38. Herbstschule für Hochenergiephysik – Maria Laach, 12^{th} September 2006

Particle Physics and Cosmology II

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DESY

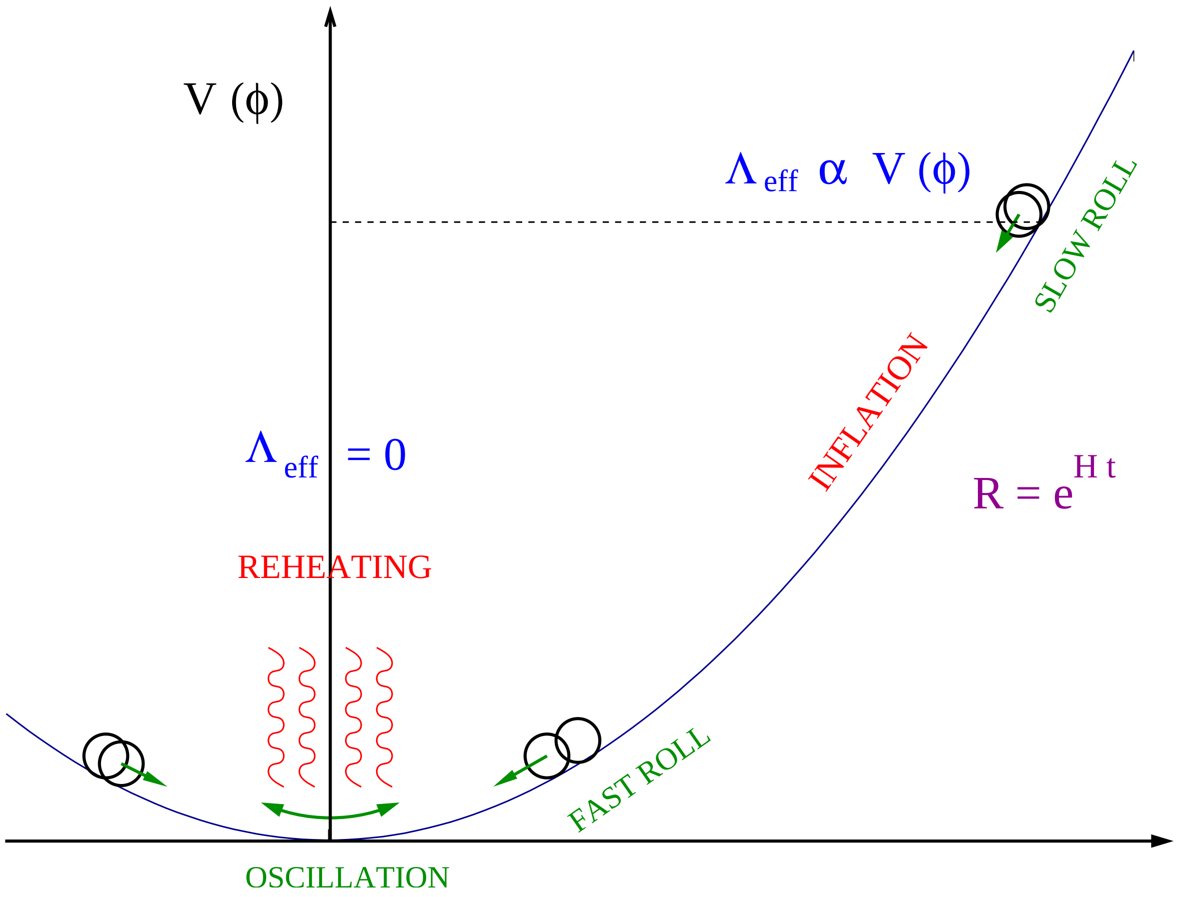
Plan of the Lectures

1. Introduction to Standard Cosmology:
   - General Relativity and Particle Physics
   - Present data and cosmological parameters
   - A short history of the Early Universe
   - Big Bang Nucleosynthesis
2. Inflation and Structure Formation:
   - Inflation and background dynamics
   - Scalar and tensor perturbations from inflation
   - Elements of perturbation theory
   - CMB and Large Scale Structure data
3. Thermal Universe and Relics
   - The Boltzmann equation in the expanding Universe
   - The number density of a thermal relic $\longrightarrow$ WIMPs
   - Supersymmetry and Dark Matter
   - Baryogenesis: EW or via leptogenesis ?

INFLATION: period of quasi-exponential expansion, explaining the FLATNESS, ISOTROPY, HOMOGENEITY of the Universe, the absence of UNWANTED RELICS and producing the initial SMALL PERTURBATIONS.

How to sustain inflation ???

$\Rightarrow$ USE THE POTENTIAL ENERGY OF A SCALAR FIELD ϕ AS AN EFFECTIVE COSMOLOGICAL CONSTANT



$\Rightarrow$ The scalar field has to slow roll in an ALMOST FLAT POTENTIAL such that

$$\ddot{\phi} \ll 3H\dot{\phi} \;\Rightarrow\; 3H\dot{\phi} = -V'$$

$\Rightarrow$ slow roll expansion

Let us discuss some of the problems...

- Flatness problem: $$\frac{d}{dt}(\Omega_{tot} - 1) = -2\frac{\ddot{a}}{aH}(\Omega_{tot} - 1)$$

  $\Omega = 1$ is a past attractor for $\ddot{a} < 0$, need instead $\ddot{a} > 0$ to converge to 1 in the future... i.e. $\rho + 3p < 0$ or $w < 1/3$!

- Horizon problem: $$d_P(t) = a(t)\int_{t_i}^{t} \frac{dt}{a(t)} \simeq \frac{3(1+w)}{1+3w} t \quad \text{for} \quad a(t) \simeq t^{-3(1+w)/2}$$

  The past light cone is finite and grows with time: every instant regions that never were in causal contact become causally connected ! On the other hand if $a(t) = e^{Ht}$, we have

$$d_P(t) = e^{Ht}\int_{t_i}^{t} e^{-Ht} dt = \frac{1}{H}\left(e^{H(t-t_i)} - 1\right)$$

$\Rightarrow$ infinite or large enough for $t_i \rightarrow -\infty$

Einstein's equation for a homogeneous scalar field

$$H^2 = \frac{1}{3M_P^2}\left(\frac{1}{2}\dot\phi^2 + V(\phi)\right) \qquad \dot H = \frac{\ddot a}{a} - H^2 = -\frac{\dot\phi^2}{2M_P^2}$$

$$\ddot\phi + 3H\dot\phi + V'(\phi) = 0$$

In general one has to solve this system of coupled equations, but things greatly simplify if we assume SLOW ROLL...: i.e. $\dot\phi^2 \ll V(\phi)$ and $\ddot\phi \ll 3H\phi$.

Then the simple solution is

$$H^2 = \frac{V(\phi)}{3M_P^2} \simeq \text{constant} \qquad \dot H \simeq 0 \rightarrow a \sim e^{Ht} \quad \text{INFLATION !}$$

While for the scalar field we have

$$\dot\phi = -\frac{V'(\phi)}{3H} \ .$$

Note that this solution is a *late-time attractor* as long as the potential is flat !

