Dispersion–Free Steering ($\rightarrow$ BBA)

for the SASE Undulators of the XFEL

(Work in Progress !)

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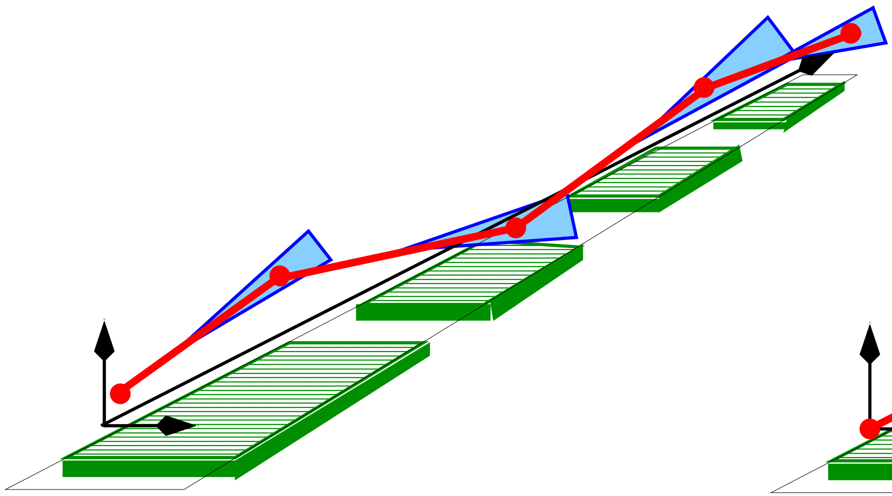
lots of input from: W.Decking, P.Castro, B.Faatz,. . .

- **Why BBA**
- **What the Heck is Dispersion–Free Steering ?**
- **The Model**
- **Preliminary Results**

Orbit Requirements for the SASE Process

- Resonant interaction of charged particle and undulator radiation
  $\Rightarrow$Particle orbit and radiation cone ($\sim 1/\gamma$) must overlap

$\longrightarrow$ Beam orbit excursion in undulator $\ll$ rms beam envelope
$\longrightarrow$longitudinal scale $\sim$ gain length

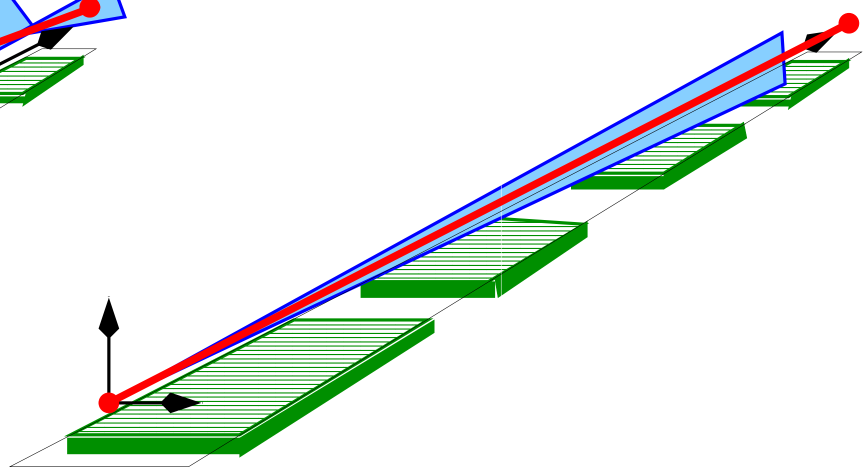


BAD ORBIT :

- Strong orbit fluctuations

$\Rightarrow$ overlap only over short ranges $\ll$ radiation length

$\Rightarrow$ **weak (or no) SASE signal**



GOOD ORBIT :

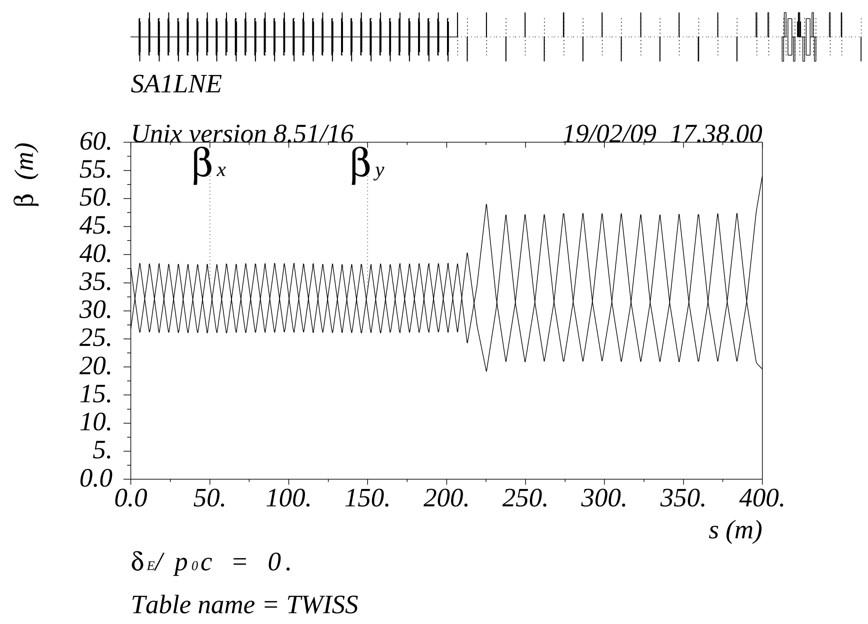
- Flat orbit

$\Rightarrow$ overlap only over most of undulator $>$ radiation length

$\Rightarrow$ **potentially: "'saturation"**

XFEL Undulator SASE-1

SASE-1 and half of T4



- misaligned quads
  ⇒perturbed orbit
- initial quad misalignment & BPM offsets ≈ 300μm (**?**)

⇒ *beam–based–alignment* (=BBA) necessary

- in SASE-1 : high resolution cavity BPMs:
  res⟶ 1μm — 3μm
- **correctors≡quad movers**
- in T4 : most likely only:
  res⟶ 20μm — 50μm

- Target for orbit : **rms (over ∼20m) < 3μm**

⇒ BBA will be tricky (mainly due to large unknown offsets)

- Option–1 : try **Dispersion–Free Steering**
  (dispersion measurement only needs a difference orbit!)

(What the heck is) Dispersion–Free Steering

- Orbit (= dipole) kicks create (spurious) dispersion

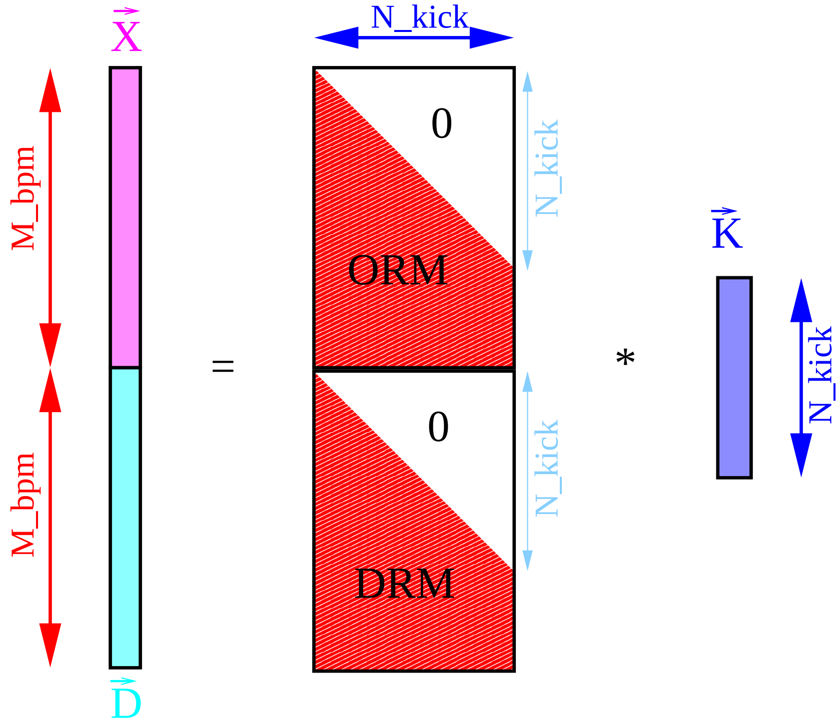
+ **given N perturbations (=correctors)** $\{K_i\}_{1\le i\le N}$ and M **BPMs**

+ **yields** M **measured orbits** $\{X_i\}_{1\le i\le M}$

+ and M **measured dispersions** $\{D_i\}_{1\le i\le M}$

+ **measured** $\vec{X}$ ←**offset** + statistical fluctuations

+ **measured** $\vec{D}$ ←statistical fluctuations **only**



↗ causality in beam line : each upper right → 0

↗ $2M$ conditions for N corrector settings ⇒

↗ **overdetermined system** :
w/o errors → conditions linearly dependent
w/ errors → **least squares solution →SVD**

Dispersion–Free Steering (2)

- Introduce **weight** w
  (**0**$\longrightarrow$ orbit-only, **1**$\longrightarrow$ dispersion-only)

$$\begin{pmatrix} (1-w)\vec{X} \\ w\vec{D} \end{pmatrix} = \begin{pmatrix} (1-w)\underline{\mathcal{O}} \\ w\underline{\mathcal{D}} \end{pmatrix} \vec{K}$$

or shorthand:

$$\vec{\Xi}(w) = \underline{\mathcal{A}}(w) \vec{K}$$

$\nearrow$ $\vec{\Xi} \in \mathbb{R}^{2M}$:= "real" orbit/dispersion,
$\underline{\mathcal{A}} \in \mathbb{R}^{2N\times M}$:=
combined orbit dispersion response matrix

- i**-th Measurement**: add systematic (const $\vec{C}$) and statistical ($\vec{S}_i$) errors

$$\vec{\xi}_i(w) = \underline{\mathcal{A}}(w)\vec{K}_i + \vec{C} + \vec{S}_i$$

- and iterate corrected dipole kicks $\longrightarrow \vec{\Phi}_i$ with error $\longrightarrow \vec{\Delta}_i$

$$\vec{K}_i = \vec{K}_{i-1} - \vec{\Phi}_i - \vec{\Delta}_i$$

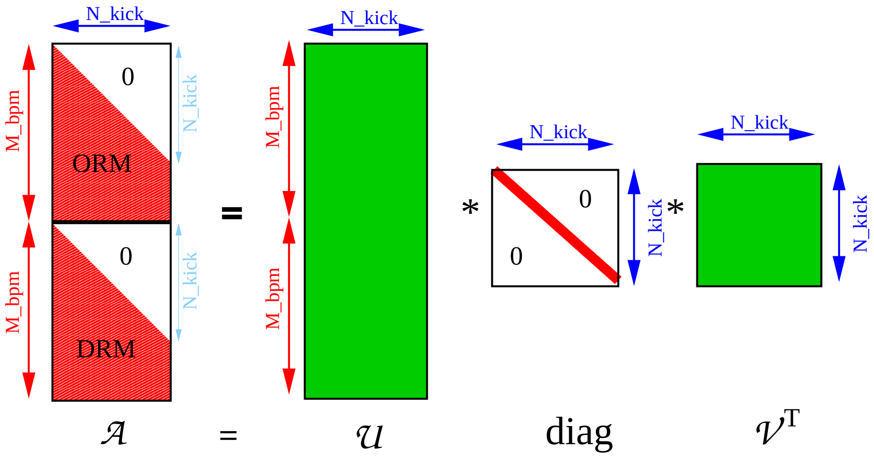
How to compute $\vec{\Phi}_i$?

