38. Herbstschule für Hochenergiephysik – Maria Laach, 13^{th} September 2006

Particle Physics and Cosmology III

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DESY

Plan of the Lectures

1. Introduction to Standard Cosmology:
   - General Relativity and Particle Physics
   - Present data and cosmological parameters
   - A short history of the Early Universe
   - Big Bang Nucleosynthesis
2. Inflation and Structure Formation:
   - Inflation and background dynamics
   - Scalar and tensor perturbations from inflation
   - Elements of perturbation theory
   - CMB and Large Scale Structure data
3. Thermal Universe and Relics
   - The Boltzmann equation in the expanding Universe
   - The number density of a thermal relic $\longrightarrow$ WIMPs
   - Supersymmetry and Dark Matter
   - Baryogenesis: EW or via leptogenesis ?

Elements of thermodynamics

In general, given the phase space distribution $f(\vec{p},\vec{x},t)$ and g the number of d. o. f., we have

$$n = g \int d^3p \; f(\vec{p}) \qquad \rho = g \int d^3p \; E(\vec{p}) f(\vec{p}) \qquad p = g \int d^3p \; \frac{\vec{p}^2}{3E(\vec{p})} f(\vec{p})$$

we can compute the number and energy density and the pressure. So indeed for radiation $E = |\vec{p}|$ and $p = \rho/3$, while for non-relativistic matter $E \sim m \left(1 + \frac{1}{2}\frac{\vec{p}^2}{m^2}\right)$ and $\rho \sim mn$, while $p \sim 0$.

Maximal entropy state $\Longrightarrow$ THERMAL EQUILIBRIUM:

- kinetic equilibrium, i.e. momentum distribution given by Fermi-Dirac or Bose-Einstein distribution, sustained by by elastic scatterings, $$f_{F/B}(\vec{p}) = \frac{1}{e^{\frac{E-\mu}{T}} \pm 1} \; ,$$ where μ is the chemical potential related to a conserved number;
- chemical equilibrium, i.e. equilibrium between different particle species, sustained by inelastic scatterings: $$i + j \leftrightarrow k + l \quad \text{gives} \quad \mu_i + \mu_j = \mu_k + \mu_l \; .$$

Some useful formulas

Relativistic limit, i.e. $|\vec{p}| \gg m, T \gg \mu$:

$$n = \frac{\zeta(3)\nu_{F/B}}{\pi^2} g T^3 \qquad \rho = \frac{\pi^2 \xi_{F/B}}{30} g T^4 \qquad p = \frac{\rho}{3}$$

for $\nu_{F/B} = 3/4, 1$ and $\xi_{F/B} = 7/8, 1$ for fermions and bosons.

Non-relativistic limit, i.e. $|\vec{p}| \ll m, T \ll \mu$ ($\longrightarrow$ Maxwell-Boltzmann):

$$n = g \left(\frac{mT}{2\pi}\right)^{3/2} e^{\frac{-(m-\mu)}{T}} \qquad \rho = mn \qquad p = 0 \; .$$

The chemical potential instead is related to a conserved charge, so particle and antiparticle have opposite sign μ and we have in the relativistic and non-relativistic case:

$$n_p - n_{\bar{p}} \to g \frac{\mu T^2}{6} + \mathcal{O}\left(\frac{\mu^3}{T^3}\right) \qquad n_p - n_{\bar{p}} \to 2g \left(\frac{mT}{2\pi}\right)^{3/2} sinh\left(\frac{\mu}{T}\right) e^{\frac{-m}{T}} \; .$$

Thermodynamics in expanding Universe

An expanding Universe is a closed system and in thermal equilibrium the entropy is conserved !

$$TdS = d(\rho V) + pdV = d((\rho + p)V) - Vdp = 0 \quad \text{since} \quad d((\rho + p)V) = Vdp$$

due to energy conservation; moreover the entropy density is $s = \frac{S}{V} = \frac{\rho+p}{T}$.

Using the expressions given above we can define the total energy and entropy density in radiation:

$$\rho_{rad} = \frac{\pi^2}{30} g_* T^4 \qquad g_* = \sum_B g_i \left(\frac{T_i}{T}\right)^4 + \sum_F \frac{7}{8} g_j \left(\frac{T_j}{T}\right)^4$$

$$s = \frac{2\pi^2}{45} g_S T^3 \qquad g_S = \sum_B g_i \left(\frac{T_i}{T}\right)^3 + \sum_F \frac{7}{8} g_j \left(\frac{T_j}{T}\right)^3$$

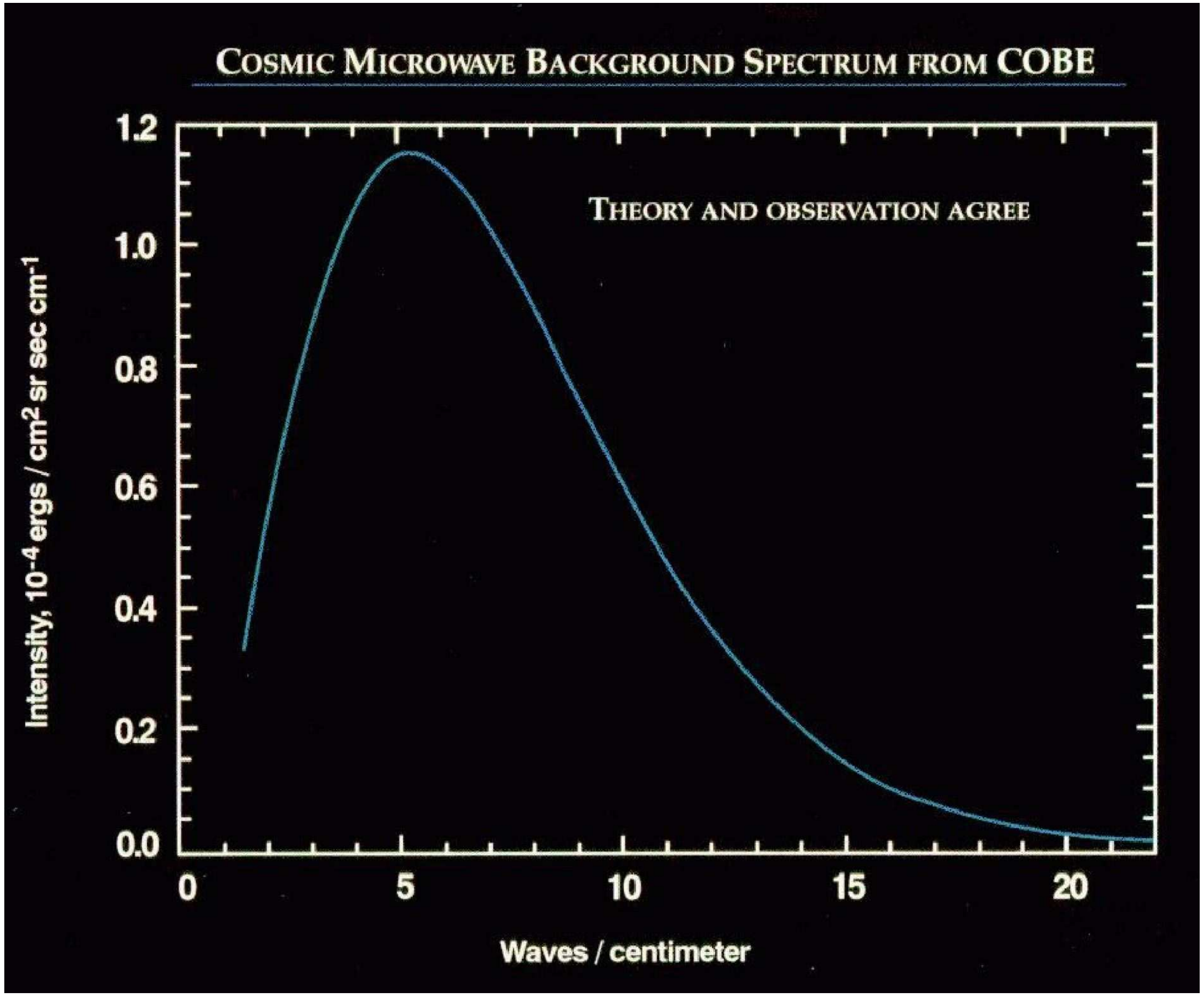
Constant entropy gives then sa^3 constant, i.e. $$\frac{ds}{s} = 3\frac{dT}{T} + \frac{dg_S}{g_S} = -3\frac{da}{a}$$

So for $dg_S = 0$ we have $T \propto a^{-1}$ adiabatic cooling !

