QFT II exercises - sheet 7

If you find a mistake in this exercise first check the website if the problem has been resolved already in a newer version. Otherwise, please email rutger.boels@desy.de.

Exercise 1

Consider

$$S_{int,3} = g f^{abc} \int d^D x \, (\partial_\mu A^a_\nu)(A^{\mu,b} A^{\nu,c})$$

a Show this term is the term with three fields from the Yang-Mills Lagrangian

b Fourier transform all fields in this term and write the result as

$$\propto \delta(p_1 + p_2 + p_3) f_{abc} A^a_\mu(p_1) A^b_\nu(p_2) A^c_\rho(p_3) D^{\mu\nu\rho}$$

for some tensor D.

c show that D can be rewritten using the symmetry properties of the structure constants as

$$D^{\mu\nu\rho} = \frac{g}{6} \left((p_1 - p_3)^\nu \eta^{\mu\rho} + (\text{cyclic}) \right)$$

Exercise 2

The BRST operator Q can be defined by its action on all the fields:

$$\begin{aligned} Q\, A^{\mu,a} &= (D_\mu c)^a \\ Q\, \psi &= i g c^a t^a \psi \\ Q\, c^a &= -\frac{1}{2} g f^{abc} c^b c^c \\ Q\, \overline{c}^a &= B^a \\ Q\, B^a &= 0 \end{aligned}$$

where Q anti-commutes with the ghost fields, $\{Q, c\} = 0$.

a Show $Q^2 = 0$ by examining its action on all the fields

b Show

$$B^a \partial_\mu A^{\mu,a} + \frac{\xi}{2} B^a B^a - \overline{c}^a \partial_\mu (D^\mu c)^a = QX$$

and compute X.

c Change X found in b) such that the resulting action has a gauge fixing term $\frac{1}{2\xi}\mathrm{tr}(\partial_\mu A^{a,\mu} t^a + \alpha (A^{a,\mu} t^a)(A^b_\mu t^b))^2$ after integrating out the auxiliary field B. α is a parameter. This gauge is called 'Gervais-Neveu' gauge.

d determine the ghost sector in this gauge.

e determine for which value of α and ξ the Feynman rules of the total action simplify.

f BONUS: the gauge fixing condition is with this choice of parameters not real: argue this is not a problem.

Exercise 3

Compute the fermion self-energy in non-Abelian Yang-Mills theory at the one loop order.