QFT II exercises - sheet 3

If you find a mistake in this exercise first check the website if the problem has been resolved already in a newer version. Otherwise, please email rutger.boels@desy.de.

Exercise 1

a For a ϕ^4 theory compute the one-loop correction to the propagator in dimensional regularisation using renormalised perturbation theory (see e.g Peshkin & Schroeder)

b Add appropriate counter terms and determine δ_Z and δ_m

c For a ϕ^3 theory consider the one-loop correction to the propagator in dimensional regularisation. There are two Feynman graphs: What is wrong with one of them?

d Consider renormalised scalar fields for the ϕ^3 theory, including shifts of the scalar fields. Show the result can be written as

$$\mathcal{L} = \frac{Z_\phi}{2}(\partial_\mu \phi)\partial^\mu - \frac{1}{2} Z_m m^2 \phi^2 + \frac{1}{6} Z_\lambda \lambda \phi^3 + Y\phi$$

e Argue that to first order in perturbation theory (tree level) $Y = 0$.

f Formulate a natural normalisation condition to fix Y (hint: the vev of a single scalar field should be zero) and use it to kill the one loop tadpole. Show this also takes care of the problem in c.

Exercise 2

The bare Lagrangian for the Yukawa theory is given by

$$\mathcal{L} = \overline{\psi}(i\partial\!\!\!/ - m_{f0})\psi + \frac{1}{2}(\partial_\mu \phi)(\partial^\mu \phi) - \frac{1}{2} m_0^2 \phi^2 - g_0 \overline{\psi}\phi\psi$$

a Classify the superficially divergent graphs in this theory. Show that there is one with 4ϕ fields.

b Add a term to the action with a coupling constant λ to absorb the latter divergence. Argue that in general one should always add all terms allowed by dimensional analysis and the renormalisability criterion.

c Express $\mathcal{L}$ in term of renormalised couplings m_f, m, g, λ, renormalised fields ϕ_r, ψ_r and appropriate counter terms (there are six). Hint: first guess the result d Write down a complete set of renormalisation conditions and indicate which counter terms they fix

e Compute explicitly the one-loop corrections of the scalar propagator and determine for an appropriate function Δ the scalar field renormalisation and mass renormalisation constants as

$$\delta_{Z_\phi} = \lim_{d\to 4} \frac{4g^2(d-1)}{(4\pi)^{d/2}} \int_0^1 dx \, \frac{x(1-x)\Gamma(2-d/2)}{(\Delta(p^2=m^2))^{2-\frac{d}{2}}}$$

$$\delta_m = \lim_{d\to 4} \frac{4g^2(d-1)}{(4\pi)^{d/2}} \int_0^1 dx \, \frac{\Gamma(1-d/2)}{(\Delta(p^2=m^2))^{1-\frac{d}{2}}} + m^2\delta_{Z_\phi}$$