Astroparticle physics I I

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DESY

Plan of the Lectures

1. Introduction to Standard Cosmology:
   - GR, particle physics and thermodynamics
   - Present data and open questions
   - A short history of the Early Universe
2. Thermal relics and Dark Matter:
   - The Boltzmann equation in the expanding Universe
   - The number density of a Thermal relic $\longrightarrow$ WIMPs
   - Supersymmetry and the neutralino DM
   - Other SUSY candidates: gravitino & Co
3. The baryon asymmetry:
   - Nucleosynthesis and baryon number of the Universe
   - Sakharov's conditions
   - Sphaleronic transitions and EW baryogenesis
   - Leptogenesis and neutrino masses

Elements of thermodynamics

In general, given the phase space distribution $f(\vec{p}, \vec{x}, t)$ and g the number of d. o. f., we have

$$n = g \int d^3p \, f(\vec{p}) \qquad \rho = g \int d^3p \, E(\vec{p}) f(\vec{p}) \qquad p = g \int d^3p \, \frac{\vec{p}^2}{3E(\vec{p})} f(\vec{p})$$

we can compute the number and energy density and the pressure. So indeed for radiation $E = |\vec{p}|$ and $p = \rho/3$, while for non-relativistic matter $E \sim m\left(1 + \frac{1}{2}\frac{\vec{p}^2}{m^2}\right)$ and $\rho \sim mn$, while $p \sim 0$.

Maximal entropy state $\Longrightarrow$ THERMAL EQUILIBRIUM:

– kinetic equilibrium, i.e. momentum distribution given by Fermi-Dirac or Bose-Einstein distribution, sustained by by elastic scatterings,

$$f_{F/B}(\vec{p}) = \frac{1}{e^{\frac{E-\mu}{T}} \pm 1} ,$$

where μ is the chemical potential related to a conserved number;

– chemical equilibrium, i.e. equilibrium between different particle species, sustained by inelastic scatterings:

$$i + j \leftrightarrow k + l \quad \text{gives} \quad \mu_i + \mu_j = \mu_k + \mu_l \, .$$

Some useful formulas

Relativistic limit, i.e. $|\vec{p}| \gg m, T \gg \mu$:

$$n = \frac{\zeta(3)\nu_{F/B}}{\pi^2} g T^3 \qquad \rho = \frac{\pi^2 \xi_{F/B}}{30} g T^4 \qquad p = \frac{\rho}{3}$$

for $\nu_{F/B} = 3/4, 1$ and $\xi_{F/B} = 7/8, 1$ for fermions and bosons.

Non-relativistic limit, i.e. $|\vec{p}| \ll m, T \ll \mu$ ($\longrightarrow$ Maxwell-Boltzmann):

$$n = g \left(\frac{mT}{2\pi} \right)^{3/2} e^{\frac{-(m-\mu)}{T}} \qquad \rho = mn \qquad p = 0 \; .$$

The chemical potential instead is related to a conserved charge, so particle and antiparticle have opposite sign μ and we have in the relativistic and non-relativistic case:

$$n_p - n_{\bar{p}} \rightarrow g \frac{\mu T^2}{6} + \mathcal{O}\left(\frac{\mu^3}{T^3} \right) \qquad n_p - n_{\bar{p}} \rightarrow 2g \left(\frac{mT}{2\pi} \right)^{3/2} sinh \left(\frac{\mu}{T} \right) e^{\frac{-m}{T}} \; .$$

Thermodynamics in expanding Universe

An expanding Universe is a closed system and in thermal equilibrium the entropy is conserved !

$$TdS = d(\rho V) + pdV = d((\rho + p)V) - Vdp = 0 \quad \text{since} \quad d((\rho + p)V) = Vdp$$

due to energy conservation; moreover the entropy density is $s = \frac{S}{V} = \frac{\rho+p}{T}$.

Using the expressions given above we can define the total energy and entropy density in radiation:

$$\rho_{rad} = \frac{\pi^2}{30} g_* T^4 \qquad g_* = \sum_B g_i \left(\frac{T_i}{T}\right)^4 + \sum_F \frac{7}{8} g_j \left(\frac{T_j}{T}\right)^4$$

$$s = \frac{2\pi^2}{45} g_S T^3 \qquad g_S = \sum_B g_i \left(\frac{T_i}{T}\right)^3 + \sum_F \frac{7}{8} g_j \left(\frac{T_j}{T}\right)^3$$