(Single field) inflationary model $\Longleftrightarrow$ Flat Potential $V(\phi)$

- SLOW ROLL CONDITIONS:

$$\begin{cases} \epsilon &= 2M_P^2 \left(\frac{V'}{V}\right)^2 \ll 1 \\ |\eta| &= M_P^2 \frac{|V''|}{V} \ll 1 \end{cases}$$

- Enough expansion:

$$\mathcal{N} = \int_{t_i}^{t_f} dtH = \int_{\phi_f}^{\phi_i} d\phi \frac{V(\phi)}{M_P^2 V'(\phi)} > 50$$

- Sufficient reheating before Big Bang Nucleosynthesis: $T_{rh} > 1$ MeV

Huge number of models in the literature:

old inflation, new inflation, chaotic inflation, hybrid inflation, smooth inflation, topological inflation, ...

$$V(\phi) = \frac{1}{2}m^2\phi^2: \qquad \begin{aligned} H^2 &= \frac{m^2\phi^2}{6M_P^2}\left(1 + \frac{\dot\phi^2}{m^2\phi^2}\right) \qquad \dot H = -\frac{\dot\phi^2}{2M_P^2} \\ \ddot\phi &+ 3H\dot\phi + m^2\phi = 0 \end{aligned}$$

The SLOW ROLL conditions require

$$\epsilon = 2M_P^2\left(\frac{2m^2\phi}{m^2\phi^2}\right)^2 = \frac{2M_P^2}{\phi^2} \ll 1 \qquad \eta = M_P^2\frac{2m^2}{m^2\phi^2} = \frac{2M_P^2}{\phi^2} \ll 1$$

$\Rightarrow$ large field values $\phi \gg M_P$, while the mass m is undetermined !

Then the solution is simply: $\dot\phi = -\frac{V'(\phi)}{3H} = -\sqrt{6M_P}\frac{m^2\phi}{3m\,\phi} = \sqrt{\frac{2M_P}{3}}m$

and the e-folding number is given by

$$\mathcal{N} = \int_{t_i}^{t_f} H dt = \int_{\phi_f}^{\phi_i} \frac{V(\phi)\,d\phi}{V'(\phi)M_P^2} = \int_{\phi_f}^{\phi_i} \frac{\phi\,d\phi}{2M_P^2} = \frac{(\phi_i^2 - \phi_f^2)}{4M_P^2}$$

Need $\phi_i \geq 15 - 17M_P$ for sufficient expansion..

The end of inflation and reheating the Universe

In single fields models, inflation ends when one of the slow roll conditions is violated, i.e.$\epsilon, \eta \sim 1$. Then, the exponential expansion slows down while the field "fast rolls" and reaches the minimum of the potential. There it still has kinetic energy and starts to oscillate with $w = 0$ as matter. In hybrid models, the end of inflation is set by a phase transition at a critical value of the inflaton field ϕ_c.

In general, the energy has to be transferred from the inflaton to other fields and particle must be produced and thermalise. Different mechanism can give rise to particle production and the thermalisation is a highly non linear and non perturbative process.

- perturbative decay: The inflaton condensate can be described as a collection of particles that decay at the time when $H \sim \Gamma$;
- preheating: a parametric resonance in the field equations due to the oscillatory behaviour of the inflaton and its relatively strong coupling enhances the production of specific modes k;
- tachyonic preheating: during a phase transition the modes with a negative mass, i.e. the low k modes, grow faster than exponentially;

In general not so straightforward to compure the reheat temperature !

But how did structure arise from an homogeneous (classical) state ?

Primordial power spectrum of the curvature perturbations $\frac{\delta\rho}{\rho}$

Quantum fluctuation during inflation $\delta\phi = \frac{H}{2\pi}$ $\Longrightarrow$ $\mathcal{P}_{\mathcal{R}}(k) \sim \frac{H^2}{2\pi\dot{\phi}} \propto k^{n-1}$ gaussian and adiabatic

with $|n-1| \propto \frac{V''}{V}, \left(\frac{V'}{V}\right)^2 < 1$ due to slow roll !

These small fluctuations in the density act as seeds for the gravitational collapse.

The evolution of the perturbations depends on the cosmological parameters $h, \Omega_{tot}, \Omega_M, \Omega_B$, the equation of state of the dominant component of the energy density and the nature of Dark Matter (Cold, Warm or Hot).

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Radiation dominance:

linear regime, oscillations

CMB anisotropies:

