wake fields in undulator

bunch shape

2d geometry, resistive effects

- round pipe
  - beam impedance, surface impedance
- arbitrary cross section
- flat pipe

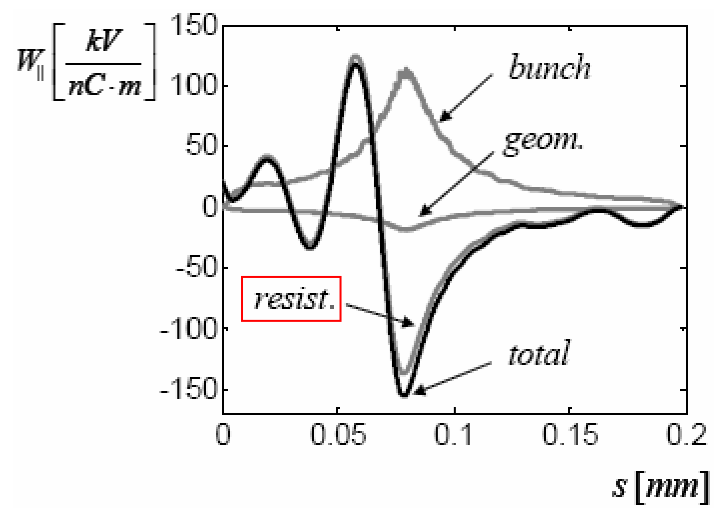
surface

- oxide layer
- roughness
- conductivity
  - material → 2d geometry
  - anomalous skin effect

3d geometry

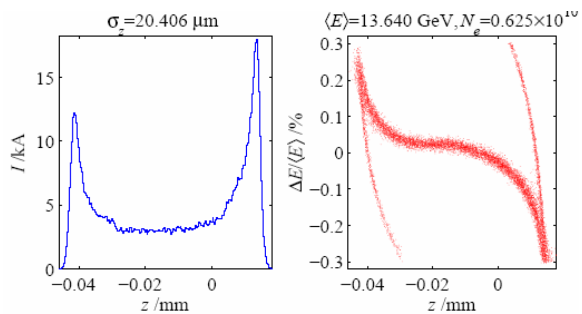
summary

European XFEL undulator wake

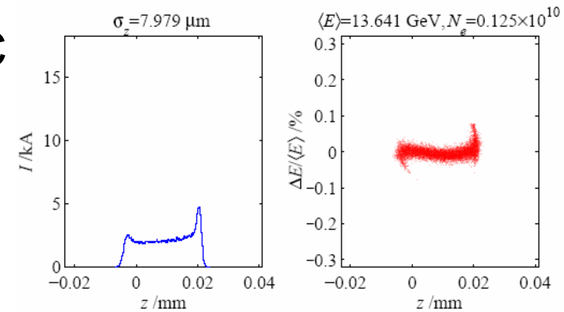


bunch shape

LCLS 1nC

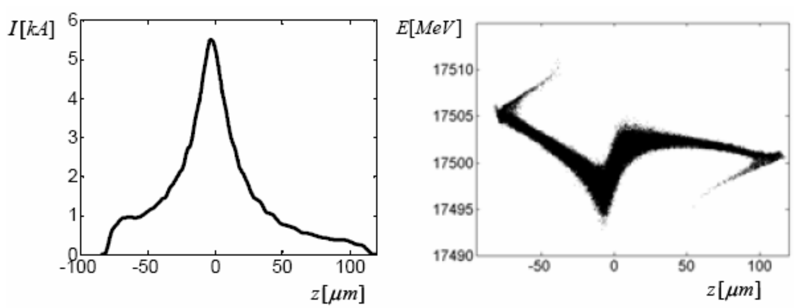


0.2nC



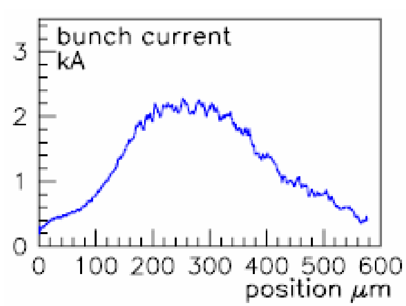
European XFEL

1nC



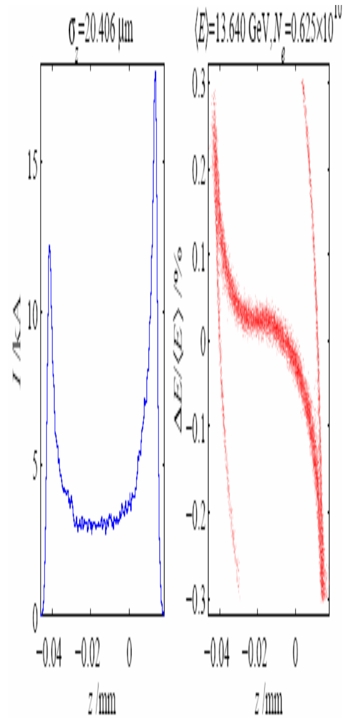
BESSY FEL

2.4nC

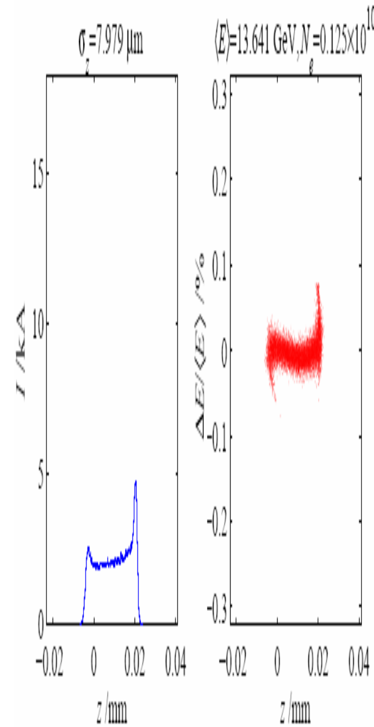


LCLS

1nC

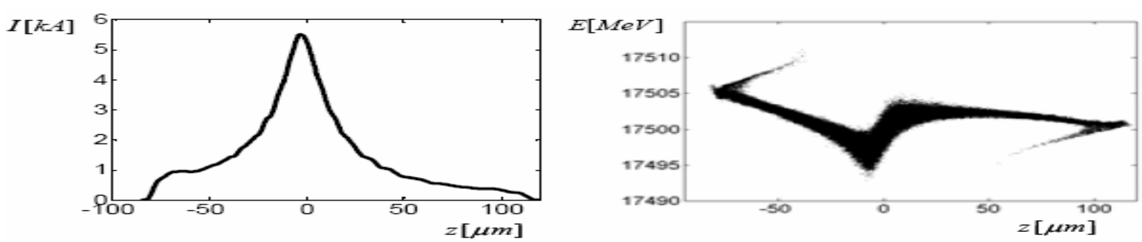


0.2nC



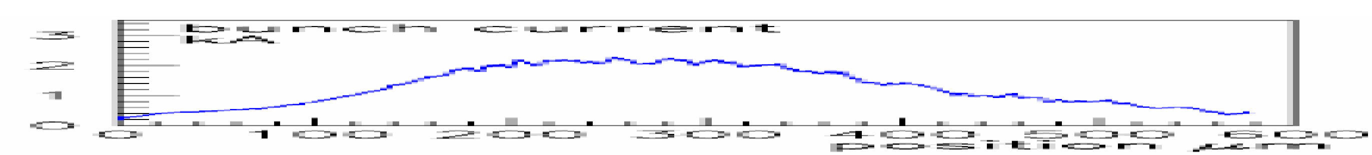
European XFEL

1nC



BESSY FEL

2.4nC



resistive wall effects:

characteristic length

$$S_{\mathrm{ch}} = \left(\frac{R^2}{2\kappa_0 Z_0} \right)^{1/3}$$

example

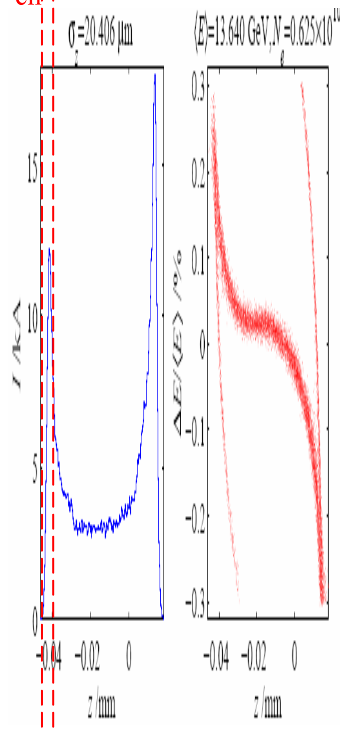
$R = 3\,\mathrm{mm}$

$\kappa_0 = 58\,\mathrm{S/m}$

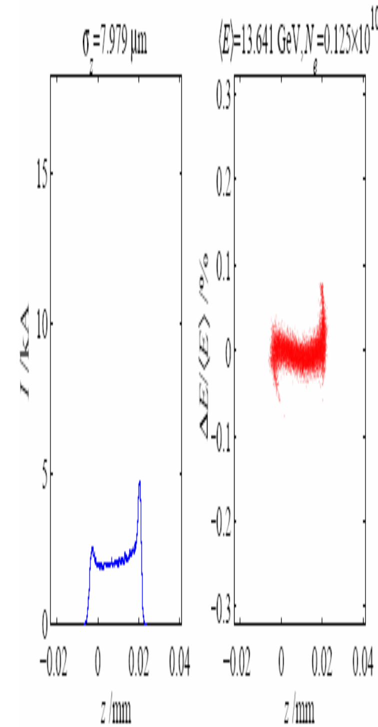
$\rightarrow S_{\mathrm{ch}} \approx 6\,\mu\mathrm{m}$

