# modeling of space charge effects and CSR in bunch compression systems

SC and CSR effects are crucial for the simulation of BC systems
CSR and related effects are challenging for EM field calculation
non-CSR effects are indispensable for design and simulation of BC systems

## part 1: CSR codes

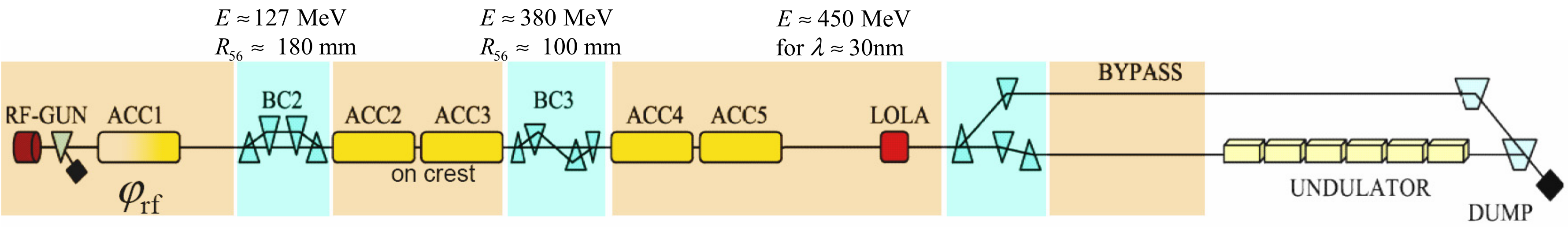
- effects: SC, CSR, shape variation ...
- approaches
  - Vlasov-Maxwell
  - paraxial approximation
  - 1d
  - sub-bunch
- Zeuthen benchmark example

## part 2: simulation of BC systems

- space charge
- codes & tools, particle distributions
- μ bunching
- non linear effects in longitudinal phase space
- example “rollover compression” (FLASH)
- example “controlled compression” (European XFEL)
- compensation in 2-bc systems
- conclusion

# effects

what is different in magnetic BC systems (compared to usual LINACS)?




CSR & SC, guiding fields dispersive

SC, guiding fields & rf linear

$r_{56}$ : there are dispersive sections with non-linear trajectories
chirp: there is a strong linear correlation between energy and longitudinal position
there is a variation of bunch shape
the ratio $I_{peak}$/Energy after compression is quite high

uniform motion(*): forces scale as $1/\gamma^2$
circular motion: some coherent effects as CSR are not suppressed with increasing $\gamma$

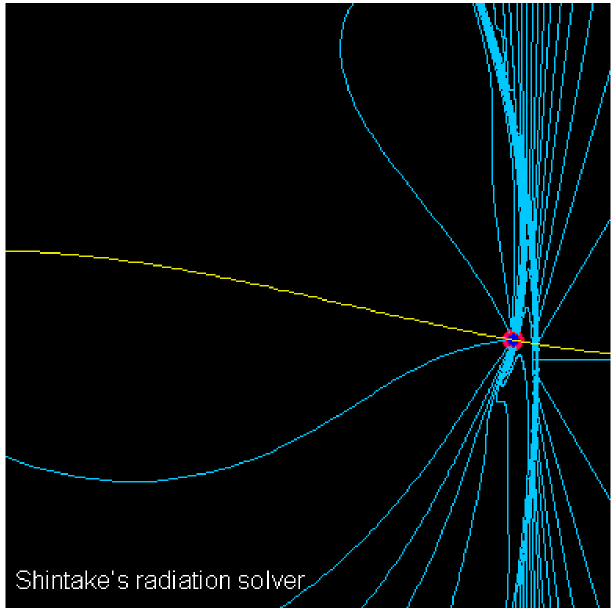
$\rightarrow$ new types of tracking codes with more general electromagnetic field solvers

*) uniform motion is an approximation for the motion of a particle distribution in a drift

