Fermion Mass Effects in $e^+e^- \to 4f\gamma$

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Plan

- Motivation.
- The method of calculating matrix elements.
- Application to $e^+e^- \to 4f\gamma$.
- The phase space integration.
- Numerical results.
- Summary and outlook.

Motivation

Necessary precision of theoretical predictions for $W^{\pm}$-pair production:
the final LEP2 data analysis – better than 1% (possibly 0.5%),
future e^+e^- colliders – probably about 0.1%.

Precise SM predictions for $e^+e^- \to 4f$ including radiative corrections are required, especially for the NLC.

Precise SM predictions for

$$e^+e^- \to 4f\gamma, 4f\gamma\gamma, \ldots$$

are needed.

For the recent status see

M. Grünewald, G. Passarino et al., "Four-Fermion Production in Electron-Positron Collisions", report of the four-fermion working group of the *LEP2 Monte Carlo workshop*, CERN, 1999/2000, hep-ph/0005309.

Motivation

Keeping the non zero fermion masses will allow for:

- The proper treatment of the collinear photons;
- Calculation of cross sections independent of angular cuts;
- Estimation of the background from undetected hard photons;
- Proper Handling of a photon exchange in the t-channel;
- Consistent incorporation of the Higgs boson effects.

Method

The helicity amplitude method introduced in

K. Kołodziej, M. Zrałek, Phys. Rev. D 43 (1991) 3619;

F. Jegerlehner, K. Kołodziej, Eur. Phys. J. C 12 (2000) 77.

The Weyl representation:

$$\gamma^\mu = \begin{pmatrix} 0 & \sigma_+^\mu \\ \sigma_-^\mu & 0 \end{pmatrix}, \qquad \sigma_\pm^\mu = (I, \pm\sigma_i) \ .$$

Chirality projectors $P_\pm = (1 \pm \gamma_5)/2$:

$$P_- = \begin{pmatrix} I & 0 \\ 0 & 0 \end{pmatrix}, \qquad P_+ = \begin{pmatrix} 0 & 0 \\ 0 & I \end{pmatrix} .$$

A contraction of any four-vector a^μ with the γ^μ

$$\not{a} = a^\mu \gamma_\mu = \begin{pmatrix} 0 & a^\mu \sigma_\mu^+ \\ a^\mu \sigma_\mu^- & 0 \end{pmatrix} = \begin{pmatrix} 0 & a^+ \\ a^- & 0 \end{pmatrix},$$

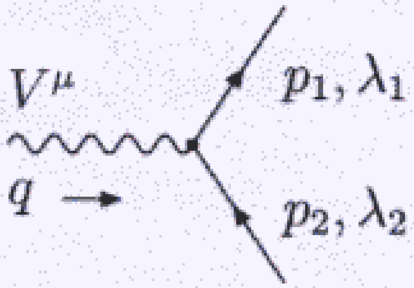
where $a^\pm$ are given by

$$a^\pm = \begin{pmatrix} a^0 \mp a^3 & \mp a^1 \pm i a^2 \\ \mp a^1 \mp i a^2 & a^0 \pm a^3 \end{pmatrix}.$$

Method

Fermion masses are kept non zero.

Building blocks of the amplitudes: parts of the Feynman graphs which contain a single uncontracted Lorentz index – the generalized polarization vectors, e.g.



$$\rightarrow \; \varepsilon_V^\mu(p_1, p_2, \lambda_1, \lambda_2)$$

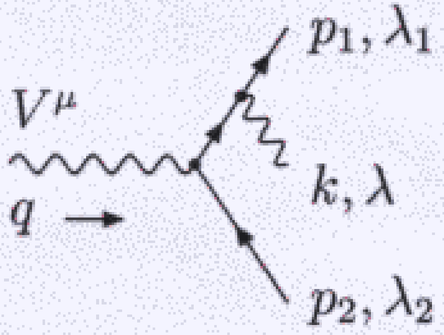
$$= D_V^{\mu\nu}(q)\bar{\psi}_1(p_1, \lambda_1)\gamma_\nu \left(g_V^{(-)} P_- + g_V^{(+)} P_+\right) \psi_2(p_2, \lambda_2)$$

where $q = \pm p_1 \pm p_2$ is the four-momentum transfer; $+(-)$ sign corresponds to an outgoing (incoming) particle or antiparticle.

