Dispersion–Free Steering (→ BBA)

for the SASE Undulators of the XFEL

(Work in Progress !)

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Mathias Vogt (DESY–MPY)
lots of input from: W.Decking, P.Castro, B.Faatz,…

• Why BBA
• What the Heck is Dispersion–Free Steering ?
• The Model
• Preliminary Results
**Orbit Requirements for the SASE Process**

- Resonant interaction of charged particle and undulator radiation
  \[\Rightarrow\text{Particle orbit and radiation cone } (\sim 1/\gamma) \text{ must overlap}\]
- Beam orbit excursion in undulator \(\ll\) rms beam envelope
  \[\Rightarrow\text{longitudinal scale } \sim\text{ gain length}\]

**BAD ORBIT:**
- Strong orbit fluctuations
  \[\Rightarrow\text{overlap only over short ranges } \ll\text{ radiation length}\]
  \[\Rightarrow\text{weak (or no) SASE signal}\]

**GOOD ORBIT:**
- Flat orbit
  \[\Rightarrow\text{overlap only over most of undulator } >\text{ radiation length}\]
  \[\Rightarrow\text{potentially: “saturation”}\]
XFEL Undulator SASE-1

SASE-1 and half of T4

- misaligned quads
  ⇒ perturbed orbit
- initial quad misalignment & BPM offsets ≈ 300μm (?)
  ⇒ beam-based-alignment (=BBA) necessary
- in SASE-1: high resolution cavity BPMs:
  res → 1μm — 3μm
- correctors ≡ quad movers
- in T4: most likely only:
  res → 20μm — 50μm

Target for orbit: \textbf{rms (over ∼20m)} < 3μm

⇒ BBA will be tricky (mainly due to large unknown offsets)

- Option-1: try Dispersion–Free Steering
  (dispersion measurement only needs a difference orbit!)
(What the heck is) Dispersion–Free Steering

- Orbit (= dipole) kicks create (spurious) dispersion

+ given $N$ perturbations
  (= correctors) $\{K_i\}_{1 \leq i \leq N}$
  and $M$ BPMs

+ yields
  $M$ measured orbits
  $\{X_i\}_{1 \leq i \leq M}$
  + $M$ measured dispersions
  $\{D_i\}_{1 \leq i \leq M}$

+ measured $\vec{X}$ ← offset +
  statistical fluctuations

+ measured $\vec{D}$ ← statistical fluctuations only

→ causality in beam line: each upper right $\rightarrow 0$

→ $2M$ conditions for $N$ corrector settings $\Rightarrow$

→ overdetermined system:
  w/o errors $\rightarrow$ conditions linearly dependent
  w/ errors $\rightarrow$ least squares solution $\rightarrow$ SVD
Dispersion–Free Steering (2)

- Introduce weight $w$
  
  $$(0 \rightarrow \text{orbit-only}, \quad 1 \rightarrow \text{dispersion-only})$$
  
  $$(\begin{pmatrix} (1 - w)\vec{X} \\ w\vec{D} \end{pmatrix}) = (\begin{pmatrix} (1 - w)\vec{Q} \\ w\vec{D} \end{pmatrix}) \vec{K}$$
  
  or shorthand:
  
  $\vec{\Xi}(w) = A(w)\vec{K}$

$\vec{\Xi} \in \mathbb{R}^{2M}$ := “real” orbit/dispersion,

$A \in \mathbb{R}^{2N \times M}$ :=
combined orbit dispersion response matrix

- $i$-th Measurement: add systematic (const $\vec{C}$) and statistical ($\vec{S}_i$) errors
  
  $\vec{\xi}_i(w) = A(w)\vec{K}_i + \vec{C} + \vec{S}_i$

- and iterate corrected dipole kicks $\rightarrow \vec{\Phi}_i$
  
  with error $\rightarrow \vec{\Delta}_i$

  $\vec{K}_i = \vec{K}_{i-1} - \vec{\Phi}_i - \vec{\Delta}_i$

How to compute $\vec{\Phi}_i$?