- **assuming NO orbit/dispersion from upstream SASE-1 !**
- iff $\vec{C} \equiv \vec{S}_i \equiv \vec{\Delta}_i \equiv 0 \ \forall i$
  (& assuming $\underline{\mathcal{A}}$ is completely known)
  $\Rightarrow \vec{\xi} \equiv \vec{\Xi} = \underline{\mathcal{A}}\vec{K}$ is fully redundant, i.e.
  $\exists \underline{\mathcal{A}}^* \in \mathbb{R}^{M\times 2N}$ with $\vec{K} \equiv \vec{\Phi} := \underline{\mathcal{A}}^*\vec{\Xi}$
- The **"pseudo–inverse"** $\underline{\mathcal{A}}^*$ can be computed using a *Singular Value Decomposition* (SVD)
- In fact SVD + "τ–regularization" allow some control over correcting the highly correlated (= potentially "real") orbit/dispn. components rather than the weakly correlated (= contaminated) components

$\Rightarrow$ ...

SVD + for DispFree Steering

$$\underline{\mathcal{A}} \quad = \quad \underline{\mathcal{U}} \quad \underline{\mathrm{diag}}(\{\sigma_k\}) \quad \underline{\mathcal{V}}^{\mathrm{T}}$$



- $\underline{\mathcal{U}} \in \mathbb{R}^{2M\times N}$, $\underline{\mathcal{U}}^{\mathrm{T}}\underline{\mathcal{U}} = \underline{1}_{N\times N}$
  $\rightarrow$ $\underline{\mathcal{U}}^{\mathrm{T}}\vec{\Xi}$:= orthogonal orbit/dispn *mode*
- $\underline{\mathcal{V}} \in \mathbf{O}(N) \rightarrow \underline{\mathcal{V}}^{\mathrm{T}}\vec{K}$:= orth. *knob* for mode
- $\{\sigma_k\}_{1\le k\le N}$, $\sigma_k \ge 0$: **singular values**
  $\rightarrow$ "knob–strengths"

- for non–degenerate phase advances $\Rightarrow \underline{\mathcal{A}}$ has full rank $\Leftrightarrow \sigma_k > 0 \ \forall k$

$\Rightarrow$ $\underline{\mathcal{A}}^* := \underline{\mathcal{V}}\ \underline{\mathrm{diag}}(\{\sigma_k^{-1}\})\ \underline{\mathcal{U}}^{\mathrm{T}}$

- if system is **underdetermined**
  $\Rightarrow$solution of $\vec{\Xi} = \underline{\mathcal{A}}\,\vec{K}$ is $\vec{K} \in \vec{K}_{\mathrm{part}} + \mathrm{kern}(\underline{\mathcal{A}})$

$\Rightarrow$ SVD gives "minimal" solution : $\left\|\underline{\mathcal{A}}^*\,\vec{\Xi}\right\|_2 = \min$

- if system is **overdetermined** $\Rightarrow$solution $\exists$ only in the "least square" sense

$\Rightarrow$ SVD yields solution with minimal residue : $\left\|\vec{\Xi} - \underline{\mathcal{A}}\,(\underline{\mathcal{A}}^*\,\vec{\Xi})\right\|_2 = \min$

τ–regularization for DispFree Steering

- **What if some $\sigma_i = 0$???**

$\rightarrow$ just **redefine** $\underline{\mathcal{A}}^* := \underline{\mathcal{V}}\,\underline{\mathrm{diag}}(\{(\sigma_k > 0)^{-1}, 0\ldots\})\,\underline{\mathcal{U}}^{\mathrm{T}}$

$\Rightarrow$ yields least square solution !

- MORE GENERAL : *condition* of $\underline{\mathcal{A}}$: $\mathrm{cond}(\underline{\mathcal{A}}) := \frac{\max_i\{\sigma_i\}}{\min_{i,\sigma_i>0}\{\sigma_i\}}$

  $\rightarrow$ large cond means that solutions $\vec{K}$ of linear system $\underline{\mathcal{A}}\,\vec{K} = \vec{\Xi}$ strongly depend on small variations ($\leftarrow$errors!) of $\vec{\Xi}$

$\rightarrow$ to improve (=decrease) condition : set $\sigma_j \rightarrow 0,\ \forall \sigma_j < \tau$ with some **regularization parameter** τ

- ... and **redefine** $\underline{\mathcal{A}}^*(\tau) := \underline{\mathcal{V}}\,\underline{\mathrm{diag}}(\{(\sigma_k > \tau)^{-1}, 0\ldots\})\,\underline{\mathcal{U}}^{\mathrm{T}}$

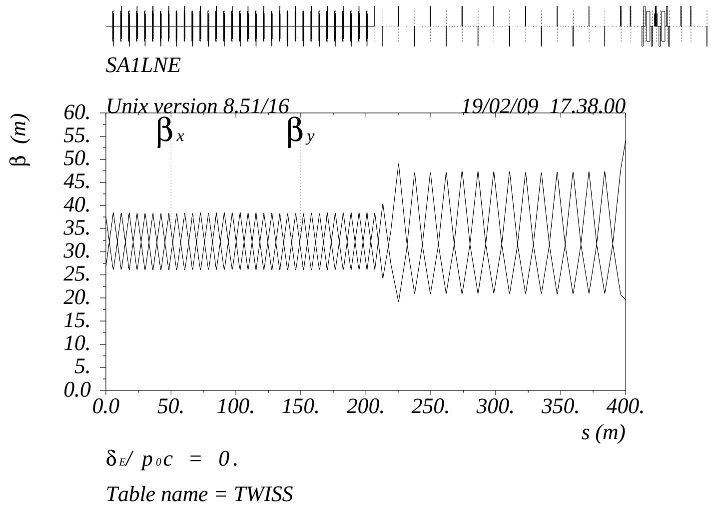
$\Rightarrow$ for **Dipersion–Free Steering :**

$\Leftrightarrow$ **use only highly correlated orbit/dispn modes !!!**

& ignore stronlgy contaminated orbit/dispn modes !!!

$\Rightarrow$ **correct orbit/dispn with:** $\Phi_i = \underline{\mathcal{A}}^*(\tau)\,\vec{\xi}_{i-1}$

Model of BBA for SASE-1



- no orbit/dispn from upstream SASE-1
- PERT: 33 misaligned quads in SASE-1
- CORR: 33 quad–movers in SASE-1
- 51 BPMs : 33 in SASE-1 + 18 in T4 upstream dispersive section
- ORM & DRM w.r.t. quad-misalignment ← `mad-8` (“`lmad`”)
- all errors ($\vec{K}_0$, $\vec{C}$, $\vec{S}_i$, $\vec{\Delta}_i$) : **independent Gaussian RV**

- initial rms quad misalignment : 300μm
- rms BPM–offset : 200μm
- rms BPM statistical error in SASE-1 : 1μm

$*$ rms BPM statistical error in T4 : 50μm

$*$ rms mover error : 1μm

- “$*$” means : **as a starting point** take BPMs in T4 as good as in SASE-1 and no mover errors
- correction method (A) : **global**, variable gain, weight, τ : $\vec{\Phi}_i = g\underline{\mathcal{A}}^*(w,\tau)\,\vec{\xi}_{i-1}$
- correction method (B) : **local** (l to m), variable gain, weight, const $\tau = 0$: $\vec{\Phi}_i\Big|_{l,m} = g\underline{\mathcal{A}}^*(w,0)\Big|_{l,m}\vec{\xi}_{i-1}$

Simulation Parameters (1-st try)

- initial quad misalignment : $\vec{\Delta}_0$-rms : 300μm
- systematic offsets : $\vec{C}\Big|_{\vec{X}}$-rms : 200μm ; $\vec{C}\Big|_{\vec{D}}$-rms : 0 $\longleftarrow$ difference orbit!
- resolution : $\vec{S}_i\Big|_{\vec{X}}$-rms $=\vec{S}_i\Big|_{\vec{D}}$-rms : 1$\mu$m $\Leftarrow$ only 3% dp/p acceptance

  $\longrightarrow$ multi–shot average to reduce $\vec{S}_i\Big|_{\vec{\mathrm{D}}}$-rms
- mover errors : $\vec{\Delta}_i = 0,\ i > 0$

Correction Sequence v001a :

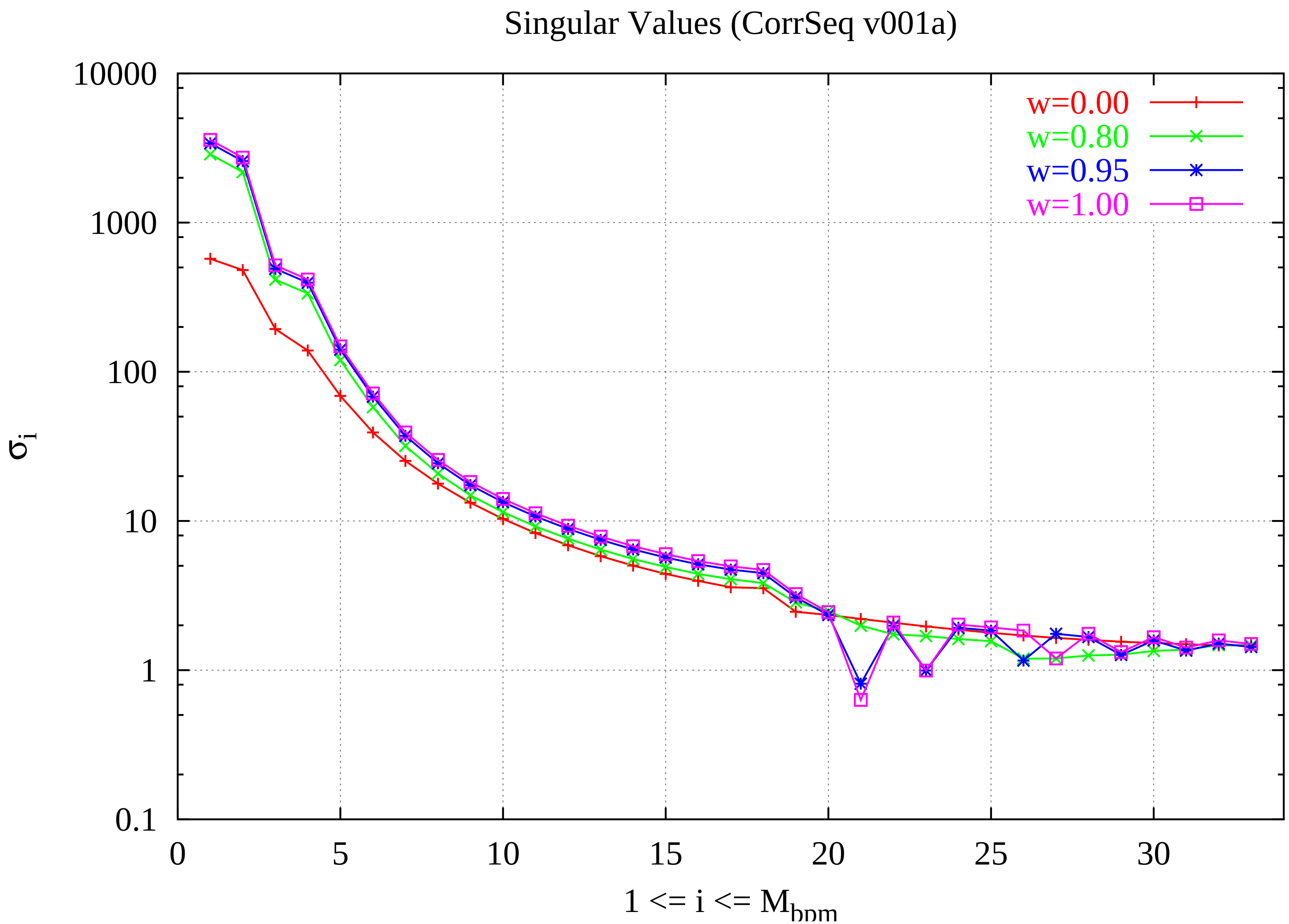
step	w	$I^{\mathrm{s.v.}}_{\mathrm{max}}$	g
1	0.00	4	1.0
2	0.80	22	1.0
3	0.95	22	1.0
4	1.00	27	1.0