Method

Other examples:

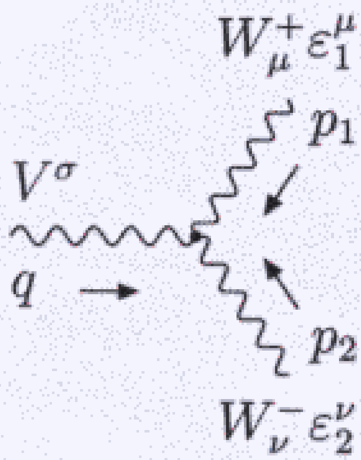
– radiation of the photon



$$\rightarrow \quad \varepsilon^{\mu}_{\gamma V}\,(p_1, p_2, k, \lambda_1, \lambda_2, \lambda)$$

$$= D_V^{\mu\nu}(q+k)\bar{\psi}_1(p_1,\lambda_1)\; g_{\gamma 1}\not{\varepsilon}(k,\lambda)\frac{\pm\not{p}_1 + \not{k} + m_1}{(\pm p_1 + k)^2 - m_1^2}$$
$$\times\gamma_\nu \left(g_V^{(-)}P_- + g_V^{(+)}P_+\right)\psi_2(p_2,\lambda_2),$$

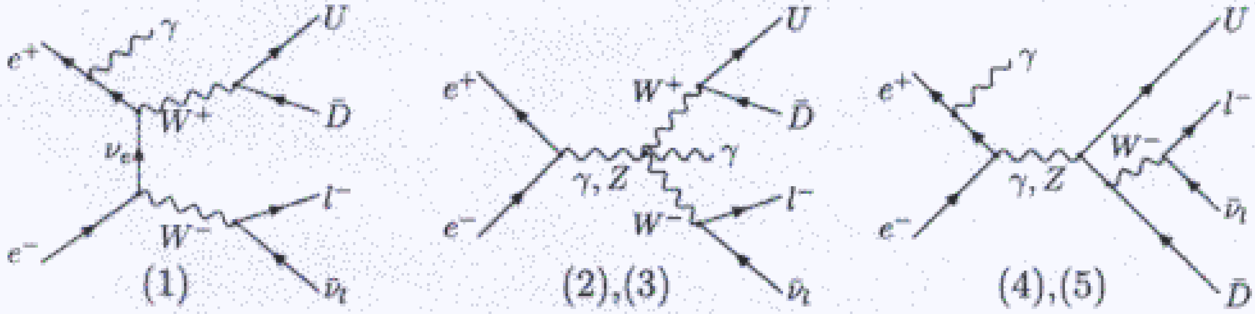
– the triple gauge boson coupling



$$\rightarrow \quad \varepsilon^{\sigma}_{V}(1,2) = D_V^{\sigma\rho}(q)\Gamma^{(WWV)}_{\mu\nu\rho}\varepsilon^\mu_1\varepsilon^\nu_2$$

$$= D_V^{\sigma\rho}(q) g_{WWV}\;[(p_1 - p_2)_\rho\, \varepsilon_1\cdot\varepsilon_2 + (p_2 - q)\cdot\varepsilon_1\, \varepsilon_{2\rho}$$
$$+ (q - p_1)\cdot\varepsilon_2\, \varepsilon_{1\rho}],$$

Application



Typical Feynman diagrams of reaction

$$e^+(p_1) + e^-(p_2) \to U(p_3) + \bar{D}(p_4) + l^-(p_5) + \bar{\nu}_l(p_6) + \gamma(p_7).$$

The corresponding amplitudes

$$M_1^\gamma = \bar{v}_1 g_{\gamma 1} \not{\varepsilon}_7 \frac{\not{p}_7 - \not{p}_1 + m_1}{(p_7 - p_1)^2 - m_1^2} g_W \not{\varepsilon}_+ P_- \frac{\not{p}_2 - \not{p}_{56}}{(p_2 - p_{56})^2} g_W \not{\varepsilon}_- P_- u_2,$$