Note: a thermal spectrum for a relativistic (massless particle) is not distorted by the expansion, just red-shifted to lower T

$\Rightarrow$ CMB photons !

Cosmic Microwave Radiation: Perfect BLACK BODY at $T = 2.7\,\mathrm{deg}$



picture from http://map.gsfc.nasa.gov

Boltzmann equation in an expanding Universe

How can we establish if we have thermal equilibrium and until when ? The time evolution of $f(\vec{p}, \vec{x}, t)$ is given classically by the Boltzmann equation

$$\hat{\mathcal{L}}[f] \quad = \quad \mathcal{C}[f]$$

Liouville operator Collision integral

In GR $\hat{\mathcal{L}}$ is given by $\hat{\mathcal{L}}[f] = p^\alpha \frac{\partial f}{\partial x^\alpha} - \Gamma^\alpha_{\beta\gamma} p^\beta p^\gamma \frac{\partial f}{\partial p^\alpha} = E\dot{f} - H\vec{p}^2 \frac{\partial f}{\partial E}$

for a homogeneous and isotropic distribution function $f(E, t)$.
Integrating over the momentum p, we obtain the equation for the number density

$$\dot{n} + 3Hn = \int \frac{d^3p}{2E} \frac{d^3k}{2E_k} .... \frac{d^3q}{2E_q} ... \delta^4(p + k... - q...) \left(|M(p + k \to q)|^2 f_p f_k ... - |M(q \to p + k)|^2 f_q ... \right)$$

Assume that there is no CP or T violation: $|M(p+k \to q)|^2 = |M(q \to p+k)|^2$

and that the particle q's are in equilibrium, i.e. $f_q \cdots = f_p^{eq} f_k^{eq} \ldots$

Then we have

$$\dot{n} + 3Hn = \int \frac{d^3p}{2E} \frac{d^3k}{2E_k} \ldots (f_p f_k \ldots - f_p{}^{eq} f_k^{eq} \ldots) \frac{d^3q}{2E_q} \ldots \delta^4(p+k\ldots - q\ldots) |M(p+k \to q)|^2$$

$$= \int \frac{d^3p}{2E} \frac{d^3k}{2E_k} .. (f_p f_k \ldots - f_p{}^{eq} f_k^{eq} \ldots)\, \sigma(p+k \to q) v$$

$$= (n n_k \ldots - n^{eq} n_k^{eq} \ldots) \langle \sigma(p+k \to q) v \rangle$$

thermally averaged cross-section

where we have assumed $f = n \frac{f^{eq}}{n^{eq}}$ for all particles involved.

Then a species is in equilibrium if its interaction rates $\langle \sigma v \rangle$ are efficient enough !

Primordial abundance of stable massive species

[see e.g. Kolb & Turner '90]

The number density of a stable particle X in an expanding Universe is given by the Bolzmann equation

$$\frac{dn_X}{dt} + 3Hn_X = \langle \sigma(X + X \rightarrow \text{anything}) v \rangle \left(n_{eq}^2 - n_X^2 \right)$$

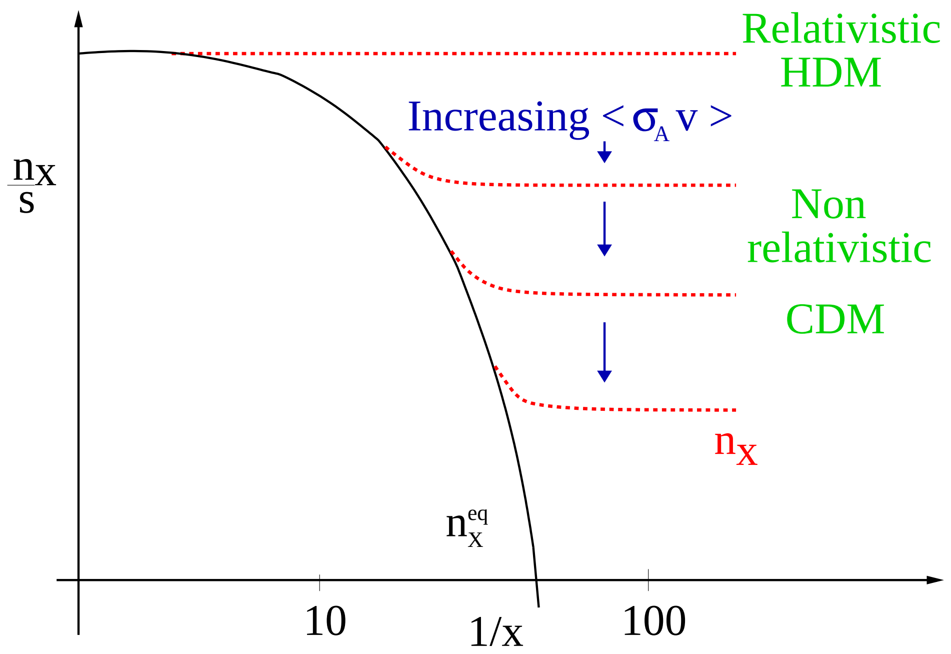
Hubble expansion   Collision integral

The particles stay in thermal equilibrium until the interactions are fast enough, then they freeze-out at $x_f = m_X / T_f$

defined by $n_{eq} \langle \sigma_A v \rangle_{x_f} = H(x_f)$ and that gives

$$\Omega_X = m_X n_X (t_{now}) \quad \propto \quad \frac{1}{\langle \sigma_A v \rangle_{x_f}}$$

Abundance $\Leftrightarrow$ Particle properties



For $m_X \simeq 100$ GeV a WEAK cross-section is needed !

Weakly Interacting Massive Particle

For weaker interactions need lighter masses HOT DM !

Approximate solution of the Boltzmann equation

Rewrite the equation in terms of $Y = \frac{n}{s}$ and $\frac{d}{dt} = Hx\frac{d}{dx}$ for $x = \frac{m_X}{T}$:

$$\frac{dY_X}{dx} = -\frac{s\langle\sigma(X + X \to \text{anything})v\rangle}{xH}\left(Y_X^2 - Y_{eq}^2\right)$$

Until x_f we have $Y_X = Y_{eq}$, after that we can neglect Y_{eq} that decreases exponentially and then

$$\frac{dY_X}{Y_X^2} = -\frac{s(x)\langle\sigma(X + X \to \text{anything})v\rangle(x)}{xH(x)}dx$$

which has the solution

$$Y_X(x) = \frac{Y_X(x_f)}{1 + Y_X(x_f)\frac{s(m_X)}{H(m_X)}\int_{x_f}^{x}\frac{dx}{x^2}\langle\sigma(X + X \to \text{anything})v\rangle(x)}$$

so when σ is sufficiently large after freeze-out

$$Y_X(x) \simeq \frac{1}{\frac{s(m_X)}{H(m_X)}\int_{x_f}^{x}\frac{dx}{x^2}\langle\sigma(X + X \to \text{anything})v\rangle(x)}$$

very weakly dependent on x_f; otherwise $Y_X(x) = Y_X(x_f)$.

