Astroparticle physics I

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DESY

Plan of the Lectures

1. Introduction to Standard Cosmology:
   - GR, particle physics and thermodynamics
   - Present data and open questions
   - A short history of the Early Universe
2. Thermal relics and Dark Matter:
   - The Boltzmann equation in the expanding Universe
   - The number density of a Thermal relic $\longrightarrow$ WIMPs
   - Supersymmetry and the neutralino DM
   - Other SUSY candidates: gravitino & Co
3. The baryon asymmetry:
   - Nucleosynthesis and baryon number of the Universe
   - Sakharov's conditions
   - Sphaleronic transitions and EW baryogenesis
   - Leptogenesis and neutrino masses

Standard cosmology

Isotropic and homogeneous universe on the scales larger or equal to 100 Mpc
$\Longrightarrow$ Friedmann-Robertson-Walker metric:

$$ds^2 = dt^2 - a^2(t) \left(\frac{dr^2}{1 - kr^2} + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2 \right) = g_{\mu\nu} dx^\mu dx^\nu$$

SCALE FACTOR 3D CURVATURE: $k = \pm 1, 0$ for hyperbolical/spherical/flat

open/closed or flat 3D space

Comoving coordinates r, θ, ϕ, physical coordinates are rescaled by $a(t)$...

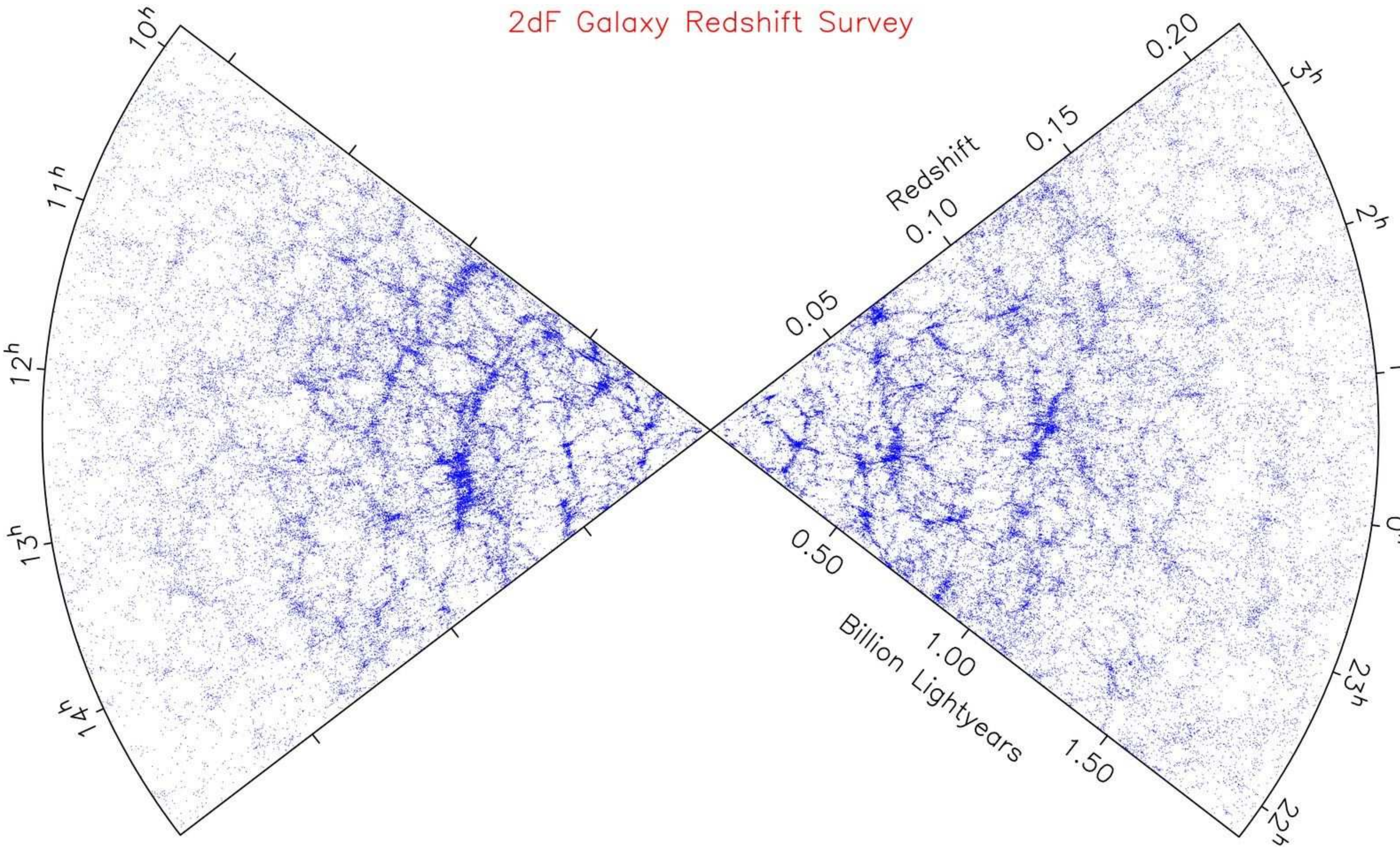
In general way we can write $d\chi^2 = \frac{dr^2}{1-kr^2}$ so that

$$r = S_k(\chi) = \begin{cases} \sin(\chi) & k = 1 \\ \chi & k = 0 \\ \sinh(\chi) & k = -1 \end{cases}$$