$$M_{2,3}^\gamma = g_{\gamma VWW} \left(\varepsilon_V \cdot \varepsilon_+ \; \varepsilon_7 \cdot \varepsilon_- + \varepsilon_V \cdot \varepsilon_- \; \varepsilon_7 \cdot \varepsilon_+ - 2\varepsilon_V \cdot \varepsilon_7 \; \varepsilon_+ \cdot \varepsilon_- \right),$$

$$M_{4,5}^\gamma = \bar{u}_3 \not{\varepsilon}_{\gamma V} \left(g_{V3}^{(-)} P_- + g_{V3}^{(+)} P_+ \right) \frac{-\not{p}_4 - \not{p}_{56} + m_3}{(-p_4 - p_{56})^2 - m_3^2} g_W \not{\varepsilon}_- P_- v_3,$$

where $p_{12} = p_1 + p_2$, $p_{34} = p_3 + p_4$ and $p_{56} = p_5 + p_6$.

The diagrams which differ only in contributions from the photon and Z propagators can be calculated simultaneously.

Application

The photon propagator $D_\gamma^{\mu\nu}(q)$ is taken in the Feynman gauge and the propagators of the massive gauge bosons $D_V^{\mu\nu}(q)$, $V = W, Z$, are defined in the unitary gauge.

The constant widths Γ_W, Γ_Z are introduced through the complex mass parameters

$$M_V^2 = m_V^2 - im_V\Gamma_V$$

in the propagators.

The electroweak mixing parameter is kept real. No obstacle for having it complex

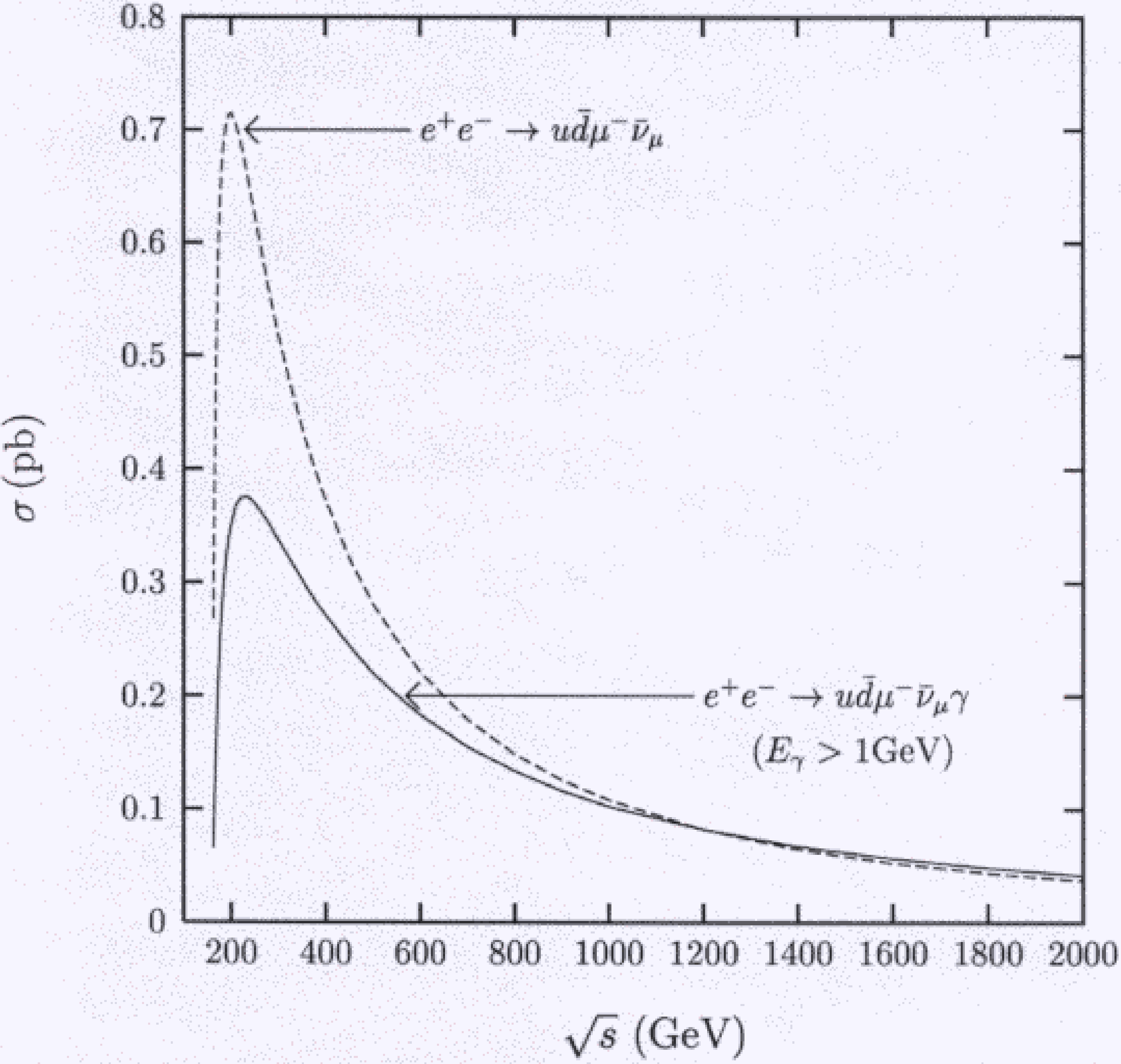
The electromagnetic gauge invariance is preserved with the non-zero fermion masses even if the widths Γ_W and Γ_Z are treated as two independent parameters.
This has been checked numerically and for some final states analytically.