S_{ch}

LCLS 1nC

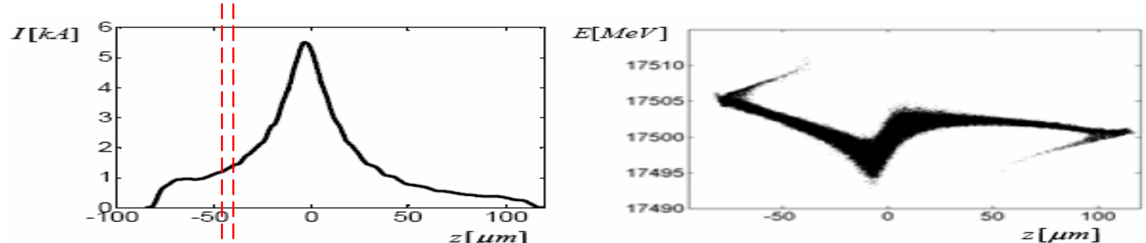


0.2nC



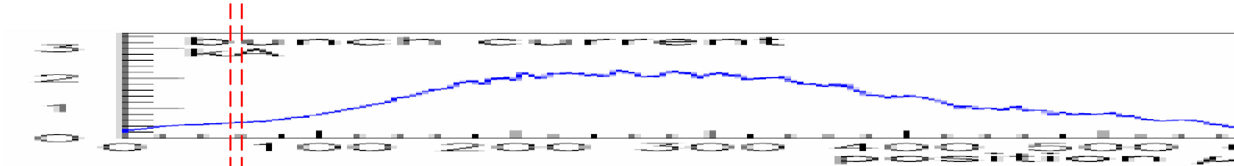
European XFEL

1nC



BESSY FEL

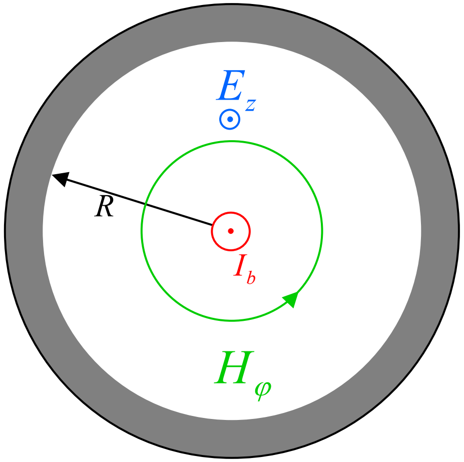
2.4nC



2d geometry: round pipe

beam impedance and surface impedance

$$Z_b' = -\left.\frac{E_z}{I_b}\right|_{r\to 0} \qquad Z_s = -\left.\frac{E_z}{H_\varphi}\right|_{r=R}$$



$$H_\varphi = \frac{I}{2\pi r}\exp(i\omega(t-z/c)) + A\frac{r}{2}\exp(i\omega(t-z/c))$$

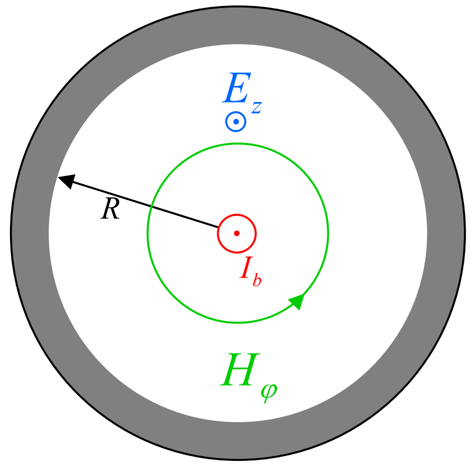
$$E_r = Z_0\frac{I}{2\pi r}\exp(i\omega(t-z/c)) + Z_0 A\frac{r}{2}\exp(i\omega(t-z/c))$$

$$E_z = 0 - \frac{A}{i\omega\varepsilon_0}\exp(i\omega(t-z/c))$$

beam | boundary

beam impedance and surface impedance

$$Z'_b = -\left.\frac{E_z}{I_b}\right|_{r\to 0} \qquad Z_s = -\left.\frac{E_z}{H_\varphi}\right|_{r=R}$$



$$Z'_b(\omega) = \frac{Z_s(\omega)}{2\pi R}\,\frac{1}{1 + i\dfrac{\omega}{c}\dfrac{R}{2}\dfrac{Z_s(\omega)}{Z_0}}$$

metallic conductor (κ):
(plane wave approximation)

$$Z_s^{(\kappa)} = \sqrt{\frac{j\omega\mu}{\kappa(\omega) + j\omega\varepsilon}} \qquad Z_s^{(\kappa)} \approx \sqrt{\frac{j\omega\mu}{\kappa(\omega)}} \qquad \kappa(\omega) \approx \frac{\kappa_0}{1 + i\omega\tau}$$

asymptotic limit

$$Z_b'(\omega) = \frac{Z_s(\omega)}{2\pi R} \frac{1}{1 + i\frac{\omega}{c}\frac{R}{2}\frac{Z_s(\omega)}{Z_0}}$$

$$|\cdots| = 1 \quad \rightarrow \omega_{\mathrm{ch}} = 2c\left(\frac{\kappa_0 Z_0}{2R^2}\right)^{1/3} \qquad S_{\mathrm{ch}} = \sqrt[3]{2}\,\frac{c}{\omega_{\mathrm{ch}}}$$

---

low frequency $\omega << \omega_{\mathrm{ch}}$

$$Z_b'(\omega) \approx \frac{Z_s(\omega)}{2\pi R} \approx \frac{1}{2\pi R}\sqrt{\frac{i\omega\mu}{\kappa_0}}$$

$\sim \sqrt{\omega}$

$\sim 1/\sqrt{\kappa}$

$\sim 1/R$

---

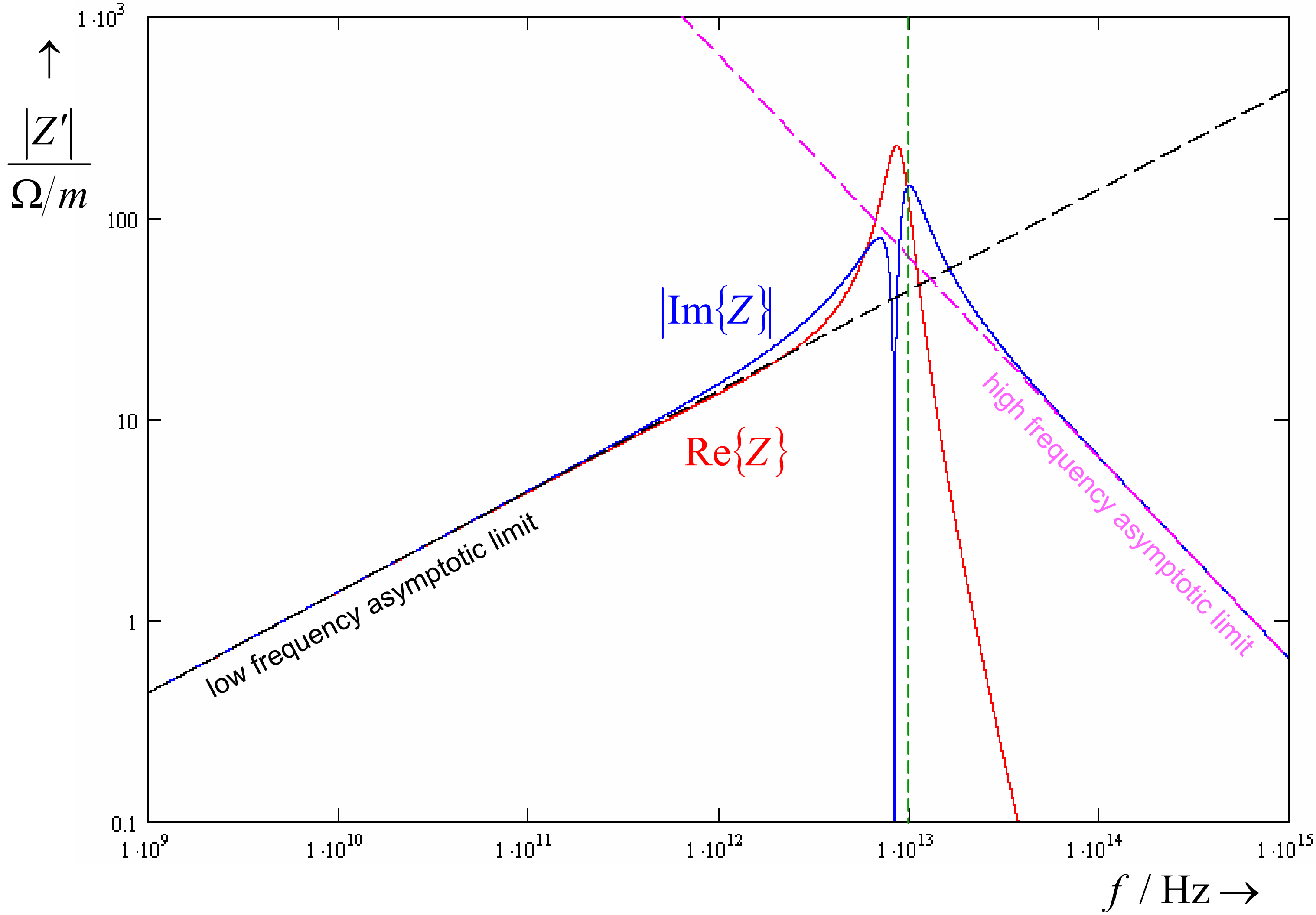
high frequency $\omega >> \omega_{\mathrm{ch}}$