- assuming NO orbit/dispersion from upstream SASE-1!
  
  iff $\vec{C} \equiv \vec{S}_i \equiv \vec{\Delta}_i \equiv 0 \forall i$

  (& assuming $A$ is completely known)
  
  $\Rightarrow \vec{\xi} \equiv \vec{\Xi} = A\vec{K}$ is fully redundant, i.e.
  
  $\exists A^* \in \mathbb{R}^{M \times 2N}$ with $\vec{K} \equiv \vec{\Phi} := A^*\vec{\Xi}$

- The “pseudo–inverse” $A^*$ can be computed using a Singular Value Decomposition (SVD)

- In fact SVD + “$\tau$–regularization” allow some control over correcting the highly correlated (= potentially “real”) orbit/dispn. components rather than the weakly correlated (= contaminated) components

$\Rightarrow \ldots$
SVD $+$ for DispFree Steering

$$A = U \text{ diag}(\{\sigma_k\}) V^T$$

- for non-degenerate phase advances $\Rightarrow A$ has full rank
  $\iff \sigma_k > 0 \ \forall k$

  $\Rightarrow A^* := V \text{ diag}(\{\sigma_k^{-1}\}) U^T$

- if system is underdetermined

  $\Rightarrow$ solution of $\vec{\Xi} = A \vec{K}$ is
  $\vec{K} \in \vec{K}_{\text{part}} + \text{ker}(A)$

  $\Rightarrow$ SVD gives “minimal” solution: $\|A^* \vec{\Xi}\|_2 = \min$

- if system is overdetermined $\Rightarrow$ solution $\exists$ only in the
  “least square” sense

  $\Rightarrow$ SVD yields solution with minimal residue:
  $\|\vec{\Xi} - A (A^* \vec{\Xi})\|_2 = \min$
\( \tau \)-regularization for DispFree Steering

- **What if some** \( \sigma_i = 0 \) ???
- \( \rightarrow \) just redefine \( \mathbf{A}^* := \mathbf{V} \text{diag}(\{(\sigma_k > 0)^{-1}, 0 \ldots\}) \mathbf{U}^T \)
- \( \Rightarrow \) yields least square solution !

- **MORE GENERAL**: condition of \( \mathbf{A} \): \( \text{cond}(\mathbf{A}) := \frac{\max_i \{\sigma_i\}}{\min_i, \sigma_i > 0 \{\sigma_i\}} \)
  - \( \rightarrow \) large cond means that solutions \( \mathbf{K} \) of linear system \( \mathbf{A} \mathbf{K} = \mathbf{E} \) strongly depend on small variations (←errors!) of \( \mathbf{E} \)
  - \( \rightarrow \) to improve (=decrease) condition: set \( \sigma_j \rightarrow 0, \forall \sigma_j < \tau \) with some 
  - **regularization parameter** \( \tau \)
- ... and redefine \( \mathbf{A}^*(\tau) := \mathbf{V} \text{diag}(\{(\sigma_k > \tau)^{-1}, 0 \ldots\}) \mathbf{U}^T \)
- \( \Rightarrow \) for **Dispersion–Free Steering**:
  - \( \Leftrightarrow \) use only highly correlated orbit/dispn modes !!!
  & ignore strongly contaminated orbit/dispn modes !!!
- \( \Rightarrow \) **correct orbit/dispn with**: \( \Phi_i = \mathbf{A}^*(\tau) \mathbf{\tilde{\xi}}_{i-1} \)
Model of BBA for SASE-1

- initial rms quad misalignment: 300μm
- rms BPM–offset: 200μm
- rms BPM statistical error in SASE-1: 1μm
- rms BPM statistical error in T4: 50μm
- rms mover error: 1μm
- “*” means: as a starting point take BPMs in T4 as good as in SASE-1 and no mover errors

- correction method (A): global, variable gain, weight, τ:
  \[ \vec{\Phi}_i = gA^*(w, \tau) \xi_{i-1} \]

- correction method (B): local (l to m), variable gain, weight, const τ = 0:
  \[ \vec{\Phi}_i|_{l,m} = gA^*(w, 0)|_{l,m} \xi_{i-1} \]