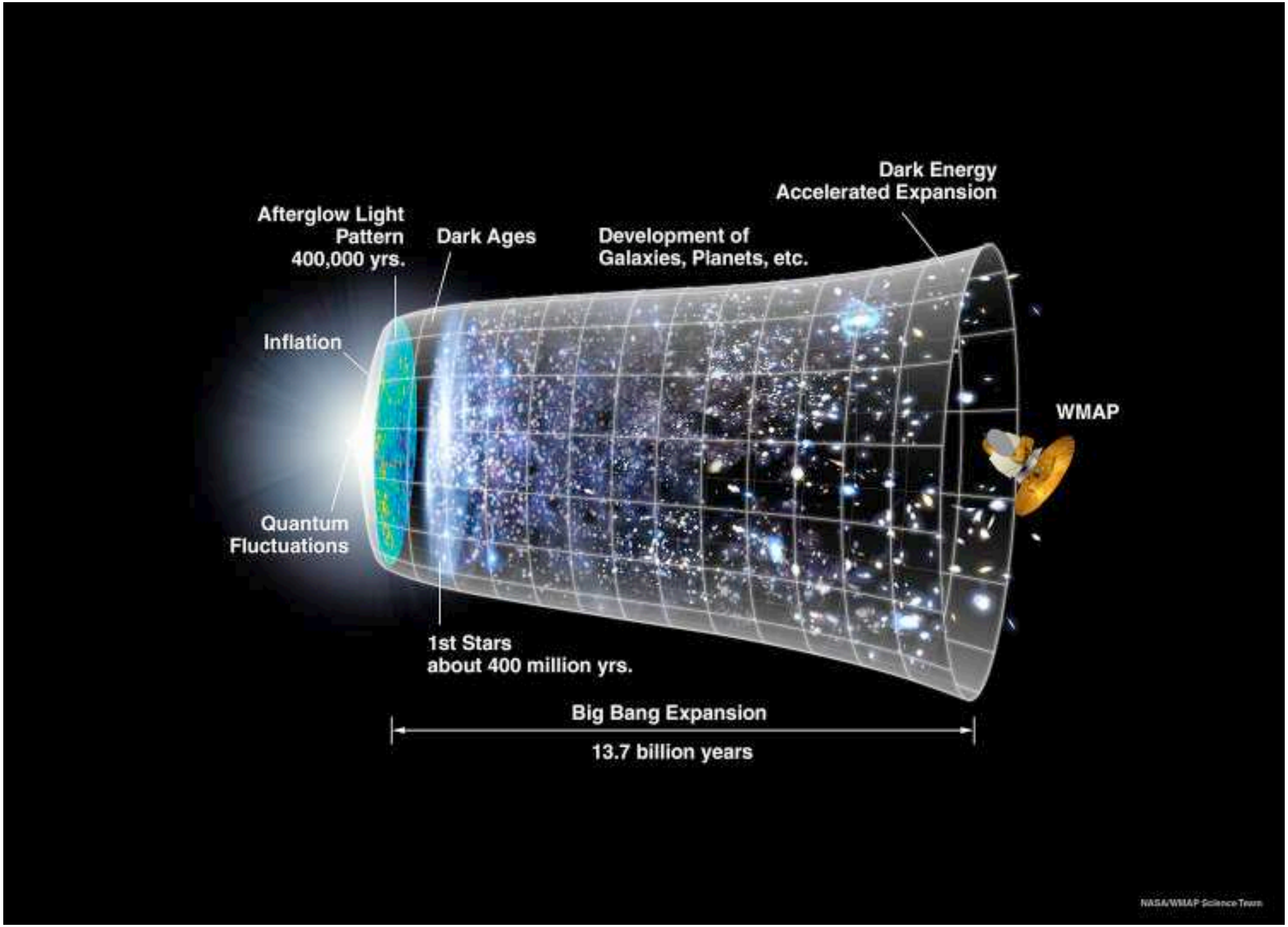
acoustic peaks

Matter dominance:

non linear regime, gravitational collapse

Large Scale Structure

peculiar velocities, etc...


Dark Energy
Accelerated Expansion
Afterglow Light
Pattern
400,000 yrs.
Dark Ages
Development of
Galaxies, Planets, etc.
Inflation
WMAP
Quantum
Fluctuations
1st Stars
about 400 million yrs.
Big Bang Expansion
13.7 billion years
NASA/WMAP Science Team

Quantum scalar field in de Sitter background

Let us consider a quantum scalar field ϕ in a flat de Sitter background $a(t) = e^{Ht}$: we can write the field as $\varphi = \phi + \delta\phi$, where ϕ is the classical part ($\langle\varphi\rangle$) while $\delta\phi$ describes the quantum fluctuations.

The equations of motion read

$$\ddot{\phi} - \Delta\phi + 3H\dot{\phi} + V'(\phi) + \ldots = 0$$
$$\delta\ddot{\phi} - \Delta\delta\phi + 3H\delta\phi + V''(\phi)\delta\phi + \ldots = 0$$

Expressing the field $\delta\phi$ in Fourier components one has

$$\delta\ddot{\phi}_k + \frac{k^2}{a^2}\delta\phi_k + 3H\delta\dot{\phi}_k + V''(\phi)\delta\phi_k = 0$$

Use as a variable $u_k = \delta\phi_k / a$ and change time to conformal time η:

$$u_k'' + \left(k^2 + V''(\phi)a^2 - \frac{a''}{a}\right) u_k = 0$$

Harmonic oscillator with a negative time-dependent mass !!!

Quantum scalar field in de Sitter background II

Two different regimes (neglecting $V''(\phi)$ for the moment):

- $k \gg \frac{a''}{a}$: the modes have "positive frequency" and oscillate as usual but in conformal time as

$$u_k \propto c_+ e^{ik\eta} + c_- e^{-ik\eta}$$

- $k \ll \frac{a''}{a}$: the modes have "negative frequency" and one solution grows; actually that is simply $u_k \propto a$ and the general solution is:

$$u_k = c_{++} a(\eta) + c_{--} \frac{1}{a(\eta)} \quad \left(\text{remember: } a(\eta) = \frac{-1}{H\eta} \right)$$

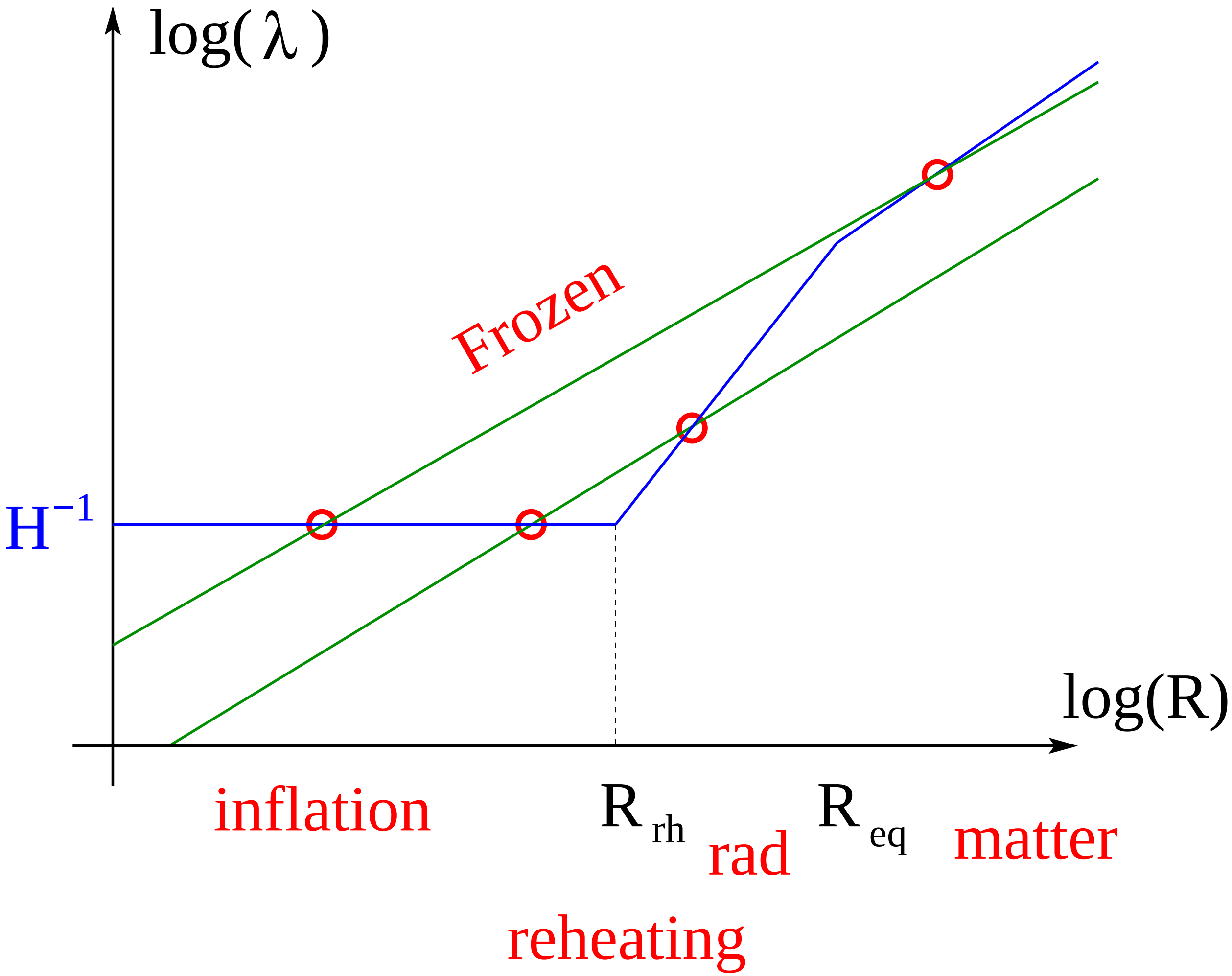
One growing and one decaying solution, but since $\phi_k = u_k/a$, ϕ_k remains constant for small k...