$$Z_b'(\omega) \rightarrow \frac{1}{\pi R^2 \varepsilon_0}\frac{1}{i\omega}$$

capacitive impedance
material independent
$\sim 1/R^2$

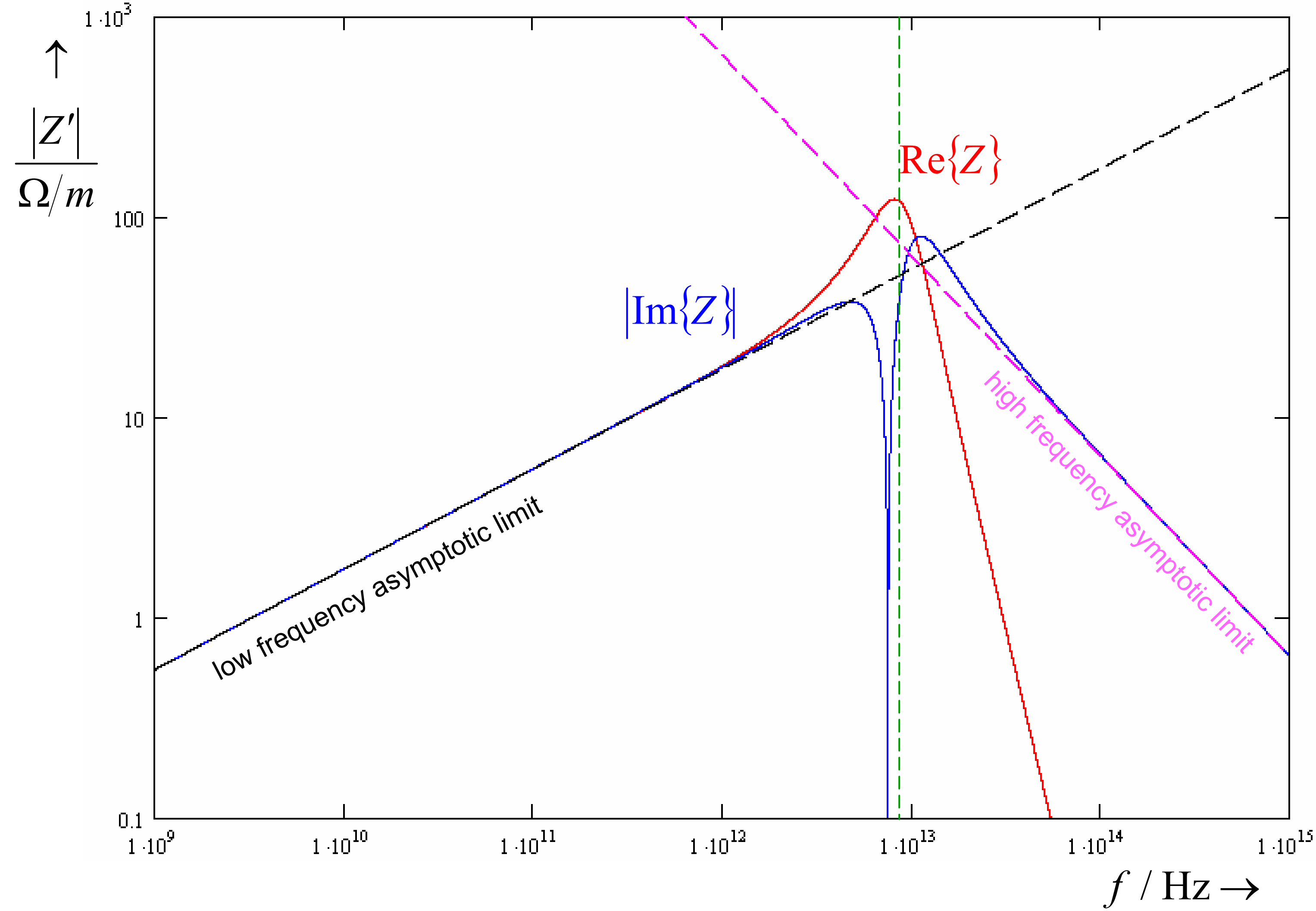
copper, R = 3mm

$\kappa_0 = 58 \cdot 10^6 \dfrac{1}{\Omega \mathrm{m}}$ $\tau = 2.46 \cdot 10^{-14}\,\mathrm{s}$ $f_{\mathrm{ch}} = 1.02 \cdot 10^{13}\,\mathrm{Hz}$

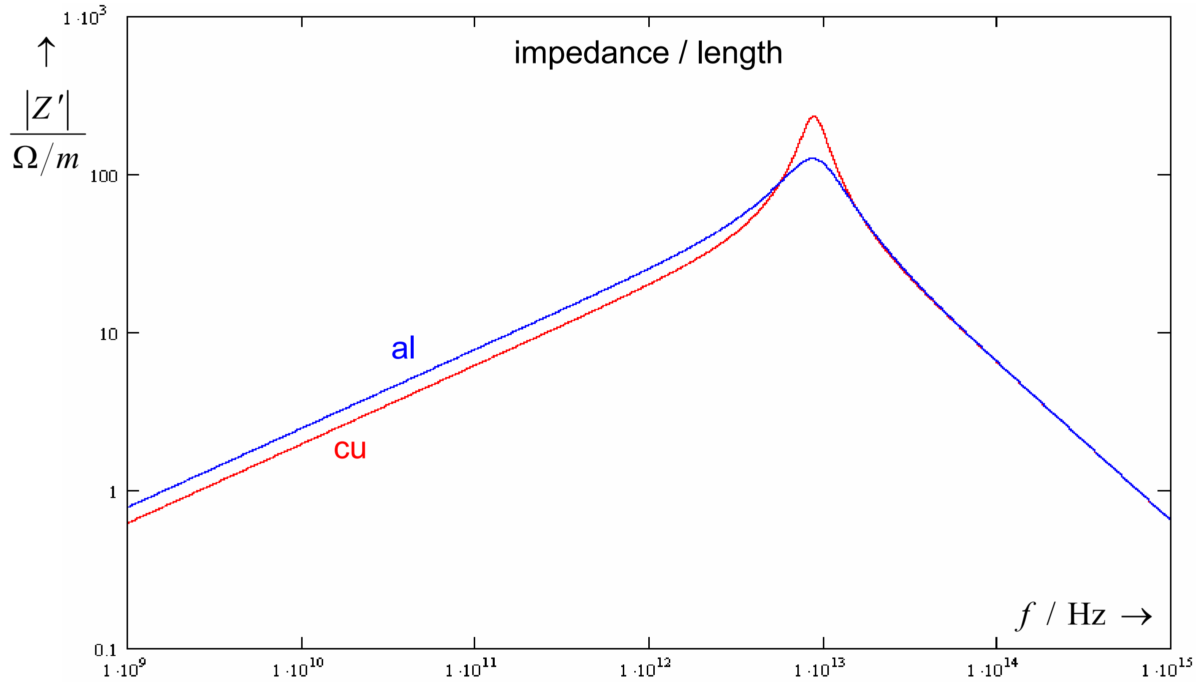


alu, R = 3mm

$$\kappa_0 = 36.6 \cdot 10^6 \frac{1}{\Omega\mathrm{m}} \qquad \tau = 0.71 \cdot 10^{-14}\,\mathrm{s} \qquad f_{\mathrm{ch}} = 0.87 \cdot 10^{13}\ \mathrm{Hz}$$