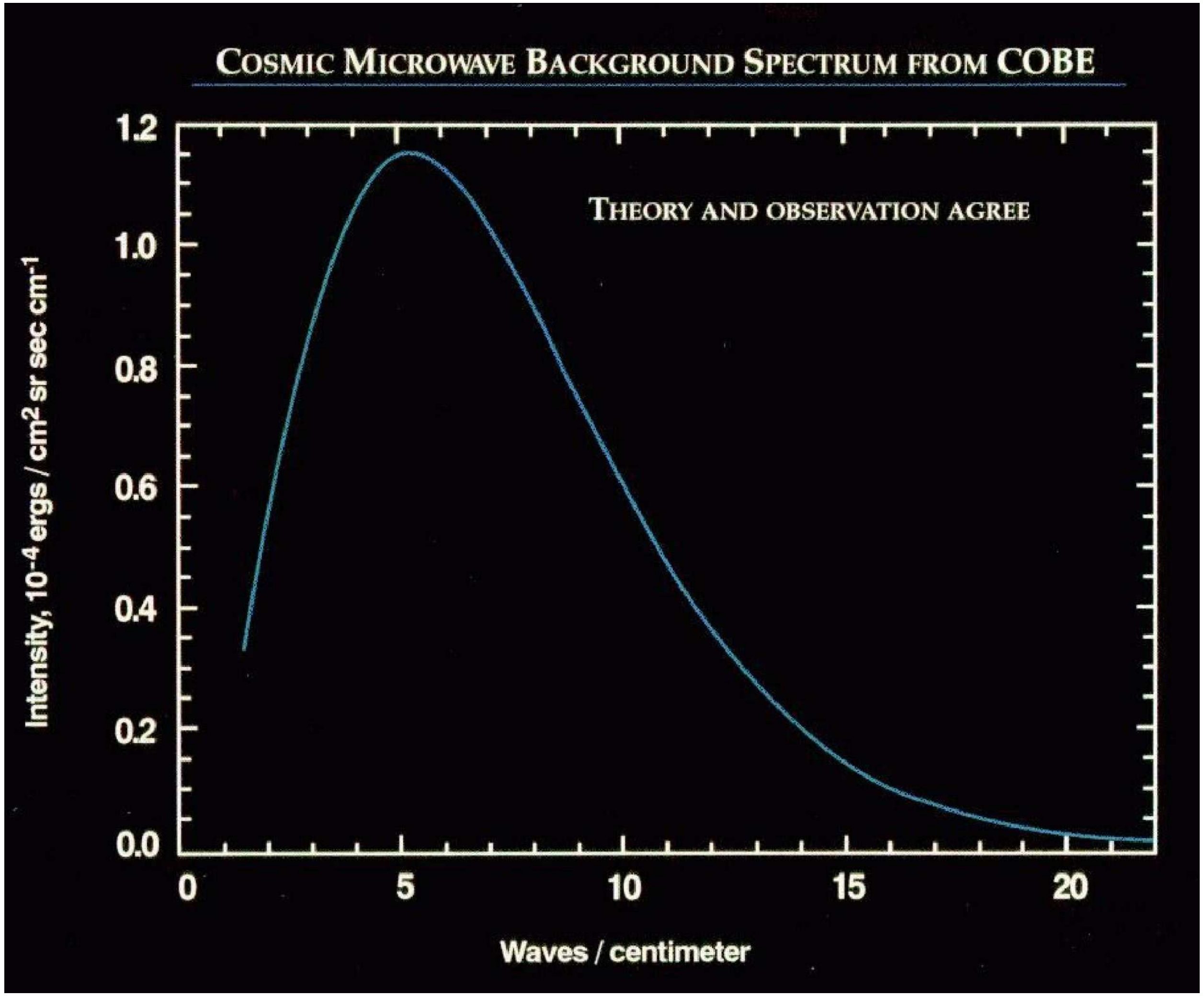
Constant entropy gives then sa^3 constant, i.e. $$\frac{ds}{s} = 3\frac{dT}{T} + \frac{dg_S}{g_S} = -3\frac{da}{a}$$

So for $dg_S = 0$ we have $T \propto a^{-1}$ adiabatic cooling !

Note: a thermal spectrum for a relativistic (massless particle) is not distorted by the expansion, just red-shifted to lower T

$\Rightarrow$ CMB photons !

Cosmic Microwave Radiation: Perfect BLACK BODY at $T = 2.7\,\mathrm{deg}$



picture from http://map.gsfc.nasa.gov

Boltzmann equation in an expanding Universe

How can we establish if we have thermal equilibrium and until when ? The time evolution of $f(\vec{p}, \vec{x}, t)$ is given classically by the Boltzmann equation

$$\hat{\mathcal{L}}[f] = \mathcal{C}[f]$$

Liouville operator   Collision integral

In GR $\hat{\mathcal{L}}$ is given by $\hat{\mathcal{L}}[f] = p^\alpha \frac{\partial f}{\partial x^\alpha} - \Gamma^\alpha_{\beta\gamma} p^\beta p^\gamma \frac{\partial f}{\partial p^\alpha} = E\dot{f} - H\vec{p}^2 \frac{\partial f}{\partial E}$

for a homogeneous and isotropic distribution function $f(E, t)$.
Integrating over the momentum p, we obtain the equation for the number density

$$\dot{n} + 3Hn = \int \frac{d^3p}{2E} \frac{d^3k}{2E_k} .... \frac{d^3q}{2E_q} ...\delta^4(p + k... - q...) \left(|M(p + k \rightarrow q)|^2 f_p f_k ... - |M(q \rightarrow p + k)|^2 f_q ...\right)$$

Assume that there is no CP or T violation: $|M(p+k\to q)|^2 = |M(q\to p+k)|^2$

and that the particle q's are in equilibrium, i.e. $f_q \cdots = f_p^{eq} f_k^{eq} \ldots$