For a massless field in pure de Sitter, with free initial conditions, the full solution is actually known:

$$u_k = \frac{e^{-ik\eta}}{\sqrt{2k}} \left(1 + \frac{i}{k\eta} \right) \Rightarrow \mathcal{P}_k = \frac{k^3}{2\pi^2} |\delta\phi_k|^2 = \frac{H^2}{4\pi^2} \left(1 + k^2\eta^2\right)$$

SCALE INVARIANT POWER SPECTRUM (i.e. INDEPENDENT OF k) !

For the inflaton fluctuations though, the background is not pure de Sitter and the potential not vanishing as also the inflaton velocity...



If H and $\dot{\phi}$ were exactly constant during inflation, all the wavelengths would have exactly the same amplitude

$\Rightarrow$ SCALE INVARIANT SPECTRUM

(Harrison-Zeldovich)

nearly so in slow roll !!!

In this limit, the power spectrum is determined by the inflaton potential:

$$\frac{H^2}{\dot{\phi}} \propto \frac{V^{3/2}}{V' M_P^3}$$

Initial condition for structure formation !

Testing inflation: Single field inflation $\Longleftrightarrow$ Flat Potential $V(\phi)$

The scalar power spectrum is given by

$$\mathcal{P}_{\mathcal{R}}(k) = \frac{1}{12\pi^2 M_P^6} \frac{V^3}{V'^2}\bigg|_{k=aH} \qquad \propto k^{n-1}$$

and its spectral index is:

$$n(k)-1 = \frac{d\log(\mathcal{P}_{\mathcal{R}})}{d\log(k)}\bigg|_{k=aH} = 2\eta - 6\epsilon + \ldots$$

For gravity waves the situation is simpler since in fact they are perfectly massless... : the gravity waves are generated by fluctuations in the metric, i.e. $h_{ij} = \delta g_{ij}$.

The tensor power spectrum is given by

$$\mathcal{P}_{grav}(k) = \frac{1}{6\pi^2} \frac{V}{M_P^4}\bigg|_{k=aH}$$

and its spectral index is

$$n_{grav}(k) = \frac{d\log(\mathcal{P}_{grav})}{d\log(k)}\bigg|_{k=aH} = -2\epsilon + \ldots$$

What happens after such perturbations "re-enter" the horizon ?

In the Newtonian limit we have for the density perturbations of a matter fluid $\delta = \frac{\delta\rho}{\rho}$

$$\ddot{\delta}_k + 2H\dot{\delta}_k + \left(\frac{{c_s}^2 k^2}{a^2} - 4\pi G\rho\right)\delta_k = 0,$$

where $c_s = \delta p/\delta\rho$ is the sound speed in the plasma. Again a linear equation with a negative "mass" term... The fluctuations with negative mass grow and those have k below k_J ,i.e. a physical wavelength larger than the Jeans length:

$$\lambda_J = \frac{2\pi a}{k} = c_s\sqrt{\frac{\pi}{G\rho}} \simeq \frac{c_s}{H} \quad \text{sound horizon}$$

How strongly do they grow ? The growing solution is

$$\delta_k \sim C_1 H \int \frac{dt}{a^2 H^2} + C_2 H \sim C_1 t^{2/3} + C_2 t^{-1} \quad \text{for matter dominance}$$

NOTE: much weaker than exponential due to the expansion friction term $\propto H$! Also if the expansion is dominated by radiation, the growth is inhibited and at most only logarithmic in time. We need a long time of matter dominance to make initial fluctuations become large...

How can we measure these perturbations ???

We cannot see directly the Dark Matter density perturbations, but we can see the perturbations in the baryon-photon plasma at decoupling ! $\Rightarrow$ Cosmic Microwave Background !

The baryon and photon plasma has also density perturbations coming from the same origin (we will assume initial adiabatic perturbations, that distribute equally on every species) and they obey similar equations, apart from the fact that the photon pressure stops the gravitational collapse.

So Dark Matter start collapsing on scales larger than the Jeans scale after radiation-matter equality, while the baryon-photon plasma shows acoustic oscillations as can be seen in the CMB anisotropies.

$$\frac{\delta T}{T} \propto \frac{\delta \rho_\gamma}{\rho_\gamma} \sim 10^{-4}!!!$$

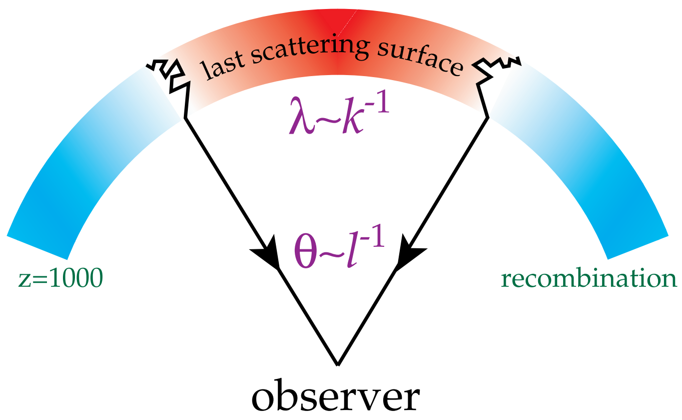
But the oscillations are strongly affected by the background metric and the Dark Matter density, so the exact position and height of the peaks give us informations on the cosmological parameters (not only on the primordial spectrum and inflationary model).

CMB anisotropy and the power spectrum

CMB experiments measure the fluctuations in the temperature of the radiation coming from the last scattering surface. $\longrightarrow$ sky maps

Describe the sky in terms of spherical harmonics:

$$\Delta T(\theta, \phi) = \sum_{\ell, m} a_{\ell m} Y_{\ell m}(\theta, \phi)$$



and obtain the temperature anisotropies as

$$C_\ell \equiv \frac{1}{2\ell + 1} \sum_m |a_{\ell m}|^2 \Leftrightarrow \quad P(k)$$

Note: ℓ gives to k on the last scattering surface !

picture from W. Hu, http://background.uchicago.edu/∼whu/

WMAP Combined map for 1st year (Monopole/Dipole and Milky way subtracted)