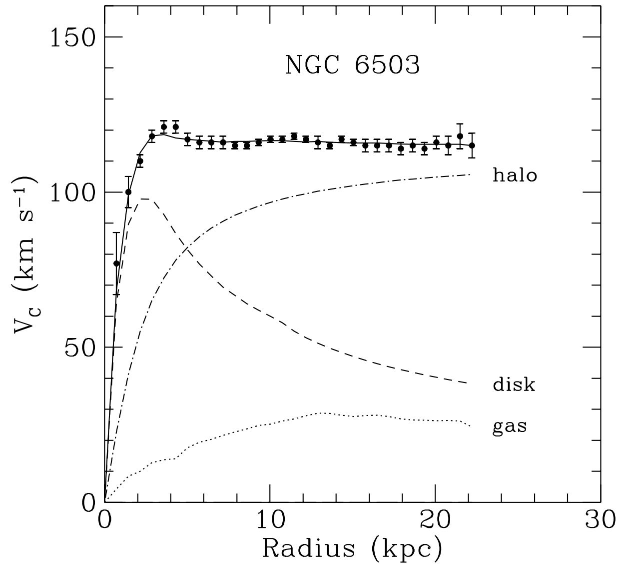
THE MATTER CONTENT

The clumpy energy density/matter divides into

Particles	$\Omega_i(t_{\text{now}})h^2$ (WMAP)	Type
Baryons	0.0224	Cold
Massive ν	$6.5 \times 10^{-4} - 0.01$	Hot
???	$\sim 0.1 - 0.13$	COLD

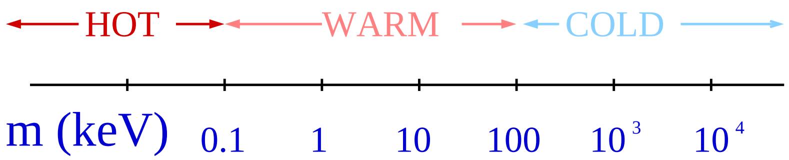
DARK matter !

[Begeman, Broeils & Sanders '91]



Note that DM was first discovered in local systems from the galaxies rotational curves...

Structure formation requires COLD Dark Matter, otherwise the structure formation on scales smaller than its free-streaming length at t_{eq} is suppressed.



Do we really NEED Dark Matter ???

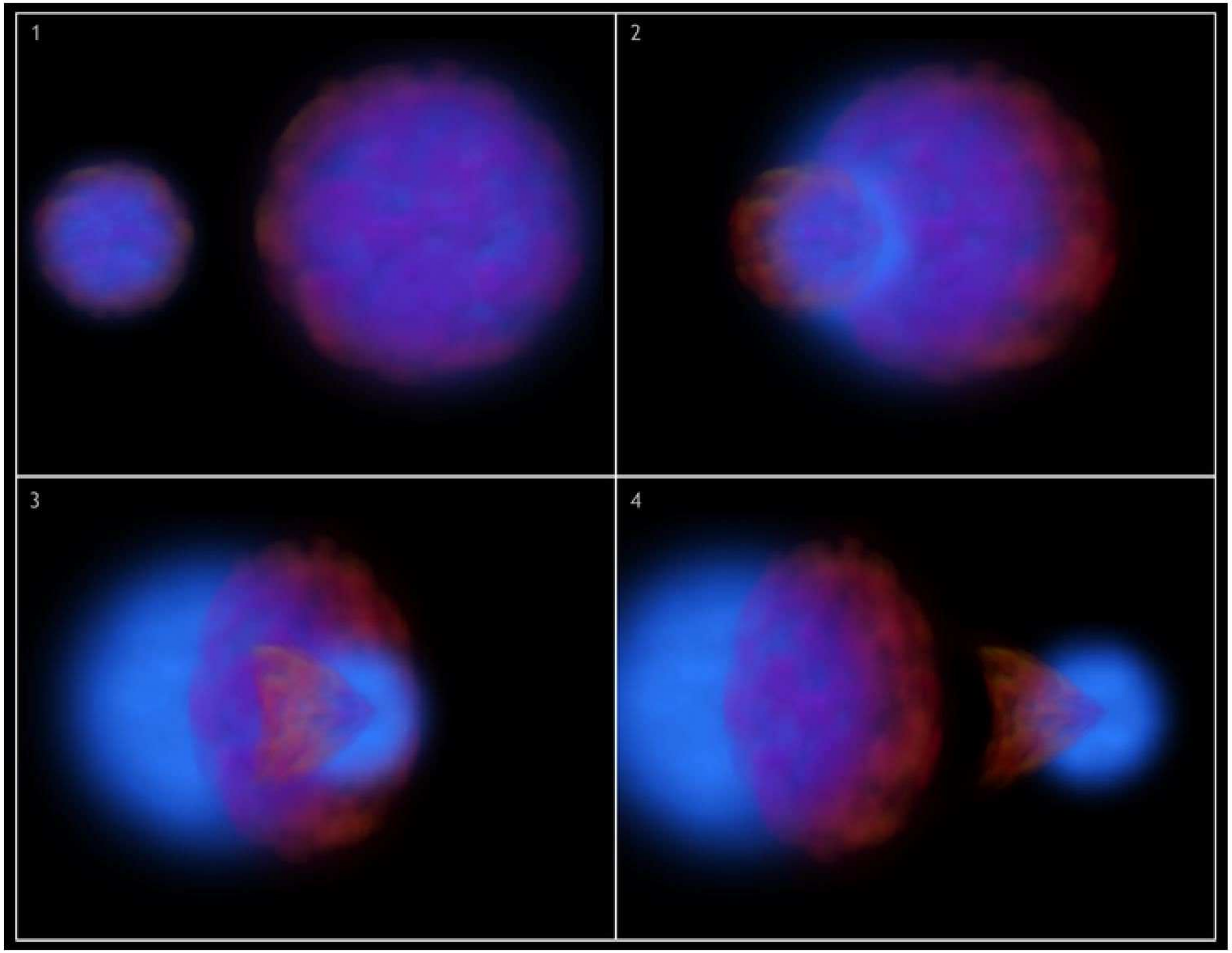
Astronomers actually have found methods to fit the rotational curves of galaxies without DM, by modifying gravity $\Rightarrow MOND(MOdifiedNewtonianDynamics)$!

Very simple scheme: set a minimal acceleration a_0. BUT it solves only part of the troubles...

A fully relativistic formulation of MOND as a scalar-vector-tensor field theory has been proposed by Bekenstein, but still it is not clear if it will allow for a successful cosmological model, in particular for STRUCTURE FORMATION. DM generates the potential wells for baryons to fall into during the time they still were in thermal equilibrium with radiation... A universe with only baryonic matter starts structure formation later and would have less power at small scales !

Also difficult for MOND to explain STRONG GRAVITATIONAL LENSING and events like the one observed recently by CHANDRA...

Real Dark Matter collider... Galaxy cluster 1E 0657-56



from http://chandra.harvard.edu/press/06_releases/press_082106.html

Astrophysical properties of Dark Matter

- electrically neutral: **DARK !**

  Can it be colored ??? Not really, it would get trapped into hadrons and give rise to exotic nuclei; bounds for such states are very strong up to masses of order 10 TeV. In general it must have decoupled early from the primordial plasma, i.e. photons and baryons, to avoid erasing power on small scales...

- stable on cosmological times, i.e. $\tau \gg 13.7$ Gyrs or in more familiar units $\Gamma \ll 1.5 \times 10^{-42}$ GeV. Note that assuming a decay of the type m^5/M_{Pl}^4, this gives just the bound $m \ll 0.8 \times 10^7$ GeV.

- it must be massive and NON-relativistic for structure formation $\longrightarrow$ Cold or at most Warm DM !

Otherwise (nearly) any mass-scale would do as long as $0.095 \leq \Omega_{CDM} h^2 \leq 0.13$

What are the possible candidates ??? The Standard Model (SM) does not offer any suitable particle, the SM neutrinos are at most Hot DM, so we are obliged to look beyond...

A natural candidate emerges as a thermal relic with weak couplings and mass around the electroweak scale in supersymmetric extensions of the SM. $\longrightarrow$ WIMP scenario !

Supersymmetry and the Constrained MSSM

Supersymmetry: boson $\Leftrightarrow$ fermion

- cancels quadratic divergencies and protects scalar masses
- predicts for every SM particle a superpartner with different spin
- gives unification of gauge couplings

SM Particles	Superpartners
q_L, u_R, d_R, l_L, e_R	$\tilde{q}_L, \tilde{u}_R, \tilde{d}_R, \tilde{l}_L, \tilde{e}_R$
$g, \gamma, Z, W^{\pm}$	$\tilde{g}, \tilde{\gamma}, \tilde{Z}, \tilde{W}^{\pm}$
$H_u, H_d \rightarrow h, H, A, H^{\pm}$	$\tilde{H}_u, \tilde{H}_d$

SUPERSYMMETRY is not observed in nature, it is broken softly $\longrightarrow$ massive superpartners !