Then we have

$$\begin{aligned}
\dot{n} + 3Hn &= \int \frac{d^3p}{2E}\frac{d^3k}{2E_k}\ldots\left(f_p f_k \ldots - f_p{}^{eq} f_k^{eq}\ldots\right)\frac{d^3q}{2E_q}\ldots\delta^4(p+k\ldots-q\ldots)|M(p+k\to q)|^2 \\
&= \int \frac{d^3p}{2E}\frac{d^3k}{2E_k}..\left(f_p f_k \ldots - f_p{}^{eq} f_k^{eq}\ldots\right)\sigma(p+k\to q)v \\
&= \left(n n_k \ldots - n^{eq} n_k^{eq}\ldots\right)\langle\sigma(p+k\to q)v\rangle
\end{aligned}$$

thermally averaged cross-section

where we have assumed $f = n\frac{f^{eq}}{n^{eq}}$ for all particles involved.

Then a species is in equilibrium if its interaction rates $\langle\sigma v\rangle$ are efficient enough !

Primordial abundance of stable massive species

[see e.g. Kolb & Turner '90]

The number density of a stable particle X in an expanding Universe is given by the Bolzmann equation

$$\frac{dn_X}{dt} + 3Hn_X = \langle \sigma(X + X \to \text{anything}) v \rangle \left(n_{eq}^2 - n_X^2\right)$$

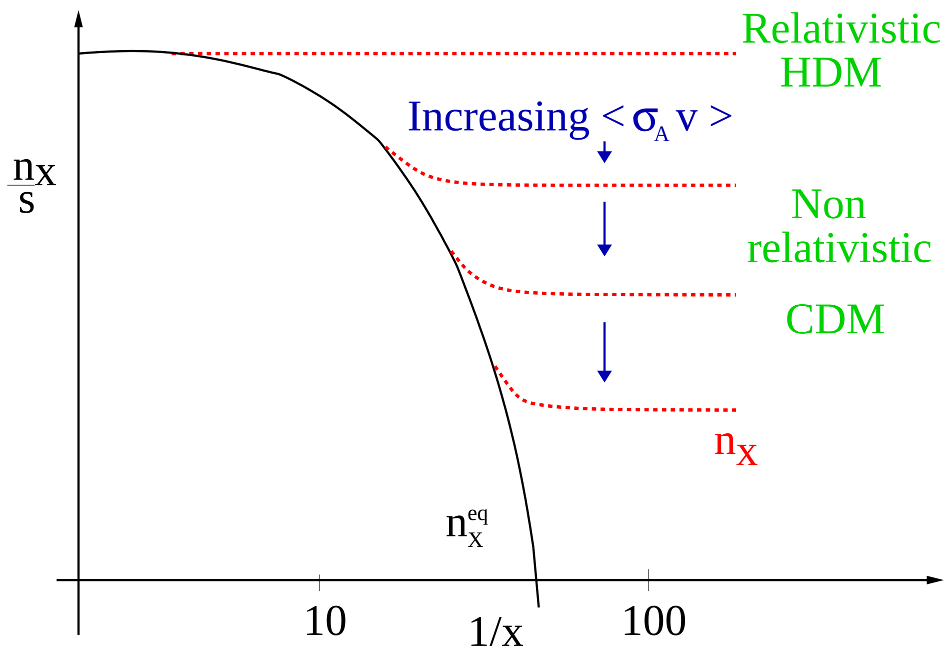
Hubble expansion   Collision integral

The particles stay in thermal equilibrium until the interactions are fast enough, then they freeze-out at $x_f = m_X/T_f$

defined by $n_{eq} \langle \sigma_A v \rangle_{x_f} = H(x_f)$ and that gives

$$\Omega_X = m_X n_X(t_{now}) \propto \frac{1}{\langle \sigma_A v \rangle_{x_f}}$$

Abundance $\Leftrightarrow$ Particle properties



For $m_X \simeq 100$ GeV a WEAK cross-section is needed !

Weakly Interacting Massive Particle

For weaker interactions need lighter masses HOT DM !

Approximate solution of the Boltzmann equation

Rewrite the equation in terms of $Y = \frac{n}{s}$ and $\frac{d}{dt} = Hx\frac{d}{dx}$ for $x = \frac{m_X}{T}$:

$$\frac{dY_X}{dx} = -\frac{s\langle\sigma(X + X \to \text{anything})v\rangle}{xH}\left(Y_X^2 - Y_{eq}^2\right)$$

Until x_f we have $Y_X = Y_{eq}$, after that we can neglect Y_{eq} that decreases exponentially and then

$$\frac{dY_X}{Y_X^2} = -\frac{s(x)\langle\sigma(X + X \to \text{anything})v\rangle(x)}{xH(x)}dx$$

which has the solution

$$Y_X(x) = \frac{Y_X(x_f)}{1 + Y_X(x_f)\frac{s(m_X)}{H(m_X)}\int_{x_f}^{x}\frac{dx}{x^2}\langle\sigma(X + X \to \text{anything})v\rangle(x)}$$

so when σ is sufficiently large after freeze-out

$$Y_X(x) \simeq \frac{1}{\frac{s(m_X)}{H(m_X)}\int_{x_f}^{x}\frac{dx}{x^2}\langle\sigma(X + X \to \text{anything})v\rangle(x)}$$

very weakly dependent on x_f; otherwise $Y_X(x) = Y_X(x_f)$.