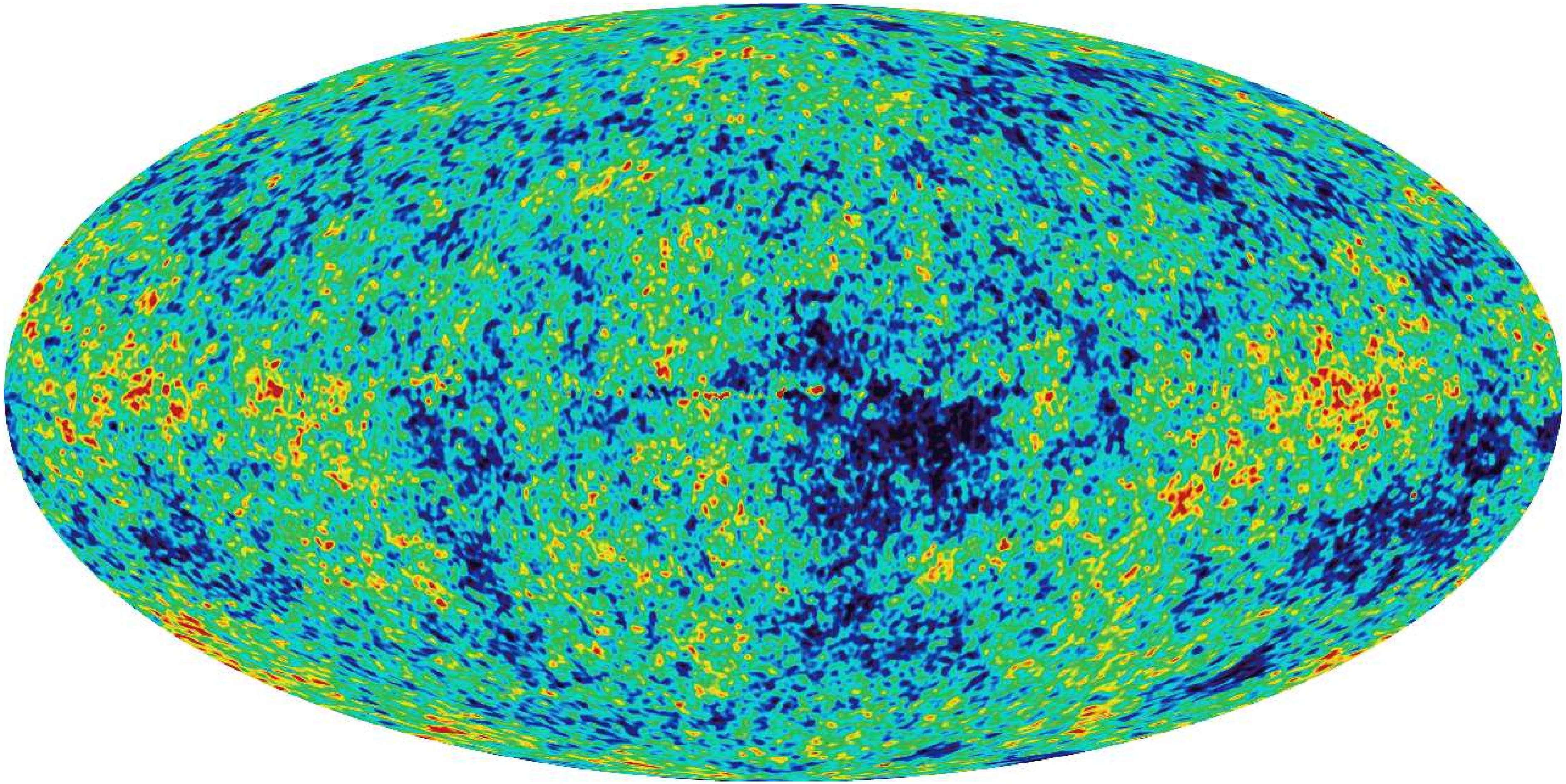
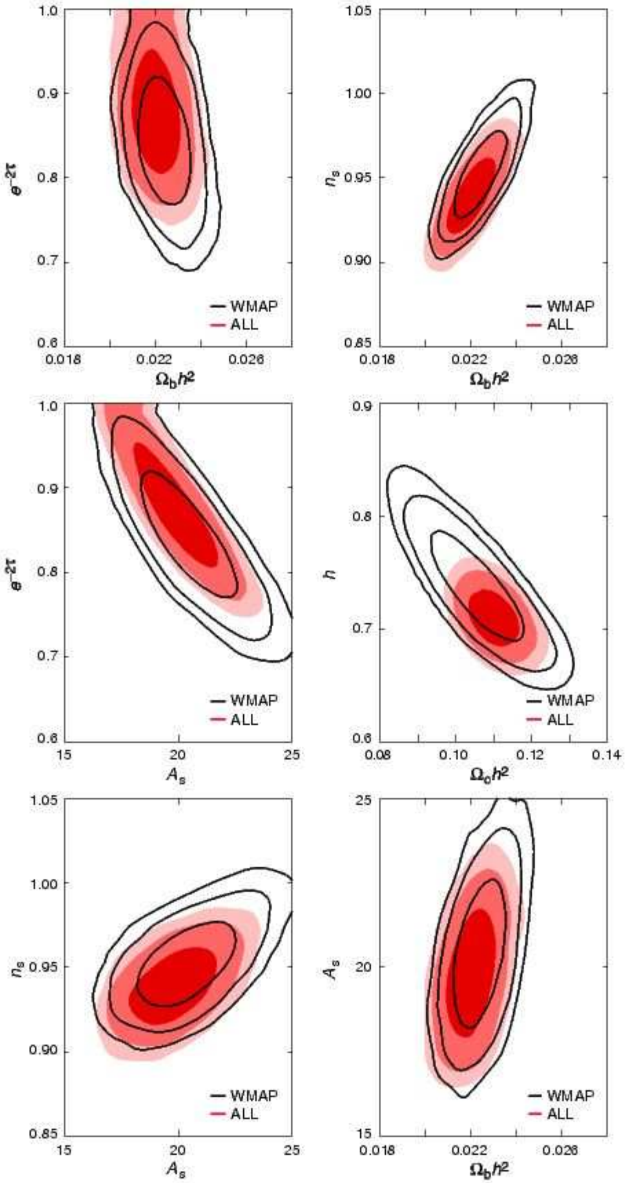
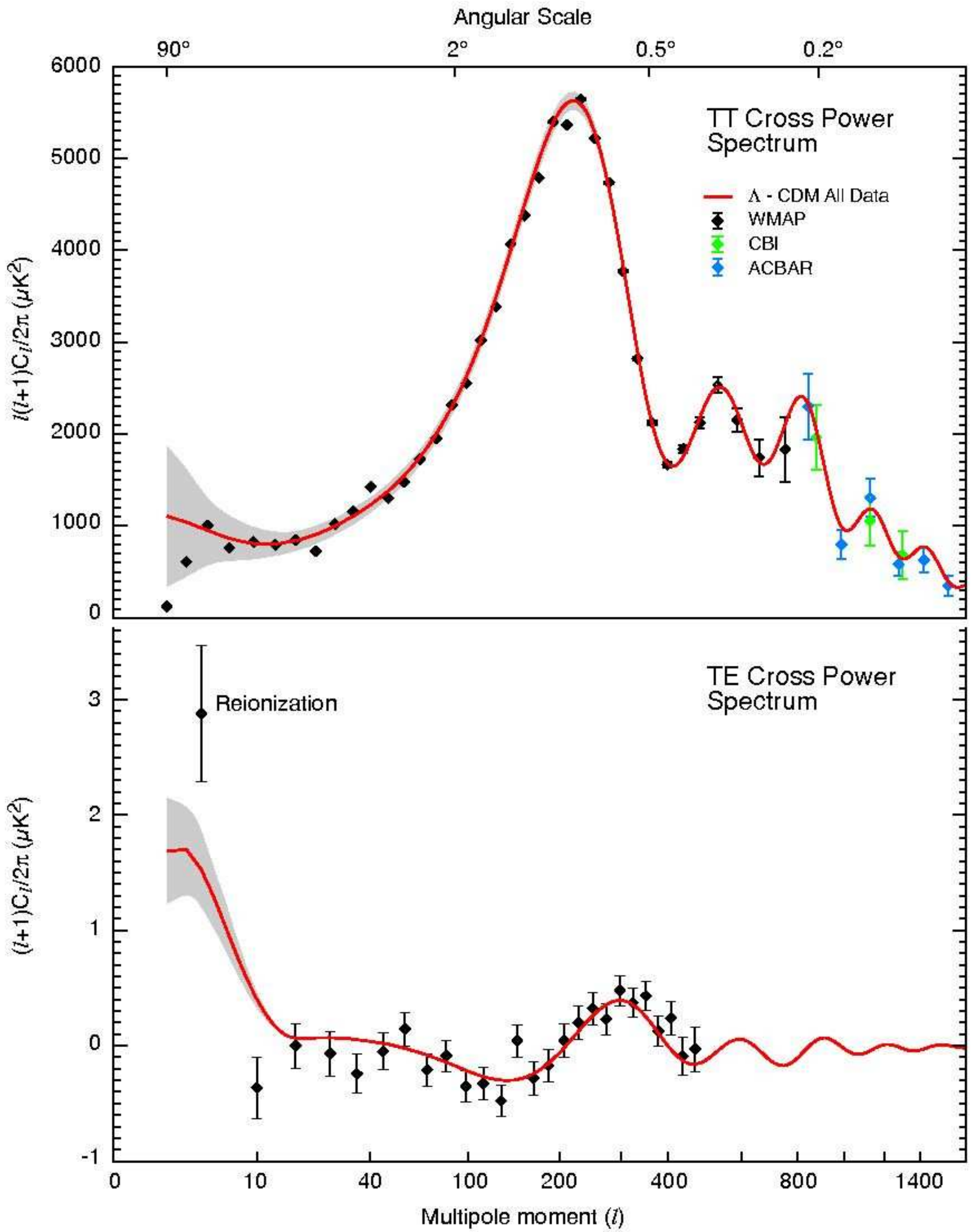


