated with complex M_W
- the tensor coefficients are IR finite

- complex e_i + boxes with real masses $\longrightarrow$
  $\longrightarrow$ IR divergent tensor coefficients

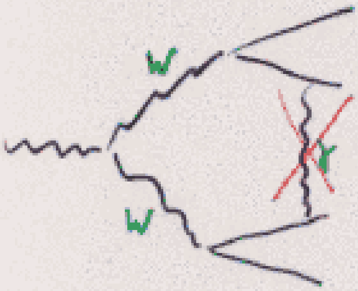
- IR divergent boxes (Beenakker, Denner 1990) extended to the case with 1 complex mass

- Using this new routine $\to$ tensor coefficient IR finite

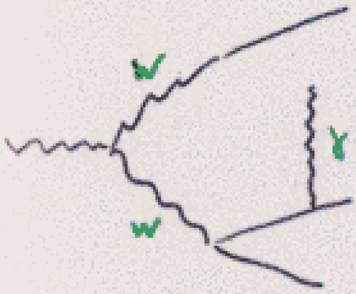
- Analogously gauge cancellations might be spoiled leading to wrong scalar and tensor 5pt integrals

$\Longrightarrow$ a consistent, exact treatment is mandatory.

STILL MISSING :



OFF-SHELL W's (ON-SHELL W's)



EXACT (DOUBLE POLE APPROX.)

$\sim \log \frac{\Gamma_W^2}{m_W^2}$

FIRST NUMERICAL TRIALS

1 pt in phase-space 0.86 s $5^7 = 78125$

Tests with 50'000 and 100'000 points, 6-10 iterations

Born, counterterms, self-energies, vertices, 4-point diagrams } OK

- stable central values
- error essentialy statistical
- double resonant importance sampling improves the convergence

5- and 6-point diagrams UNSTABLE

EXAMPLE: arbitrary splitting in 3 subsets of the amplitude (indicative numbers)

$\sqrt{s} = 200$ GeV

cross-section (pb)

Born + S.E. + V + ct + bremsstrahlung + "on-shell W-pair" boxes $\sim 0.500 \pm 0.001$

new boxes $- 1.253 \pm 0.003$

5- and 6-pt diag's $+ 1.2 \ldots \pm 0.3$

- huge cancellation between subsets of diagrams
- instability { inconsistent treatment of 5-point integrals; bugs?

- in phase-space the double resonant region dominates

nevertheless

- individual single-resonant diagrams are numerically sizeable

  → one would need a different importance sampling

- in the total amplitude these contributions cancel to a large extent → double resonant imp. sampling OK

but

- spoiling gauge cancellation might leave large spurios (wrong) contributions which severely disturb the convergence of the Montecarlo

- present tests done diagram by diagram for debugging

but

- real calculation using the sum of all diagrams exact gauge cancellation → smaller numbers → → smaller fluctuations → better accuracy

CONCLUSIONS

- AUTOMATIC SOFTWARE WITH A PHYSICALLY MOTIVATED STRUCTURE
- GOOD UNDERSTANDING OF ALL NUMERICAL CONTRIBUTIONS EXCEPT FOR 5- AND 6-PT DIAGRAMS
- IR CANCELLATIONS ARE <u>REALLY</u> UNDER CONTROL
- GAUGE CANCELLATIONS, INCLUDING THE DECAY WIDTH, ARE STILL UNCLEAR

- SOLVE TECHNICAL PROBLEMS WITH 5- AND 6-POINT DIAGRAMS
- EVALUATE THE CROSS-SECTION (PROBLEMS WITH THE DECAY WIDTH PRESCRIPTION?)
- FIRST PHYSICAL PREDICTIONS

- EXPLOIT OPT. 2 (AMPLITUDE IN TERMS OF CURRENTS) TO SENSIBLY SIMPLIFY ANALITICALLY THE FORMULAE

- THE CROSS-SECTION AT 0.1% IS NOT TRIVIAL

- THERE IS NO REASON *a priori* TO ESTIMATE THE CORRECTIONS TO BE SMALLER THAN 0.5%