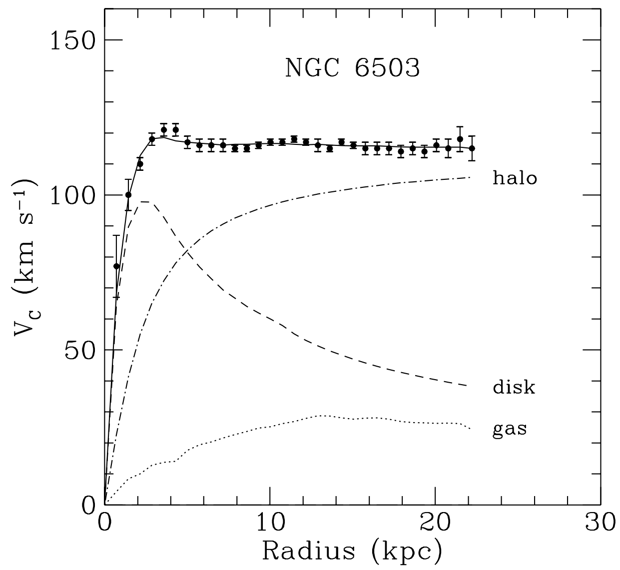
THE MATTER CONTENT

The clumpy energy density/matter divides into

Particles	$\Omega_i(t_{\text{now}})h^2$ (WMAP)	Type
Baryons	0.0224	Cold
Massive ν	$6.5 \times 10^{-4} - 0.01$	Hot
???	$\sim 0.1 - 0.13$	COLD

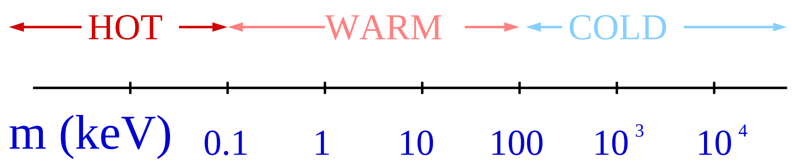
DARK matter !

[Begeman, Broeils & Sanders '91]



Note that DM was first discovered in local systems from the galaxies rotational curves...

Structure formation requires COLD Dark Matter, otherwise the structure formation on scales smaller than its free-streaming length at t_{eq} is suppressed.



Do we really NEED Dark Matter ???

Astronomers actually have found methods to fit the rotational curves of galaxies without DM, by modifying gravity $\Rightarrow MOND(MOdifiedNewtonianDynamics)$!
Very simple scheme: set a minimal acceleration a_0. BUT it solves only part of the troubles...

A fully relativistic formulation of MOND as a scalar-vector-tensor field theory has been proposed by Bekenstein, but still it is not clear if it will allow for a successful cosmological model, in particular for STRUCTURE FORMATION.
In fact DM also generates the potential wells for baryons to fall into even during the time they still were in thermal equilibrium with radiation... A universe with only baryonic matter starts structure formation too late and would have less power on the small scales !

Astrophysical properties of Dark Matter

- electrically neutral: **DARK !**

  Can it be colored ??? Not really, it would get trapped into hadrons and give rise to exotic nuclei; bounds for such states are very strong up to masses of order 10 TeV. In general it must have decoupled early from the primordial plasma, i.e. photons and baryons, to avoid erasing power on small scales...

- stable on cosmological times, i.e. $\tau \gg 13.7$ Gyrs or in more familiar units $\Gamma \ll 1.5 \times 10^{-42}$ GeV. Note that assuming a decay of the type m^5/M_{Pl}^4, this gives just the bound $m \ll 0.8 \times 10^7$ GeV.

- it must be massive and NON-relativistic for structure formation $\longrightarrow$ Cold or at most Warm DM !

Otherwise (nearly) any mass-scale would do as long as $0.095 \leq \Omega_{CDM} h^2 \leq 0.13$

What are the possible candidates ??? The Standard Model (SM) does not offer any suitable particle, the SM neutrinos are at most Hot DM, so we are obliged to look beyond...

A natural candidate emerges as a thermal relic with weak couplings and mass around the electroweak scale in supersymmetric extensions of the SM. $\longrightarrow$ WIMP scenario !

Supersymmetry and the Constrained MSSM

Supersymmetry: boson $\Leftrightarrow$ fermion

- cancels quadratic divergencies and protects scalar masses
- predicts for every SM particle a superpartner with different spin
- gives unification of gauge couplings

SM Particles	Superpartners
q_L, u_R, d_R, l_L, e_R	$\tilde{q}_L, \tilde{u}_R, \tilde{d}_R, \tilde{l}_L, \tilde{e}_R$
$g, \gamma, Z, W^{\pm}$	$\tilde{g}, \tilde{\gamma}, \tilde{Z}, \tilde{W}^{\pm}$
$H_u, H_d \rightarrow h, H, A, H^{\pm}$	$\tilde{H}_u, \tilde{H}_d$

SUPERSYMMETRY is not observed in nature, it is broken softly $\longrightarrow$ massive superpartners !