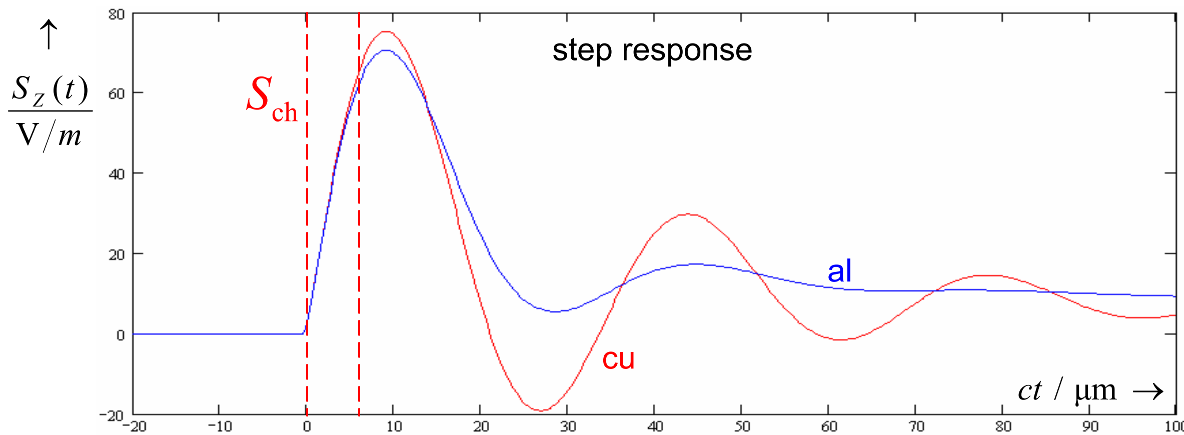


copper & alu, R = 3mm

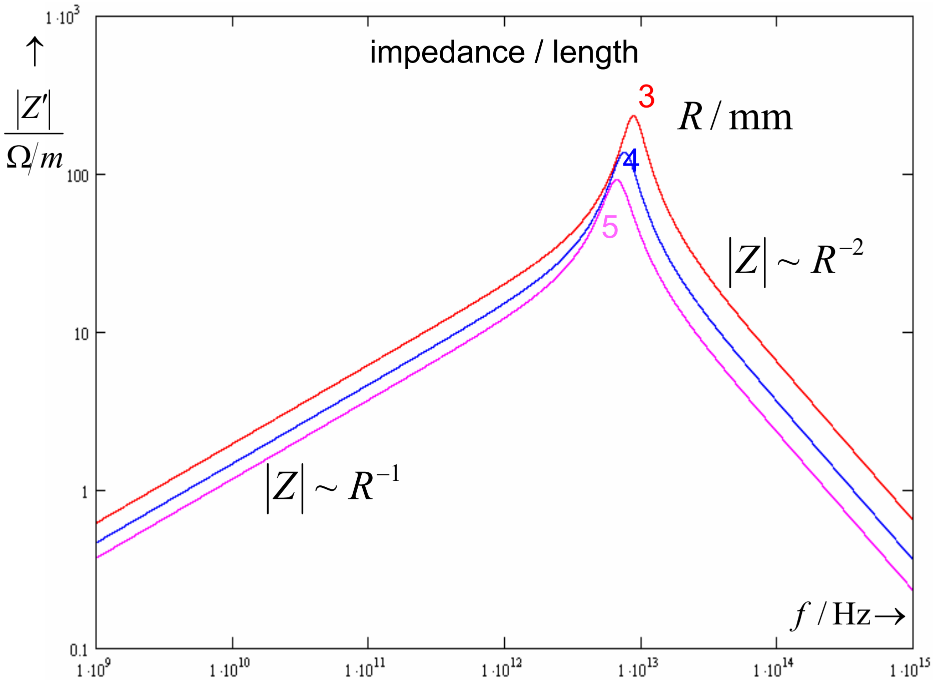


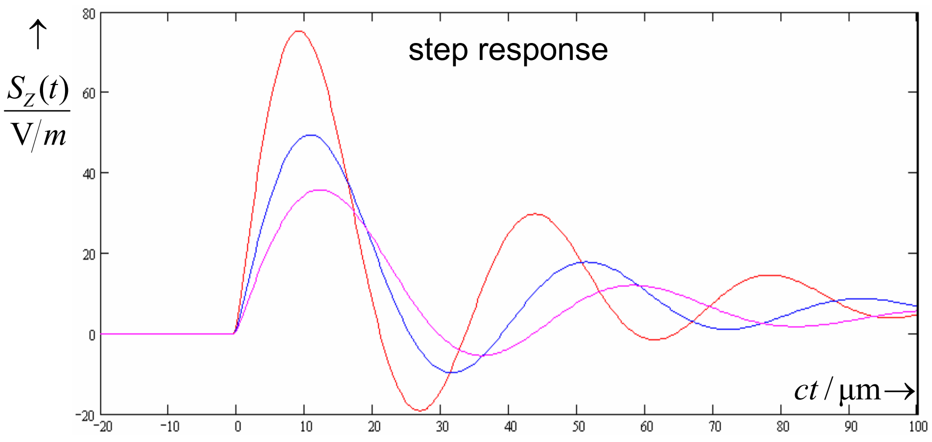
$$S(t) = \begin{cases} 0 & \text{if } t < 0 \\ 1\text{A} & \text{otherwise} \end{cases}$$

$$S_Z(t) = S(t) \otimes F^{-1}\{Z'\}$$

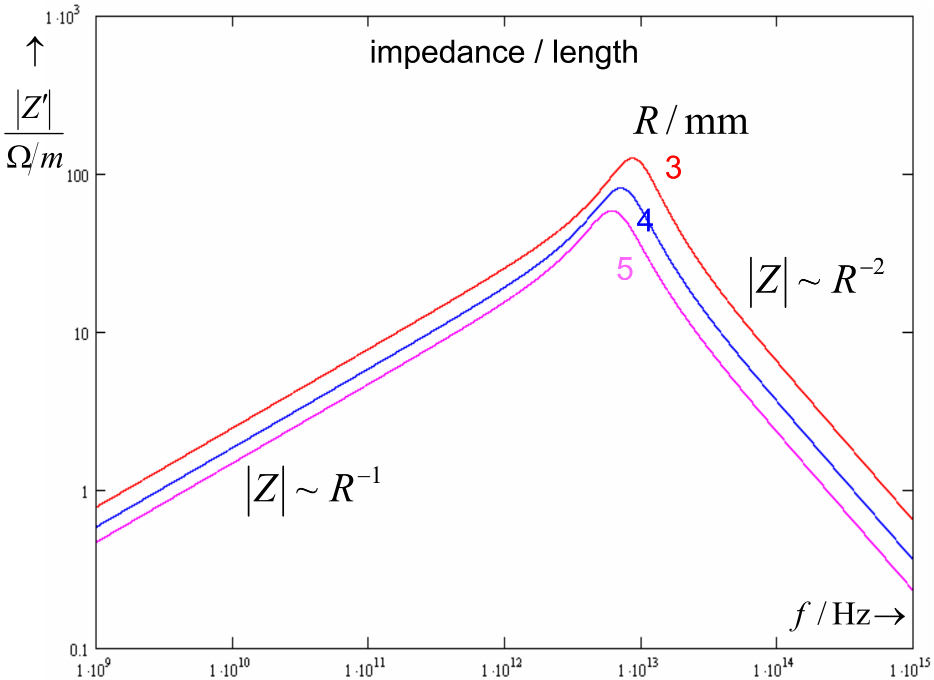


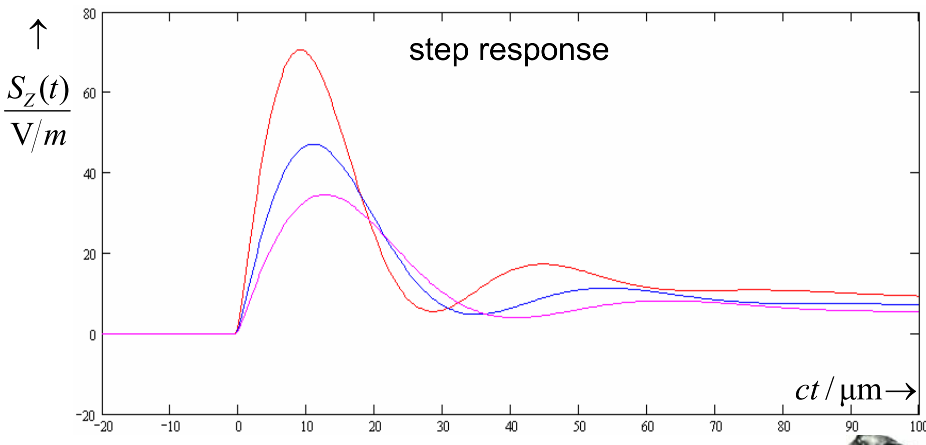
copper


impedance / length
$\frac{|Z'|}{\Omega/m}$
R / mm
3
4
5
$|Z| \sim R^{-2}$
$|Z| \sim R^{-1}$
f / Hz→
0.1
1
10
100
$1\cdot10^3$
$1\cdot10^9$
$1\cdot10^{10}$
$1\cdot10^{11}$
$1\cdot10^{12}$
$1\cdot10^{13}$
$1\cdot10^{14}$
$1\cdot10^{15}$

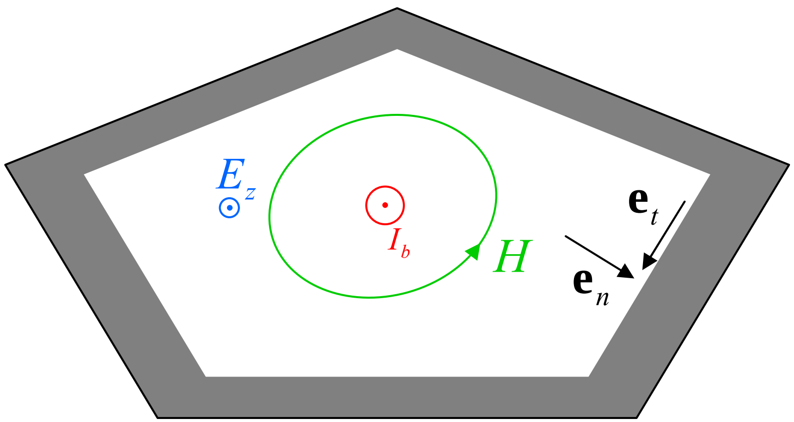

step response
$\frac{S_Z(t)}{V/m}$
ct / µm→
-20
0
20
40
60
80
-20
-10
0
10
20
30
40
50
60
70
80
90
100

alu


impedance / length
$\frac{|Z'|}{\Omega/m}$
R / mm
3
4
5
$|Z| \sim R^{-2}$
$|Z| \sim R^{-1}$
f / Hz→
0.1
1
10
100
$1\cdot10^3$
$1\cdot10^9$
$1\cdot10^{10}$
$1\cdot10^{11}$
$1\cdot10^{12}$
$1\cdot10^{13}$
$1\cdot10^{14}$
$1\cdot10^{15}$


step response
$\frac{S_Z(t)}{V/m}$
ct / µm→
-20
0
20
40
60
80
-20
-10
0
10
20
30
40
50
60
70
80
90
100

2d geometry: arbitrary cross-section



$$\mathbf{E} = \tilde{\mathbf{E}}(x,y)\exp(i\omega(t - z/c))$$
$$\rho = \tilde{\rho}(x,y)\exp(i\omega(t - z/c))$$
$$\mathbf{J} = \rho c\,\mathbf{e}_z$$

2d differential equation:

$$\nabla_t^2 \tilde{\mathbf{E}}_\perp = \frac{1}{\varepsilon}\nabla_t \rho$$

$$\tilde{E}_z = \frac{c}{\omega}\left(\nabla_t \tilde{\mathbf{E}}_\perp - \frac{\tilde{\rho}}{\varepsilon}\right)$$

boundary condition (surface impedance)