Correction Sequence v003a :

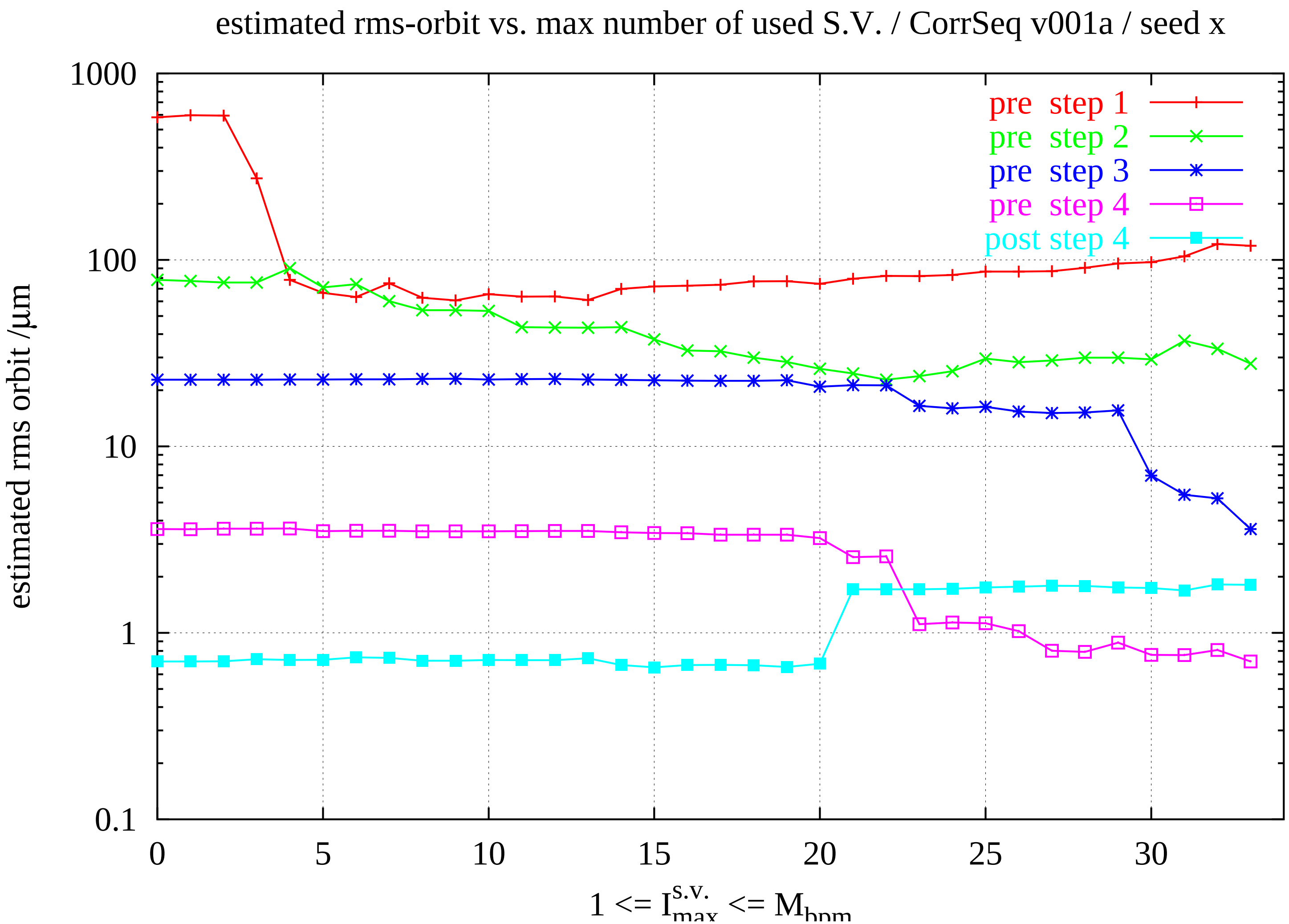
step	w	$I^{\mathrm{s.v.}}_{\mathrm{max}}$	g
1	0.000	33*	1.0
2	0.950	33*	1.0
3	0.900	21	1.0
4	0.999	4	1.0

*: **all** singular values !

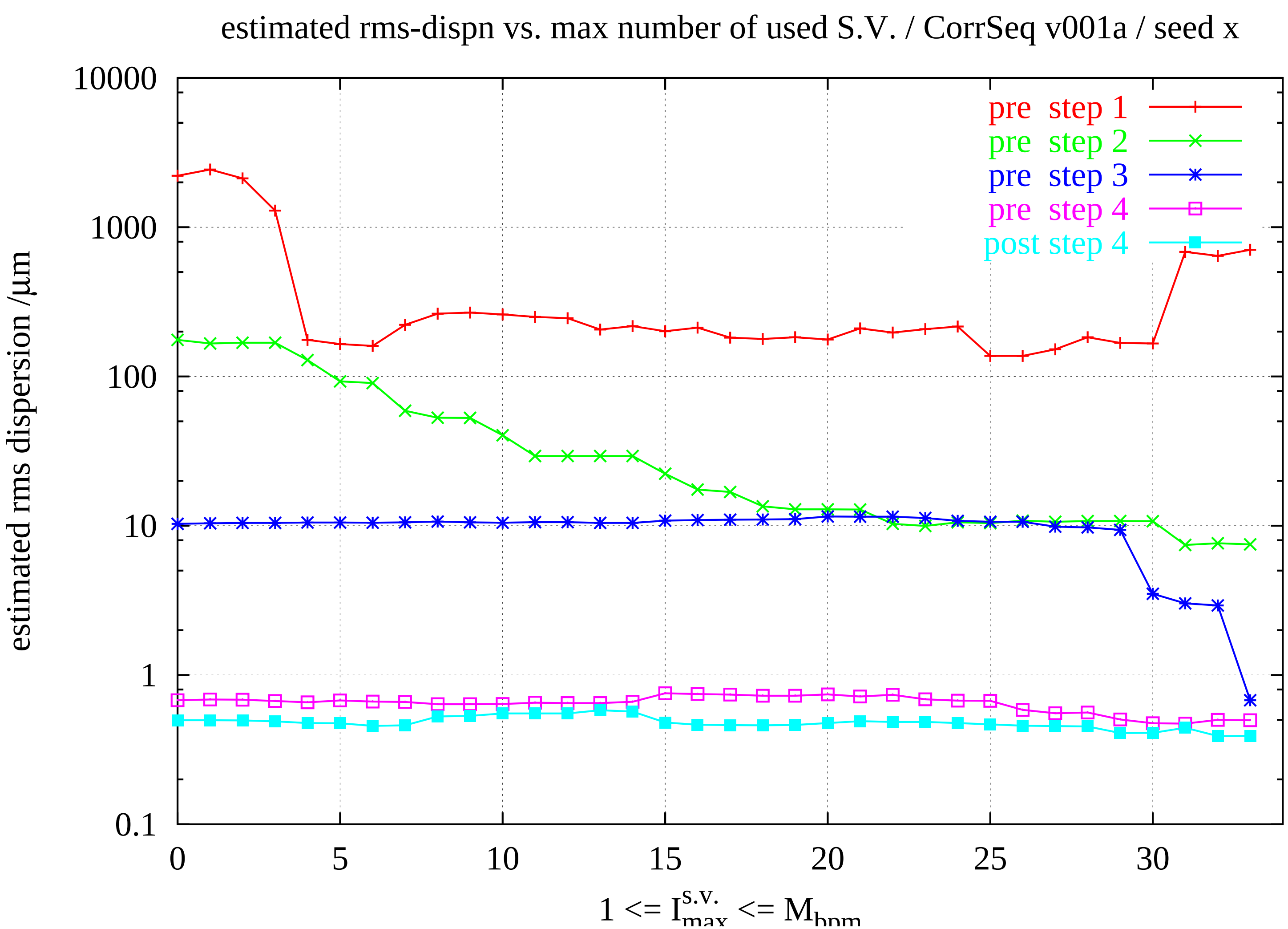
Singular Values for chosen w / **v001a**



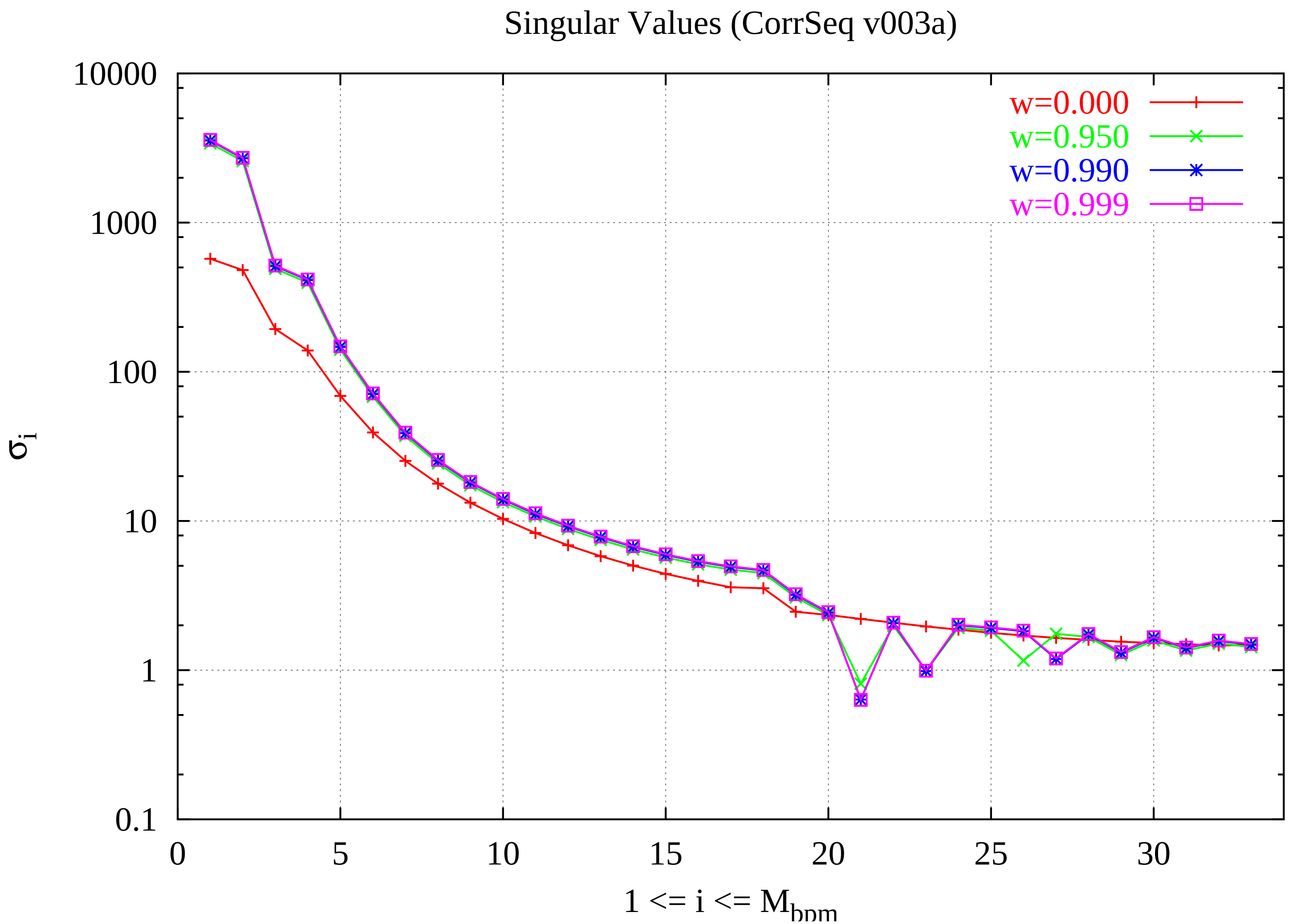
Finding the Right $I^{s.v.}_{\max}$ for Each Step / v001a orbit