$\longrightarrow$ 105 parameters !

Assuming universality at the GUT scale, then we have only 5 additional parameters:

SUSY parameters: $\tan\beta = \dfrac{\langle H_u \rangle}{\langle H_d \rangle}, \quad \mu$

scalar/gaugino mass, scalar trilinear: $m_0, m_{1/2}, A_0$

EW symmetry is broken radiatively: $|\mu|$ is fixed ! $\Rightarrow$ Constrained Minimal Supersymmetric SM

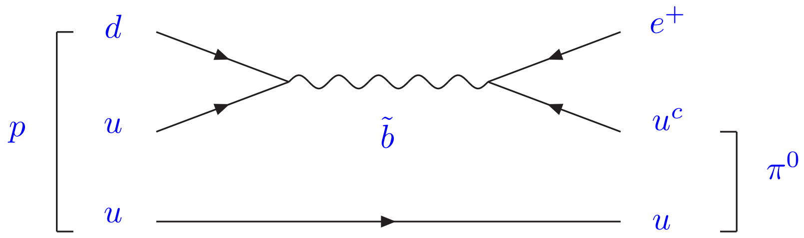
R-parity conservation

The SM accidentally preserves baryon number and therefore the proton is stable. (Actually $U(1)_{B+L}$ is anomalous and L number is violated if the neutrinos are Majorana fermions, but such effects are important only at high temperature) $\longrightarrow$ BARYOGENESIS ?

In a supersymmetric model we can have additional renormalizable couplings that do violate explicitely baryon and lepton number.

$$W = \lambda L L E^c + \lambda' L Q D^c + \lambda'' U^c D^c D^c + \mu_i L_i H_2$$

$\Rightarrow$ Dimension 4 proton decay operators



$$\propto \frac{1}{m_{\tilde{b}}^2}$$

To avoid fast proton decay, impose a discrete symmetry called R-parity, which forbids these terms:

$$q_L, u_R, d_R, l_L, e_R, g, \gamma, Z, W^\pm, H_u, H_d \quad \longrightarrow \quad +q_L, u_R, d_R, l_L, e_R, g, \gamma, Z, W^\pm, H_u, H_d$$

$$\tilde{q}_L, \tilde{u}_R, \tilde{d}_R, \tilde{l}_L, \tilde{e}_R, \tilde{g}, \tilde{\gamma}, \tilde{Z}, \tilde{W}^\pm, \tilde{H}_u, \tilde{H}_d \quad \longrightarrow \quad -\tilde{q}_L, \tilde{u}_R, \tilde{d}_R, \tilde{l}_L, \tilde{e}_R, \tilde{g}, \tilde{\gamma}, \tilde{Z}, \tilde{W}^\pm, \tilde{H}_u, \tilde{H}_d$$

So couplings only involve pairs of superpartners and therefore the supersymmetric particles can only be produced or destroyed in pairs ! $\Rightarrow$ The Lightest Susy Particle is stable !

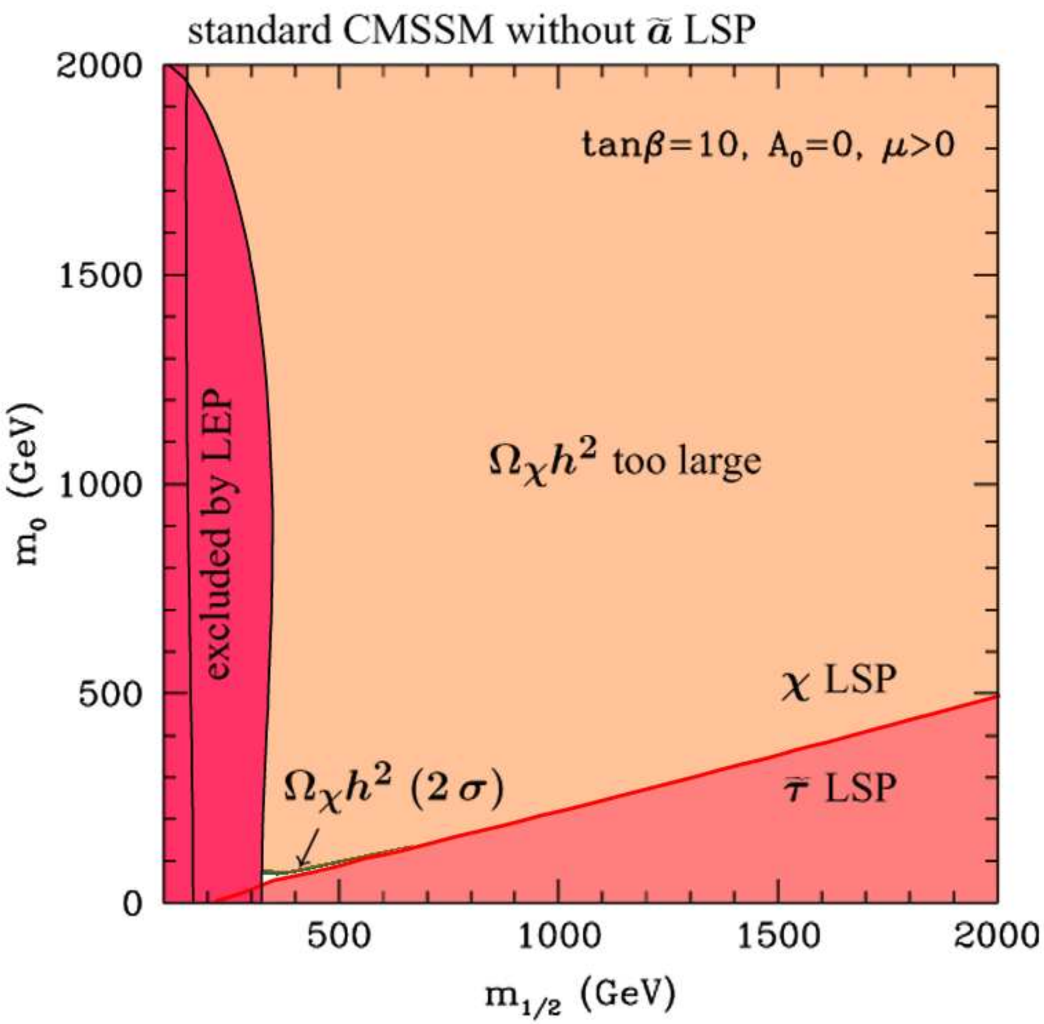
Which are suitable WIMPs ?