$$\tilde{E}_z = -\frac{Z_s}{Z_0}\tilde{E}_n \qquad \tilde{E}_t = 0$$

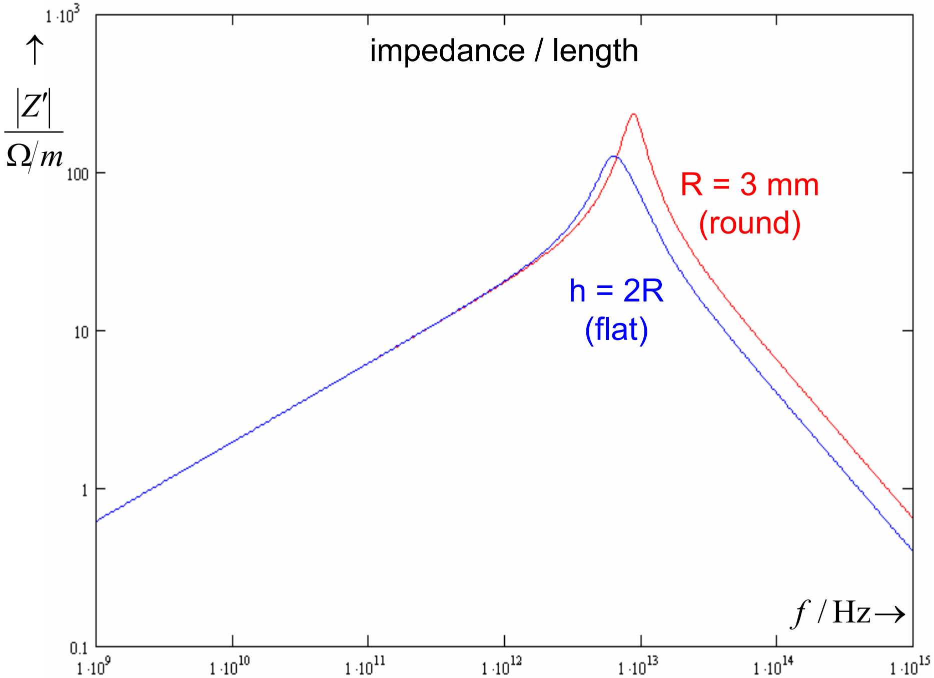
beam impedance

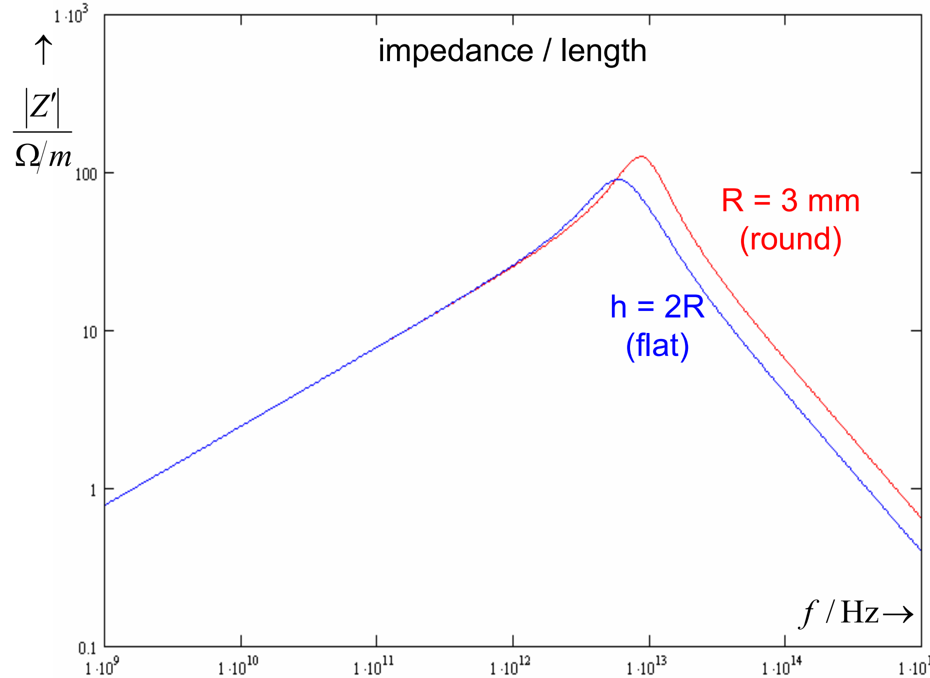
$$Z_b' = -\left.\frac{\tilde{E}_z}{\tilde{I}_b}\right|_{\mathbf{r}\to\mathbf{r}_b}$$

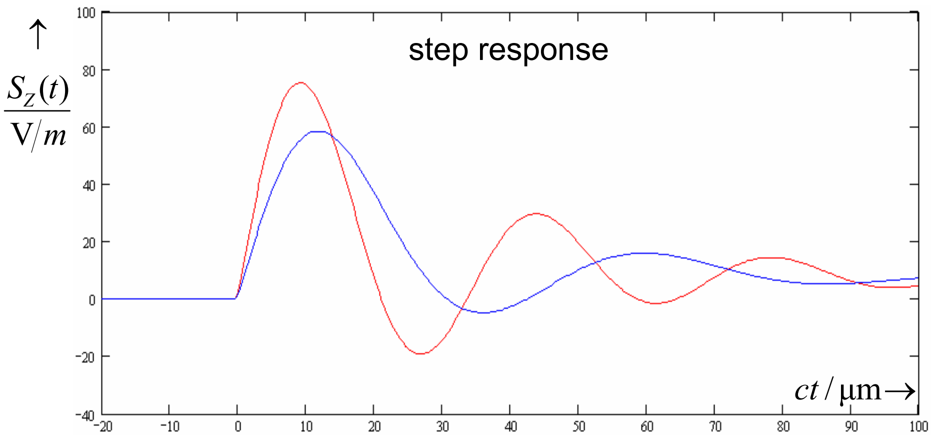
more:
K. Yokoya: Resistive Wall impedance of beam pipes of general cross section
Part. Acc., 1993, Vol. 41, pp 221-248

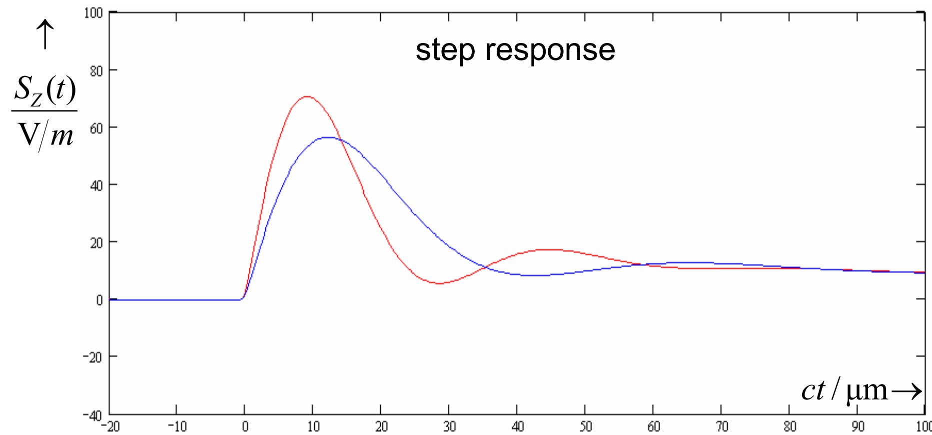
2d geometry: flat beam pipe

copper | alu









surface: oxide layer

surface impedance of thin dielectric layer (ε_r) on perfect conductor:

$$Z_s^{(\varepsilon)} \approx j\omega L \quad \text{with} \quad L = \Delta\mu \frac{\varepsilon_r - 1}{\varepsilon_r}$$

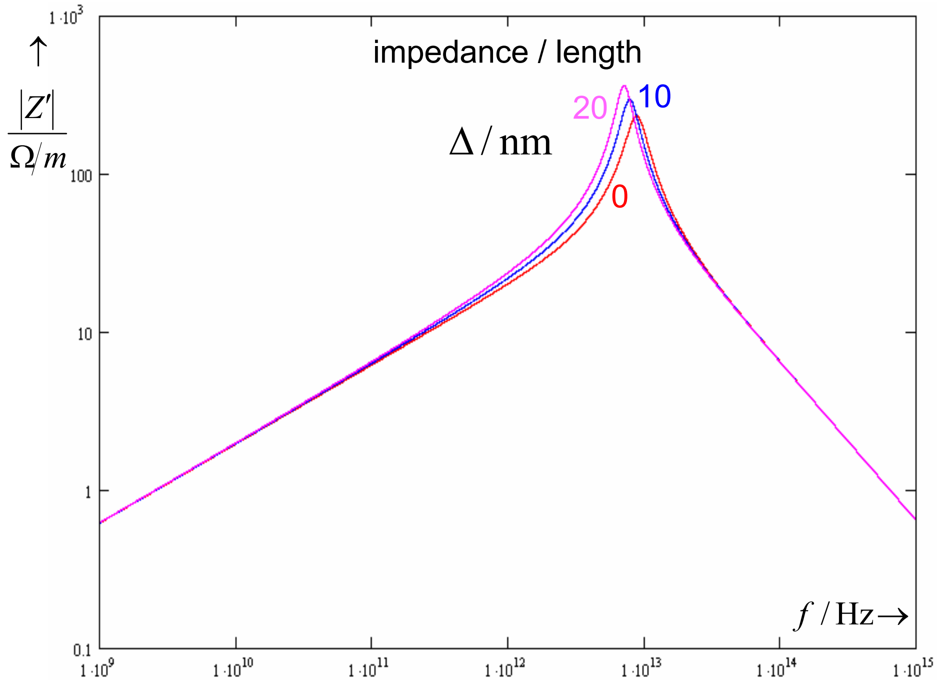
$$L \to \Delta\mu \quad \text{for} \quad \varepsilon_r >> 1$$

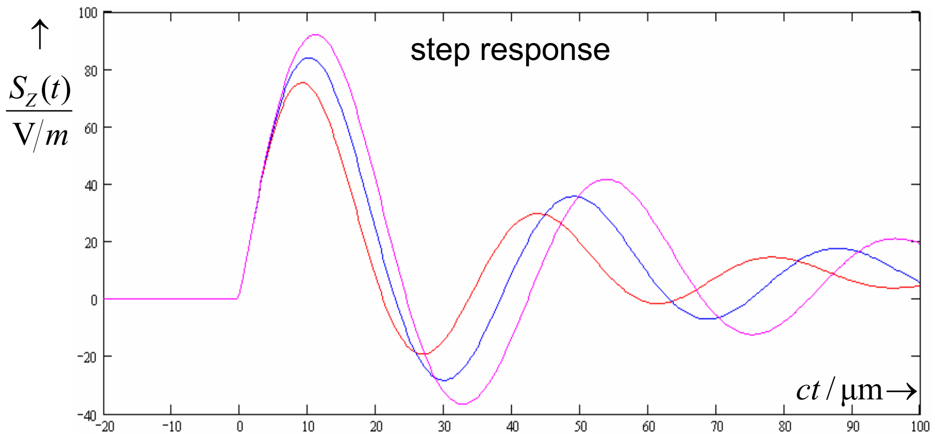
surface impedance of thin dielectric layer (ε_r) on metallic conductor (κ):

$$Z_s = Z_s^{(\kappa)} + Z_s^{(\varepsilon)}$$

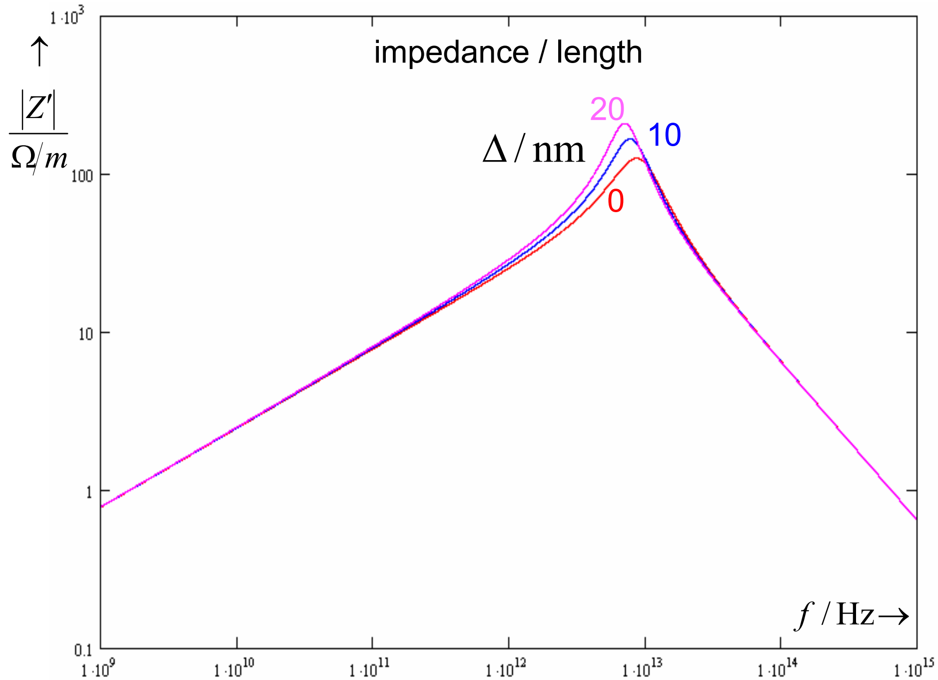
R=3mm & oxide layer (ε_r=2)