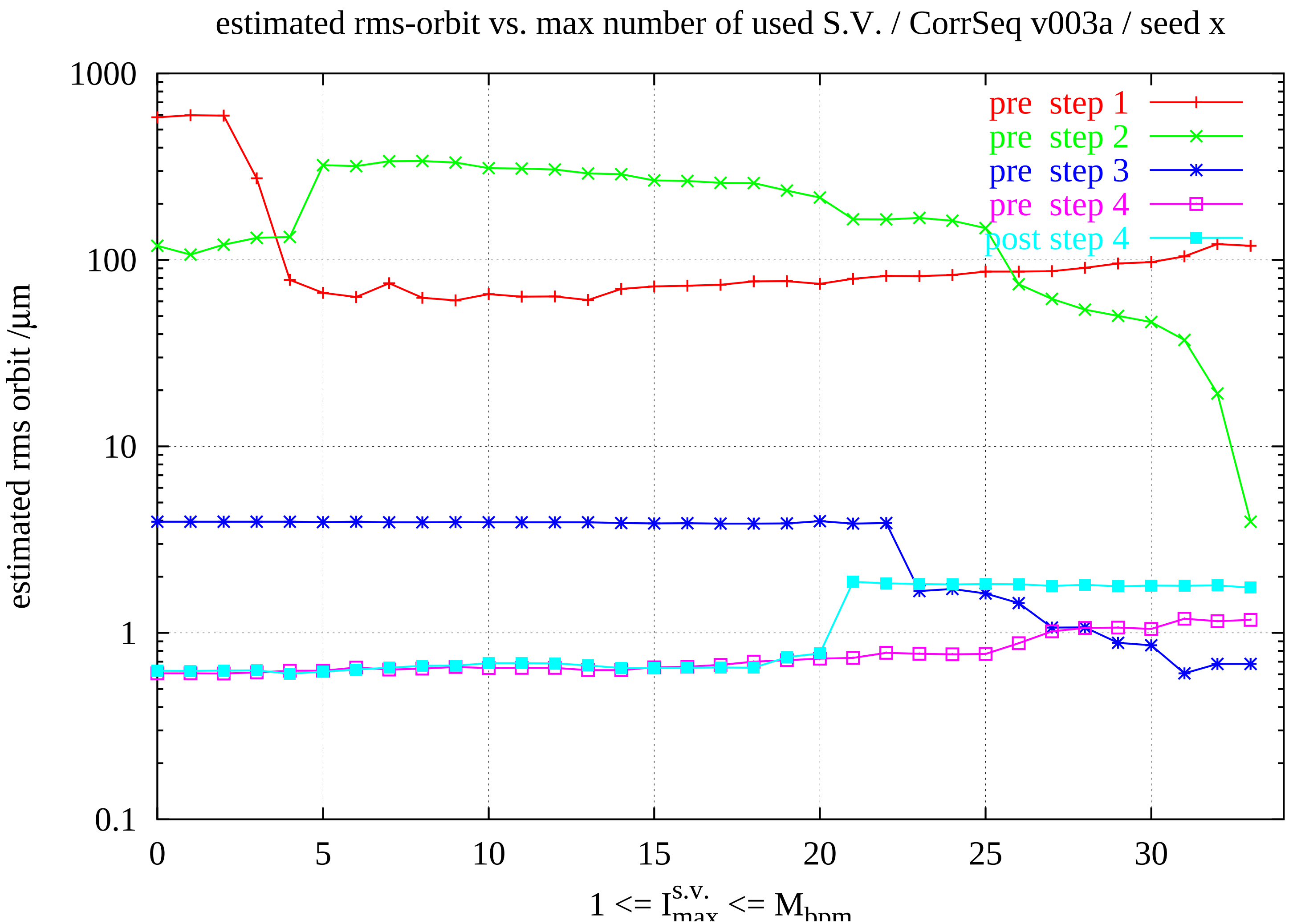
Finding the Right $I^{s.v.}_{\max}$ for Each Step / **v001a dispn**



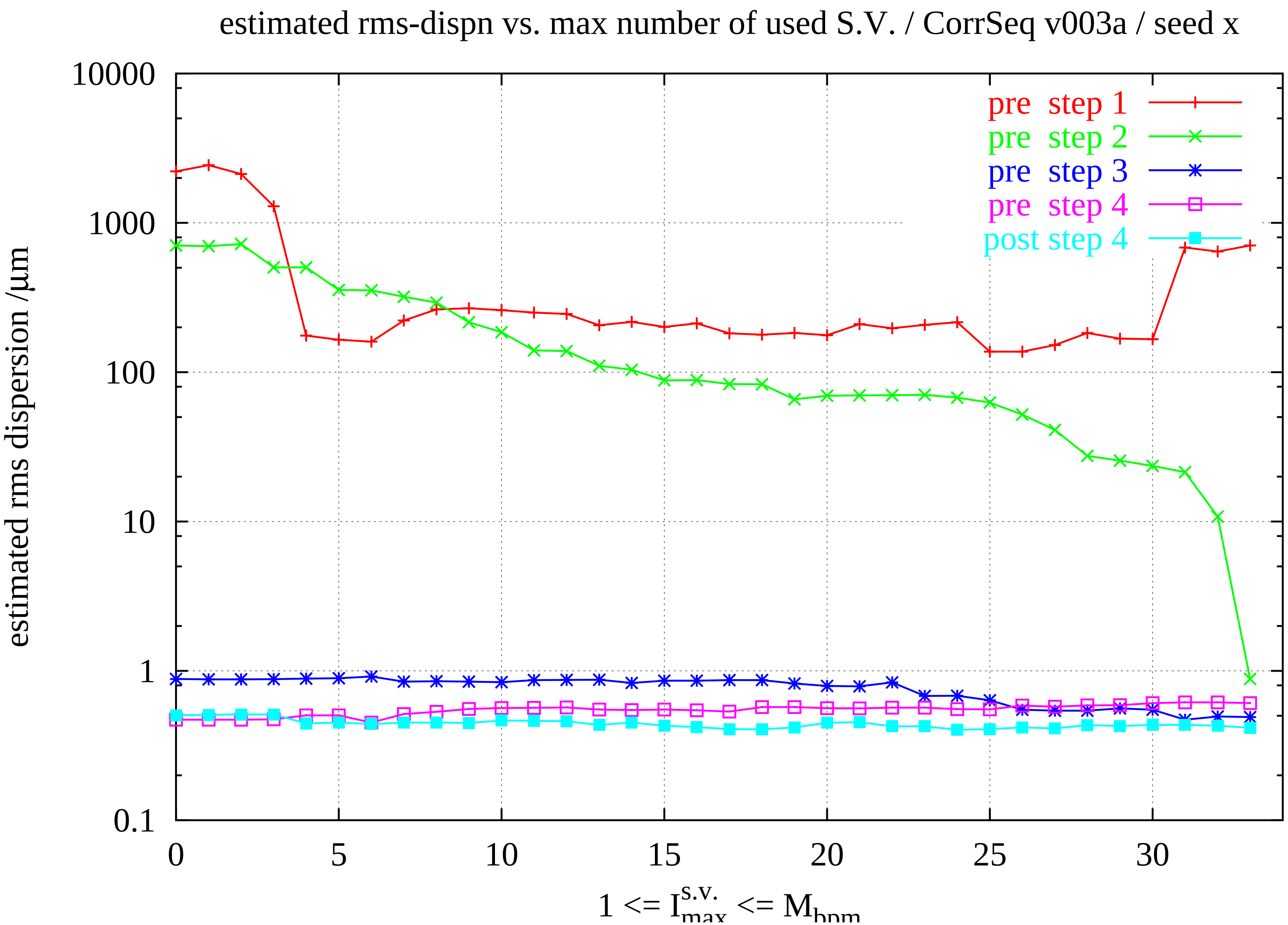
Singular Values for chosen w / **v003a**



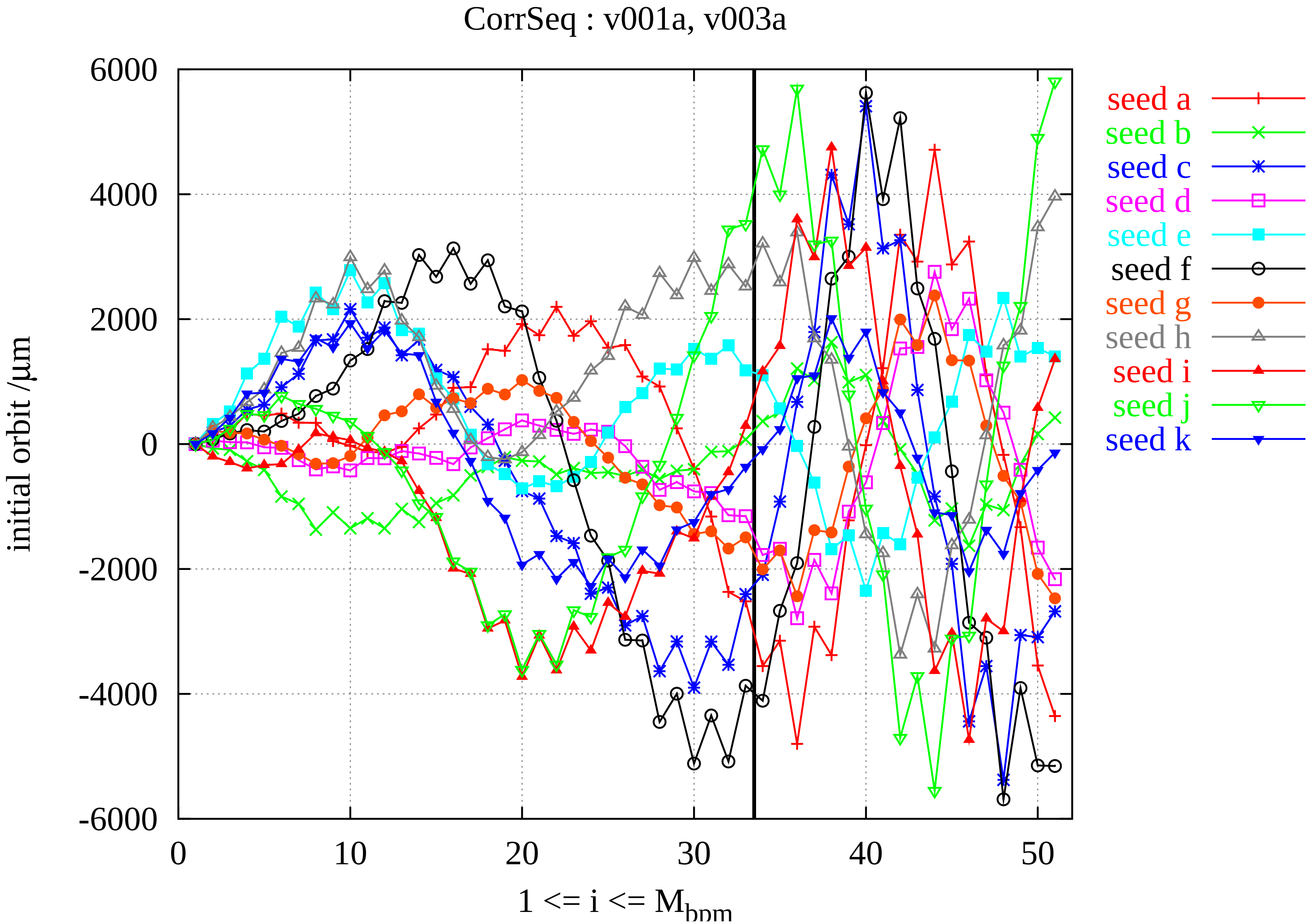
Finding the Right $I^{s.v.}_{\max}$ for Each Step / v003a orbit



