QFT II exercises - sheet 4

If you find a mistake in this exercise first check the website if the problem has been resolved already in a newer version. Otherwise, please email rutger.boels@desy.de.

This set of exercises is to be handed in on Monday 12th of May, at the start of the lecture in order to qualify for the bonus. Email submission (of scanned pages) will also be accepted.

Exercise 1

Consider adding a so-called Fermi interaction term to the free Dirac Lagrangian,

$$\mathcal{L} = \overline{\psi}(i\partial\!\!\!/ - m)\psi + G(\overline{\psi}\psi)^2$$

a Compute the superficial degree of ultraviolet divergence for Feynman graphs of this theory as a function of number of external fermion lines and number of vertices directly from the Feynman graphs as well as from dimensional analysis. (The superficial degree of divergence is the 'D' in Λ^D when using a momentum cutoff Λ).

b Argue that this theory is non-powercounting-renormalizable in four dimensions: it seems to require an infinite number of counter terms. Show that at one loop only a few graphs diverge. Why do these statements not contradict each other?

c Show that in two dimensions the theory is power-counting renormalizable.

Non-powercounting-renormalizability is a sign the theory needs essential new information to be well-defined in the UV.

d Argue that in the limit that the mass of the scalar becomes much larger than the scale Λ, Yukawa theory leads through renormalisation group flow to a Fermi-type interaction term. Make this explicit to leading order in perturbation theory and show the Fermi-coupling constant is a natural combination of m_ϕ and the Yukawa coupling λ

e Show Yukawa-theory is power counting renormalizable.

f BONUS: Argue that calculating any one-loop correction in the effective theory (free Dirac Lagrangian with a Fermi-interaction term) is an approximation to a corresponding one-loop correction in the full Yukawa-theory, in the limit the scalar mass is much larger than the typical momentum invariants in the correlation function. Hint: argue some of the diagrams also appear in the renormalisation group flow argument.

Exercise 2

Consider computing the mass renormalisation constant in QED.

a Write down the steps needed to compute this constant to the first non-vanishing order. Don't do any integrals!

b What is the superficial degree of divergence for the two-point correlator of fermionic fields in four dimensions?

c Is this realised in the actual one-loop integral? (Hint: fix an appropriate gauge for the gauge field and focus on the divergent part only)

d Argue that the integrals which saturate the superficial degree of divergence always contain an odd number of loop momenta in the numerator. Use this to argue that the actual degree of divergence for the mass renormalisation constant is logarithmic to all orders in perturbation theory.

Exercise 3

Compute the β function of QED to first non-trivial order. Hint: use the fact that the charge renormalisation constant and the fermion field renormalisation constant are equal by the Ward identity to reduce the problem to the computation of the photon self-energy. Using results from outside sources is allowed (e.g. for an integral), but only with explicit references!