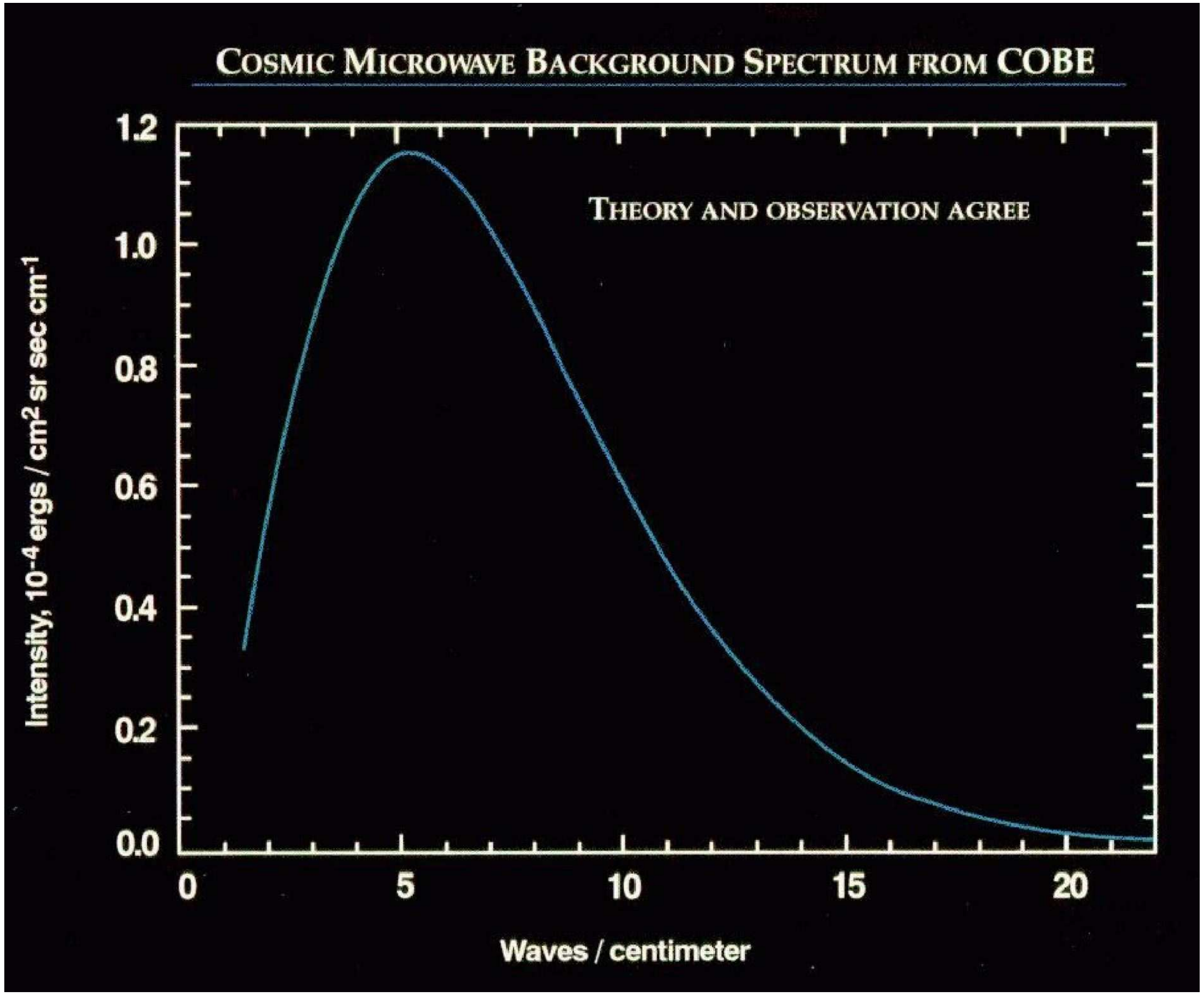
Also this metric is locally conformal to the Minkowski metric, in fact we can define the conformal time $d\eta = dt/a(t)$ and then we can write

$$ds^2 = a^2(t) \left(d\eta^2 - d\chi^2 - S_k^2(\chi) d\Omega^2 \right)$$

At large red-shift the Universe seems to become homogeneous...



Cosmic Microwave Radiation: Perfect BLACK BODY in all directions !!!



picture from http://map.gsfc.nasa.gov

From such a metric *without any assumption on the function* $a(t)$, it directly comes the Hubble law:

$$v = \frac{dD}{dt} = \frac{d(a(t)r)}{dt} = \dot{a}r = \frac{\dot{a}}{a}ar = H(t)D$$

the velocity of recession is proportional to the distance, with "constant" of proportionality given by $H(t) = \dot{a}/a$ the Hubble parameter. $\longrightarrow$ Hubble flow discovered by Hubble in 1920's.

Another direct consequence is the red- or blue-shifting of light travelling on geodesics $d\eta = d\chi$: the conformal period is constant so we have

$$\frac{\Delta t_{em}}{a(t_{em})} = \frac{\Delta t_{obs}}{a(t_{obs})} \quad \Rightarrow \quad \frac{\lambda_{obs}}{\lambda_{em}} = \frac{a(t_{obs})}{a(t_{em})} = 1 + z$$

the light wavelength is stretched/squeezed by the change in scale factor. We can use the redshift z to keep track of the expansion of the Universe.

Let us see also what determines the dynamics of $a(t)$... GENERAL RELATIVITY!