figure from WMAP at http://lambda.gsfc.nasa.gov/

NEW Results of the WMAP 3 years release early this year:



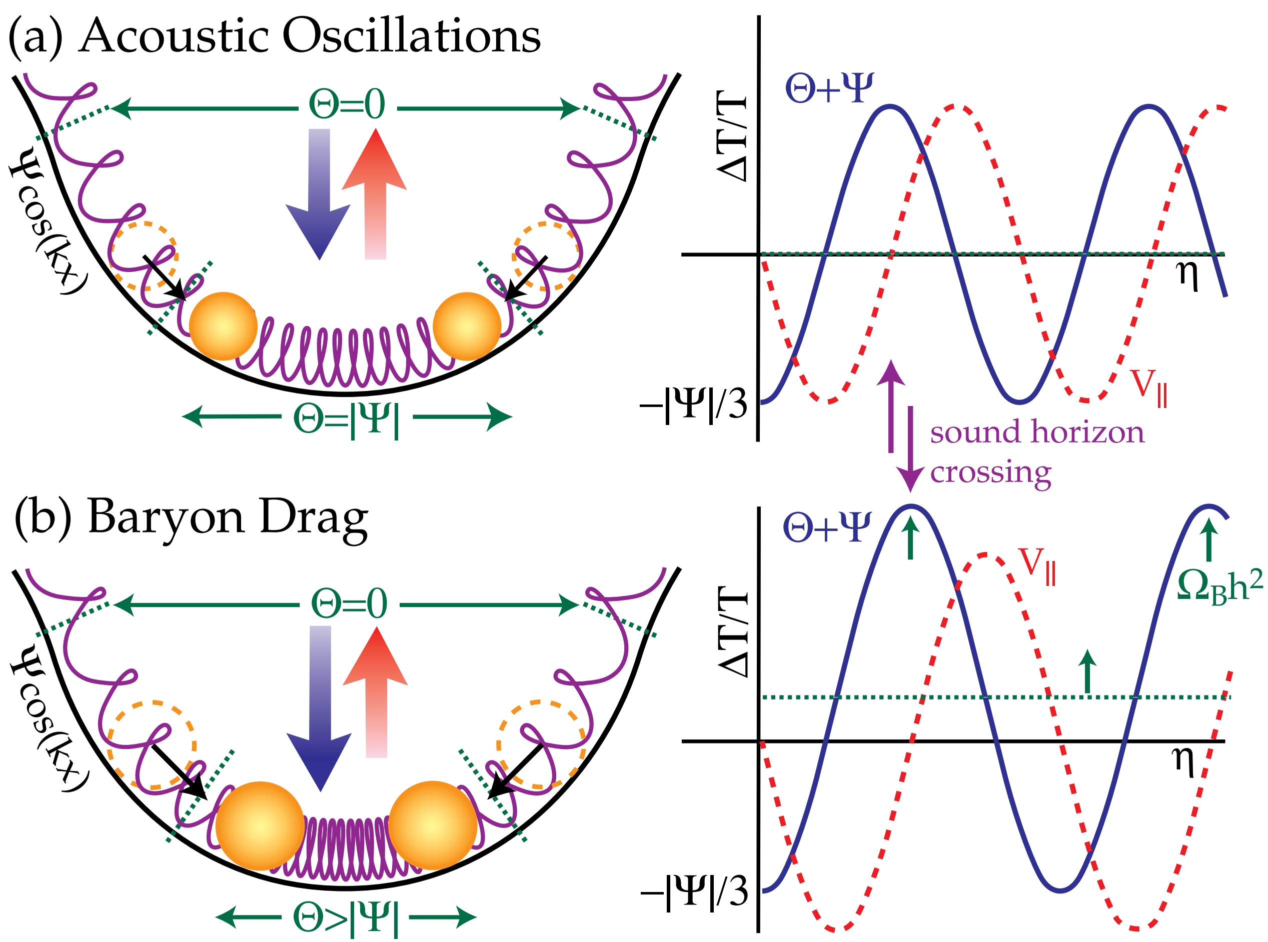


The Physics of the CMB

detailed review by W. Hu at http://background.uchicago.edu/∼whu/

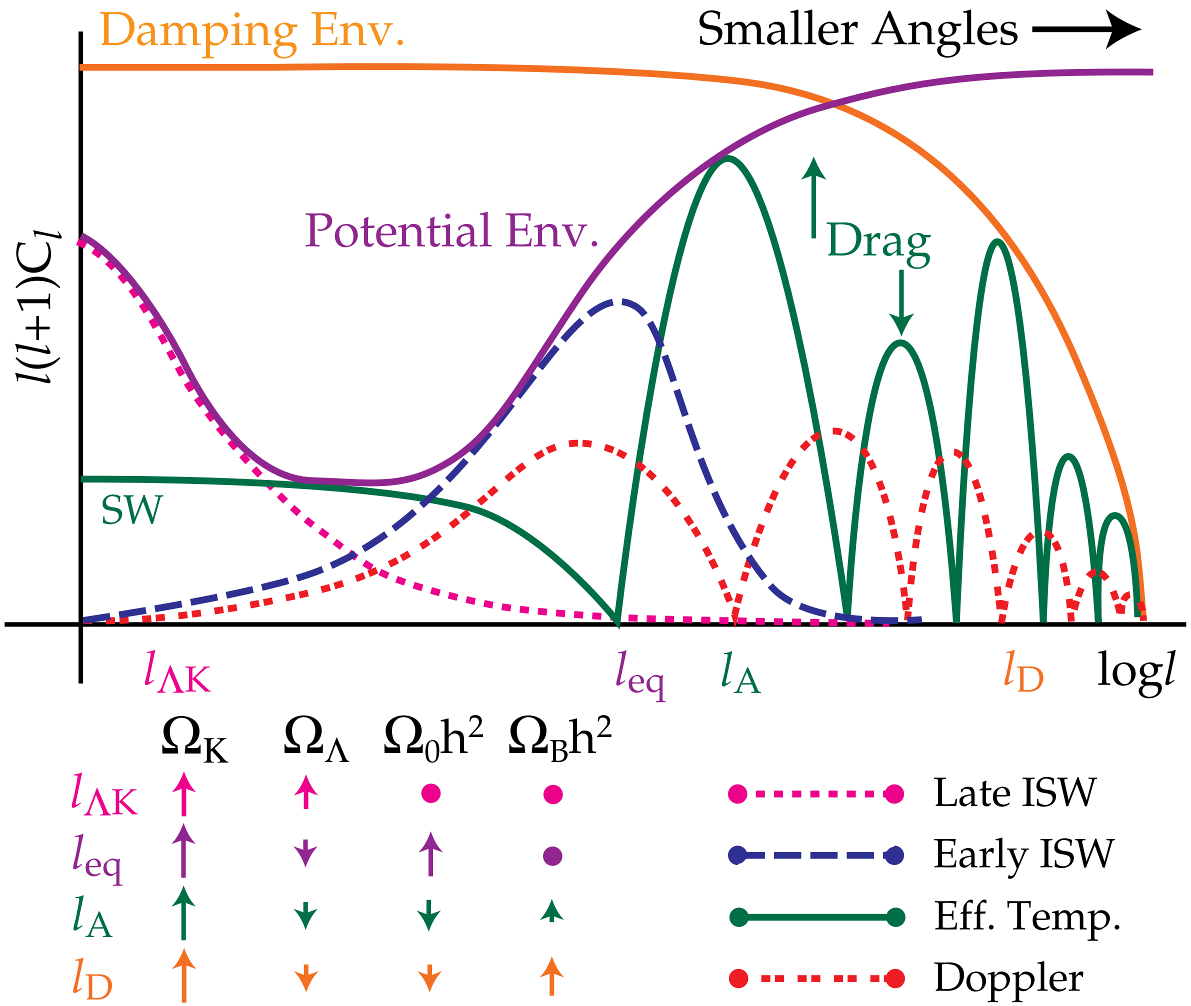
As soon as $\lambda \le c_s H^{-1}$, the mode starts to oscillate: first peak corresponds exactly to a maximal compression at wave-length equal to the horizon ! The even peaks instead are rarefaction peaks.

Baryons off-shift the oscillations since they increase the mass of the oscillator.