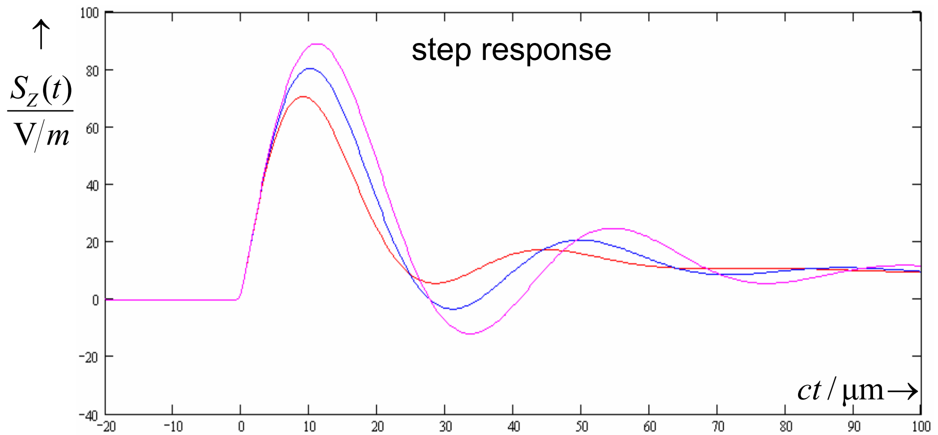
copper





alu

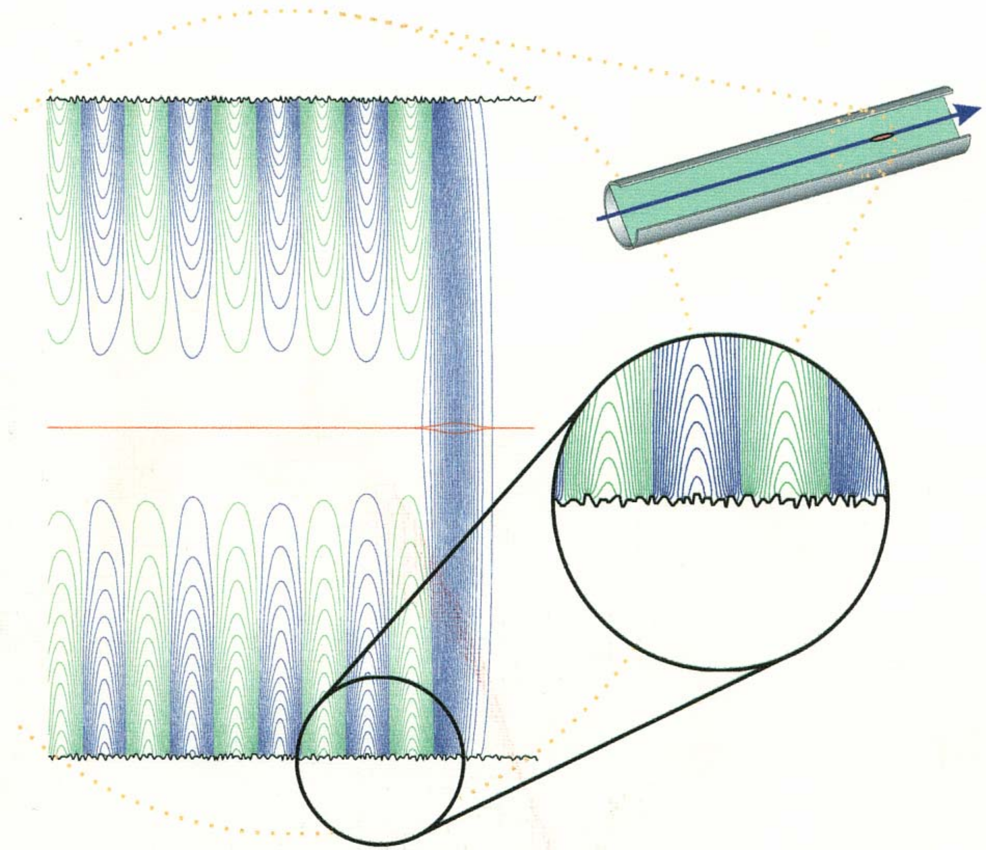




surface: roughness

(very incomplete list of models)

Novokhatski, Timm, Weiland



rough surface behaves as dielectric layer
→ synchronous mode

$$Z_b'(\omega) = \frac{Z_s(\omega)}{2\pi R} \frac{1}{1 + i\frac{\omega}{c}\frac{R}{2}\frac{Z_s(\omega)}{Z_0}}$$

with $Z_s(\omega) = i\omega L$

$$L = \mu_0 \Delta_{eff} \frac{\varepsilon_r - 1}{\varepsilon_r}$$

$$\Delta_{eff} \approx 0.5\Delta_{rms} \qquad \varepsilon_r \approx 2$$

synchronous mode:

$$1 + i\frac{\omega}{c}\frac{R}{2}\frac{Z_s(\omega)}{Z_0} = 0$$

Stupakov I $\quad Z_b'(\omega) = \frac{i\omega L}{2\pi R}$ $\quad L$ from ACF of surface roughness $L \approx \mu_0 \Delta_{rms} \frac{\varepsilon_r - 1}{\varepsilon_r}$ $\quad \Delta_{eff} \approx 10^{-2} \Delta_{rms}$

Stupakov II $\quad$ sinusoidal surface $R(z) = R_0 + \Delta \cos(2\pi z/\lambda)$ $\quad$ with $\Delta << \lambda/2\pi$ → no synchronous mode

Dohlus $\quad$ sinusoidal surface, with $\Delta < \lambda/2\pi$, includes transition sync. mode → no synchronous mode
sinusoidal & resistive surface

April 2001, TESLA 2001-26

Impedance of Beam Pipes with Smooth Shallow Corrugations

M. Dohlus
Deutsches Elektronen Synchrotron
Notkestr. 85, D-22603 Hamburg, Germany

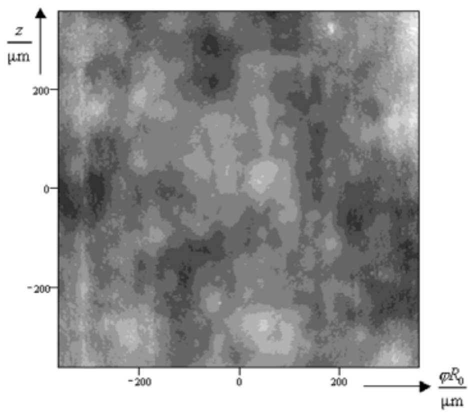


Fig. 23: Surface function $\delta r(y,z) = \delta r(y,z) - (q_0 + q_y y + q_{y2} y^2 + q_z z)$ after extraction of the curvature (in azimuthal direction) and slope (in z direction). The gray scale ranges from -2 to 2.8 µm.

rms value of the roughness $\delta r_{rms} = \sqrt{R_{c,2D}(0,0)} = 0.58\,\mu m$

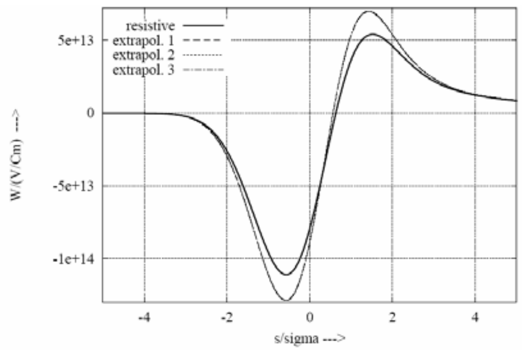


Fig. 28: Wake potential calculated by the LB-N,2nd method for three extrapolations of the ACF (thin lines), resistive wall wake (thick line) of a perfect pipe. Configuration: aluminum pipe, $R_0 = 3\text{mm}$, random surface, gaussian bunch with $\sigma = 25\,\mu m$.