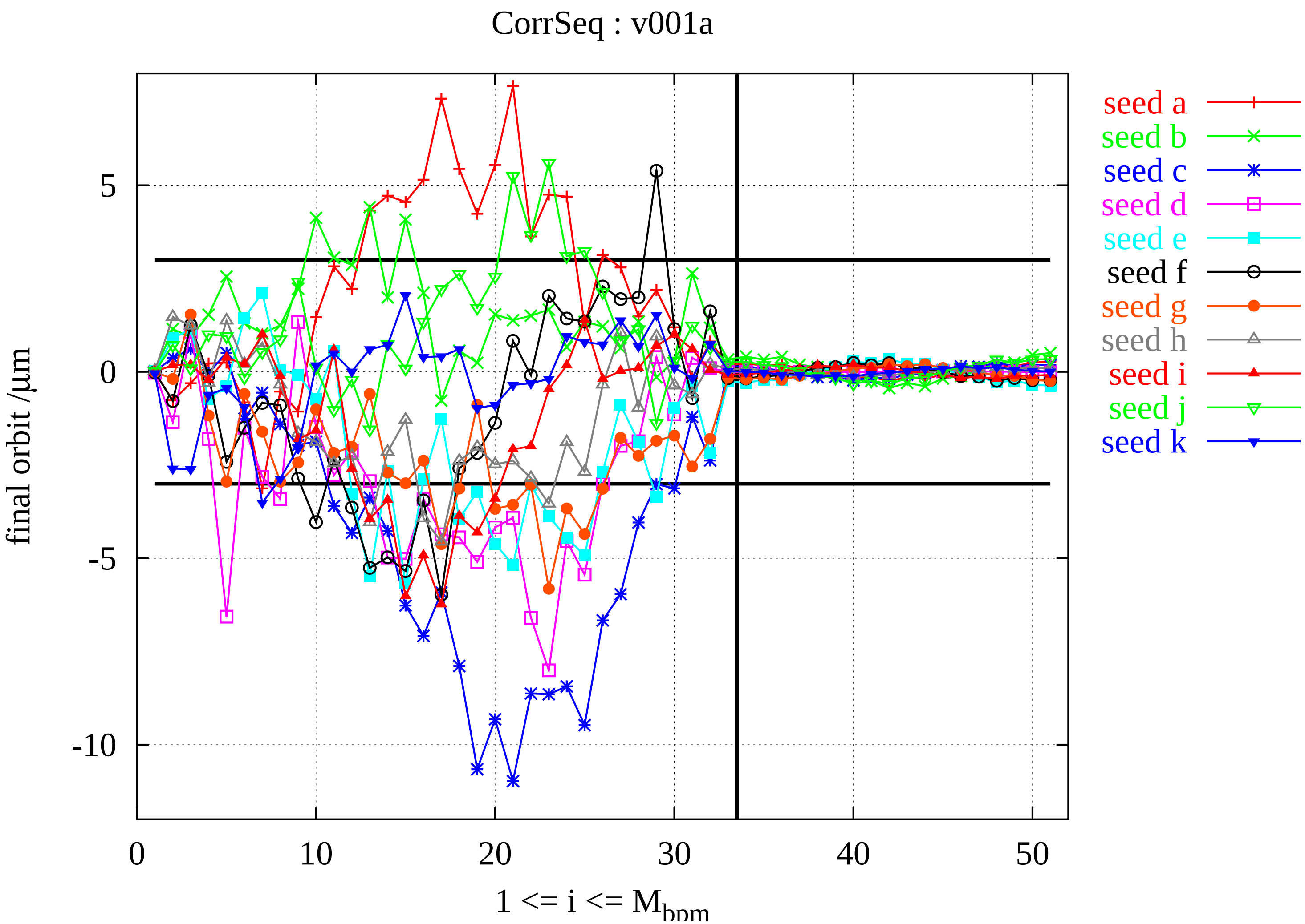
Finding the Right $I^{s.v.}_{\max}$ for Each Step / v003a dispn



