Start2End Simulations for Micro-Bunching Experiments at FLASH

“reloaded” :-(

19.11.2007

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- Two Slides of Theory...
- A Revised Set Up (thanx to N.G. & V.B.)
- Scans & Evaluation
- A New Candidate...
- Double-Humps
A Simple Purely Longitudinal Model of Micro–Bunching (1)

- long. phasespace $\mathbb{R}^2 : v := (z, p_z)$
- ps–density $\Psi(z, p_z), \int \Psi \, d^2 v = 1$
- (linear!) projection operator $\hat{Q} : 
  \Psi \mapsto \rho := \hat{Q} \Psi = \int \Psi \, dp_z$
- ultra–relativistic $\Rightarrow \rho(z) = \text{const}$, except in BunchCompressor
- cavity, space charge (any long. wake) : KICKS
- all kicks commute $\Leftrightarrow$ cav+SC :
  $(z, p_z) \mapsto (z, p_z + \text{cav}(z) + (g_{sc} \ast \rho)(z))$
  $g_{sc} \ast \rho := \int g_{sc}(z, z')\rho(z')dz'$
- collective kick : $K[\rho] = Id + \Delta[\rho] :$
  $(z, p_z) \mapsto (z, p_z + (g \ast \rho)(z))$
  Property: $K[\rho_1 + \rho_2] = K[\rho_1] + \Delta[\rho_2]$
  with $K^{-1}[\rho_1 + \rho_2] = K^{-1}[\rho_1] - \Delta[\rho_2]$
- BunchCompressor :
  (generalized) DRIFT with $R_{56}/p_0$ as “length”
- FEL w/o undulator := Cascade :
  (ACC$\rightarrow$BC$\rightarrow)^n \Rightarrow$
  $D_n \circ K_n[\rho_{n-1}] \circ \ldots D_1 \circ K_1[\rho_0]$
  ( FLASH : $n = 2$ )
- $\Leftarrow$ all the former maps are measure-preserving !!!

$\Rightarrow \Psi_k = \Psi_{k-1} \circ K_{k}^{-1}[\hat{Q}\Psi_{k-1}] \circ D_k^{-1}$

- $\Leftarrow$ linear operator $M[\rho] :$
  $\Psi \mapsto M[\rho] \Psi := \Psi \circ K^{-1}[\rho] \circ D^{-1}$
  $\Psi_k = M[\hat{Q}\Psi_{k-1}]\Psi_{k-1}$

time–discrete Vlasov system, nonlinear integro-difference-eqn.
A Simple Purely Longitudinal Model of Micro–Bunching (2)

- Now assume we already now
  \[ \Psi_1 := \mathcal{M}[\hat{Q}\Psi_0] \Psi_0 \] (\(\Psi_0\) suff. smooth)
- \(\ldots\) and add a tiny modulation:
  \[ \Psi_0 \to \Psi_0 + \epsilon \Phi_0, \quad \epsilon \ll 1, \quad \int \Phi_0 \, d^2v = 0 \]
  \[ \Rightarrow \quad \tilde{\Psi}_1 := \mathcal{M}[\hat{Q}(\Psi_0 + \epsilon \Phi_0)] (\Psi_0 + \epsilon \Phi_0) \] (\(\epsilon\) is small enough)
  \(\Leftarrow\) **NONLINEAR EVOLUTION!**
  \(\Leftarrow\) can lead to increasing amplitudes for certain wavelengths \(\Rightarrow\) **GAIN**

  can be \(\gg 1 \Rightarrow\) micro–bunching

  **Gain Functions:**

- evolution eqn. (\(\ast\)) can in principle be **completely** studied numerically using so–called 2-D Perron–Frobenius codes
  (for PF see e.g. papers by Bassi, Ellison, Sobol, Venturini, Vogt, Warnock)

- **M. Dohlus:** quasi analytic model of modulation:
  \[ z \mapsto z/\Pi_c + \Re\{a(\delta p_z)e^{ikz}\} + c\delta p_z \]
  \[ p_z \mapsto p_0 + \chi z/\Pi_c + \Re\{b(\delta p_z)e^{ikz}\} + d\delta p_z \]
  with iteration procedure for all parameters for transport through **Cavity**, **BunchCompressor** and **SpaceCharge**