Elements of General Relativity

The dependence of the scale factor from time is determined by Einstein's equation:

$$\mathcal{R}_{\mu}^{\nu} - \frac{1}{2}\delta_{\mu}^{\nu}\mathcal{R} \quad = \quad 8\pi G T_{\mu}^{\nu} + \Lambda\delta_{\mu}^{\nu}$$

GEOMETRY of Space-time Energy-momentum tensor: physics content

G is the —Newton constant, $\mathcal{R}_{\mu}^{\nu}$ the Ricci tensor, while $\mathcal{R}$ is its trace, the Ricci scalar (4D curvature), and they are given as a function of the Crystoffel symbols and the metric by

$$\mathcal{R}_{\mu\nu} = \Gamma^{\alpha}_{\mu\nu,\alpha} - \Gamma^{\alpha}_{\alpha\nu,\mu} + \Gamma^{\alpha}_{\mu\nu}\Gamma^{\beta}_{\alpha\beta} - \Gamma^{\alpha}_{\mu\beta}\Gamma^{\beta}_{\alpha\nu}$$

for

$$\Gamma^{\alpha}_{\beta\gamma} = \frac{1}{2}g^{\alpha\delta}\left[g_{\delta\gamma,\beta} + g_{\beta\delta,\gamma} - g_{\beta\gamma,\delta}\right]$$

where the comma in the indices indicates differentiation, i.e. $\phi_{,\mu} = \frac{d\phi}{dx^{\mu}}$ and $g^{\alpha\beta}$ is the inverse of the metric, i.e. $g^{\alpha\beta}g_{\beta\gamma} = \delta^{\alpha}_{\gamma}$.

For the metric $g_{\alpha\beta} = diag(1, -\frac{a^2(t)}{1-kr^2}, -a^2(t)r^2, -a^2(t)r^2\sin^2\theta) \quad \Rightarrow \quad g^{\alpha\alpha} = \frac{1}{g_{\alpha\alpha}}$.

So the only non-zero components of Γ have at most one index zero and two equal spatial ones, and are

$$\Gamma^0_{ii} = -H(t)g_{ii} \quad \Gamma^i_{0i} = \Gamma^i_{i0} = H(t)$$

while the others are time independent:

$$\Gamma^i_{jj} = -\frac{1}{2}l^i l_{j,i} \quad \Gamma^i_{ij} = \frac{1}{2}l^i l_{i,j}$$

where $l_i = -g_{ii}a^{-2}, l^i = -g^{ii}a^2$.

Then also $\mathcal{R}$ has an easy expression: $\mathcal{R}^0_0 = -3(\dot{H} + H^2) \qquad \mathcal{R}^i_i = -\dot{H} - H^2 - \frac{2k}{a^2}$

and the Ricci scalar is $\mathcal{R} = -6\left(\dot{H} + 2H^2 + \frac{k}{a^2}\right)$ curved space in 4D also for $k = 0$!

Since we have only one single dynamical variable $a(t)$, we have two independent equations.

From the $0-0$ component of Einstein equation:

$$3\left(H^2 + \frac{k}{a^2}\right) = 8\pi G T^0_0 + \Lambda$$

while the $i-i$ components are equal and give

$$2\frac{\ddot{a}}{a} + H^2 + \frac{k}{a^2} = 8\pi G T^i_i + \Lambda$$

or summing them together and using the 1st equation:

$$2\frac{\ddot{a}}{a} = -8\pi G\left(T^0_0 - \sum_i T^i_i\right) + 2\Lambda\ .$$

Now the question is: what is $T^{\mu\nu}$??? And Λ ????

Perfect fluid approximation

$$T^{\mu}_{\nu} = (\rho + p)u^{\mu}u_{\nu} - p\delta^{\mu}_{\nu}$$

where ρ and p are the fluid density and pressure, while u is the fluid 4-velocity. So in the rest-frame of the fluid, where $u = (1, \vec{0})$, i.e. assuming that the fluid is at rest in the Universe, we have

$$T^{\mu}_{\nu} = \begin{pmatrix} \rho & 0 & 0 & 0 \\ 0 & -p & 0 & 0 \\ 0 & 0 & -p & 0 \\ 0 & 0 & 0 & -p \end{pmatrix}$$

Moreover the energy-momentum tensor is covariantly conserved:

$$\mathcal{D}_{\mu} T^{\mu\nu} = 0 \quad \rightarrow \quad \dot{\rho} + 3H(\rho + p) = 0$$

continuity equation

This can be solved if we know the equation of state $p(\rho) = w\rho$ then

$$\frac{\dot{\rho}}{\rho} = -3(1+w)H \quad \Rightarrow \quad \rho \propto a^{-3(1+w)}$$

So the different energy types are modeled by perfect fluids with equation of state $w_i = p_i/\rho_i$.

Energy-momentum tensor in Quantum Field Theory

For a general lagrangian $\mathcal{L}$ for a quantum field φ we have that the energy-momentum tensor is given by

$$T_{\mu\nu} = \frac{d\mathcal{L}}{d\partial^\mu \varphi} \partial_\nu \varphi - g_{\mu\nu} \mathcal{L}$$

so we have for example for a scalar field, when we can neglect space-gradients:

$$\rho = \frac{1}{2}\dot{\phi}^2 + V(\phi) \qquad p = \frac{1}{2}\dot{\phi}^2 - V(\phi)$$

so we have
$$-1 \leq w = -1 + \frac{\dot{\phi}^2}{\frac{1}{2}\dot{\phi}^2 + V(\phi)} \leq 1$$

In particular $w = -1$ for $\dot{\phi} \simeq 0$ or $w \sim 0$ for $\dot{\phi}^2 \sim 2V(\phi)$.

For a massless gauge field like the photon instead we have always $w = \frac{1}{3}$.

N.B. The Einstein equation is classical, so we should actually consider in the r.h.s. the vacuum expectation value of $T_{\mu\nu}$, but for a quantum field $\langle T_{\mu\nu} \rangle$ gets contribution from all vacuum fluctuations and *diverges* !