$\longrightarrow$ 105 parameters !

Assuming universality at the GUT scale, then we have only 5 additional parameters:

SUSY parameters: $\tan\beta = \dfrac{\langle H_u \rangle}{\langle H_d \rangle}, \quad \mu$

scalar/gaugino mass, scalar trilinear: $m_0, m_{1/2}, A_0$

EW symmetry is broken radiatively: $|\mu|$ is fixed ! $\Rightarrow$ Constrained Minimal Supersymmetric SM

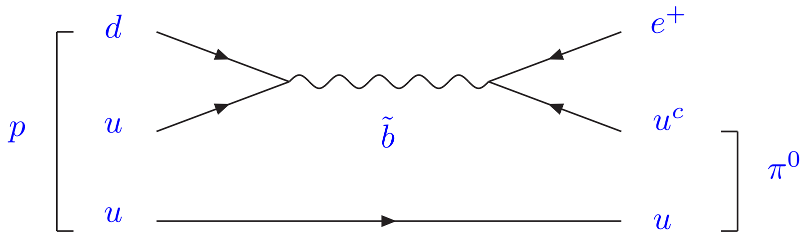
R-parity conservation

The SM accidentally preserves baryon number and therefore the proton is stable. (Actually $U(1)_{B+L}$ is anomalous and L number is violated if the neutrinos are Majorana fermions, but such effects are important only at high temperature) $\longrightarrow$ BARYOGENESIS ?

In a supersymmetric model we can have additional renormalizable couplings that do violate explicitely

baryon and lepton number. $$W = \lambda LLE^c + \lambda' LQD^c + \lambda'' U^c D^c D^c + \mu_i L_i H_2$$

$\Rightarrow$ Dimension 4 proton decay operators



$\propto \frac{1}{m_{\tilde{b}}^2}$

To avoid fast proton decay, impose a discrete symmetry called R-parity, which forbids these terms:

$$q_L, u_R, d_R, l_L, e_R, g, \gamma, Z, W^\pm, H_u, H_d \quad \longrightarrow \quad +q_L, u_R, d_R, l_L, e_R, g, \gamma, Z, W^\pm, H_u, H_d$$
$$\tilde{q}_L, \tilde{u}_R, \tilde{d}_R, \tilde{l}_L, \tilde{e}_R, \tilde{g}, \tilde{\gamma}, \tilde{Z}, \tilde{W}^\pm, \tilde{H}_u, \tilde{H}_d \quad \longrightarrow \quad -\tilde{q}_L, \tilde{u}_R, \tilde{d}_R, \tilde{l}_L, \tilde{e}_R, \tilde{g}, \tilde{\gamma}, \tilde{Z}, \tilde{W}^\pm, \tilde{H}_u, \tilde{H}_d$$

So couplings only involve pairs of superpartners and therefore the supersymmetric particles can only be produced or destroyed in pairs ! $\Rightarrow$ The Lightest Susy Particle is stable !

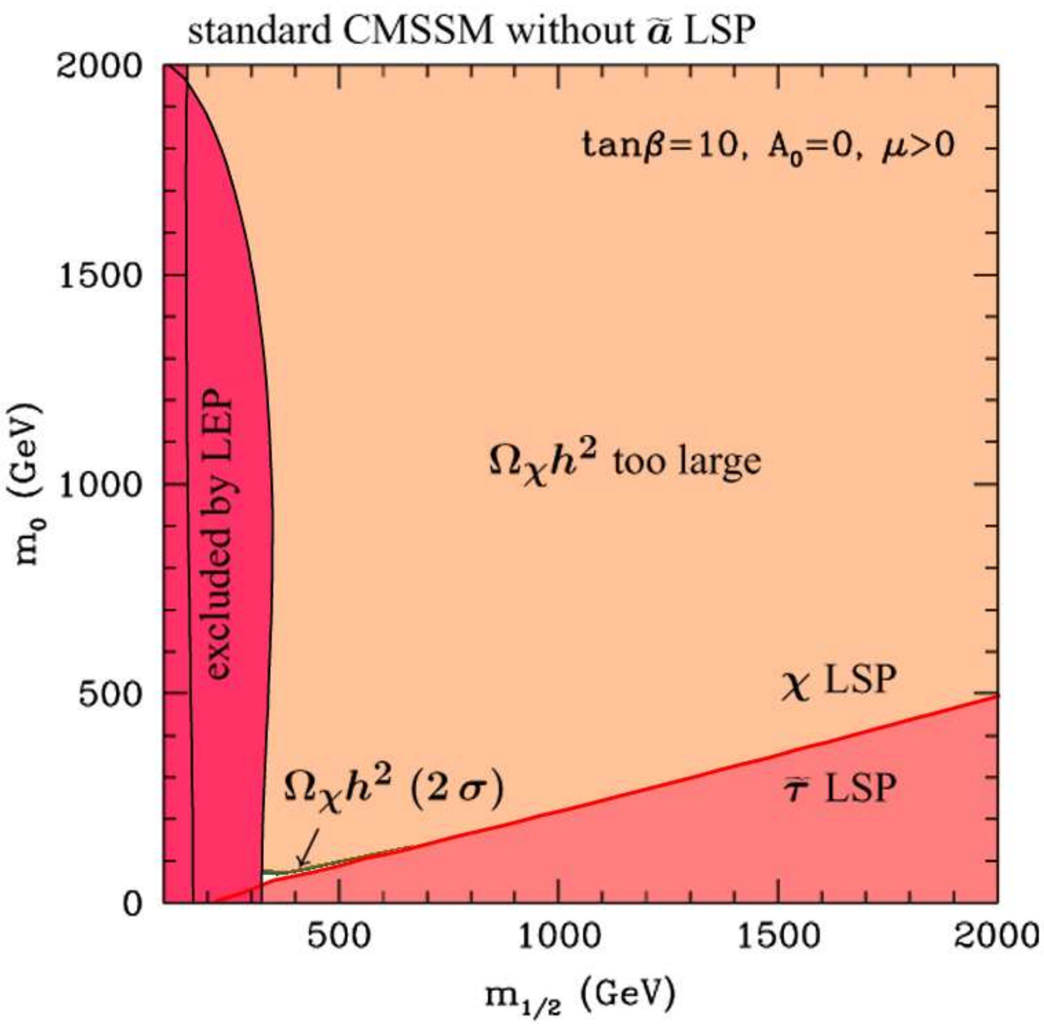
Which are suitable WIMPs ?

- neutralinos: in the CMSSM the LSP is typically a neutralino (WIMP !), but its abundance depends strongly on the sparticle masses

$\longrightarrow$ it is too large apart when enhanced by some mechaanisms, i.e. coannihilation, Higgs resonance... The natural "bulk region" already excluded by LEP.

$\longrightarrow$ fine–tuning ???

- sneutrinos: unfortunately excluded already by the large cross-section via Z boson that would have been already observed in DM experiments...



The neutralino composition, mass and its couplings change strongly depending on the SUSY breaking parameters and allow to span about 5 orders of magnitude in Ωh^2...

In particular in the CMSSM the neutralino cross-section turns out to be too weak (or its mass is too heavy) in most of the parameter space, apart in four scenarios:

- efficient annihilation into SM fermions (bulk region): needs light sfermions and it is nearly completely excluded by LEP bounds...