  \(\Leftarrow\) **USED IN THIS STUDY !!!**

- to linear order in \(\epsilon\), (\(\ast\)) gives
  (for smooth \(\Psi_0\) and \(\text{gain} \times \epsilon \ll 1\))
  \[ \tilde{\Psi}_1 = \Psi_1 + \epsilon \Phi_1 + O(\epsilon^2) \]
  with
  \[ \Phi_1 = \mathcal{M}[\hat{Q}\Psi_0] \Phi_0 - (\nabla \Psi_0 \cdot \Delta [\hat{Q}\Phi_0]) eD^{-1} \]

- spectral analysis seems at least possible.
  - treatment of short–wavelength modulations is hardly possible in 6-D collective simulations. However, indications for “micro–bunching” effects exist in S2E simulations
Revised Set Up

S2E–range

- BC2: \( \rho_2 = 1.76, 1.82m \)
  (lattice: \( \rho_2 = 1.62m \))
  \( R^{(2)} = -0.15, -0.14m \)
  (lattice: -0.25m)

- BC3: \( \rho_3 = 5.7m - 7.7m \)
  (lattice: \( \rho_3 = 7.5m \))
  \( R^{(3)} = -0.09m - 0.05m \)
  (lattice: -0.05m)

<table>
<thead>
<tr>
<th>( \phi_1 ) (gun)</th>
<th>( \phi_2 )</th>
<th>( \phi_3 )</th>
<th>( \phi_4 )</th>
<th>( \phi_5 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>–0.55°</td>
<td>…</td>
<td>–90° - –105°</td>
<td>0°</td>
<td>–4°, –5°</td>
</tr>
<tr>
<td>VB!!</td>
<td>ACC1.1</td>
<td>ACC1.2-4</td>
<td>ACC1.5-8</td>
<td>ACC2&amp;3</td>
</tr>
<tr>
<td>fixed</td>
<td>DONE ⇒ –96°</td>
<td>fixed</td>
<td>scan</td>
<td>scan</td>
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</tbody>
</table>

with **long, Gaussian** bunch from cathode
Scanning $\phi_4$, $\phi_5$, $\rho_2$ and $\rho_3$

Goal of S2E Scans:

- $I(z)$ moderately large over sufficiently large length
- ...separated from spike!
- transv. ps: not first priority
- check $\mu$–bunching gain (model & spread–sheet by M.Dohlus)

Evaluation:

- scan of $\phi_2$ (see talk from 24.09.07)
  - not affected by revised setup $\Rightarrow$ $\phi_2 = -96^\circ$
- for different choices of $\phi_4$ and $\rho_2$
- look at length scales supporting various currents as function of $\phi_5$ and $\rho_3$

EXAMPLE: $\phi_2 = -98^\circ$, $\rho_3 = 3.8 \text{m (after BC3)}$
fract. of bunch with I > 300A excl. peak (after BC3)
\( \phi_{5,8} = 4\text{deg} \Theta_2 = 16\text{deg} \)

fract. of bunch with I > 300A excl. peak (after BC3)
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fract. of bunch with I > 300A excl. peak (after BC3)
\( \phi_{5,8} = 5\text{deg} \Theta_2 = 16.5\text{deg} \)
fract. of bunch with $I > 400\,\text{A}$ excl. peak (after BC3)

$\phi_{3.8}=4\,\text{deg}$ $\Theta_2=16\,\text{deg}$

$L_{>400\,\text{A}} \geq 0.1\,\text{mm}$
$L_{>400\,\text{A}} < 0.1\,\text{mm}$

fract. of bunch with $I > 400\,\text{A}$ excl. peak (after BC3)

$\phi_{3.8}=5\,\text{deg}$ $\Theta_2=16.5\,\text{deg}$

$L_{>400\,\text{A}} \geq 0.1\,\text{mm}$
$L_{>400\,\text{A}} < 0.1\,\text{mm}$
averaged (I>200A) normalized slice-emittance (after BC3)
\[ \phi_{1,8}^\text{deg} = 4\text{deg} \quad \Theta_2 = 16\text{deg} \]

\[ \epsilon^\text{n,sy}_\gamma^{\mu m} \]

\[ \phi_{1,8}^\text{deg} = 5\text{deg} \quad \Theta_2 = 16.5\text{deg} \]

\[ \epsilon^\text{n,sy}_\gamma^{\mu m} \]
A New Candidate...