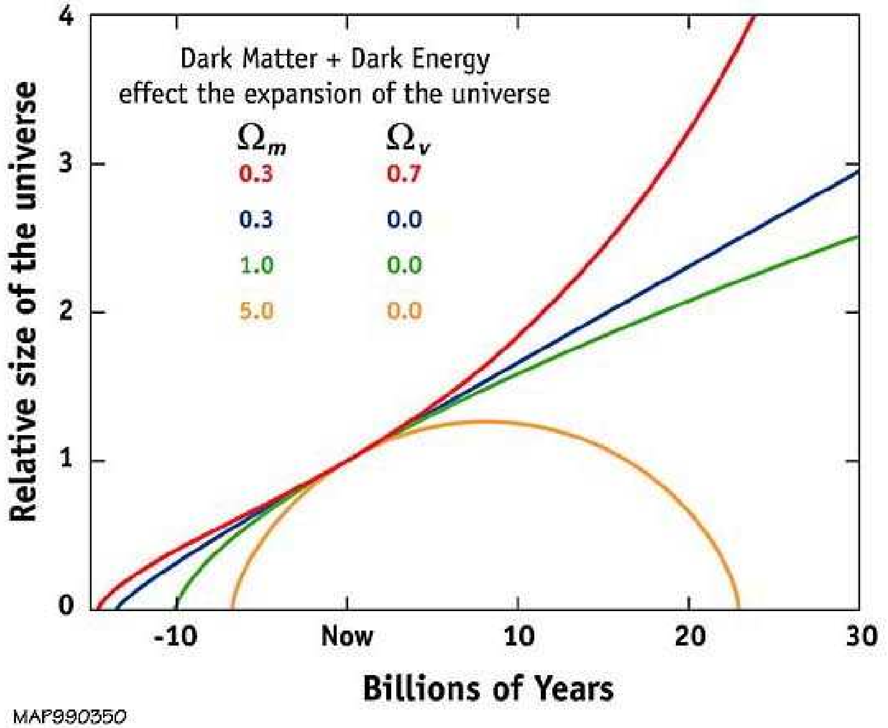
One part of the cosmological constant problem...

Apart for the continuity equation, we need only another equation to solve for the scale factor $a(t)$.

Usually one takes the Friedmann's equation:

$$H^2 \equiv \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi}{3M_P^2}\rho - \frac{k}{a^2} + \Lambda$$

Hubble parameter — energy density — curvature — cosmological constant



picture from http://map.gsfc.nasa.gov

Or dividing by the present value of H_0^2 one has

$$\frac{H^2}{H_0^2} = \frac{8\pi}{3M_P^2 H_0^2}\rho - \frac{k}{a^2 H_0^2} + \frac{\Lambda}{H_0^2}$$

Rescale the densities by the critical density as $\rho_c = \frac{3M_P^2}{8\pi} H_0^2 \Rightarrow \Omega_i = \frac{\rho_i}{\rho_c} \propto a^{-3(w_i+1)/2}$

H_0 is measured presently as h 100 km/s/Mpc, so $\Omega_i h^2$ are physical energy densities in units of $\rho_c/h^2 = 1.879 \times 10^{-29}$ g/cm$^3 = 1.054 \times 10^4$ eV/cm^3.

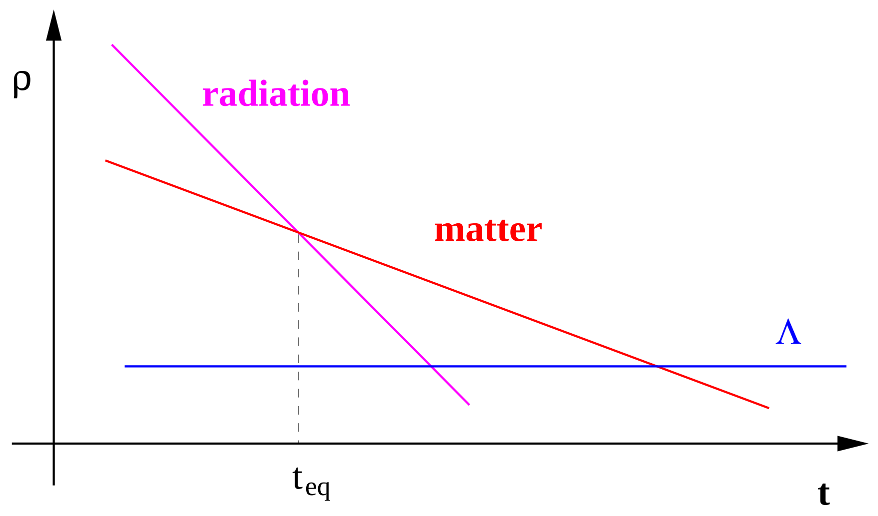
Since $k = a^2 H^2(\Omega_{tot} - 1)$, the equation is simply $H = H_0 \left(\Omega_{tot} - (\Omega_{tot} - 1)a^{-2}\right)^{1/2}$