Numerical results

Checks

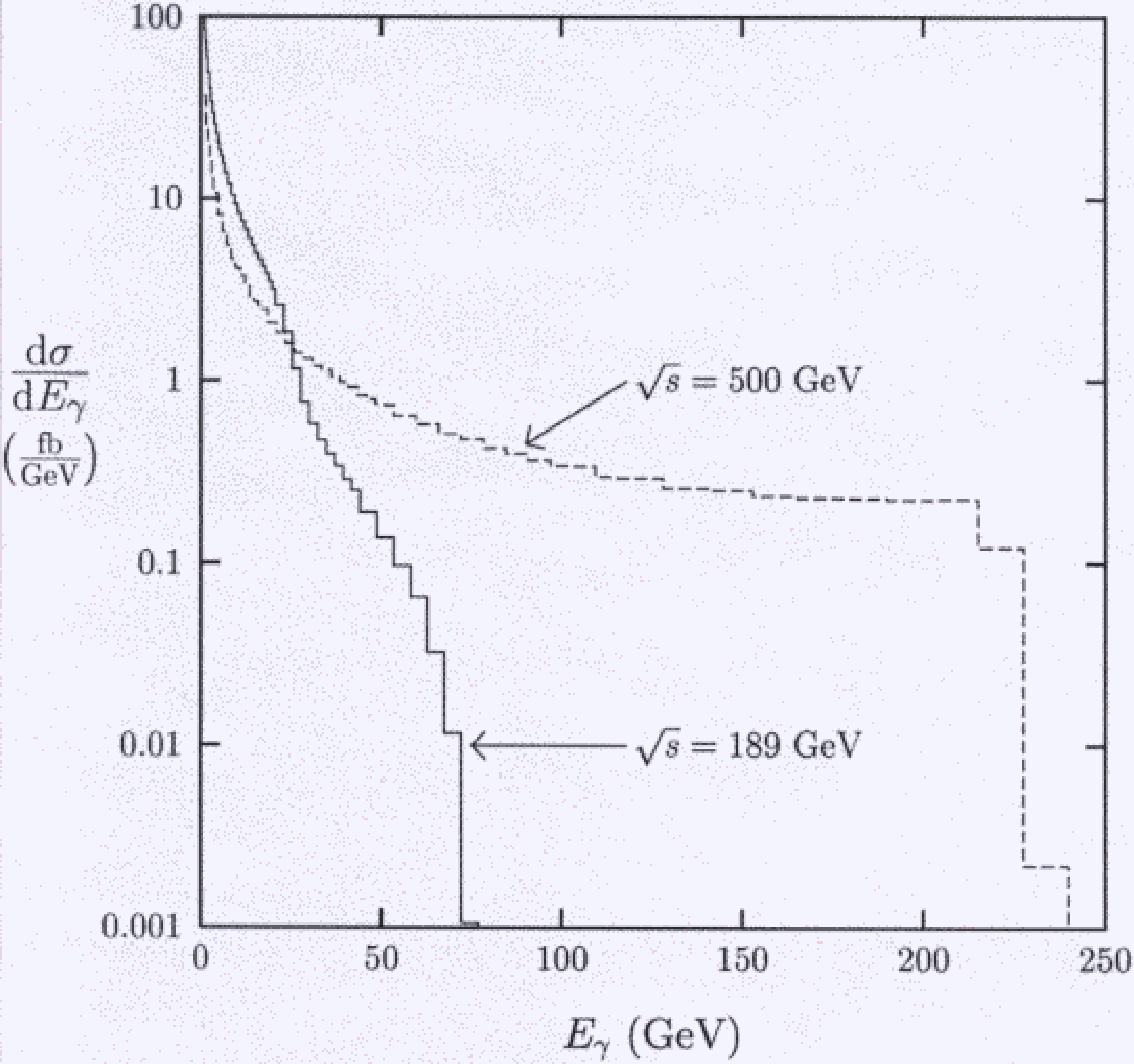
- The matrix elements have been checked against MADGRAPH (T. Stelzer, W.F. Long).
- The electromagnetic gauge invariance of the matrix element of the bremsstrahlung process, both analytically and numerically.
- The phase space integrals have been checked against their asymptotic limits obtained analytically.
- The cut-independence of $\sigma_\gamma = \sigma_s + \sigma_h$.
- Comparison against EXCALIBUR (F.A. Berends, R. Pittau, R. Kleiss).
- Comparison against A. Denner, S. Dittmaier, M. Roth, D. Wackerroth.