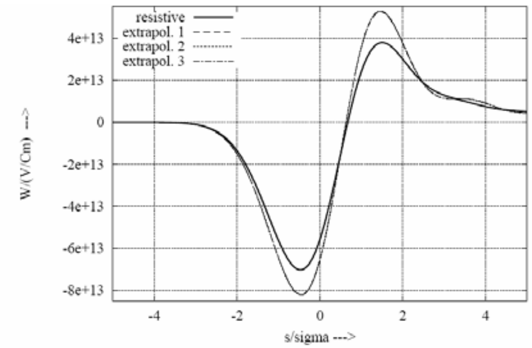
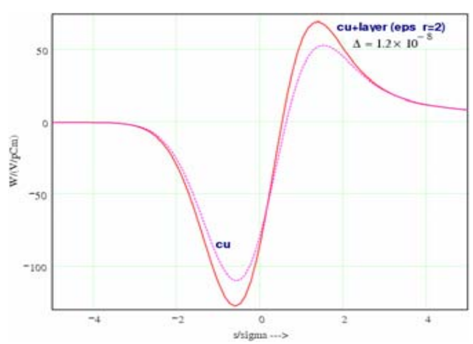
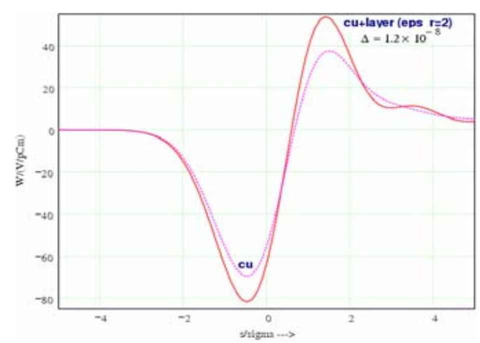


Fig. 29: Wake potential calculated by the LB-N,2nd method for three extrapolations of the ACF (thin lines), resistive wall wake (thick line) of a perfect pipe. Configuration: aluminum pipe, $R_0 = 5\text{mm}$, random surface, gaussian bunch with $\sigma = 25\,\mu m$.

$R_0 = 3\text{mm}$ gaussian bunch	$\min(W^\sigma)$ V/pCm	$\max(W^\sigma)$ V/pCm	$\langle W^\sigma \rangle$ V/pCm	$\text{rms}(W^\sigma)$ V/pCm
resistive (cu)	-111	54.1	-44.9	56.7
resistive+rough	-129	69.6	-47.9	68.7

$R_0 = 5\text{mm}$ gaussian bunch	$\min(W^\sigma)$ V/pCm	$\max(W^\sigma)$ V/pCm	$\langle W^\sigma \rangle$ V/pCm	$\text{rms}(W^\sigma)$ V/pCm
resistive (cu)	-70.3	38.0	-29.0	36.5
resistive+rough	-82.0	52.6	-31.7	45.1





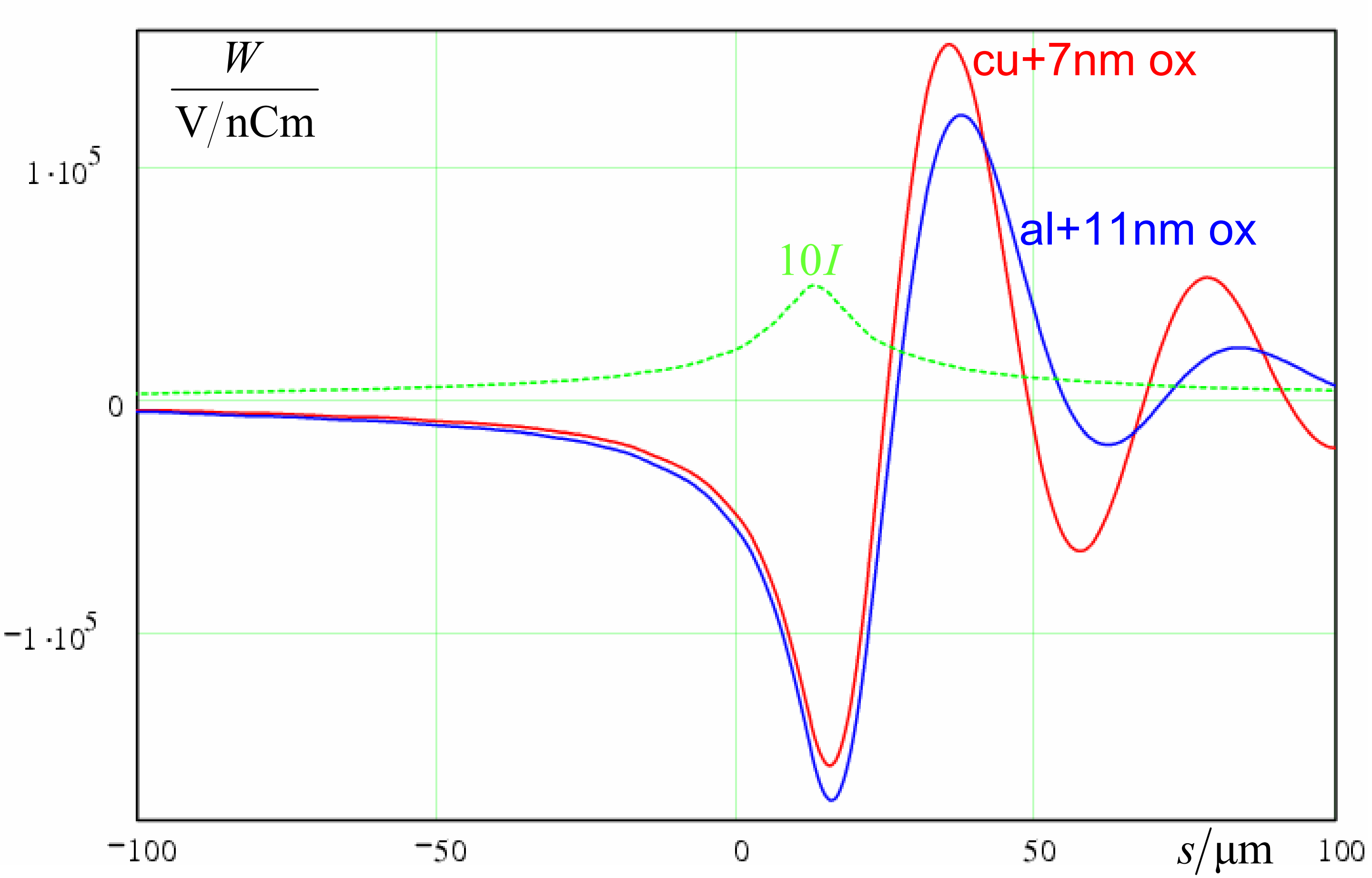
		av	rms
cu		$kcu = -4.441\times 10^{13}$	$rmscu = 5.597\times 10^{13}$
cu+layer (eps_r=2)	$\Delta = 1.2\times 10^{-8}$	$k = -4.559\times 10^{13}$	$rms = 6.873\times 10^{13}$

		av	rms
cu		$kcu = -2.871\times 10^{13}$	$rmscu = 3.602\times 10^{13}$
cu+layer (eps_r=2)	$\Delta = 1.2\times 10^{-8}$	$k = -3.031\times 10^{13}$	$rms = 4.551\times 10^{13}$

conclusion: 0.58 µm roughness behaves similar as 12 nm $\varepsilon_r = 2$ oxide layer

300nm roughness → "≈ 6nm oxide layer, ε_r=2"

$$R_{\mathrm{pipe}} = 3.8\ \mathrm{mm}$$



cu: 7nm ox = 1nm (real oxide) + 6nm (roughness)

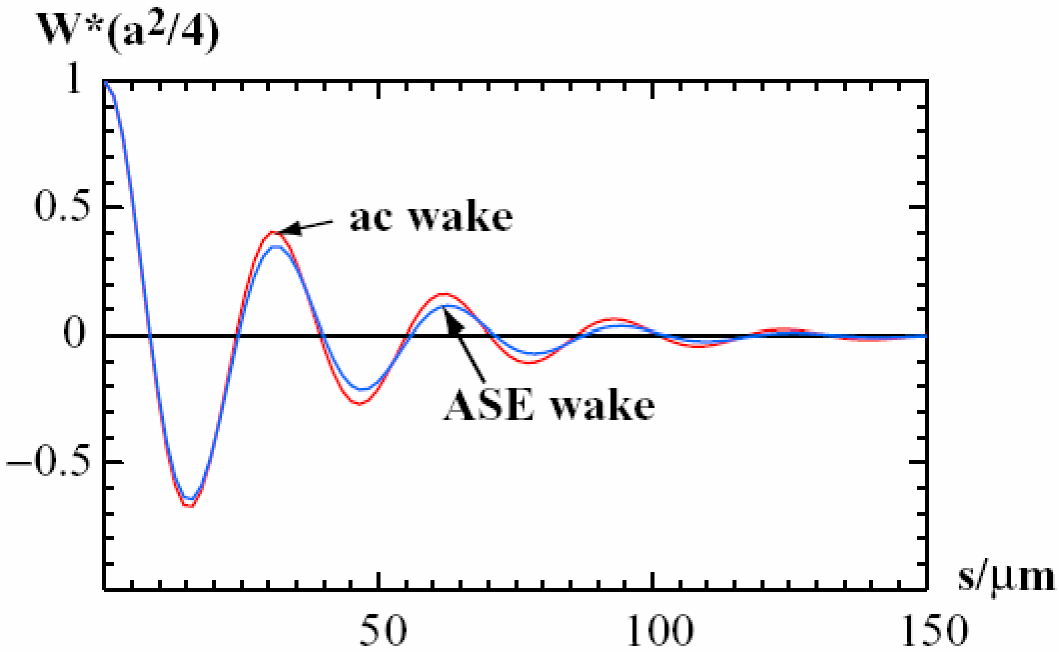
al: 11nm ox = 5nm (real oxide) + 6nm (roughness)

surface: anomalous skin effect

'ac' conductivity of copper: $\kappa(\omega) \approx \frac{\kappa_0}{1 + i\omega\tau}$ $\kappa_0 = 58 \cdot 10^6 \frac{1}{\Omega \mathrm{m}}$ $\tau = 2.46 \cdot 10^{-14}\,\mathrm{s}$

Resistive wall wakefield in the LCLS undulator, Bane, Stupakov; SLAC-PUB-11227, May 2005

