1D-FEL Without Approximations

motivation, capabilities

1D theory → 1D-solver for waves

implementation (without and with Lorentz transformation)

excitation of waves (single particle)

without self effects

one and few particles with self effects

mystery
Motivation, Capabilities

FEL codes use many approximations, as averaged equation of motion, local periodical approximation, EM-field calculation by paraxial approximation, one or several harmonics.

These approximations are questionable for ultra-short bunches or bunches with extreme energy modulation.

It is easy to implement complete FEL effects in 1D.

1D model can be used to verify approximations.

Capabilities of 1D model: ultra broadband (not split into harmonics, excitation and radiation), no local periodic approximation, complete 1D field computation, seems possible: particle = macro particle, LT method can be tested.
1D Theory

**EM Fields**

a) undulator field

b) external wave

c) longitudinal self field

\[ \varepsilon \partial E_z / \partial z = \rho(z,t) - \frac{q}{A} \sum_v \delta(z - z_v) \]

no principle problem, but neglected

d) transverse self field

\[ J_x(z,t) - \frac{q}{A} \sum_{v_x,v} \delta(z - z_v) \]

\[
E_{x,z} = -B_{y,t} \\
-B_{y,z} = \mu J_x + c^2 E_{x,t}
\]

\[
\downarrow
\]

\[ E_x = (L + R)/2 \quad \text{L and R are waves to the left and to the right} \]

\[
\downarrow
\]

\[ R(U + ct,t) = \tilde{R}(U,t) \quad \text{with} \quad \partial_t \tilde{R} = -\varepsilon^{-1} J(U + ct,t) \]

\[ L(V - ct,t) = \tilde{L}(V,t) \quad \partial_t \tilde{L} = -\varepsilon^{-1} J(V - ct,t) \]
waves to the left and to the right
Implementation

Equation of Motion

\[
\frac{d}{dt} \begin{bmatrix} \mathbf{r}_v \\ \mathbf{p}_v \end{bmatrix} = \begin{bmatrix} \mathbf{v}_v \\ \mathbf{f}_v \end{bmatrix}
\]

with

\[
\mathbf{v}_v = \frac{\mathbf{p}_v}{m_0 \gamma} = c \frac{\mathbf{p}_v}{\sqrt{p_v^2 + (m_0 c)^2}}
\]

\[
\mathbf{f}_v = q_0 (\mathbf{E} + \mathbf{v} \times \mathbf{B})
\]

horizontal plane, only x and z components:

\[
\begin{pmatrix} F_x \\ F_z \end{pmatrix}_v = q_0 \begin{pmatrix} E_{x,e} - v_z B_{y,e} \\ v_x B_{y,e} \end{pmatrix}_v + \frac{q_0}{2} \begin{pmatrix} \tilde{R} + \tilde{L} - \beta_z (\tilde{R} - \tilde{L}) \end{pmatrix}_v
\]

Coupled Problem

\[
J_x(z,t) \leftarrow \frac{q}{A} \sum_v v_{x,v} \delta(z - z_v)
\]

\[
\frac{d}{dt} \begin{bmatrix} \mathbf{r}_v \\ \mathbf{p}_v \\ \mathbf{L}(V,t) \\ \mathbf{R}(U,t) \end{bmatrix} = \begin{bmatrix} \mathbf{v}_v \\ \mathbf{f}_v \\ -\varepsilon^{-1} J(V - ct,t) \\ -\varepsilon^{-1} J(U + ct,t) \end{bmatrix}
\]

← binning and smoothing (on equi. mesh) needs spatial resolution

← PDE solver (f.i. RK4) needs time resolution
without Lorentz transformation

integration of PDE (by rk4) needs small time step for left wave

criterion: slip between source and wave $< dz/c$

$\rightarrow dt \approx dz/c$, number of time steps $\sim$ undulator length / photon wavelength !!!

solution a: neglect L, $\max(L) < \approx \frac{\max(R)}{4\gamma^2}$

solution b: the part of the left wave, seen by the bunch, is determined by near interaction; use $J_x(z,t) \approx J_x(z + \bar{v}\tau, t + \tau)$

solution c:

with Lorentz transformation

differenced between length scales are shrunk

huge external fields (from undulator) $\left( \begin{array}{c} E_{x,u} \\ B_{y,u} \end{array} \right)' = \gamma_{LT} \left( \begin{array}{c} -c\beta_{LT} \\ 1 \end{array} \right) B_{y,u}$

same magnitude of left and right wave

it is possible, it is applicable even in 3D!
Excitation of Waves (single particle)

left and right wave

\[ \lambda_w = 6 \text{ cm} \]

\[ \lambda_w = 77 \text{ nm}, 3 \times 77 \text{ nm}, 5 \times 77 \text{ nm} \ldots \]
left and right wave with Lorentz transformation

less power in fwd. direction!

higher harmonics:

\[ m = 2n + 1 \]

\[ JJ = J_n \left( \frac{mK^2}{4 + 2K^2} \right) - J_{n+1} \left( \frac{mK^2}{4 + 2K^2} \right) \]

\[ \lambda_w = 0.67 \text{ mm} \]
Example: Without Self Effects

$B_u = 1T$
$\lambda_u = 3 \text{ cm}$
$\mathcal{E} = 500 \text{ MeV}$
$N_u = 10$
$\downarrow$
$K = 2.80$
$\lambda_w = 77 \text{ nm}$
same parameters, with Lorentz transformation
frame = initial velocity

\[ \frac{\mathcal{E}}{\mathcal{E}_0} = 978 = \gamma_{LT} \]
same parameters, with Lorentz transformation. The frame = average velocity.

\[ \gamma_{av} = \gamma_{LT} = 441 \]
Why are left and right waves asymmetric?
One and Few Particles with Self effects

parameters as before

- particle after undulator

left and right waves

left wave out of window

longitudinal momentum

excitation of waves

pictures from solutions (a) and (b) cannot be distinguished by eye!
parameters as before, with Lorentz transformation to frame = av. velocity

- particle after undulator
parameters as before, with Lorentz transformation to frame = av. velocity

transformation back to lab-frame

direct calculation in lab frame:

2700 time steps

9600 time steps
parameters as before, with Lorentz transformation to frame = av. velocity
two particles, separated by one photon wavelength
parameters as before, with Lorentz transformation to frame = av. velocity two particles, separated by half photon wavelength

\[ \begin{align*}
L' & \quad \text{left wave vs. } z: \\
R' & \quad \text{right wave vs. } z:
\end{align*} \]
Mystery

In the frame “av. undulator velocity” the energy loss to both waves (left and right) is about equal. It seems the effect from both waves to the one-particle dynamic is similar.

In the rest frame the effect of the left wave seems negligible.

What happens if we neglect the left wave in the frame “av. undulator velocity”? 
example as before, with Lorentz transformation to frame = av. velocity

no stimulation of left wave:

$$\frac{d}{dt} \begin{bmatrix} r_v \\ p_v \\ \tilde{L}(V,t) \\ \tilde{R}(U,t) \end{bmatrix} = \begin{bmatrix} v_v \\ f_v \\ 0 \\ -\varepsilon^{-1} J(U + ct, t) \end{bmatrix}$$

for comparison: complete calculation
Why is L negligible in the frame “av. undulator velocity”? 