QFT II exercises - sheet 1

Exercise 1

a Show
$$\det\left(e^{A}\right) = e^{\mathrm{tr}A}$$
for a symmetric matrix A.

b use the result of a) to compute
$$\frac{\partial}{\partial B_{ij}} \det\left(B\right)$$

c Compute
$$\int dx_1 \dots dx_n e^{-f(x_1,\dots,x_n)}$$
by expanding the real function f around its minimum to second order in x^i

Exercise 2

The so-called generating functional $Z[J]$ for a free scalar field theory is
$$Z[J] = \int \mathcal{D}\phi e^{i\int d^4x(\mathcal{L}_0 + J\phi)} = Z[0]e^{-\frac{1}{2}\int d^4x d^4y J(x) D_F(x-y) J_y}$$
with $\mathcal{L}_0 = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}m^2\phi^2$ and $D_F(x-y) = \mathrm{i}\int \frac{e^{-\mathrm{i}k\cdot(x-y)}}{k^2-m^2}$. It can be used to compute correlation functions.

a Compute by explicit integration as done in wednesday's lecture
$$\langle 0|T\left\{\phi(x_1)\dots\phi(x_2)\right\}|0\rangle = \frac{1}{Z[0]}\int \mathcal{D}\phi \left(\phi(x_1)\dots\phi(x_4)\right) e^{i\int d^4x\, \mathcal{L}_0}$$
in the free scalar field theory.

b Argue that the expression on the right hand side appears at fourth order in the Taylor expansion of the generating functional with respect to J.

c Compute this same quantity by taking four functional derivatives of $Z[J]$ with respect to J and compare the results to those obtained under a).

d Argue that both the direct integration method, as well as the generating functional method predict that all vacuum expectation values of an odd number of fields vanish.