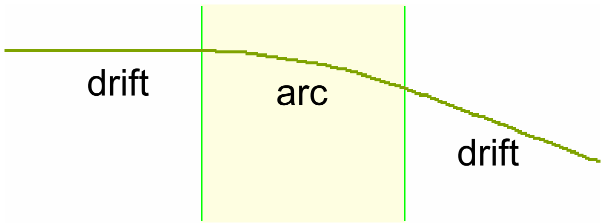
# … effects

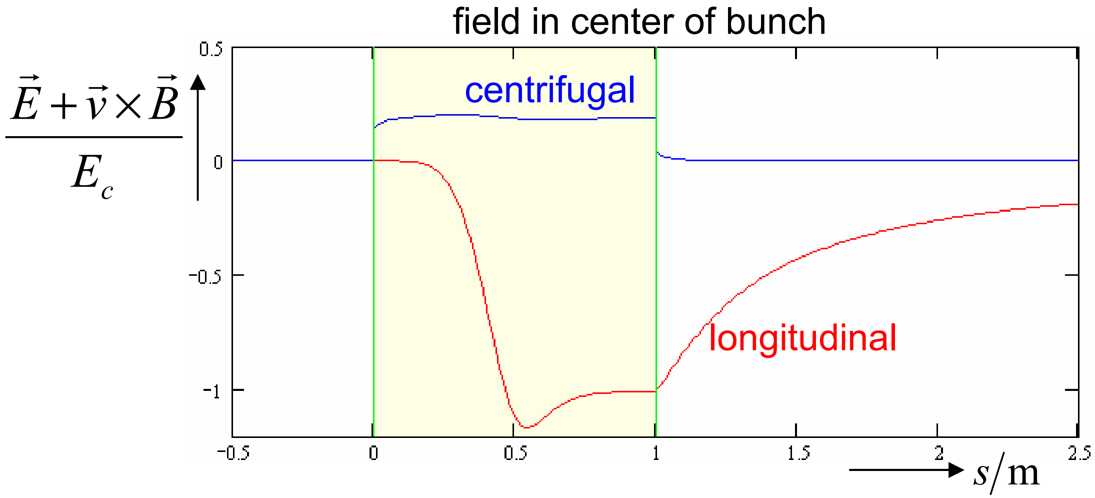
radiation effects

overtaking




long transients:

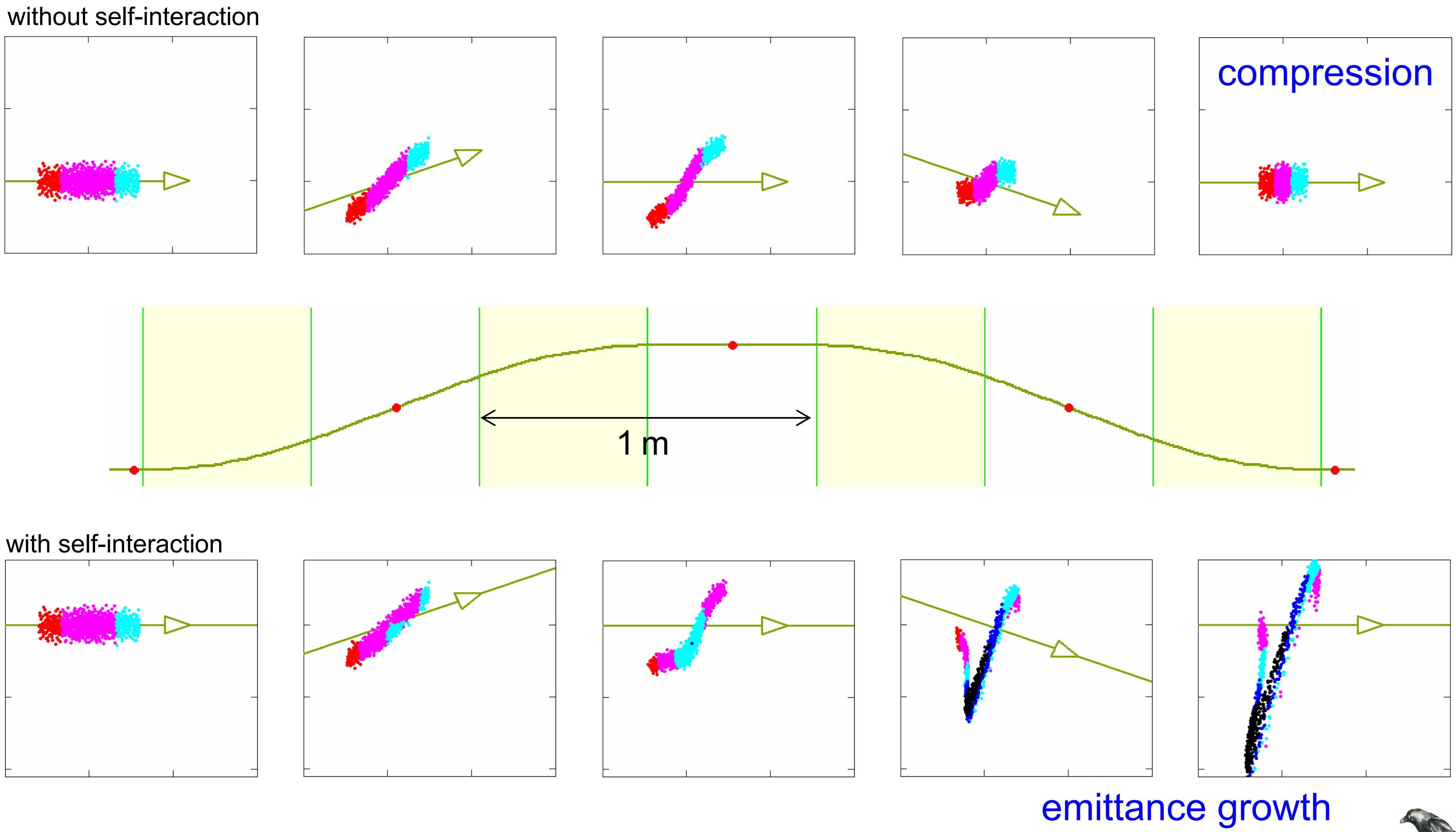


# … effects

shape variation

top view (horizontal plane), color = energy




# approaches

| | | |
|---|---|---|
| 1d (or projected) | (1) | solution of Maxwell problem based on retarded sources |
| sub-bunch approach | (2) | solves integral equation |
| Maxwell-Vlasov | (3) | → "natural" boundary condition: open<br>shielding: mirror charges → two plane model |
| paraxial approximation | (4) | Maxwell equations on grid<br>curvilinear coordinates<br>solves PDE<br><br>→ "natural" boundary condition: closed<br>efficient for strong shielding<br>generalization: resistivity, geometric wakes |

(1) Schneidmiller, Stupakov, Emma, Borland, Dohlus, …; ELEGANT, CSRtrack, …
(2) R.Li, Kabel, Dohlus, Limberg, Giannessi, Quattromini; ???, TrafiC$^4$, CSRtrack, TREDI
(3) Warnock, Bassi, Ellison
(4) Agoh, Yokoya

# Vlasov-Maxwell

Warnock, Bassi, Ellison: Progress on Vlasov Treatment …, PAC2005

4d Vlasov equation in beam frame: $\frac{\partial f}{\partial s} + z' \frac{\partial f}{\partial z} + p'_z \frac{\partial f}{\partial p} + x' \frac{\partial f}{\partial x} + p'_x \frac{\partial f}{\partial p_x} = 0$ with $f = f(\mathbf{r}, \mathbf{p}, s)$
(horizontal)

3d charge and current density distributions:
(in lab frame)

$$\rho_L(\mathbf{R}, Y, ct) = Q \cdot H(Y) \cdot \int f(\mathbf{r}, \mathbf{p}, \beta ct) d\mathbf{p}$$
$$\mathbf{J}_L(\mathbf{R}, Y, ct) = \ldots$$

with $Q$ = bunch charge
$H(Y)$ = fixed vertical profile (including mirror charges)
$Y$ = vertical coordinate

2d ($Y$-averaged) electromagnetic fields:
(fields by retarded source integration)

$$\mathbf{E}(\mathbf{R}, ct) = \langle \mathbf{E}(\mathbf{R}, Y, ct) H(Y) \rangle_{Y \in \mathrm{gap}}$$
$$B(\mathbf{R}, ct) = \ldots$$

EoM in beam frame:

$$z' = -\kappa(s) x \qquad p'_z = F_z$$
$$x' = p_x \qquad p'_x = \kappa(s) p_z + F_x$$