- neutralinos: in the CMSSM the LSP is typically a neutralino (WIMP !), but its abundance depends strongly on the sparticle masses

$\longrightarrow$ it is too large apart when enhanced by some mechaanisms, i.e. coannihilation, Higgs resonance... The natural "bulk region" already excluded by LEP.

$\rightarrow$ fine–tuning ???

- sneutrinos: unfortunately excluded already by the large cross-section via Z boson that would have been already observed in DM experiments...



The neutralino composition, mass and its couplings change strongly depending on the SUSY breaking parameters and allow to span about 5 orders of magnitude in Ωh^2...

In particular in the CMSSM the neutralino cross-section turns out to be too weak (or its mass is too heavy) in most of the parameter space, apart in four scenarios:

- efficient annihilation into SM fermions (bulk region): needs light sfermions and it is nearly completely excluded by LEP bounds...

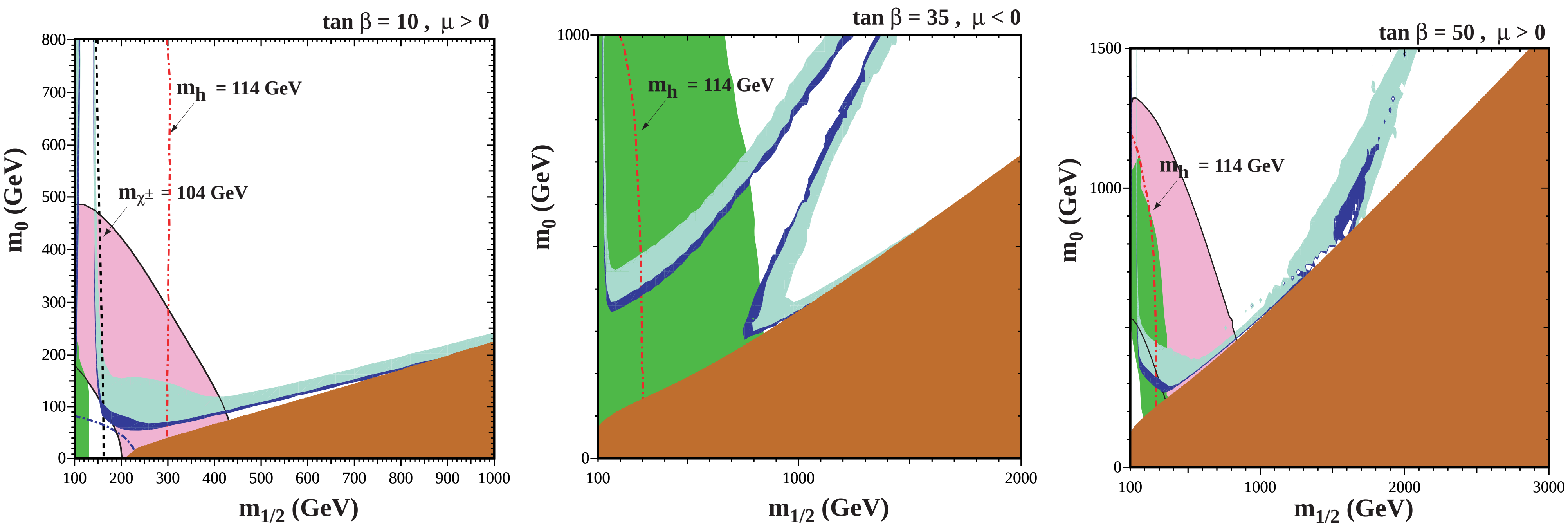
- efficient coannihilation with another superparticle, in the CMSSM the lightest stau , but in general also stops, sneutrinos, etc.. Note that in this case the number density is strongly dependent on the mass difference between the two particles (exponentially...);

-enhanced annihilation near a Higgs resonance, again the mass of the neutralino has to be very near to e.g. half the A-mass and Ωh^2 strongly depends on the mass difference and Γ_A;

- large Higgsino component (focus point region): then the channel of annihilation into WW is open and reduces the number density sufficiently;

The different regions would give completely different spectra at LHC !