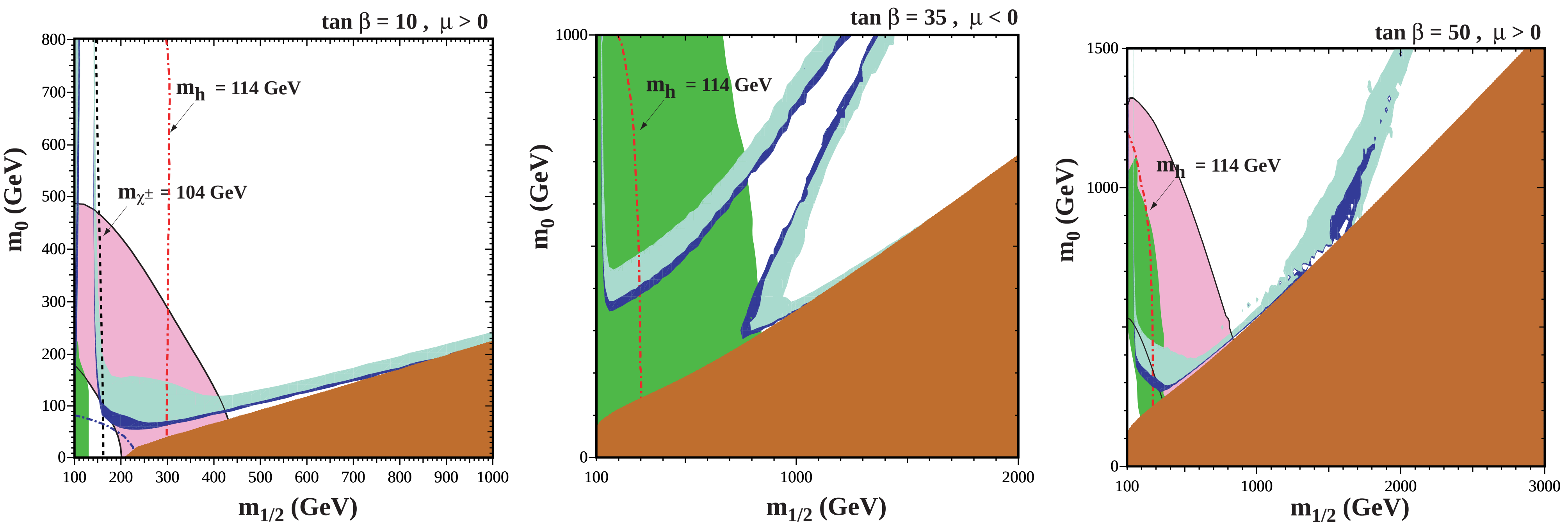
- efficient coannihilation with another superparticle, in the CMSSM the lightest stau , but in general also stops, sneutrinos, etc.. Note that in this case the number density is strongly dependent on the mass difference between the two particles (exponentially...);

-enhanced annihilation near a Higgs resonance, again the mass of the neutralino has to be very near to e.g. half the A-mass and Ωh^2 strongly depends on the mass difference and Γ_A;

- large Higgsino component (focus point region): then the channel of annihilation into WW is open and reduces the number density sufficiently;

The different regions would give completely different spectra at LHC !

Allowed regions for different $\tan\beta$ after WMAP [Ellis, Olive, Santoso & Spanos '03]

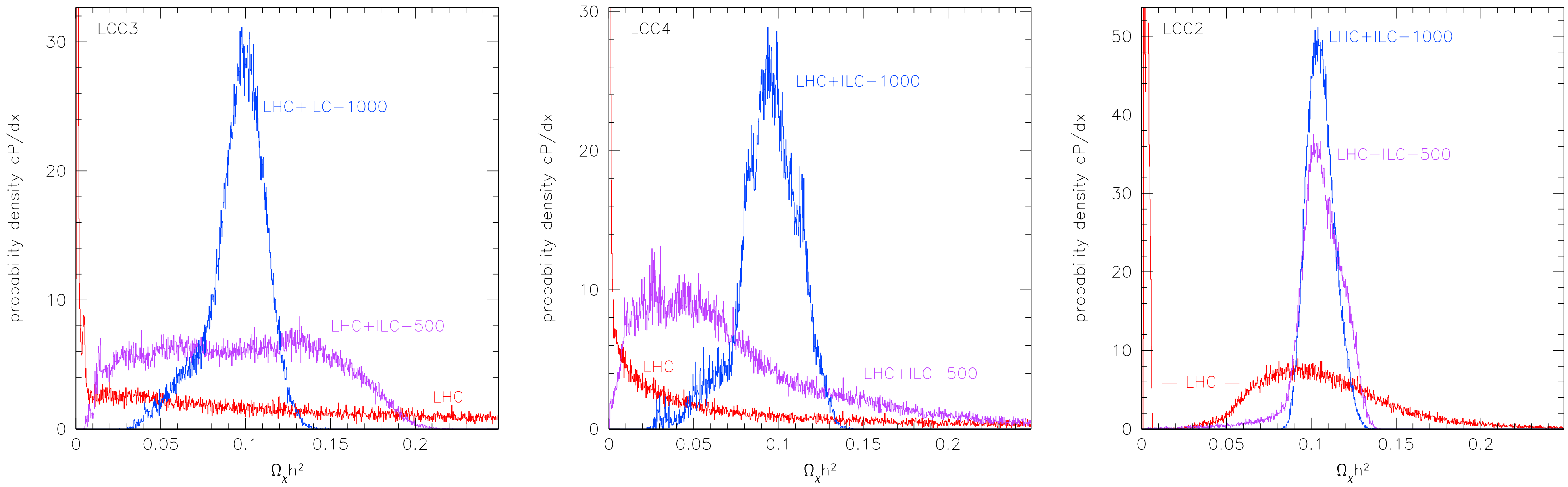


Different constraints: $b \to s\gamma$, $g_\mu - 2$, $\Omega h^2 = 0.094 - 0.129$.

No focus point region for large m_{top} according to Ellis et al. Note the different scales.

HOW WELL CA WE RECONSTRUCT Ωh^2 from LHC data ?

[Baltz, Battaglia, Peskin & Wizansky '06]



coannihilation region resonant region Higgsino-like -region

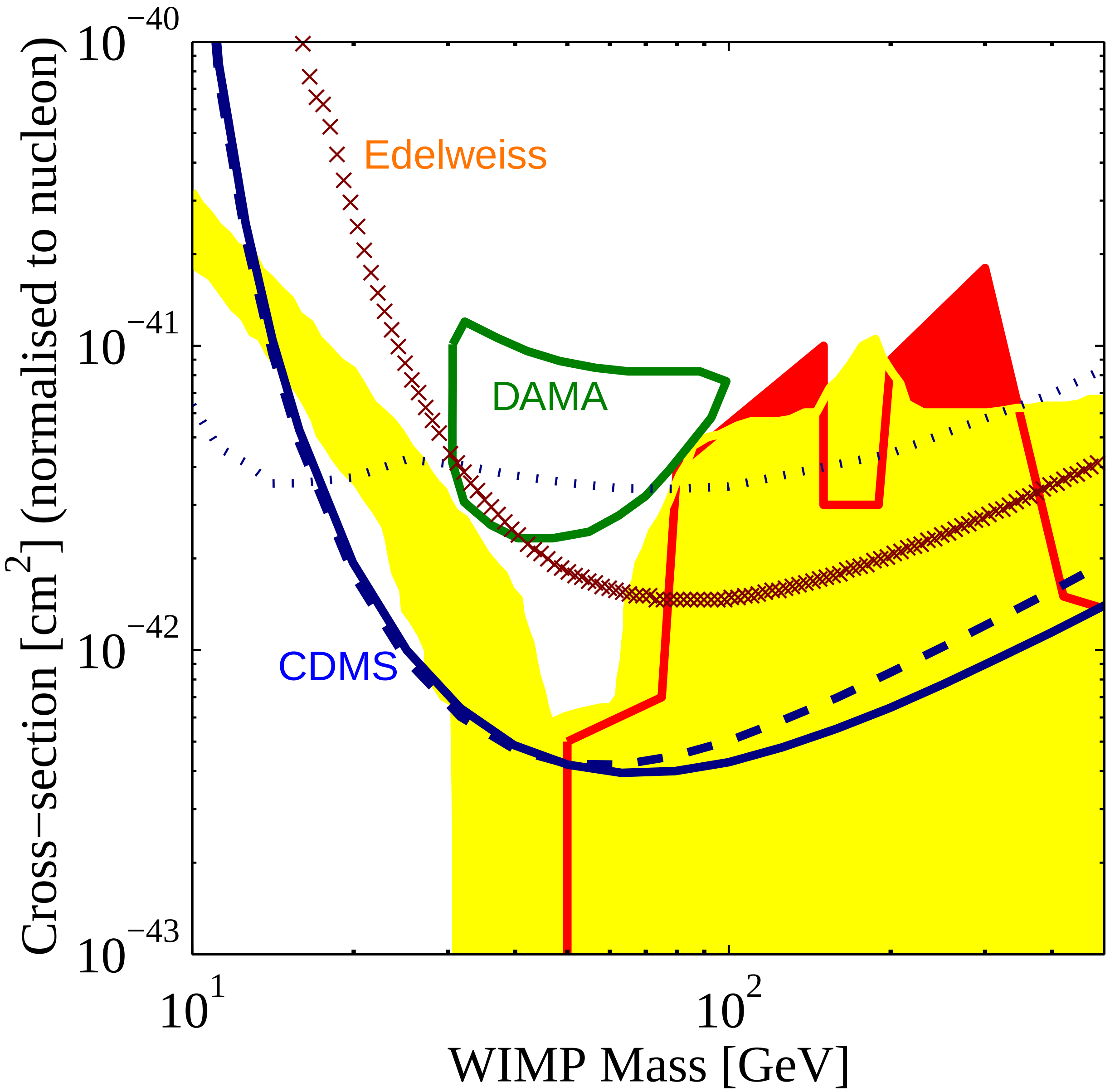
Not very well unfortunately due to the strong sensitivity to the parameters... In the bulk region (LCC1) things look more promising.