Dependence from cosmological parameters

picture from W. Hu, http://background.uchicago.edu/∼whu/

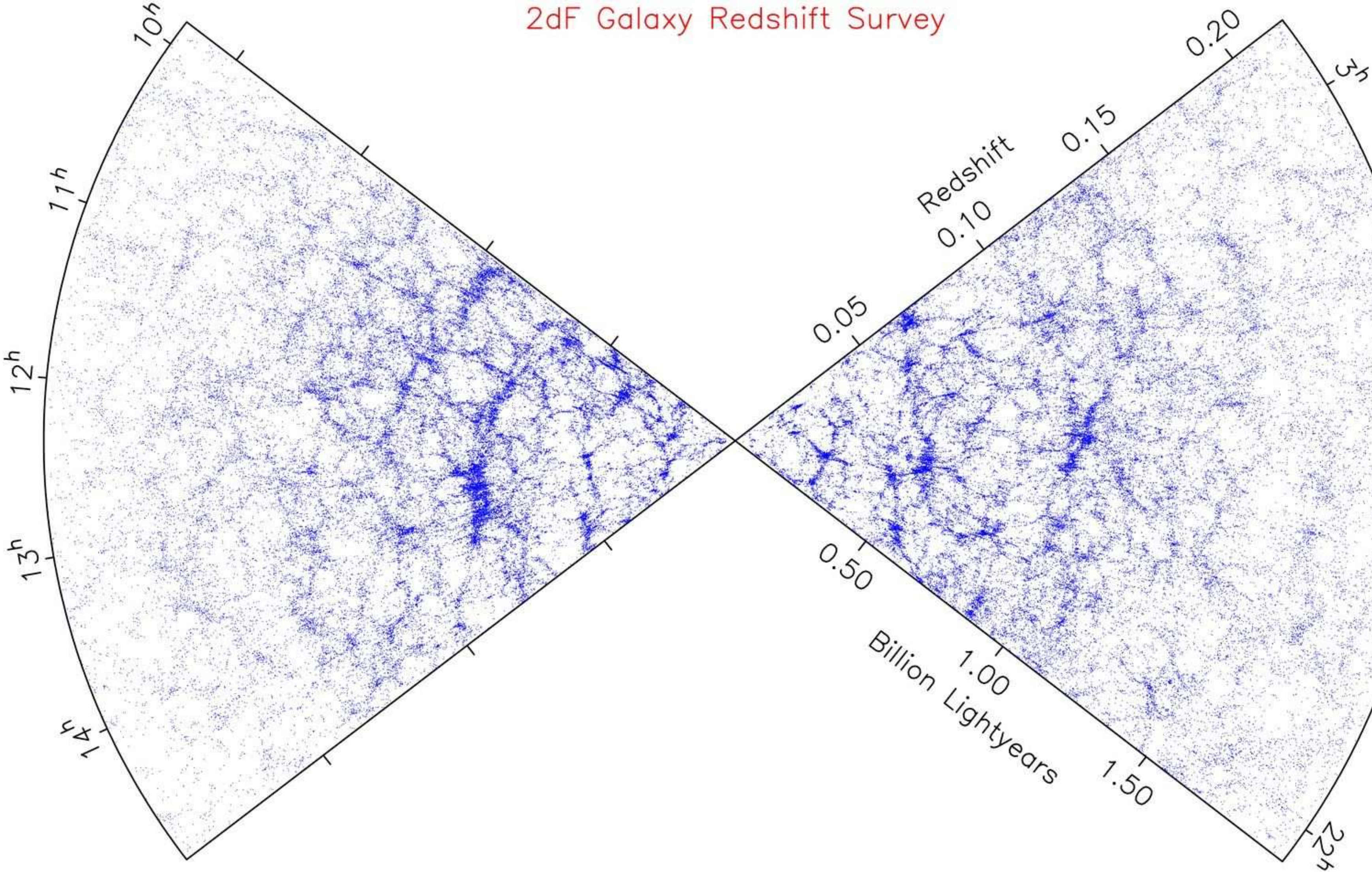


Dipendence of the cosmological parameters

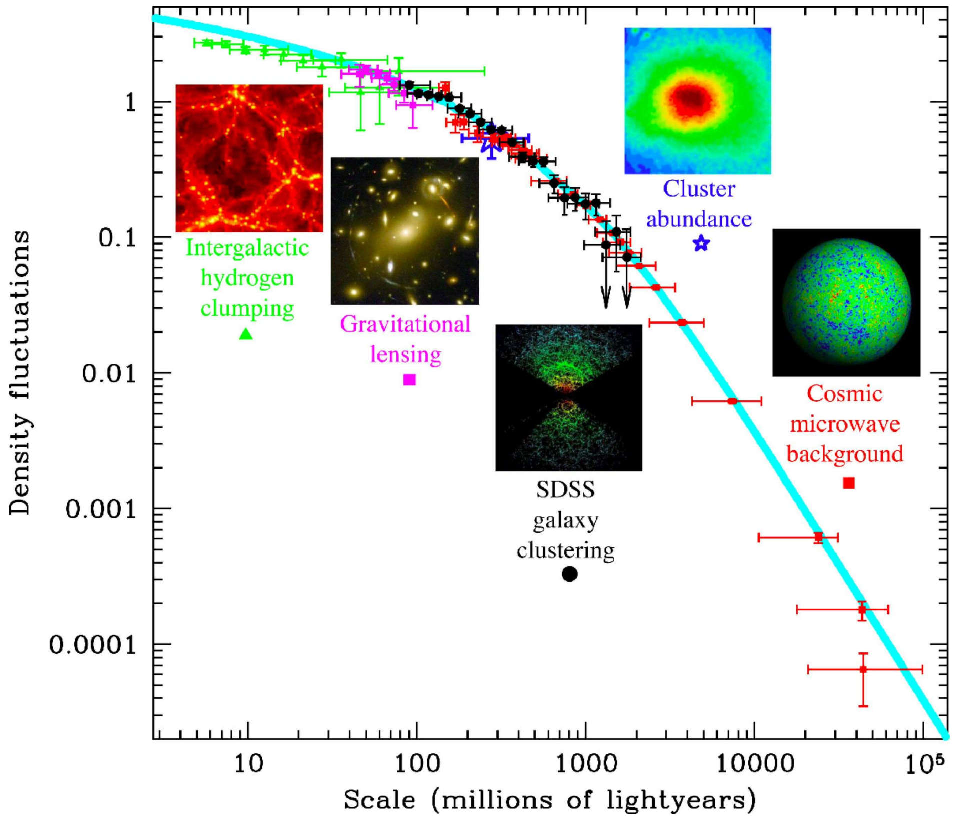
- Plateau at small ℓ: affected only by Sachs-Wolfe effect and more directly connected to the promordial spectrum; $\longrightarrow$ spectrum normalization $\sim 10^{-5}$
- Position of 1st peak: mostly dependent on the total density Ω_{tot} and the value of H at recombination; the depence on Ω_m and $\Omega_b h^2$ is weaker; $\longrightarrow \Delta\theta$
- Height of the 1st peak: becomes larger with decreasing $\Omega_m h^2$ and increasing $\Omega_b h^2$;