Numerical results



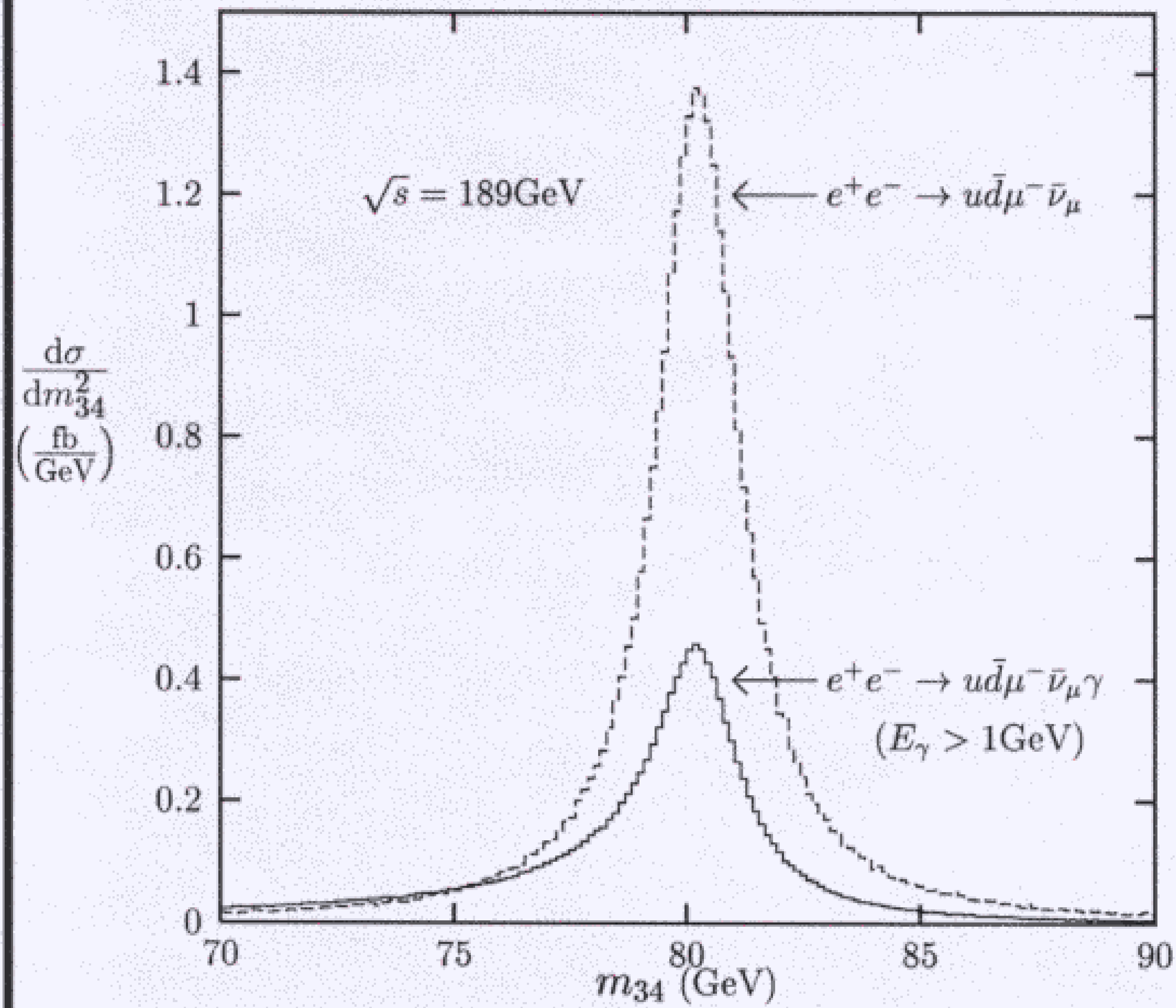
The energy dependence of the total cross sections.