Dark Matter nature still unknown und undetected as particle....

Many particle physics candidates are viable and WIMPs probably will be identified in the next decade at colliders and/or DM experiments !

See here on the right the present bounds from WIMP direct detection from CDMS.

[astro-ph/0405033]



Baryogenesis

Both from BBN and from CMB measurements we obtain $\Omega_B \simeq 0.05$. Is it possible to explain such a number in a symmetric Universe ? Not really: from thermal decoupling 'a la WIMP' we would expect a freeze-out value $\Omega_B \simeq 10^{-10}$ instead... Either need a chemical potential μ_B or a mechanism to separate sufficiently matter and antimatter to avoid annihilation.

Suppose we live in a matter patch in an otherwise symmetric Universe, how large is such a patch ???

We have no evidence of a boundary, that would give either energetic γ's from π^0 decay and deform the γ rays background, or some antinuclei. We observe antiprotons in cosmic rays, but they are consistent by being produced in spallation processes.

Conclusion: our matter patch is at least as large as the observable Universe ! No causal mechanism can separate particles on scales larger than the horizon so the Universe must be asymmetric !

Sakharov's conditions

Sakharov studied already in 1967 the question of how it is possible to generate a baryon number from an initial symmetric phase ! He found that 3 conditions have to be satisfied:

- B violation: actually need $B - L$ violation since $B + L$ is violated in the SM by the chiral anomaly

$$\partial_\mu J^\mu_{B+L} = 2n_f \frac{g^2}{32\pi^2} F_{\mu\nu}\tilde{F}^{\mu\nu}$$

- C and CP violation: otherwise b and $\bar{b}$ would be annihilated/created with the same rate;

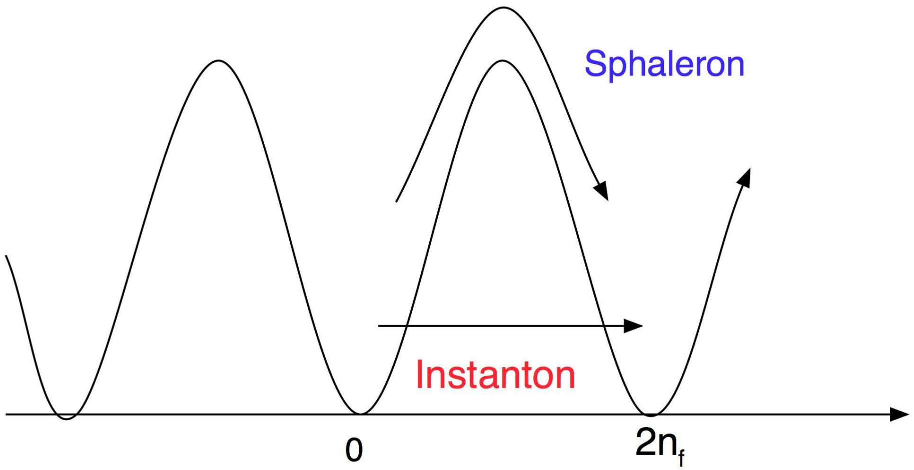
- violation of thermal equilibrium: otherwise the maximal entropy state is with $\mu_B = 0$ or

$$\begin{aligned}\langle B\rangle_T &= \mathrm{Tr}(Be^{-\beta H}) = \mathrm{Tr}(CPT(CPT)^{-1}Be^{-\beta H}) \\ &= \mathrm{Tr}((CPT)^{-1}B\, CPT\; e^{-\beta H}) = -\mathrm{Tr}(Be^{-\beta H}) = -\langle B\rangle_T\end{aligned}$$

since $[CPT, H] = 0$, no B generated without a "time arrow".

$B + L$ violation in the Standard Model

In the SM the global $U(1)_{B+L}$ is anomalous. This is related to the complex vacuum structure of the theory, which contains vacua with different configurations of the gauge fields and different topological number. Non-perturbative transitions between the vacua change $B + L$ by $2n_f$.



– $T = 0$: tunneling and is suppressed by $e^{-\frac{4\pi}{\alpha_W}} \ll 1$

$\rightarrow B$ & L practically conserved!

– $T > 0$: the transition can happen via a sphaleron

with rate $\Gamma_{sph}(T) \sim \left(\frac{M_W}{\alpha_W T}\right)^3 M_W^4 e^{-E_{sph}/T}$

So at temperatures $T \geq 100$ GeV sphaleronic transitions are in equilibrium in the Universe $\rightarrow B + L$ erased if $B - L = 0$, otherwise

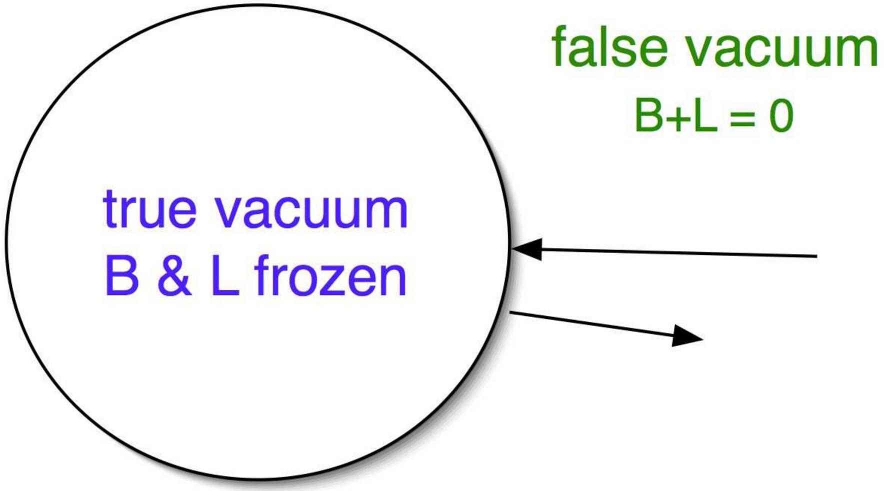
$$B = \frac{8n_f + 4n_H}{22n_f + 13n_H}(B - L)$$

A $B - L$ number is reprocessed into B number !

Is it possible to have baryogenesis in the SM ???

We have B violation via sphalerons, C and CP are present due to the phase in the CKM matrix, what about departure from thermal equilibrium ??? This also happens in the SM if the electroweak phase transition is strongly 1^{st} order. $\longrightarrow$ Bubble nucleation !

Electroweak baryogenesis



The strength of the transition depends on the height of the barrier between the true and false vacua v/T_c and so on the Higgs mass. Lattice studies have shown that the phase transition in the SM is first order only for masses $m_H \leq 40$ GeV, while now we know that $m_H \geq 114$ GeV: the mechanism does not work in the Standard Model !!! Still it could in the MSSM and extended model:

- stronger phase transition: 1^{st} order until $m_H \sim 120$ GeV for one light stop;
- more CP violating phases, while in SM $J \sim 10^{-20}$ perhaps too small.