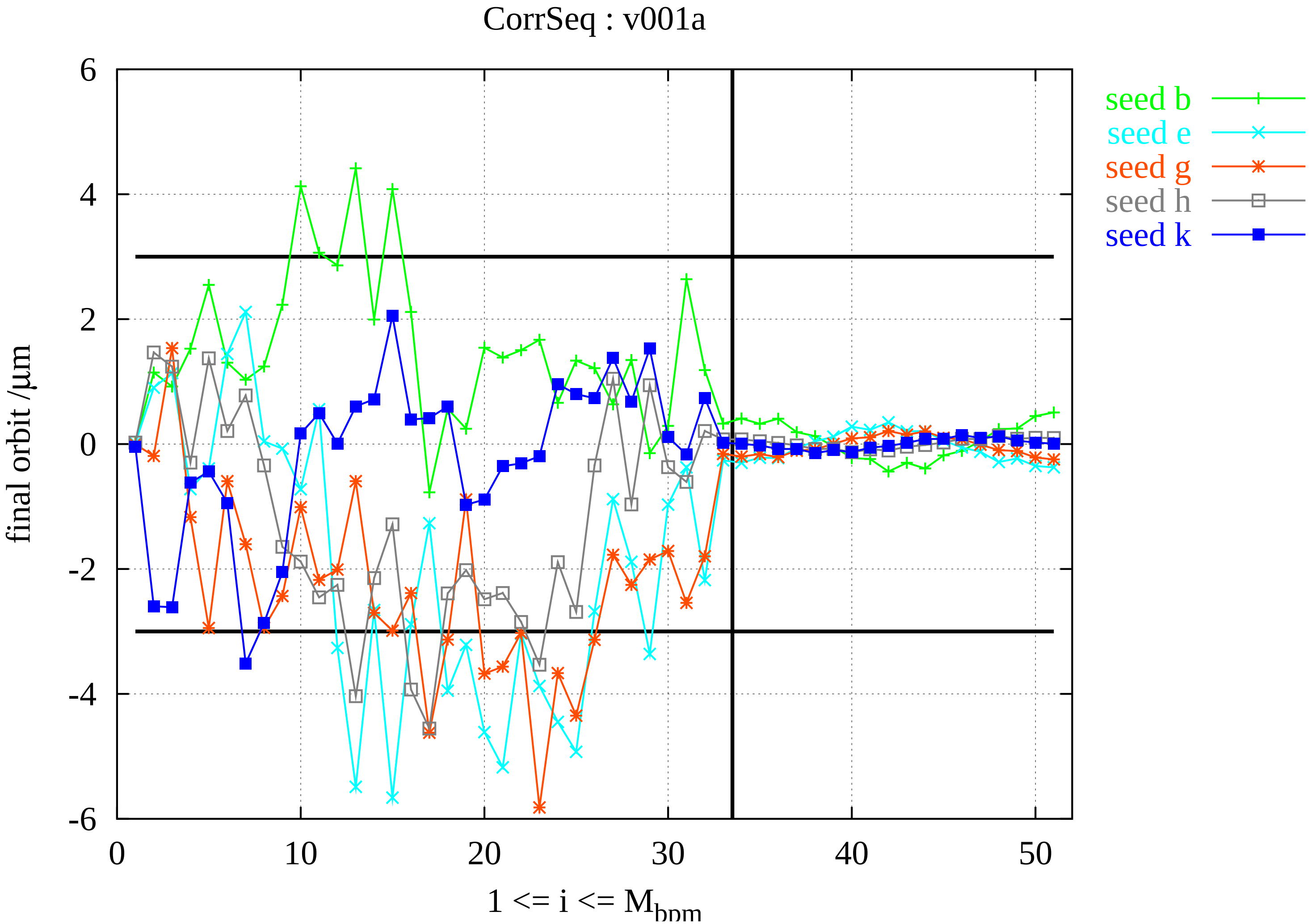
Initial Orbits / **all seeds**



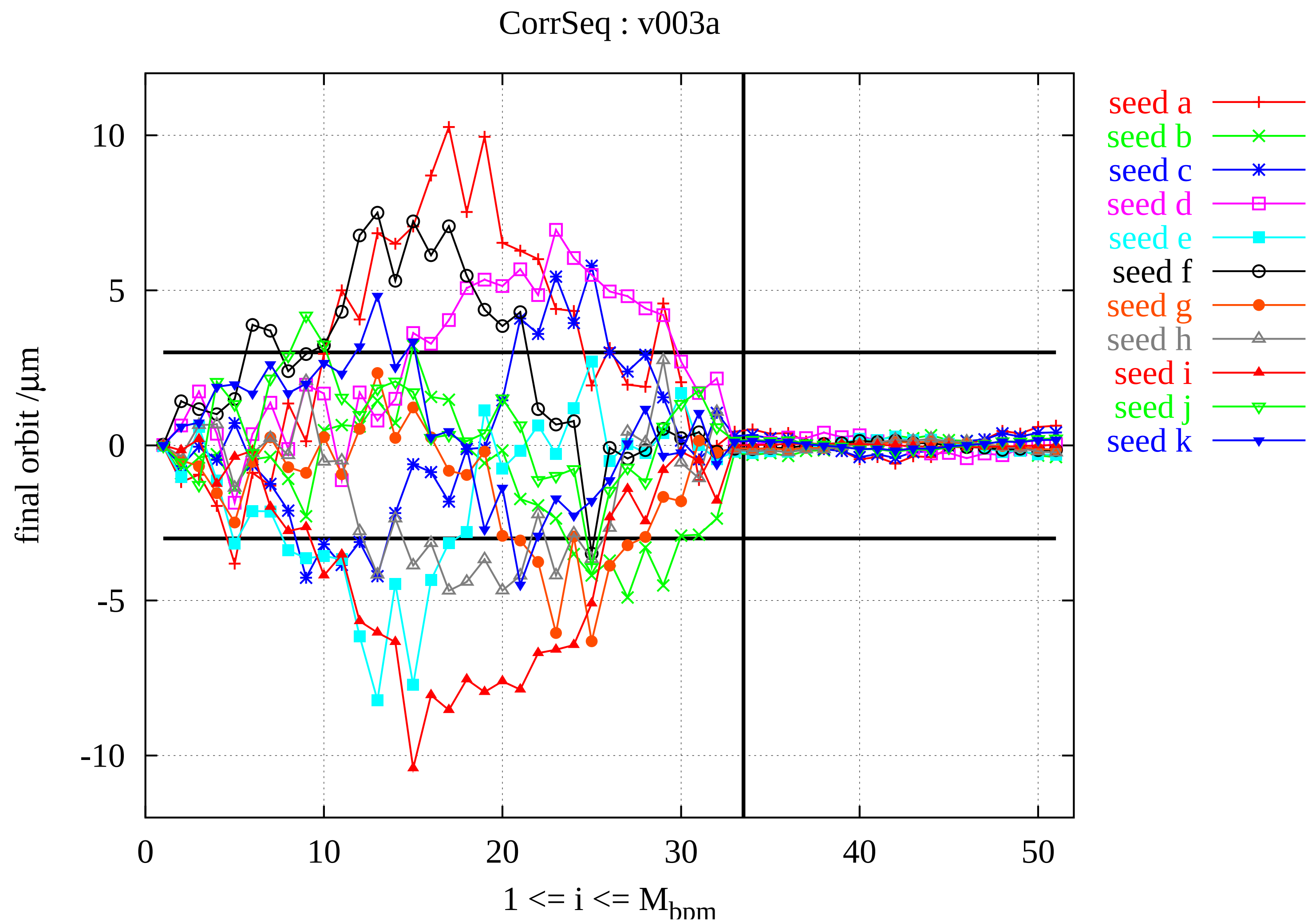
Result of Correction Sequence **v001a**



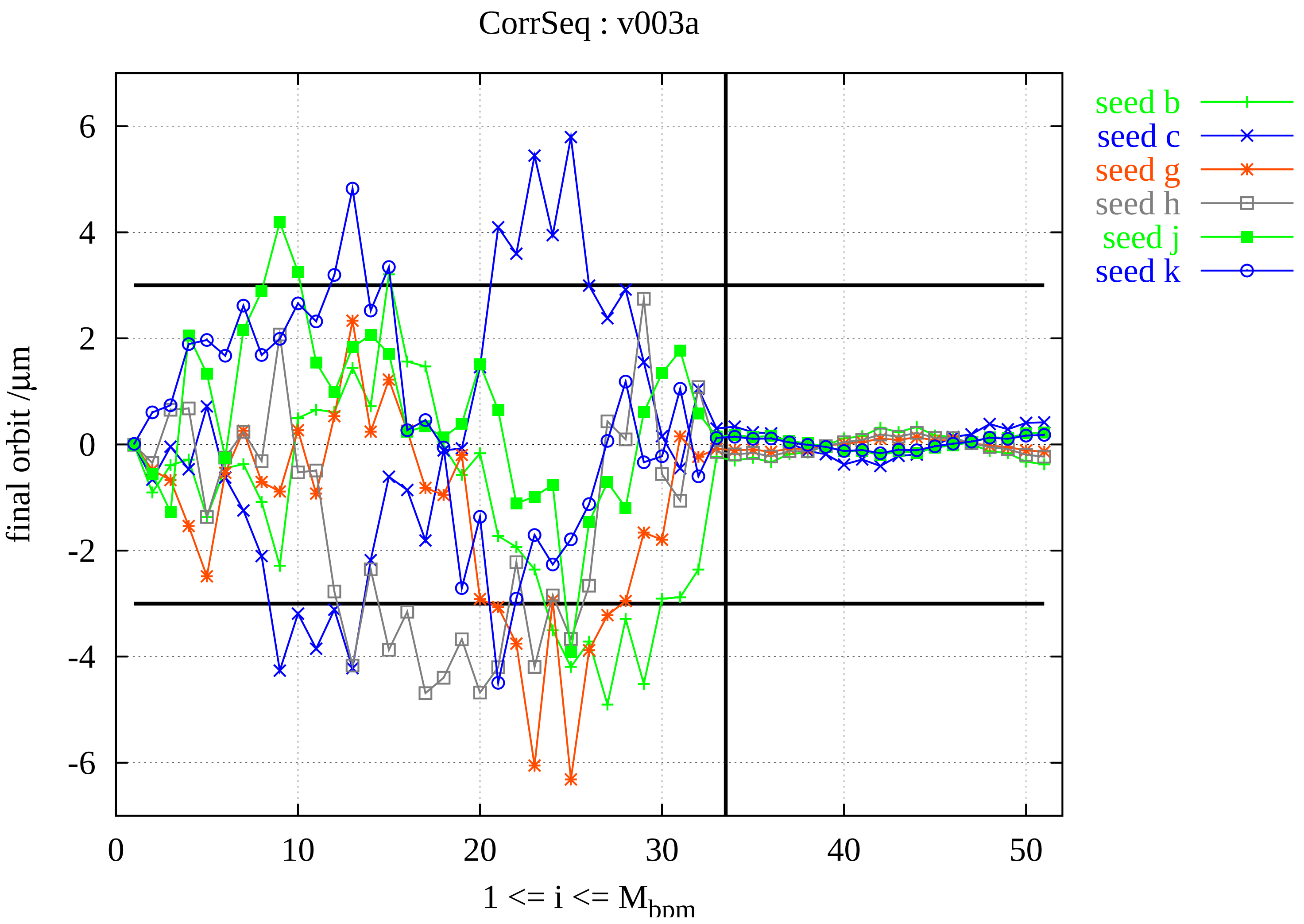
Result of Correction Sequence **v001a** (BEST)



Result of Correction Sequence **v003a**



Result of Correction Sequence **v003a** (BEST)



A More Realistic Example . . .

Parameters :

- initial quad misal. : $\vec{\Delta}_0$-rms : 300μm
- systematic offsets : $\vec{C}\Big|_{\vec{X}}$-rms : 300μm

  but $\vec{C}\Big|_{\vec{D}}$-rms : 0 $\longleftarrow$ difference orbit!
- resolution : $\vec{S}_i\Big|_{\vec{X}}^{\mathrm{SASE1}}$-rms : 1$\mu$m

  $\longrightarrow$ $\vec{S}_i\Big|_{\vec{X}}^{\mathrm{T4}}$-rms : 20$\mu$m $\Leftarrow$ cheaper BPMs

  $\longrightarrow$ $\vec{S}_i\Big|_{\vec{D}}^{\mathrm{SASE1}}$-rms : 20$\mu$m $\Leftarrow$ only 3% dp/p

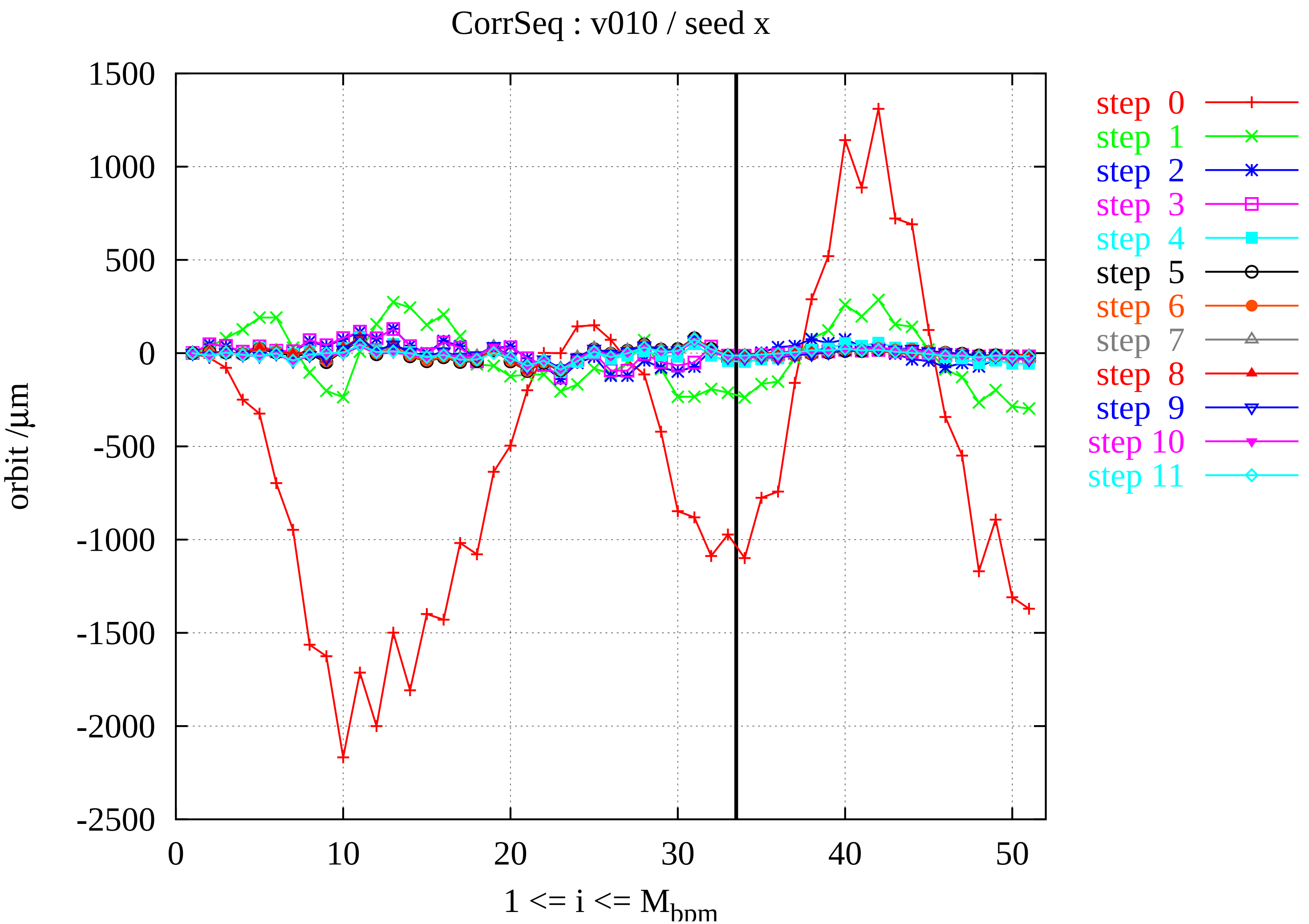
  $\longrightarrow$ $\vec{S}_i\Big|_{\vec{D}}^{\mathrm{T4}}$-rms : 400$\mu$m $\Leftarrow$ acceptance & BPMs
- mover errors : $\vec{\Delta}_i$-rms : 1μm, $i > 0$

CorrSeq v010 (**1-st attempt = yesterday!!**) :

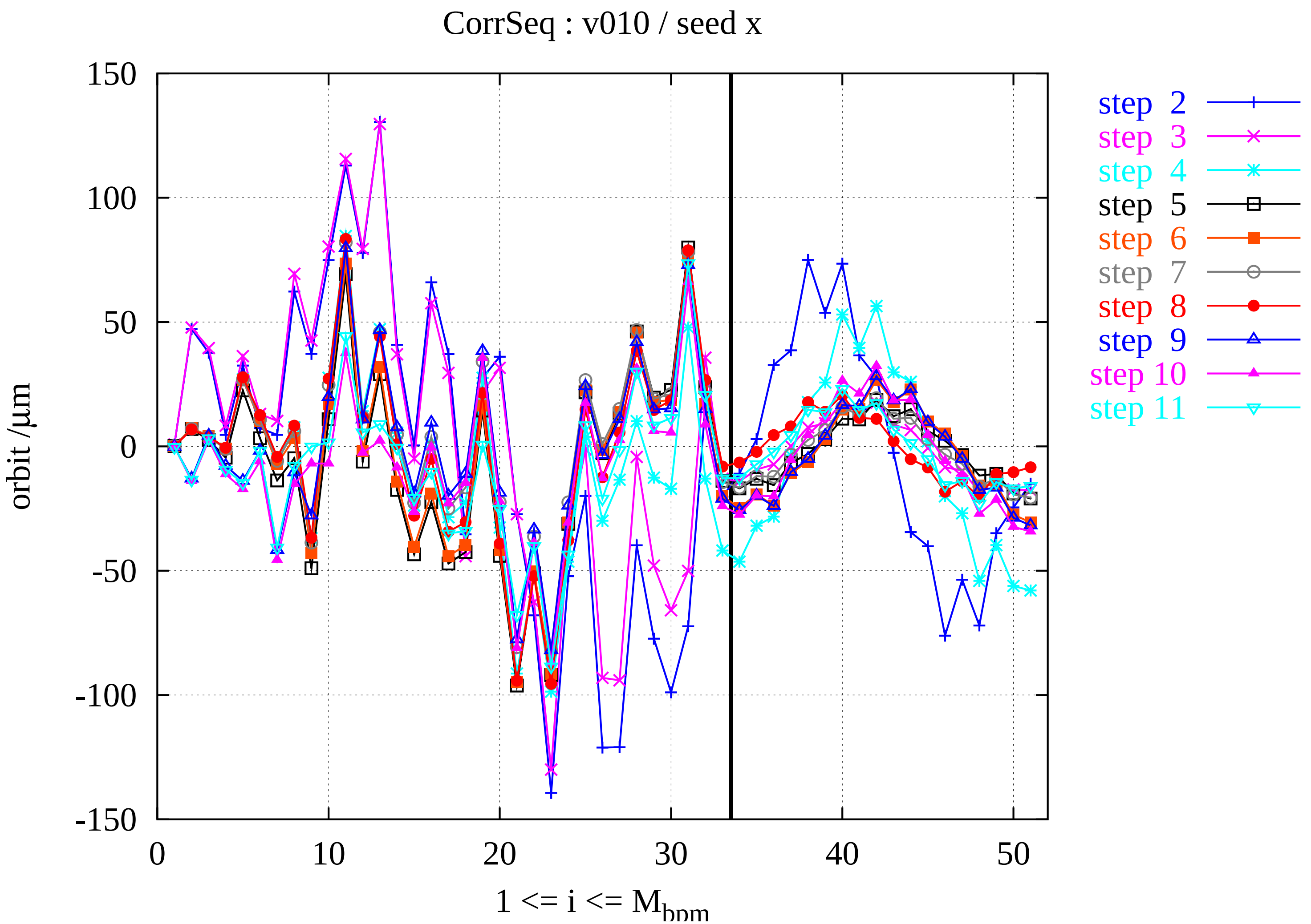
step	range	w	$I^{\mathrm{s.v.}}_{\mathrm{max}}$	g
1	1 — 33	0.00	5	1.0
2	1 — 33	0.80	17	1.0
3	1 — 33	0.95	3	1.0
4	1 — 33	0.95	22	1.0
5	1 — 33	0.99	2	1.0
6	1 — 33	1.00	5	0.5
7	1 — 33	1.00	5	0.5
8	1 — 33	1.00	3	0.5
9	1 — 10	1.00	10^*	0.5
10	8 — 17	1.00	10^*	0.5
11	15 — 24	1.00	10^*	0.5

*: **all** singular values !