Or we can use red-shift and set the boundary conditions today and have

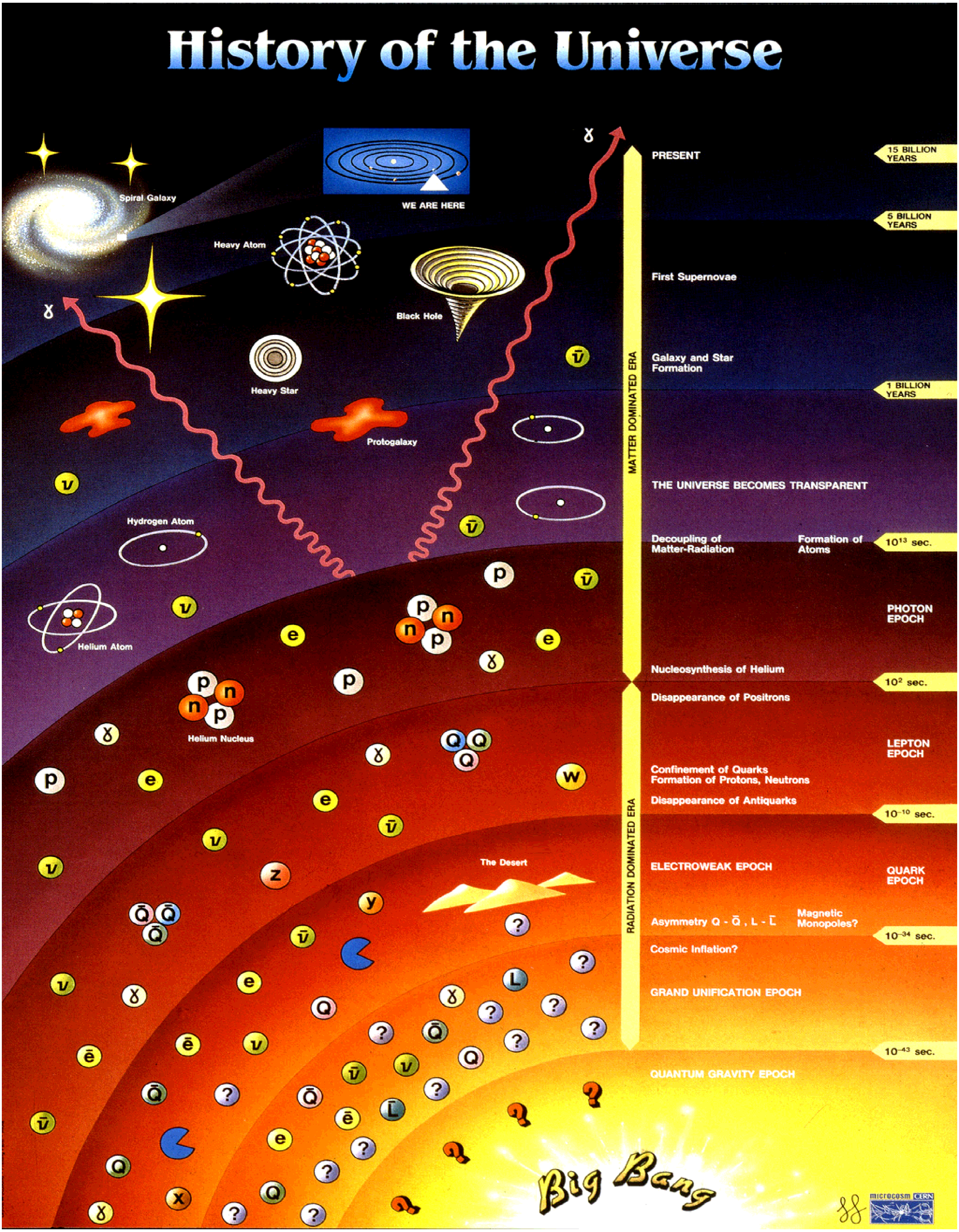
$$H = H_0 \sqrt{\sum_i \Omega_{i,0}(1+z)^{3(1+w_i)} - \left(\sum_i \Omega_{i,0} - 1\right)(1+z)^2} \sim H_0 \Omega_{D,0}^{1/2} a^{-3(w_D+1)/2}$$

Depending on the dominant component with equation of state $p = w\rho$, we have

Type	w	$\rho(a)$	$a(t)$	$\Omega_i(t_{\text{now}})$
Radiation	$1/3$	$\propto a^{-4}$	$\propto t^{1/2}$	$\sim 10^{-5}$
Matter	0	$\propto a^{-3}$	$\propto t^{2/3}$	$\sim 1/3$
Λ	-1	const.	e^{Ht}	$\sim 2/3$
Curvature	$-1/3$	$\propto a^{-2}$	$\propto t^1$	~ 0



$\Rightarrow$ Different epochs of expansion of the Universe


History of the Universe
Spiral Galaxy
WE ARE HERE
Heavy Atom
Black Hole
Heavy Star
Protogalaxy
Hydrogen Atom
Helium Atom
Helium Nucleus
The Desert
Big Bang
PRESENT
15 BILLION YEARS
5 BILLION YEARS
First Supernovae
Galaxy and Star Formation
1 BILLION YEARS
MATTER DOMINATED ERA
THE UNIVERSE BECOMES TRANSPARENT
Decoupling of Matter-Radiation
Formation of Atoms
10^{13} sec.
PHOTON EPOCH
Nucleosynthesis of Helium
10^{2} sec.
Disappearance of Positrons
LEPTON EPOCH
Confinement of Quarks
Formation of Protons, Neutrons
Disappearance of Antiquarks
10^{-10} sec.
RADIATION DOMINATED ERA
ELECTROWEAK EPOCH
QUARK EPOCH
Asymmetry $Q - \bar{Q}$, $L - \bar{L}$
Magnetic Monopoles?
10^{-34} sec.
Cosmic Inflation?
GRAND UNIFICATION EPOCH
10^{-43} sec.
QUANTUM GRAVITY EPOCH
microcosm CERN

How can we measure the expansion history ?

Exploit some Standard Candles and measure the luminosity vs redshift relation !