Baryogenesis via leptogenesis

[Fukugita & Yanagida '86]

Since sphalerons reprocess $B + L$ number, we actually can do baryogenesis even by generating *lepton number* ! Extend the SM to include RH neutrinos with a Majorana mass

$$W = Y_\nu L H_2 N + \frac{1}{2} M_R N N$$

Then after EW symmetry breaking, the mass matrix for neutrinos become

$$M_\nu = \begin{pmatrix} 0 & Y_\nu v_2 \\ Y_\nu^T v_2 & M_R \end{pmatrix} \Rightarrow m_\nu \sim Y_\nu^T M_R^{-1} Y_\nu v_2^2 \ll 1 \text{ for } M_R \gg v_2$$

$\longrightarrow$ seesaw !

This is a good explanation of why the neutrinos are very light ! But note that here we have also L violation due to the Majorana mass term and the RH neutrinos can decay as

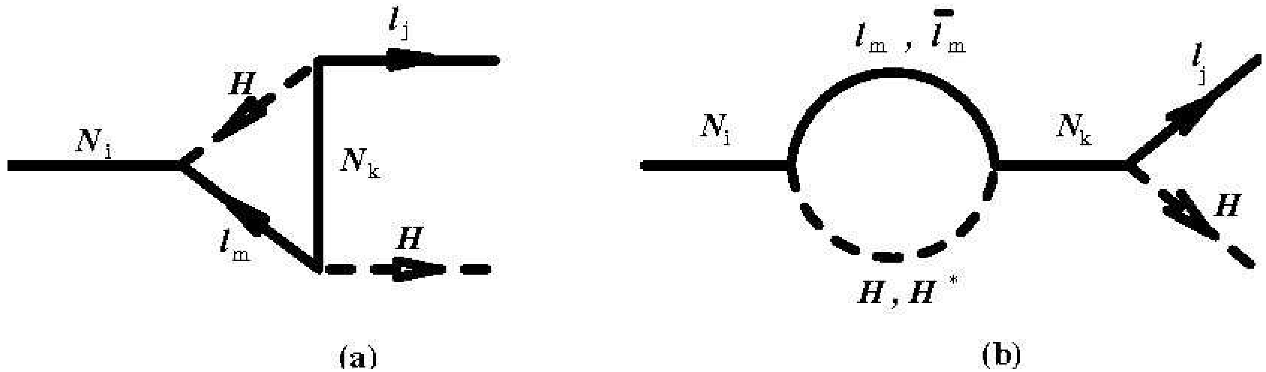
$$N \to L H_2 \qquad N \to \bar{L} \bar{H}_2$$

Possible to exploit N for baryogenesis ?

CP violation in N decay

We have CP in the decay of N if the couplings are complex.

CP violation always arises from an interference: tree + one-loop diagrams



We can define

$$\epsilon_i = \frac{\Gamma(N_i \to L) - \Gamma(N_i \to \bar{L})}{\Gamma(N_i \to L) + \Gamma(N_i \to \bar{L})} = -\frac{3}{16\pi} \sum_{i \neq j} \frac{M_i}{M_j} \frac{\Im[(Y_\nu^\dagger Y_\nu)_{ji}^2]}{(Y_\nu^\dagger Y_\nu)_{ii}} \text{for} M_i \ll M_j$$

$\longrightarrow$relation to neutrino masses via Y_ν...

Out of equilibrium decay

To generate the lepton asymmetry we need also departure from thermal equilibrium: out of equilibrium decay of the lightest N. This happens if $\Gamma_1 \leq H$ at $T \sim M_1$.

$$\Gamma_1 = \frac{(Y_\nu^\dagger Y_\nu)_{11}}{16\pi} M_1 \leq H = \sqrt{\frac{\pi^2 g_*}{90}} \frac{M_1^2}{M_P}$$

$\Rightarrow M_1 \geq \sqrt{\frac{90}{\pi^2 g_*}} \frac{(Y_\nu^\dagger Y_\nu)_{11}}{16\pi} M_P$, i.e. the RH neutrino have to be sufficiently massive. Or one can refrase it as

$$\tilde{m}_1 = \frac{(Y_\nu^\dagger Y_\nu)_{11} v^2}{M_1} \leq \sqrt{\frac{\pi^2 g_*}{90}} \frac{v^2}{M_P} \sim 10^{-3}\text{eV}$$

If this condition is satisfied, then it is trivial to see that every N gives an ϵ amount of lepton number and the final asymmetry is simply

$$\frac{n_L}{s} = \frac{n_{B-L}}{s} = \frac{135\zeta(3)g}{8\pi^4 g_S}\epsilon_1 \simeq 4 \times 10^{-3}\epsilon_1 \quad \rightarrow \frac{n_B}{s} \sim -1.5 \times 10^{-3}\epsilon_1$$

Otherwise one has to solve a couple of Boltzmann equations...

Boltzmann equation for the $B - L$ number

Remember, earlier we have taken CP conserved in the Boltzmann equation, but in this case that is not true... We had

$$\dot{n} + 3Hn = \int \frac{d^3p}{2E}\frac{d^3k}{2E_k}\dots\frac{d^3q}{2E_q}\dots\delta^4(p+k\dots-q\dots)\left(|M(p+k\to q)|^2 f_p f_k\dots - |M(q\to p+k)|^2 f_q\dots\right)$$

In this case we have to consider that the matrix elements are slightly different:

$$\Gamma(N\to L) \sim \frac{1}{2}\Gamma_N(1+\epsilon) \qquad \Gamma(N\to \bar{L}) \sim \frac{1}{2}\Gamma_N(1-\epsilon)$$

Then we have

$$\dot{n}_N + 3Hn_N = -\int \frac{d^3p}{2E}\frac{d^3k_L}{2E_L}\frac{d^3k_H}{2E_H}\left(f_p - f_p{}^{eq}\right)\frac{1}{2}(1+\epsilon+1-\epsilon)\langle\Gamma_N\rangle + \dots$$

$$\dot{n}_L + 3Hn_L = \int \frac{d^3p}{2E}\frac{d^3k_L}{2E_L}\frac{d^3k_H}{2E_H}\left(f_p - f_p{}^{eq}\right)\frac{1}{2}(1+\epsilon-1+\epsilon)\langle\Gamma_N\rangle + \dots$$

where we have assumed that the leptons and higgses are in equilibrium.

Taking into account also other processes than the decay we then have

$$\frac{dY_N}{dx} = -\left(\langle\Gamma\rangle + \langle\sigma_{\Delta N=1}v\rangle\right)\left(Y_N - Y_N^{eq}\right)$$
$$\frac{dY_{B-L}}{dx} = -\epsilon_1\langle\Gamma\rangle(Y_N - Y_N^{eq}) - \langle W\rangle Y_{B-L}$$