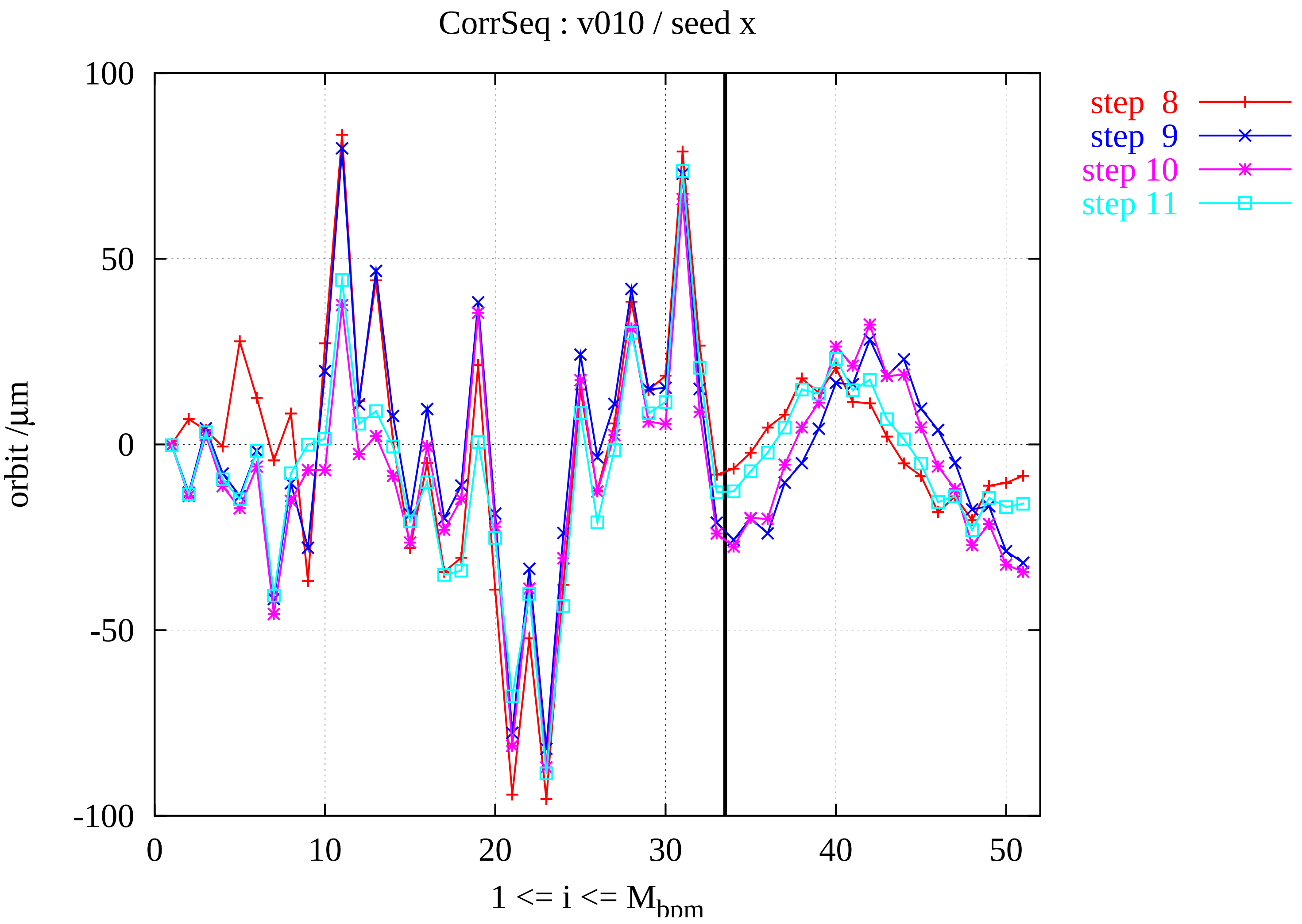
All BPMs in T40 : 20× worse Resolution (**1-st attempt = yesterday!!**)



All BPMs in T40 : 20× worse Resolution (**1-st attempt = yesterday!!**)



All BPMs in T40 : 20× worse Resolution (**1-st attempt = yesterday!!**)



A Slightly More Expensive Example . . .

Parameters :

- initial quad misal. : $\vec{\Delta}_0$-rms : 300μm
- systematic offsets : $\vec{C}\Big|_{\vec{X}}$-rms : 300μm

  but $\vec{C}\Big|_{\vec{D}}$-rms : 0 $\longleftarrow$ difference orbit!
- resolution :

  $\longrightarrow \vec{S}_i\Big|_{\vec{X}}^{\text{SASE1 + 1–st 5 in T4}}$-rms : 1$\mu$m

  $\longrightarrow \vec{S}_i\Big|_{\vec{X}}^{\text{T4 (rest)}}$-rms : 20$\mu$m

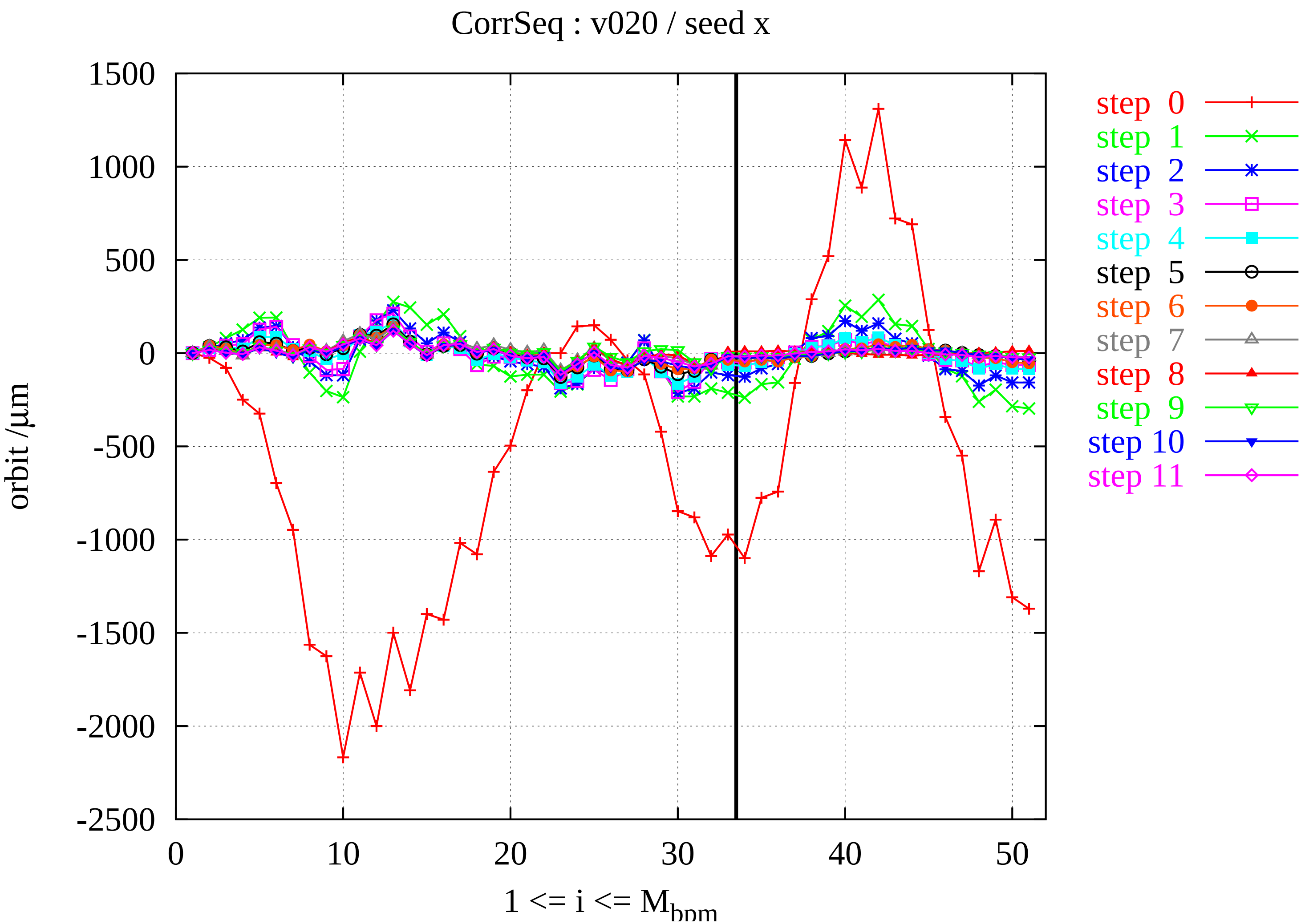
  $\longrightarrow \vec{S}_i\Big|_{\vec{D}}^{\text{SASE1 + 1–st 5 in T4}}$-rms : 20$\mu$m

  $\longrightarrow \vec{S}_i\Big|_{\vec{D}}^{\text{T4 (rest)}}$-rms : 400$\mu$m
- mover errors : $\vec{\Delta}_i = 1,\ i > 0$