We can define the luminosity distance as $D_L^2 = \frac{L}{4\pi\Phi}$ — L: Intrinsic luminosity; Φ: measured flux

Using the expressions:

$$\Delta E_{em} = L a_{em} \Delta\eta \qquad \Delta E_{obs} = \Delta E_{em} \frac{a_{em}}{a_{obs}}$$

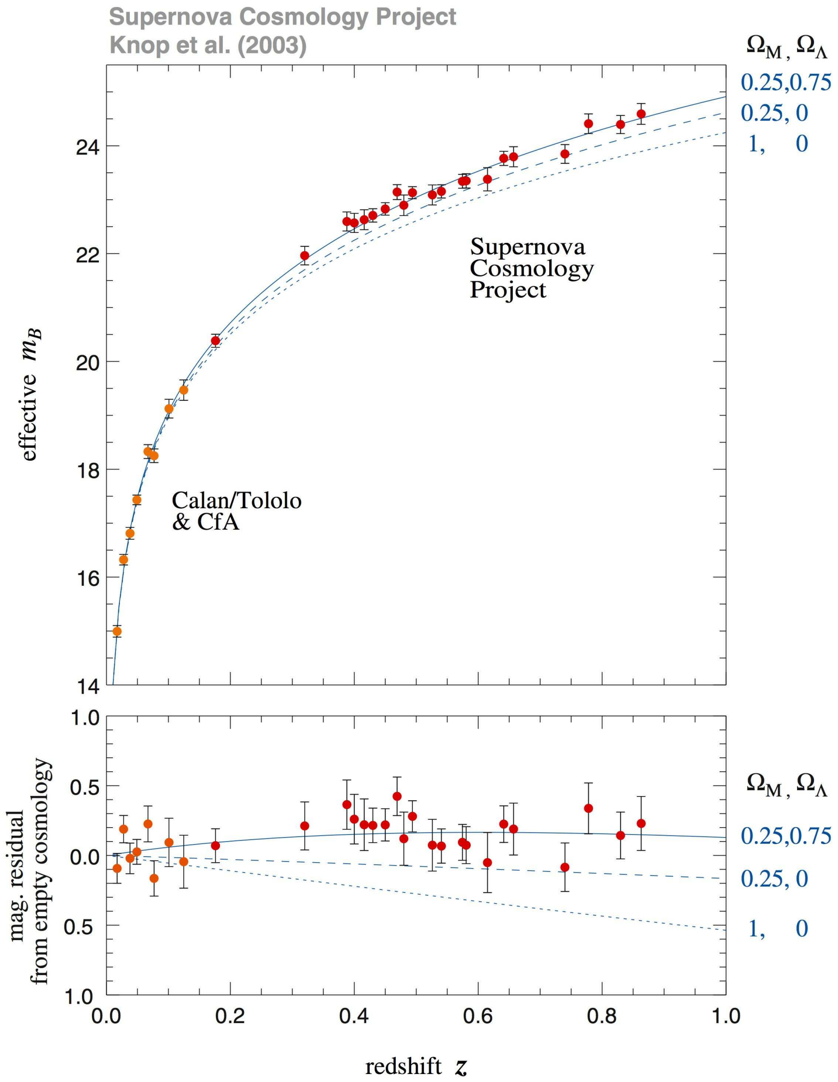
and since the surface is $S = 4\pi a_{obs}^2 \Delta\chi^2$ we have

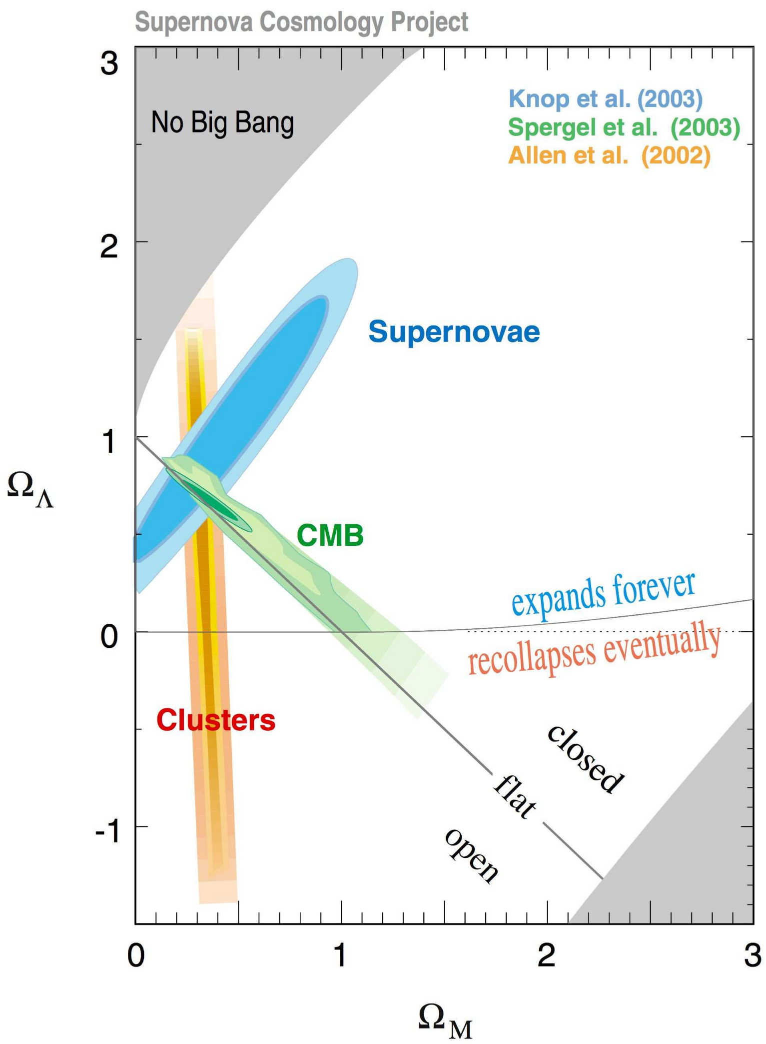
$$D_L = a_{obs} \Delta\chi (1+z) = (1+z) \int_0^z \frac{dz}{H(z)}$$

where for $k = 0$ we have $H = H_0 \sqrt{\sum_i \Omega_{i,0} (1+z)^{3(1+w_i)}}$.

