QFT II exercises - sheet 9

If you find a mistake in this exercise first check the website if the problem has been resolved already in a newer version. Otherwise, please email rutger.boels@desy.de.

Exercise 1

This exercise is based on section 2.1 of arXiv:hep-th/9702094, lecture notes by Michael Peskin. You probably want to find this[1]!

a By decomposing a Dirac spinor into left and right Weyl spinors (easiest using the chiral representation), show that the massless limit of QCD with only the three flavours u, d and s enjoys a $U(3) \times U(3)$ global symmetry. Hint: the three flavours fill out the fundamental representation of this symmetry: the left and right chiral spinors transform separately.

b Check that a mass term would spoil the symmetry.

c Argue that since $U(3) \sim SU(3) \times U(1)$, there are two global $U(1)$ transformations: a chiral ("involving γ_5") and a non-chiral one. For complicated reasons, in the following the chiral one can be disregarded.

d Argue that a vev for a scalar operator

$$\langle \bar{q}_L^f q_R^{f'} \rangle \Delta \delta^{ff'}$$

breaks the global symmetry $SU(3) \times SU(3) \times U(1)$ down to $SU(3) \times U(1)$.

e count the number of 'broken' generators and argue using Goldstone's theorem that there must be 8 massless Goldstone bosons. Verify these fit into the adjoint of the unbroken symmetry.

f Argue for the correctness of equation 7 in arXiv:hep-th/9702094, in the approximation that the momenta of all particles is very small.

g Verify that if you take the momentum of a pion to be soft, the corresponding tree level amplitude in this theory vanishes (This is known as an Adler zero).

Exercise 2

Consider ϕ^4 theory,

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)\partial^\mu \phi + \frac{1}{2}\mu^2 \phi^2 - \frac{\lambda}{4!}\phi^4$$

with $\mu > 0$ and $\lambda > 0$. This theory has a spontaneously broken $U(1)$ symmetry.

[1] for instance, at http://arxiv.org/abs/hep-th/9702094

a Introduce new coordinates for the vacuum for which $\langle\phi\rangle = +\sqrt{\frac{6\mu}{\lambda}}$ gives the 'vacuum' vet and derive the Lagrangian

b Taking inspiration from Peskin and Schroeder, introduce a set of renormalisation conditions.

c Can all UV divergences be absorbed into field, mass (μ) and coupling constant (λ) redefinitions to the one loop order?