Allowed regions for different $\tan\beta$ after WMAP [Ellis, Olive, Santoso & Spanos '03]

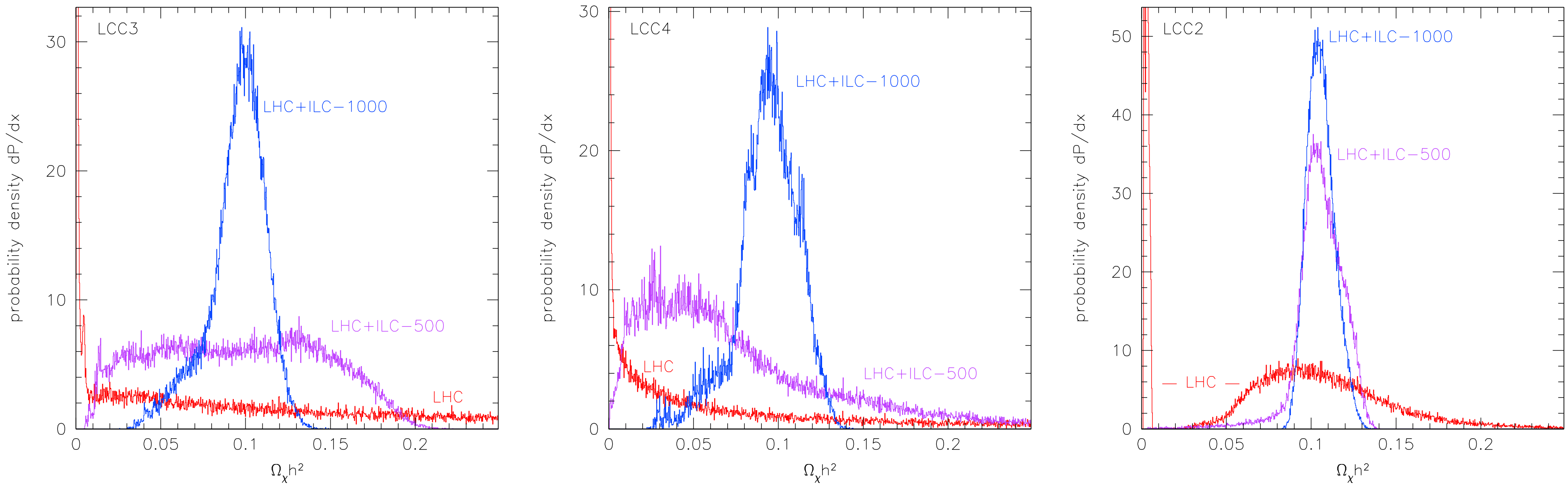


Different constraints: $b \to s\gamma$, $g_\mu - 2$, $\Omega h^2 = 0.094 - 0.129$.

No focus point region for large m_{top} according to Ellis et al. Note the different scales.

HOW WELL CA WE RECONSTRUCT Ωh^2 from LHC data ?

[Baltz, Battaglia, Peskin & Wizansky '06]



coannihilation region | resonant region | Higgsino-like -region

Not very well unfortunately due to the strong sensitivity to the parameters... In the bulk region (LCC1) things look more promising.

Can we have DM more weakly interacting than WIMPs ?

We have seen that very weakly interacting particles freeze-out with a large number density, therefore they must be light to give the same energy density $\rho = mn$... $\longrightarrow$ HOT/WARM DM !

But another possibility is that the temperature of the Universe was always too low for such particles to reach equilibrium $T_{RH} < T_D$. Then their present density is given (at least) by two mechanisms:

– thermal scattering and decays in the plasma

$$\frac{d}{dT}\frac{n_X}{s} = \frac{-1}{HTs(T)} \left[\sum_{ij}\langle\sigma(i+j \to X+...)v_{rel}\rangle n_i n_j + \sum_i \langle\Gamma(i \to X+...)\rangle n_i\right]$$

scatterings decays

strongly dependent on T_{RH} ! Note: here we can neglect backreaction !

– decay *out of equilibrium* of the NLSP:

$$\Omega_X^{NT} = \frac{m_X}{m_{\rm NLSP}}\,\Omega_{\rm NLSP}$$

BEWARE of the decay products (γs or hadrons) not spoiling Nucleosynthesis or distort the CMB !

THERMAL PRODUCTION: At high temperatures, the dominant contribution to the production come from 2-body scatterings with colored states, mediated by non-renormalizable operators:

- gravitino case: $$\Omega_{\tilde{G}}^{TH} h^2 \simeq 0.2 \left(\frac{100\text{GeV}}{m_{\tilde{G}}} \right) \left(\frac{m_{\tilde{g}}}{1\text{TeV}} \right)^2 \left(\frac{T_R}{10^{10}\text{GeV}} \right)$$

[Bolz, Brandenburg & Buchmüller '01]

- axino case: $$\Omega_{\tilde{a}}^{TH} h^2 \simeq 0.6 \left(\frac{m_{\tilde{a}}}{0.1\text{GeV}} \right) \left(\frac{10^{11}\text{GeV}}{f_a} \right)^2 \left(\frac{T_R}{10^4\text{GeV}} \right)$$

[LC, HB KIm, JE Kim & Roszkowski '01, Brandenburg & Steffen '04]

NOTE the completely different dependence on the LSP mass !!! It is due to the fact that the gravitino is produced via its Goldstino component, whose couplings are enhanced by the ratio $\frac{m_{\tilde{g}}}{m_{\tilde{G}}}$!