Measure the dependence of H from red-shift and the energy content of the Universe!

SN IA data


Supernova Cosmology Project
Knop et al. (2003)
Ω_M, Ω_Λ
0.25,0.75
0.25, 0
1, 0
Supernova Cosmology Project
Calan/Tololo & CfA
effective m_B
mag. residual from empty cosmology
redshift z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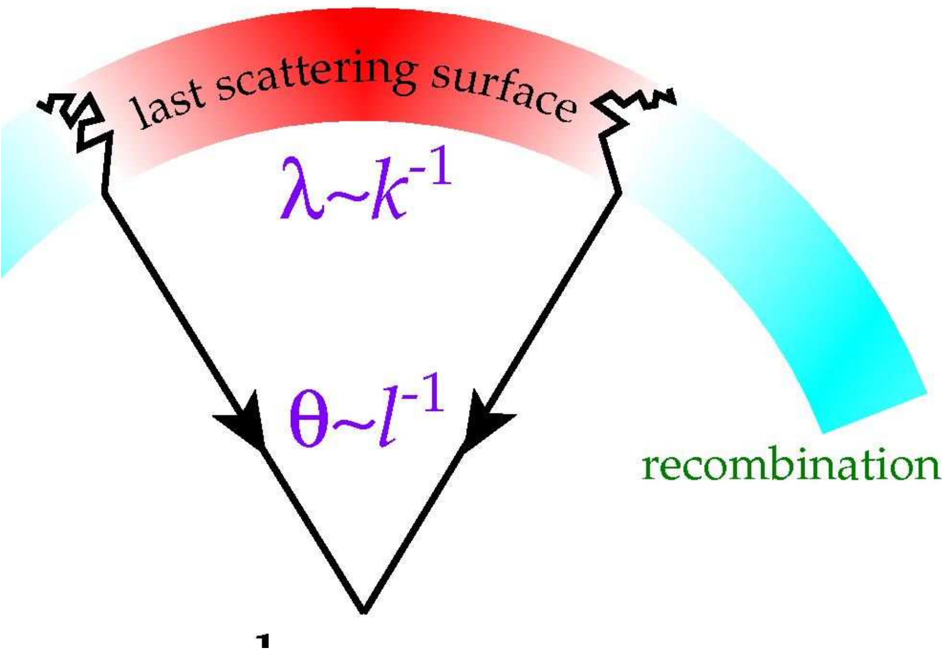
Supernovae
CMB
Clusters
expands forever
recollapses eventually
closed
flat
open
Ω_Λ
Ω_M

Another probe: the angular diameter red-shift relation

Use as a reference a Standard Ruler R and measure the angular diameter vs redshift relation !

We can define the angular diameter as
$$\Delta\theta = \frac{R}{a_{em}\chi_{em}} = (1+z)\frac{R}{\int_0^z \frac{dz}{H(z)}}$$

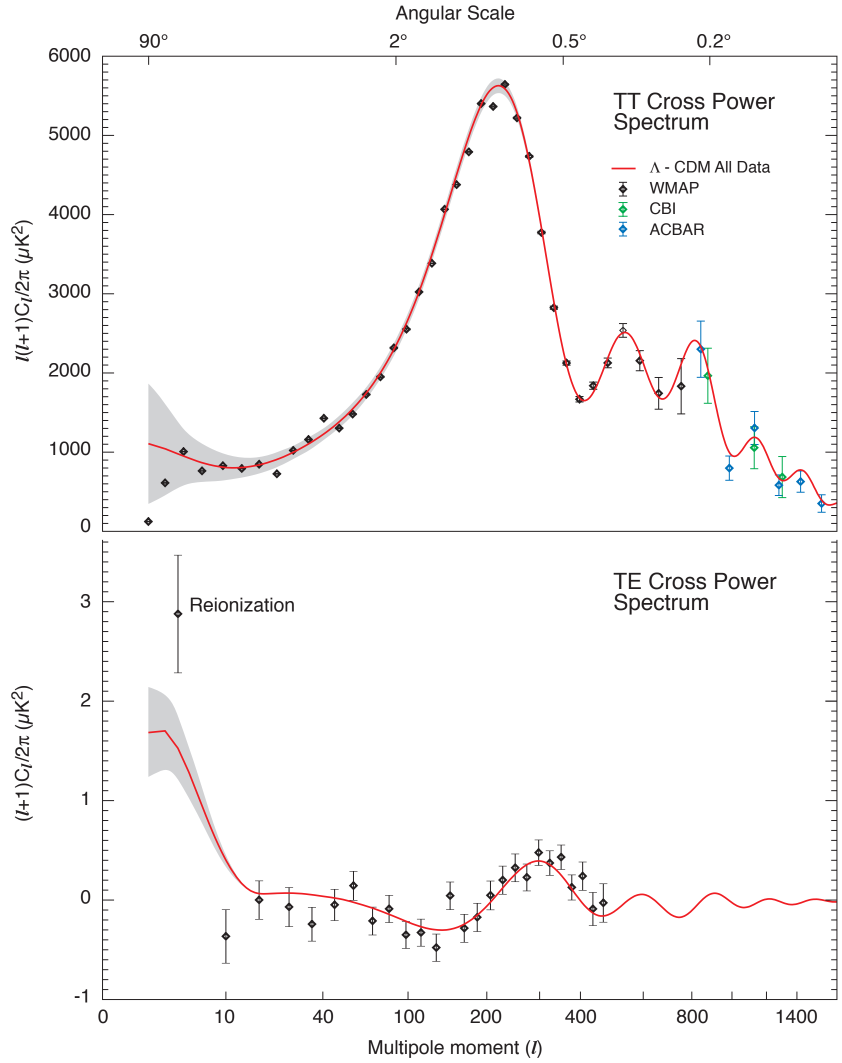
One example: sound horizon at the time of radiation decoupling...