- Height of the 2nd peak: it is enhanced by decreasing $\Omega_m h^2$, *but* suppressed by increasing $\Omega_b h^2$;
- Height of the 3rd peak: becomes larger with decreasing $\Omega_m h^2$ and increasing $\Omega_b h^2$;

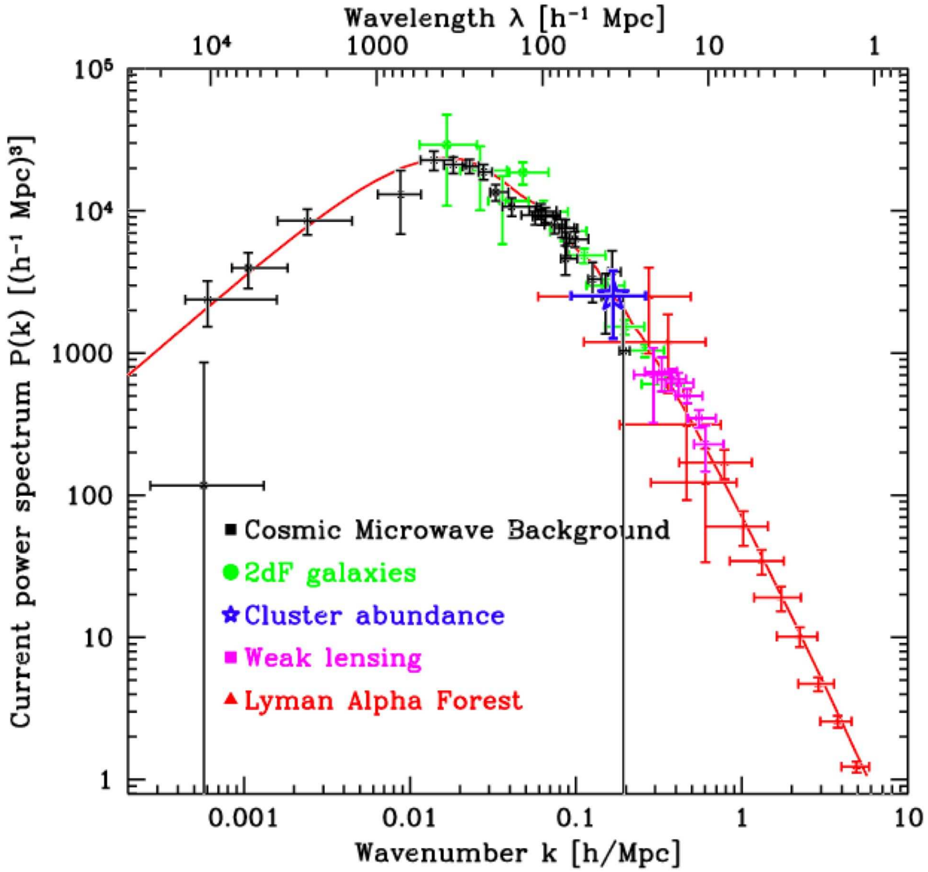
Using the structure of the whole spectrum, it is possible to disentangle most of the cosmological parameters ! Still one must beware of degeneracies...

Another handle on the perturbation is the distribution of matter itself...



The power spectrum of fluctuations can be obtained from the matter density distributions, measured via different methods...





[Figures by M. Tegmark]