Technical point: Hard Thermal loop resummation needed to regularize the gluon IR divergences.

At temperatures of the order of the superpartner masses also the effect of the sparticle decays become important, so there is a stronger dependence on the parameters of supersymmetry breaking.

OUT OF EQUILIBRIUM DECAY

An LSP population is also generated by NLSP decay after freeze-out: e.g. for neutralino we have usually $\chi \longrightarrow X\gamma$ or for staus $\tilde{\tau} \longrightarrow X\tau$.

The important parameter is the lifetime:

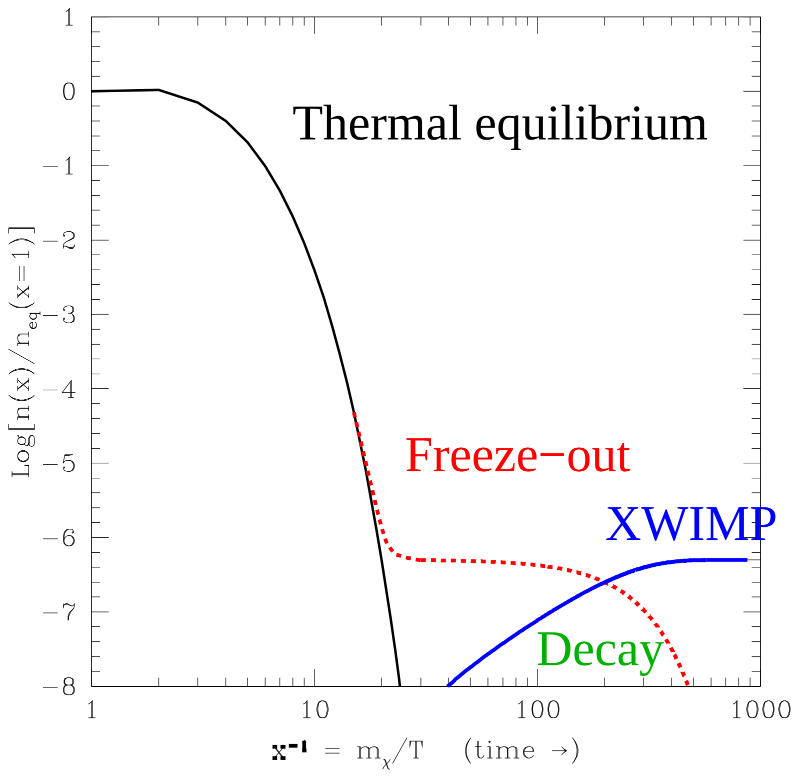
$\tau \gg 1/H(x_f)$

$\Rightarrow$ the NLSP freeze-out is not modified:

$$\Omega_X^{NT} = \frac{m_X}{m_{NLSP}} \Omega_{NLSP}$$

Still a connection to weak physics via Ω_{NLSP}!

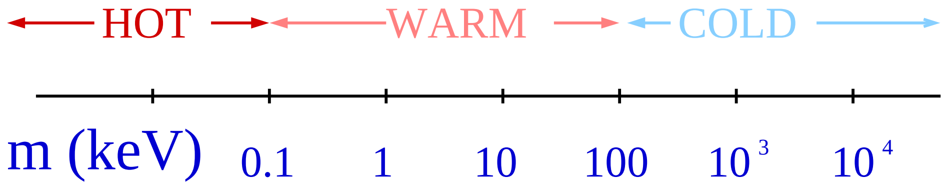
$\tau > 1$ sec $\Rightarrow$ strong BBN constraints !



Constraints on the decay scenario: the trouble of long-lived particles...

- Big Bang Nucleosynthesis: strong limits on the injection of energetic particles for $\tau > 1$ sec. At early times the stronger bounds are given by hadronic showers, later also electromagnetic showers become important. In general the bounds are much weaker
- Distortion of the CMB at late times, only important for lifetimes above 10^4 sec.
- Are these particles cold enough to be CDM ? They are produced as relativistic and with a

  non-thermal spectrum: $$p(T) \simeq \frac{m_{NLSP}}{2} \left(\frac{g_*(T)}{g_*(T_{dec})} \right)^{1/3} \frac{T}{T_{dec}}$$

  For a thermal relic one has 

  but LSPs generated by NLSP decay can be still warm at larger masses...

Not all possible NLSP and LSP's are allowed, but still the scenario can work, especially for charged NLSP ! LHC could discover a charged "stable" stau...

Dark Matter nature still unknown....

Many particle physics candidates are viable

and probably will be identified in the next decade

at colliders and/or DM experiments !