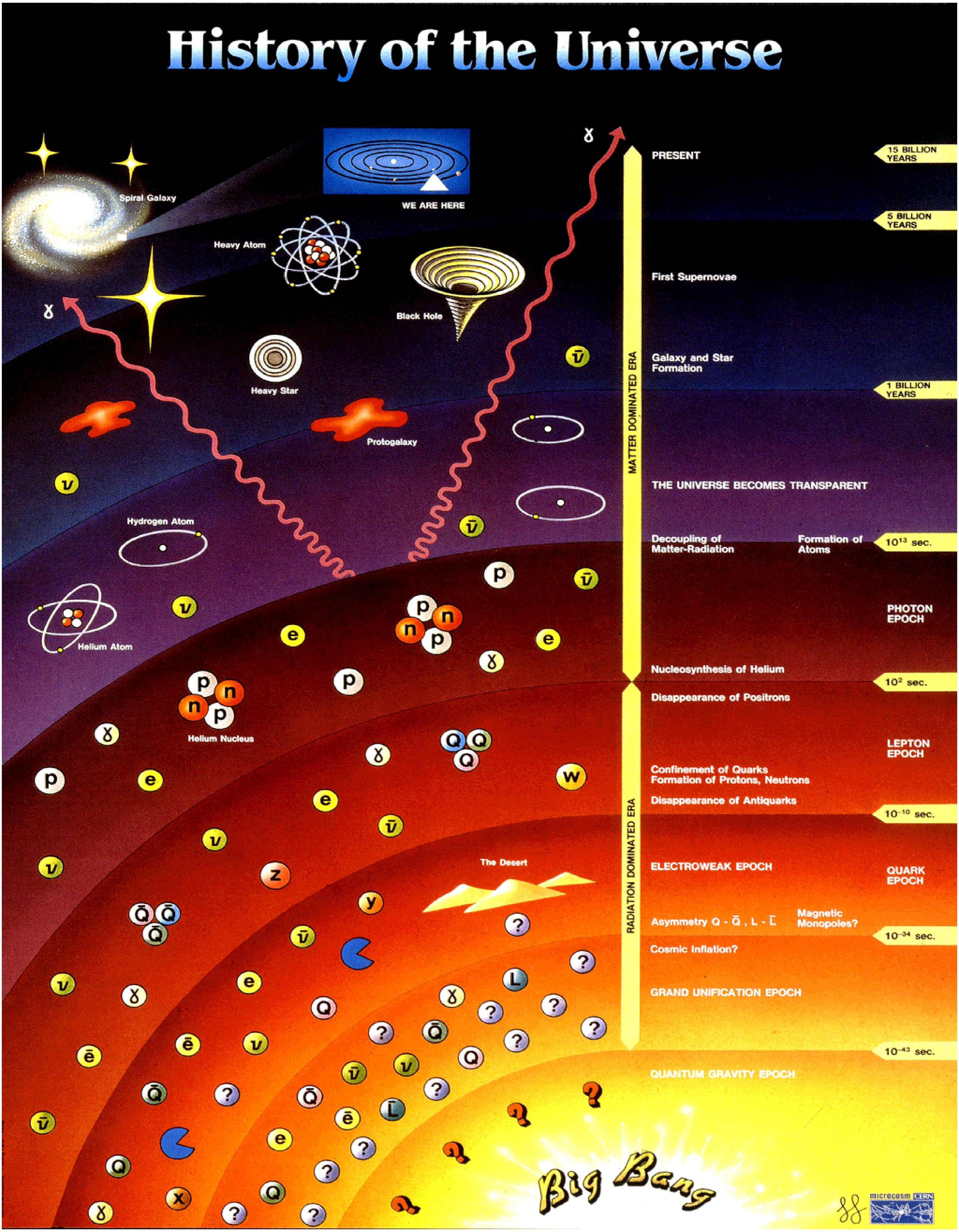


The sound horizon at $z_{dec} \simeq 1100$ is approximately $H^{-1}(z_{dec})$, so assuming matter domination we have

$$\Delta\theta_H = \frac{z_{dec}H_0\Omega_{tot}}{2H(z_{dec})} \simeq \frac{1}{2} z_{dec}^{1/2}\Omega_{tot}^{1/2}$$

Observed in the first peak in the CMB $\rightarrow \Omega_{tot} = 1$!

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A short history of the Universe

- Inflationary epoch setting the initial conditions for Standard Cosmology $\longrightarrow$ next lecture !
- Reheating/Preheating at a temperature $T_{RH} > 1$ MeV: beginning of the Radiation dominated epoch;
- Big Bang Nucleosynthesis, production of the lightest elements $^2H,^3H,^3He,^4He,^6Li,^7Li$; neutrino decoupling $\Rightarrow$ light elements abundance, neutrino background...
- Matter-radiation equality: pressure drops and structure formation by gravitational collapse begins...
- Recombination: the electrons are captured by the Nuclei and the Universe becomes transparent mbox $\Rightarrow$ CMB !