- no orbit/dispn from upstream SASE-1
- PERT: 33 misaligned quads in SASE-1
- CORR: 33 quad–movers in SASE-1
- 51 BPMs: 33 in SASE-1 + 18 in T4 upstream dispersive section
- ORM & DRM w.r.t. quad–misalignment ← mad–8 (“1mad”)
- all errors (\(\vec{K}_0, \vec{C}, \vec{S}_i, \vec{\Delta}_i\)):
  independent Gaussian RV
Simulation Parameters (1-st try)

- initial quad misalignment: $\vec{\Delta}_0$-rms: 300$\mu$m
- systematic offsets: $\vec{C}_X$-rms: 200$\mu$m ; $\vec{C}_D$-rms: 0 $\leftrightarrow$ difference orbit!
- resolution: $\vec{S}_i|_X$-rms = $\vec{S}_i|_D$-rms: 1$\mu$m $\Leftrightarrow$ only 3% dp/p acceptance
  $\rightarrow$ multi-shot average to reduce $\vec{S}_i|_D$-rms
- mover errors: $\vec{\Delta}_i = 0$, $i > 0$

**Correction Sequence** v001a :  

<table>
<thead>
<tr>
<th>step</th>
<th>$w$</th>
<th>$I_{\text{max}}^{s.v.}$</th>
<th>$g$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.00</td>
<td>4</td>
<td>1.0</td>
</tr>
<tr>
<td>2</td>
<td>0.80</td>
<td>22</td>
<td>1.0</td>
</tr>
<tr>
<td>3</td>
<td>0.95</td>
<td>22</td>
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</tr>
<tr>
<td>4</td>
<td>1.00</td>
<td>27</td>
<td>1.0</td>
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**Correction Sequence** v003a :  

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<th>step</th>
<th>$w$</th>
<th>$I_{\text{max}}^{s.v.}$</th>
<th>$g$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.000</td>
<td>33*</td>
<td>1.0</td>
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<tr>
<td>2</td>
<td>0.950</td>
<td>33*</td>
<td>1.0</td>
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<tr>
<td>3</td>
<td>0.900</td>
<td>21</td>
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</tr>
<tr>
<td>4</td>
<td>0.999</td>
<td>4</td>
<td>1.0</td>
</tr>
</tbody>
</table>

*: all singular values!
Singular Values for chosen $w / v001a$

Singular Values (CorrSeq v001a)
Finding the Right $I_{\text{max}}^{s,v.}$ for Each Step / v001a orbit

estimated rms-orbit vs. max number of used S.V. / CorrSeq v001a / seed x
Finding the Right $I_{s.v.}^{\text{max}}$ for Each Step / v001a dispn

estimated rms-dispn vs. max number of used S.V. / CorrSeq v001a / seed x
Singular Values for chosen $w / v003a$

Singular Values (CorrSeq v003a)
Finding the Right $I_{\max}^{s,v.}$ for Each Step / v003a orbit

estimated rms-orbit vs. max number of used S.V. / CorrSeq v003a / seed x

$1 \leq I_{\max}^{s,v.} \leq M_{hnm}$
Finding the Right $J_{\text{max}}^{s.v.}$ for Each Step / v003a dispn

estimated rms-dispn vs. max number of used S.V. / CorrSeq v003a / seed x

- $1 \leq J_{\text{max}}^{s.v.} \leq M_{\text{bpm}}$

Graph showing the estimated rms-dispersion/$\mu$m vs. the maximum number of used S.V. for different steps.
Initial Orbits / all seeds

CorrSeq : v001a, v003a

- seed a
- seed b
- seed c
- seed d
- seed e
- seed f
- seed g
- seed h
- seed i
- seed j
- seed k
Result of Correction Sequence v001a

![Graph showing the final orbit in µm for different seeds.](image)

CorrSeq : v001a

- seed a
- seed b
- seed c
- seed d
- seed e
- seed f
- seed g
- seed h
- seed i
- seed j
- seed k
Result of Correction Sequence v001a (BEST)

CorrSeq : v001a

-final orbit / µm

1 <= i <= M_{bpm}
Result of Correction Sequence v003a
Result of Correction Sequence v003a (BEST)

CorrSeq : v003a

final orbit [µm]

1 <= i <= M_{bpm}
A More Realistic Example . . .