Numerical results



The photon spectra at $\sqrt{s} = 189$ GeV and $\sqrt{s} = 500$ GeV.

Numerical results



The differential cross section $d\sigma/dm_{34}^2$ at $\sqrt{s} = 189$ GeV versus the invariant mass of the $u\bar{d}$ pair m_{34}.

Numerical results

Mass dependence of cross sections (in fb) for different final states
(without cuts, except for $E_\gamma > 0.1$ GeV).

$E_{\rm cm}$ GeV	$\sigma_\gamma(u\bar{d}\mu^-\bar{\nu}_\mu)$	$\sigma_\gamma(c\bar{s}\mu^-\bar{\nu}_\mu)$	$\sigma_\gamma(u\bar{d}\tau^-\bar{\nu}_\tau)$
189.0	573.4(4)	525.2(4)	522.6(4)
360.0	448.5(4)	418.4(4)	414.1(4)
500.0	322.8(4)	302.0(4)	298.1(3)
2000.0	56.48(27)	53.19(25)	52.67(13)

The hard bremsstrahlung cross section, for the energy cut $E_\gamma > 0.1$ GeV, changes by about -9% at 189 GeV and by about -6% at 2 TeV.

Numerical results (Preliminary)

Mass dependence of cross sections at $\sqrt{s} = 190$ GeV (in fb) for different final states

(with the "canonical" cuts)

$e^+e^- \to$	zero masses [DDRW]	non zero masses
$u\bar{d}e^-\bar{\nu}_e\gamma$	220.8(4)	221.0(1)
$u\bar{d}\mu^-\bar{\nu}_e\gamma$	214.5(4)	213.8(3)
$u\bar{d}s\bar{c}\gamma$	598(1)	593(2), [598(1)]

[...] corresponds to $m_u = m_d = m_c = m_s = 0.1 m_e$

There is a mass effect of $\sim 1\%$.

The "canonical" cuts:

$$\begin{aligned} \theta(l,\mathrm{beam}) &> 10^\circ, & \theta(l,l') &> 5^\circ, \\ \theta(l,q) &> 5^\circ, & \theta(\gamma,\mathrm{beam}) &> 1^\circ, \\ \theta(\gamma,l) &> 5^\circ, & \theta(\gamma,q) &> 5^\circ, \\ E_\gamma &> 0.1\,\mathrm{GeV}, & E_l &> 1\,\mathrm{GeV}, \\ E_q &> 3\,\mathrm{GeV}, & m(q,q') &> 5\,\mathrm{GeV}\,. \end{aligned} \quad (1)$$

Summary and outlook

- An efficient method for calculating photon radiation cross sections for massive fermions has been elaborated. Tests of Monte Carlo generators which work with massless fermions for the collinear regions are possible.
- Several channels of $e^+e^- \to 4f(\gamma)$ have been studied.
- Preliminary results for fermion mass effects are available for the CC processes.
- The fermion mass effects may affect the desired accuracy of 0.5% for LEP2 and 0.1% for the linear collider even in the presence of cuts.
- A study of the t-quark production is possible. Problems with the consistent treatment of Γ_t.
- The matrix elements of almost all possible four fermion final states and the corresponding bremsstrahlung reactions have been programmed and tested.