After BC3: $\phi_2=-96^\circ$, $\phi_4=-4^\circ$, $\rho_2=1.76\text{ m}$, $\rho_3=6.2\text{ m}$

200x200 bins / smooth $\sigma_{\text{bin}}=1.5$
After BC3: $\phi_2=-96^\circ$, $\phi_4=-4^\circ$, $\rho_2=1.76m$, $\rho_3=6.2m$

200x200 bins / smooth $\sigma_{bin}=1.5$
After BC3: $\phi_2=-96\text{deg}$ $\phi_4=-4\text{deg}$ $\rho_2=1.76\text{m}$ $\phi_5=-15\text{deg}$ $\rho_3=6.2\text{m}$

tot 200 const-len bins (smooth 1.5)/ suppr <500 part

\[
\langle P_z \rangle +/- \sigma_{pz}
\]

\[I(z)\]
After BC3: $\phi_2 = -96^\circ$, $\phi_4 = -4^\circ$, $\rho_2 = 1.76\,\text{m}$, $\phi_5 = -15^\circ$, $\rho_3 = 6.2\,\text{m}$

tot 200 const-len bins (smooth 1.5) suppr <500 part
Before BC2: $\phi_2=-96\text{deg} \; \phi_4=-4\text{deg}$

10 sub-ensembles / 500 part/bin (smooth 1.5)
After BC2: $\phi_2 = -96\text{deg}$, $\phi_4 = -4\text{deg}$, $\rho_2 = 1.76\text{m}$

10 sub-ensembles / 500 part/bin (smooth 1.5)
After BC3: $\phi_2=-96^\circ$, $\phi_4=-4^\circ$, $\rho_2=1.76\text{m}$, $\phi_5=-15^\circ$, $\rho_3=6.2\text{m}$

10 sub-ensembles / 500 part/bin (smooth 1.5)
Before and after BC2 and after BC3
sub-ensembles 5-7 / 500 part/bin (smooth 1.5)
Gain Curve: $\Phi_{1-8}^{2-3} = -15^\circ$ / SubEnsemble = 5 $\Rightarrow C^{bc2} = 2.3$, $C^{bc3} = 2.6$
Gain Curve: $\Phi_{1-8}^{2-3} = -15^\circ$ / SubEnsemble = 6 $\Rightarrow C^{bc2} = 2.1$, $C^{bc3} = 2.25$
Gain Curve: $\Phi_{1-8}^{2-3} = -15^\circ$ / SubEnsemble $= 7 \Rightarrow C^{bc2} = 2.0, C^{bc3} = 1.77$
Gain Curve: $\Phi_{1-8}^{2-3} = 0^\circ / \text{SubEnsemble} = 5 \Rightarrow C^{bc2} = 2.3, C^{bc3} = 1.39$
Gain Curve: $\Phi_{1-8}^{2-3} = 0^\circ$ / SubEnsemble = 6 $\Rightarrow C^{bc2} = 2.1$, $C^{bc3} = 1.10$
Gain Curve: $\Phi_{1-8}^{2-3} = 0^\circ$ / SubEnsemble = 7 $\Rightarrow C^{bc2} = 2.0$, $C^{bc3} = 1.06$
Is This Micro–Bunching in S2E-Simulations?

After BC3 (500 prt/bin) / $\phi_{5-8}^1 = -4\text{deg}$ $\rho_2 = 1.765\text{m}$ $\rho_3 = 6.2$

- $\phi_{2-3} = -10\text{deg}$
- $\phi_{2-3} = -15\text{deg}$
Double–Humped Densities

At Gun-Cathode: 2 Gaussians, $3\sigma$ separated
200x200 const bins (smooth 1.5)

After BC3: 2 Gaussians, $3\sigma$ separated
200x200 const bins (smooth 1.5)
After BC3: 2 Gaussians, $3\sigma$ separated
10 sub-ensembles / 500 part/bin (smooth 1.5)
Summary

- Proposed set of parameters

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<td>corr. chirp</td>
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⇒ decent $z$–region with high current outside spike
50$\mu$m with $I > 500$A, 220$\mu$m with $I > 300$A

⇒ decent transverse phase space

⇒ no strong mixing in high–$I$ region

⇒ estimated gain $> 1 \cdot 10^4$ for $10\mu$m $< \lambda < 100\mu$m, $> 1 \cdot 10^3$ for $\lambda < 200\mu$m

- proposed $\mu$–bunching “switch”: vary $\phi_5$ from $0^\circ$ to $-15^\circ$

- possibly try double–humped initial densities from cathode