CorrSeq v010 (1-st attempt = yesterday!!) :

Parameters:

- initial quad misal. : $\vec{\Delta}_0$-rms : 300\,$\mu$m
- systematic offsets : $\vec{C}_{\vec{X}}$-rms : 300\,$\mu$m
  but $\vec{C}_{\vec{D}}$-rms : 0 $\leftarrow$ difference orbit!
- resolution : $S_{\vec{X}}^{\text{SASE1}}$-rms : 1\,$\mu$m

\[
\begin{array}{ccc|ccc}
\text{step} & \text{range} & w & I_{\text{s.v.}}^{\max} & g \\
1 & 1 - 33 & 0.00 & 5 & 1.0 \\
2 & 1 - 33 & 0.80 & 17 & 1.0 \\
3 & 1 - 33 & 0.95 & 3 & 1.0 \\
4 & 1 - 33 & 0.95 & 22 & 1.0 \\
5 & 1 - 33 & 0.99 & 2 & 1.0 \\
6 & 1 - 33 & 1.00 & 5 & 0.5 \\
7 & 1 - 33 & 1.00 & 5 & 0.5 \\
8 & 1 - 33 & 1.00 & 3 & 0.5 \\
9 & 1 - 10 & 1.00 & 10^* & 0.5 \\
10 & 8 - 17 & 1.00 & 10^* & 0.5 \\
11 & 15 - 24 & 1.00 & 10^* & 0.5 \\
\end{array}
\]

*: all singular values!
All BPMs in T40: 20× worse Resolution (1-st attempt = yesterday!!)

CorrSeq : v010 / seed x

orbit / µm

1 <= i <= M_{bpm}
All BPMs in T40: 20× worse Resolution (1-st attempt = yesterday!!)
All BPMs in T40: 20× worse Resolution (1-st attempt = yesterday!!)

CorrSeq : v010 / seed x
A Slightly More Expensive Example . . .

CorrSeq v020 (2-nd attempt = today!!) :

Parameters :

- initial quad misal. : $\Delta_0$-rms : 300$\mu$m
- systematic offsets : $C\big|_\vec{X}$-rms : 300$\mu$m
  but $C\big|_\vec{D}$-rms : 0 ← difference orbit!

- resolution :
  - $\vec{S}_i\big|_{\text{SASE1} + 1\text{-st 5 in T4}}$-rms : 1$\mu$m
  - $\vec{S}_i\big|_{\text{T4 (rest)}}$-rms : 20$\mu$m
  - $\vec{S}_i\big|_{\text{SASE1} + 1\text{-st 5 in T4}}$-rms : 20$\mu$m
  - $\vec{S}_i\big|_{\text{T4 (rest)}}$-rms : 400$\mu$m

- mover errors : $\Delta_i = 1$, $i > 0$

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<tr>
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</tr>
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<td>8</td>
<td>0.5</td>
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All but 1-st 5 BPMs in T40: 20× worse Resolution (2-nd attempt = today!!)
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All but 1-st 5 BPMs in T40 : 20× worse Resolution \(2\)-nd attempt = today!!

CorrSeq : v020 / seed x

![Graph showing BPM resolution comparison across different steps.](image-url)
TODO:

- Larger parameter space to be scanned (including varying of BPM–distribution)
- $y$–plane !!!! & SASE-2,-3,…
- Include deviations of actual (=unknown) ODRM from design–ODRM (=known)
- Include $x/y$–coupling
- Implement also uniform RVs, etc
- Implement drifts (time domain correlations)
- Include non–linear dispersion into application of the kicks

SUMMARY:

- Work in progress!!
- Even with state of the art diagnostics : orbit constrains for SASE very tight!
- In particular : initial misalignment and BPM–offsets are tough
- Strategy : dispersion–free steering with variable weighting between orbit and dispersion, variable $\tau$ (→strongly vs. weakly correlated modes) and variable gain.
- Result so far : with realistic tolerances and reduced BPM–resolution upstream of the undulators the constraints seem extremely hard to meet!