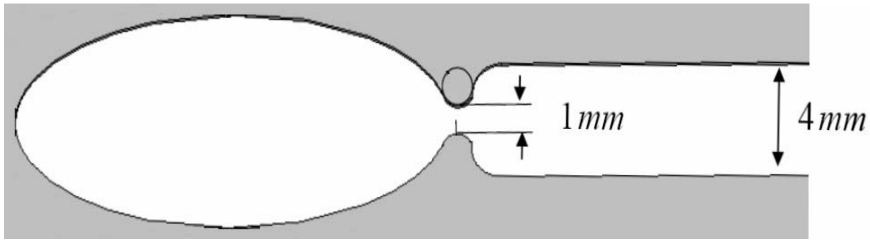
round pipe, R = 2.5mm, δ-wake



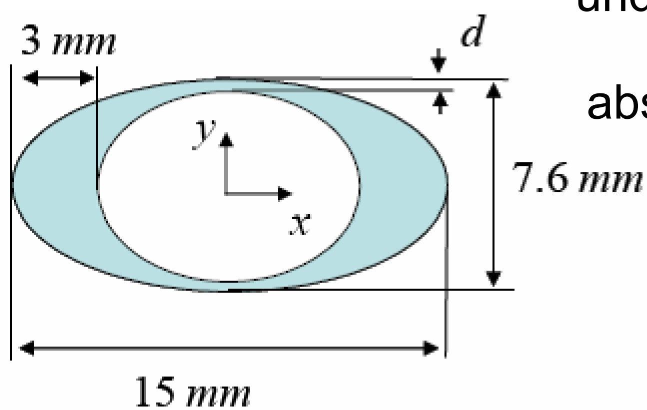
'ac' conductivity causes a slight overestimation

3d geometry

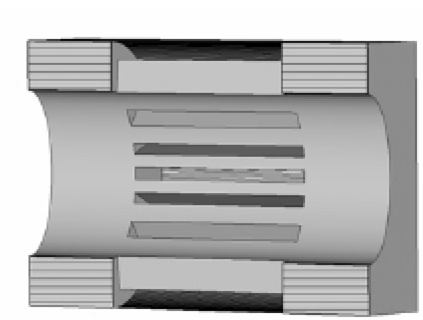
European XFEL



undulator pipe

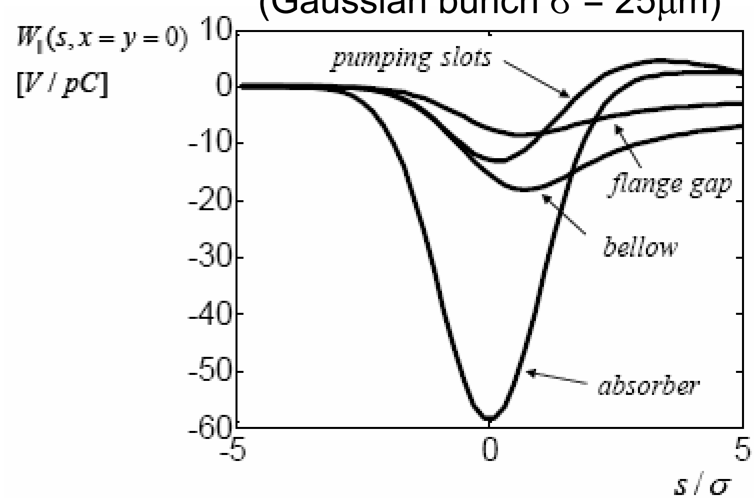


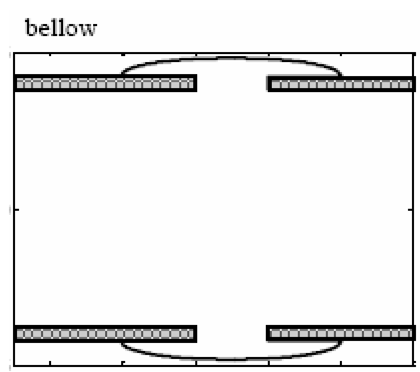
absorber

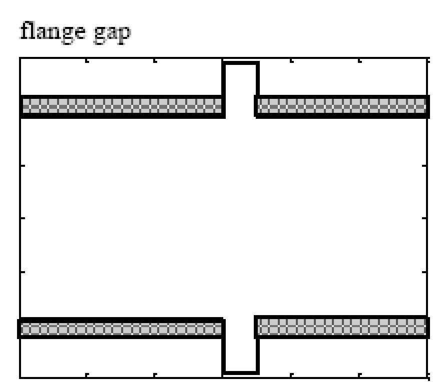


pump slots

per section (= 5m)
(Gaussian bunch σ = 25μm)



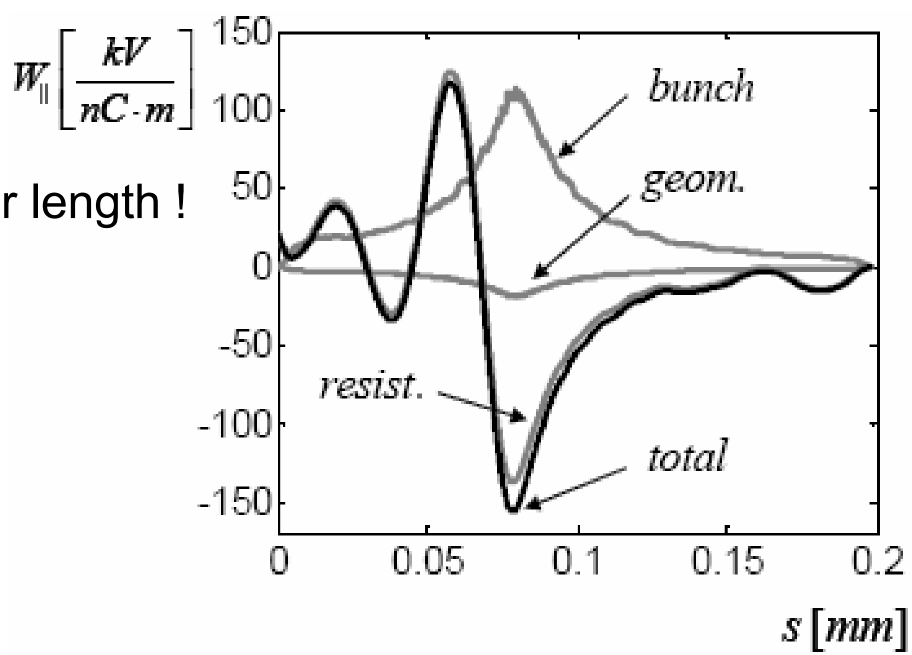




wakes in European XFEL

per section (= 5m)
(Gaussian bunch σ = 25μm)

	Loss, V/pC	Spread, V/pC	Peak, V/pC
absorber	42	16	-58
pumping slot (Fig.4)	<0.2	<0.1	>-0.3
pump (Fig.7)	9	4	-13
bellow	13	5	-18
flange gap	6	2.4	-8.5
Total geom.	**70**	**25**	**-95**



per length !

resistive wake
of round pipe !

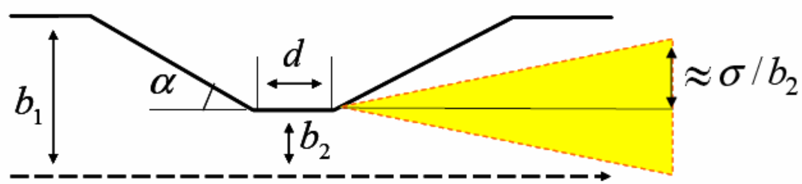
collimator wakes

Igor Zagorodnov
http://www.desy.de/xfel-beam/data/talks/talks/zagorodnov_-_colli_wakes_20060925.pdf

Outline

- Round collimators
  - Inductive regime
  - Diffractive regime
  - Near wall wakefields
  - Resistive wakefields
- 3D collimators (rectangular, elliptical)
  - Diffractive regime
  - Inductive regime
- Simulation of SLAC experiments
- XFEL collimators
  - Effect of tapering and form optimization
  - Kick dependence on collimator length

Round collimator. Regimes



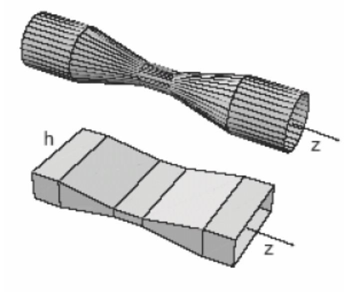
Top half of a symmetric collimator.

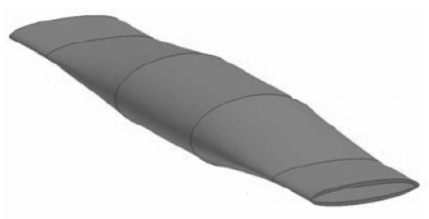
Inductive

$$\rho_1 \equiv \alpha b_2 \sigma^{-1} < 1$$

Diffractive

$$\rho_1 > 1$$





wakes (codes)

ECHO	2d,3d	no dispersion	indirect w-integration	conf. mesh	
CST-MWS	3d	dispersion	(indirect w-integration)	conf. mesh	
ABCI	2d	dispersion	indirect w-integration	staircase	
MAFIA	2d/3d	dispersion	(indirect w-integration)	staircase+	
GdfidL	3d	dispersion	indirect w-integration	staircase+	?
PBCI	3d	no dispersion	indirect w-integration	staircase	parallel
VORPAL	3d	dispersion		conf. mesh	parallel
Tau3P	3d	dispersion		conf. mesh	parallel
more		no dispersion			

$L\delta_z^2 > \sigma^3$ FDTD not sufficient

(L = length of field calculation,
δz = step width of mesh
σ = rms bunch length)

summary

bunch shape: if: $\sigma >> S_{\mathrm{ch}}$ $$Z_b'(\omega) \approx \frac{1}{2\pi R}\sqrt{\frac{i\omega\mu}{\kappa_0}} \quad \rightarrow w \propto \frac{1}{R\sqrt{\sigma\kappa_0}}$$

surface effects:
- roughness $\Delta_r \rightarrow$ dielectric layer $\Delta_d \approx \Delta_r/50$, $\varepsilon_r = 2$
- dielectric layer $\rightarrow$ inductive surface impedance

  } ≈ 10 %
- conductivity $\rightarrow$ surface impedance $\approx \sqrt{i\omega\mu/\kappa_0}$ ≈ 90 %
- effective surface impedance = superposion of surface imp.

round pipe: transformation to beam impedance $$Z_b' = \frac{Z_s}{2\pi R}\left(1 + i\frac{\omega}{c}\frac{R}{2}\frac{Z_s}{Z_0}\right)^{-1}$$

flat pipe: effects are overestimated by round pipe

geometric wakes: small compared to surface effects (for European XFEL)

acknowledgement to Igor Zagorodnov