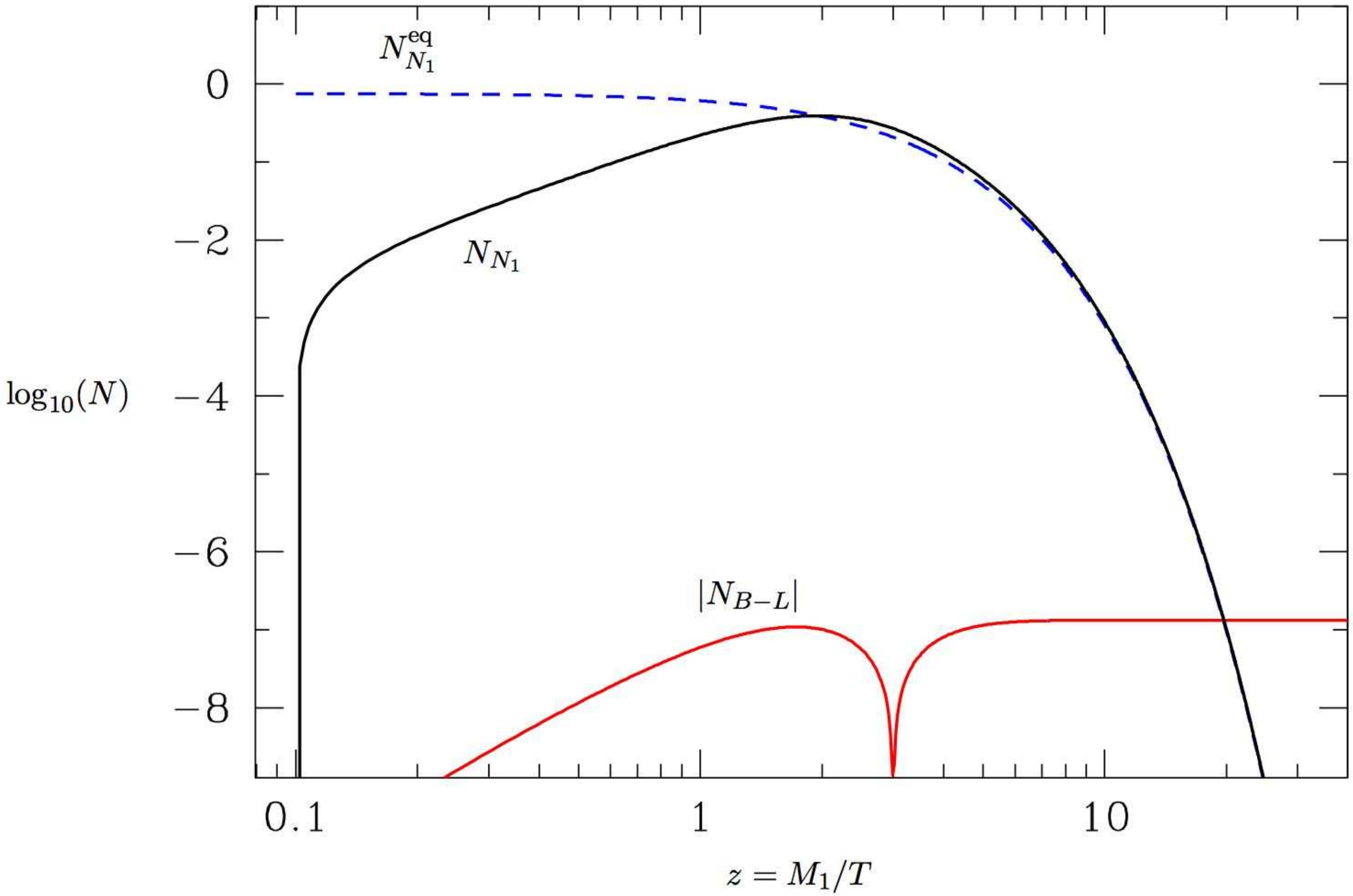
If only $\langle\Gamma\rangle$ is important we are way out of equilibrium, but in general the $\Delta N = 1$ scatterings $\langle\sigma v\rangle$ and the wash-out processe $\langle W\rangle$ are non negligible

$$\langle\Gamma\rangle, \langle\sigma_{\Delta N=1}v\rangle, \langle W(N\text{exch.})\rangle \quad \propto \quad \frac{\tilde{m}_1 M_P}{v^2}$$
$$\langle\Delta W\rangle \quad \propto \quad \frac{\bar{m}M_1 M_P}{v^4}$$

The most important parameter is $\tilde{m}_1$ and the average neutrino mass scale $\bar{m}$.

Solutions of the Boltzmann equation

[Buchmüller, Di Bari & Plümacher ’04]



General solution of the Boltzmann equation

[Buchmüller, Di Bari & Plümacher ’04]

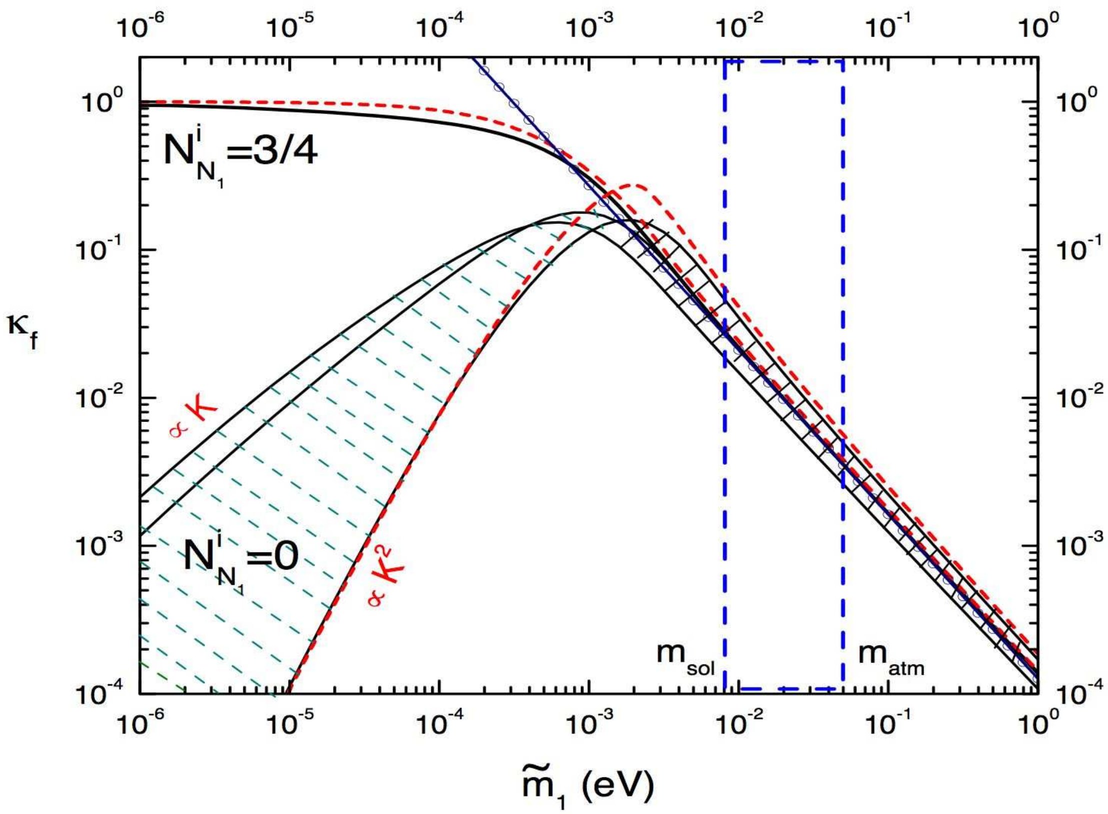
A general solution for the baryon number from leptogenesis can be given in the form

$$\eta_B \simeq 10^{-2} \epsilon_1 \kappa_f$$

where the efficiency factor κ_f comes from the solution of the Boltzmann equation.

$\longrightarrow$ light neutrino mass window

$$10^{-3}\text{eV} \leq m_1 \leq 0.1\text{eV}$$



Final remarks on leptogenesis

- possible also to realize the scenario in SUSY, but then some conflict with the limits on T_{RH} from gravitinos since one needs $T_{RH} \sim 10^9$ GeV ;
- such bounds can be relaxed in ”resonant” leptogenesis, that shows an enhanced ϵ due to nearly degenerate RH Majorana masses;
- possible also to have non-thermal leptogenesis, but then one is more sensitive to the initial conditions;
- the CP violation in leptogenesis is in general uncorrelated with the low energy CP asymmetry in the leptonic sector; only in specific mass models it is possible to relate the two, e.g. the sign or even more;
- leptogenesis can be used to test and constrain specific neutrino mass models ;
- still, baryogenesis could also be due to some other mechanism apart from the one covered in this lecture...

To finish a list of references...

- Books:

  The Early Universe by E.W. Kolb and M.S. Turner by Addison-Wesley 1990

  Physical Foundations of Cosmology by V. Mukhanov by Cambridge University Press 2005

- Reviews:

  Particle Data Group on Astrophysics and Cosmology on the web at `http://pdg.lbl.gov/`

  Particle Dark Matter:Evidence, Candidates and Constraints by

  G. Bertone, D. Hooper and J. Silk, arXiv:hep-ph/0404175

  Primordial Nucleosynthesis: Successes and Challenges by

  G. Steigman, arXiv:astro-ph/0511534

  Baryogenesis and leptogenesis by

  M. Trodden, arXiv:hep-ph/0411301

  Leptogenesis for pedestrians by

  W. Buchmüller, P. Di Bari and M. Plümacher, arXiv:hep-ph/0401240