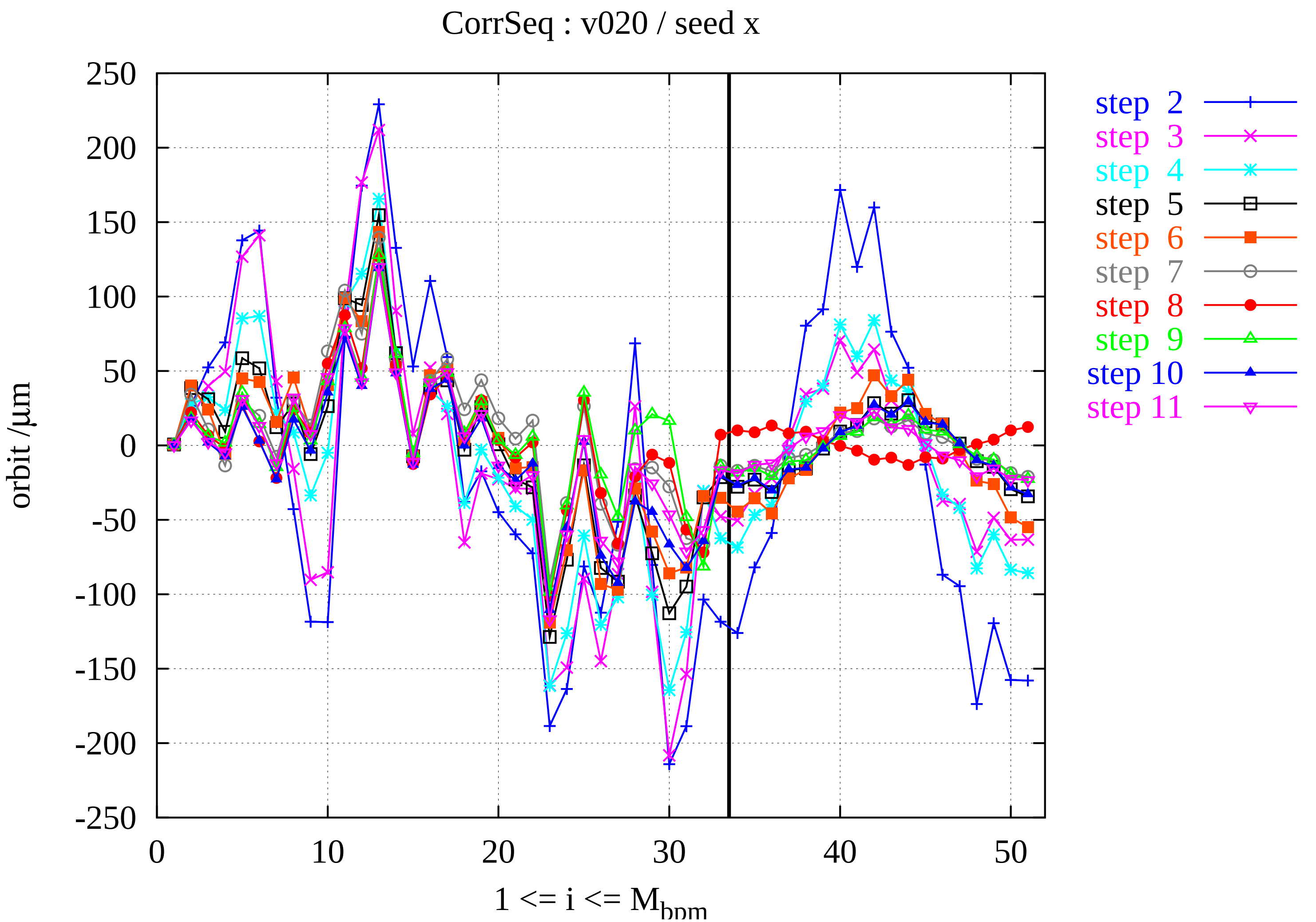
CorrSeq v020 (**2-nd attempt = today!!**) :

step	w	$I^{\text{s.v.}}_{\text{max}}$	g
1	0.00	5	1.0
2	0.80	10	0.5
3	0.80	7	0.5
4	0.80	18	0.5
5	0.80	18	0.5
6	0.80	11	0.5
7	0.80	19	0.5
8	0.80	19	0.7
9	0.95	10	0.5
10	0.95	18	0.5
11	0.95	8	0.5

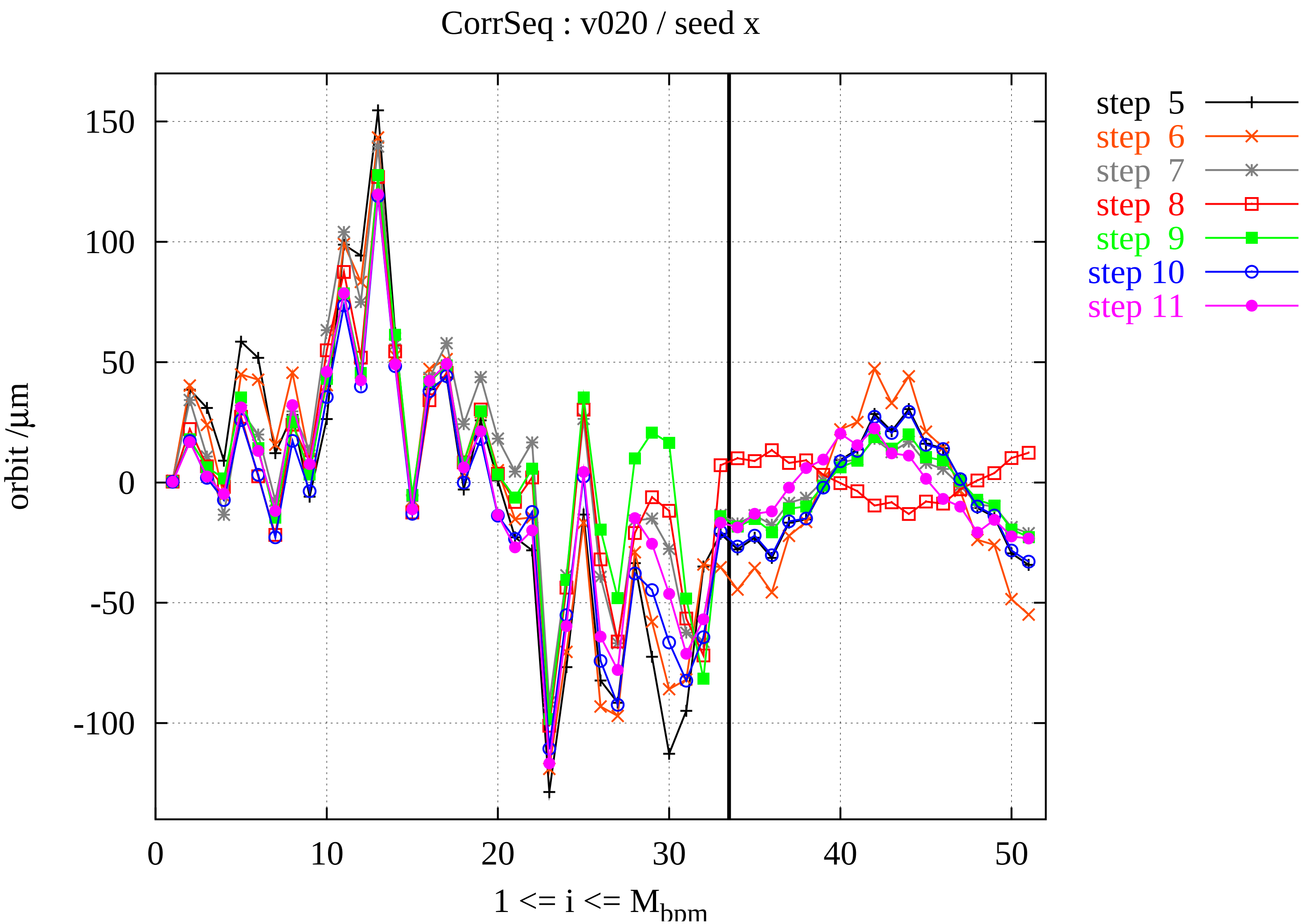
All but 1-st 5 BPMs in T40 : 20× worse Resolution (**2-nd attempt = today!!**)



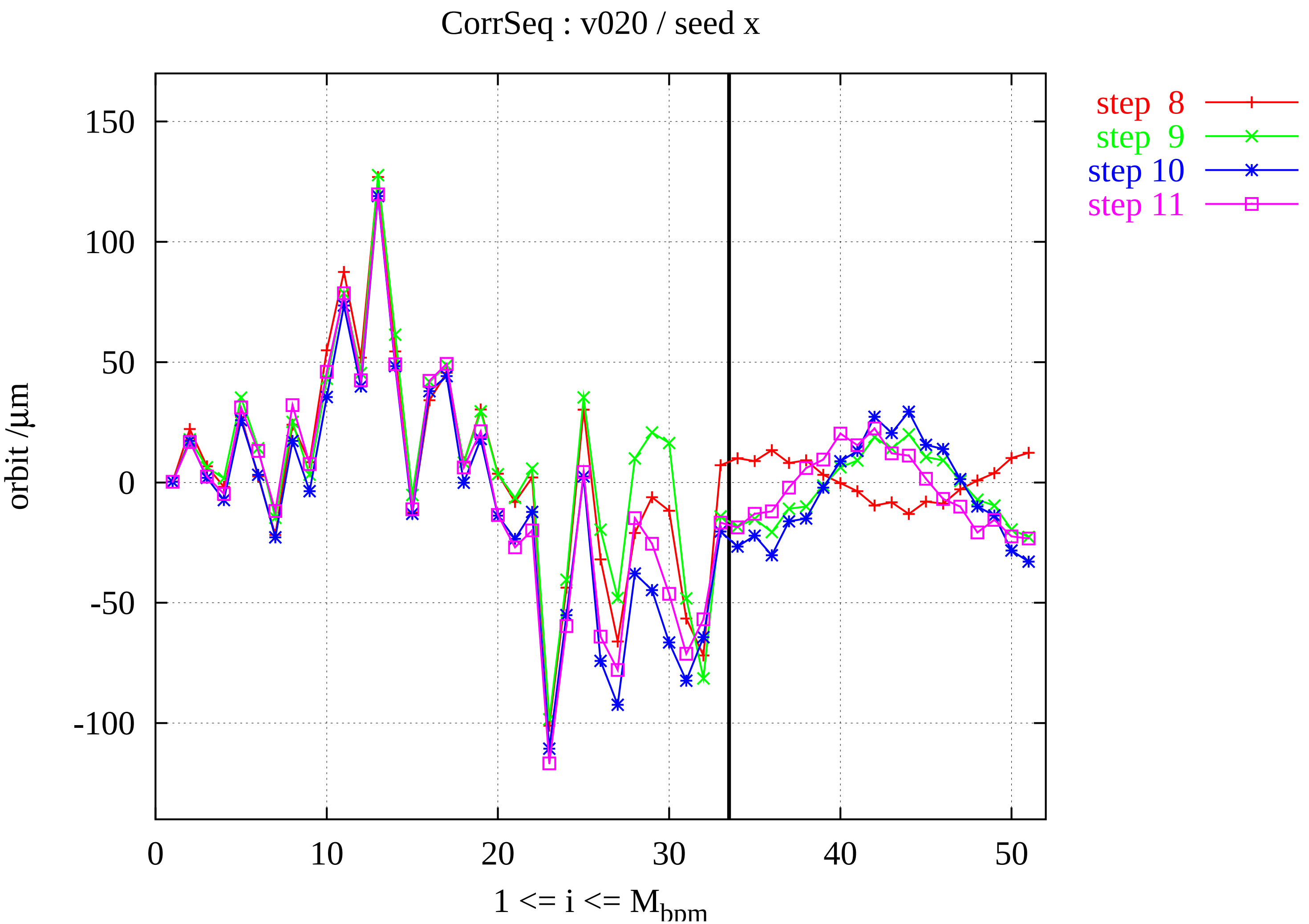
All but 1-st 5 BPMs in T40 : 20× worse Resolution (**2-nd attempt = today!!**)



All but 1-st 5 BPMs in T40 : 20× worse Resolution (**2-nd attempt = today!!**)



All but 1-st 5 BPMs in T40 : 20× worse Resolution (**2-nd attempt = today!!**)



TODO :

- Larger parameter space to be scanned (including varying of BPM–distribution)
- y–plane !!!! & SASE-2,-3,...
- Include deviations of actual (=unknown) ODRM from design–ODRM (=known)
- Include x/y–coupling
- Implement also uniform RVs, etc
- Implement drifts (time domain correlations)
- Include non–linear dispersion into application of the kicks

SUMMARY :

- **Work in progress!!**
- Even with state of the art diagnostics : orbit constrains for SASE very tight !
- In particular : initial misalignment and BPM–offsets are tough
- Strategy : dispersion–free steering with variable weighting between orbit and dispersion, variable τ ($\rightarrow$strongly vs. weakly correlated modes) and variable gain.
- **Result so far : with realistic tolerances and reduced BPM–resolution upstream of the undulators the constraints seem extremely hard to meet!**