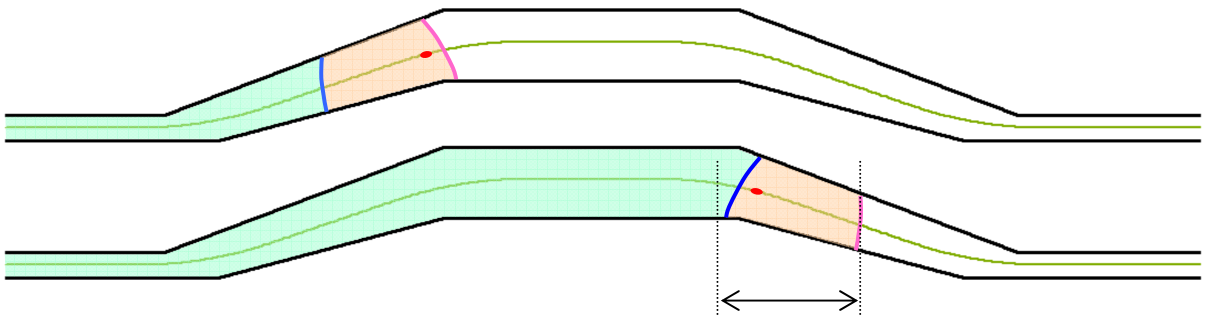
normalized Lorentz force:

$$F_x \propto \mathbf{E} \cdot \mathbf{V}(s, p_x)$$
$$F_z \propto \ldots$$

# em-field calculation with PDE?
(on a grid)

the problems (direct time domain calculation):

calculation window is much bigger than bunch




e.g. 2cm×8cm×6cm, $\sigma \approx 100\mu m \rightarrow V \approx 10^8 \sigma^3 \rightarrow$ large mesh
number of time steps $\propto$ chicane length / $\sigma \propto 10^6$
numerical dispersion

no way with explicit schemes (my personal opinion)

but: strong shielding; calculation window can be reduced
neglect backward waves; Field(x,y,s,t) is a slowly function of s-ct

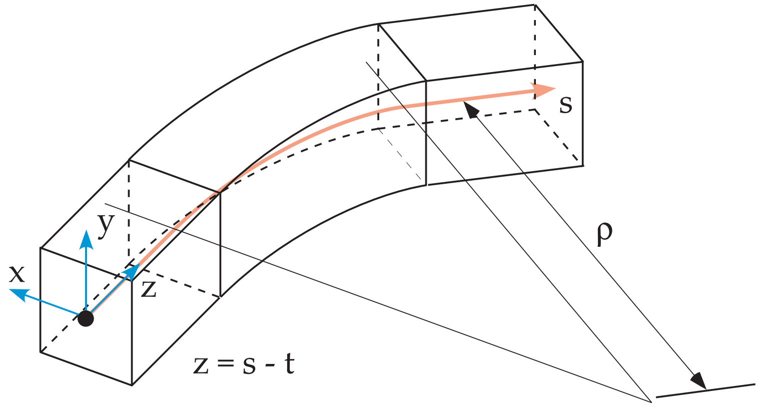
the slowly variation should allow an algorithm with large steps in s-ct
not in time domain!

# paraxial approximation

T. Agoh: PhD Thesis, Dec. 2004

$$\left(\nabla^2 - \frac{\partial^2}{\partial t^2}\right)\breve{\mathbf{E}} = \mu_0\left(\nabla \breve{J}_0 + \frac{\partial \breve{\mathbf{J}}}{\partial t}\right)$$

wave equation in time domain




"accelerator coordinates"
and Fourier transformation

$$E_\xi(x,y,k,s) = \int \breve{E}_\xi(x,y,s,t=s+\tau)e^{-ik\tau}d\tau$$

with $\xi = x,y,s$

weak s-dependence (forward propagation)

$$\frac{\partial^2 E}{\partial s^2} << 2ik\frac{\partial E}{\partial s}$$

pipe size small compared to bend radius

$$a << \rho$$

relativistic particles $\gamma >> 1$

paraxial approximation for transverse em-fields

$$\frac{\partial E_\perp}{\partial s} = \frac{i}{2k}\left[\left(\nabla_\perp^2 + \frac{2k^2 x}{\rho}\right)E_\perp - \mu_0 \nabla_\perp J_0\right] \quad \rightarrow \quad E_s = \frac{i}{k}\left[\frac{\partial^2 E_x}{\partial x^2} + \frac{\partial^2 E_y}{\partial y^2} - \mu_0 J_0\right]$$

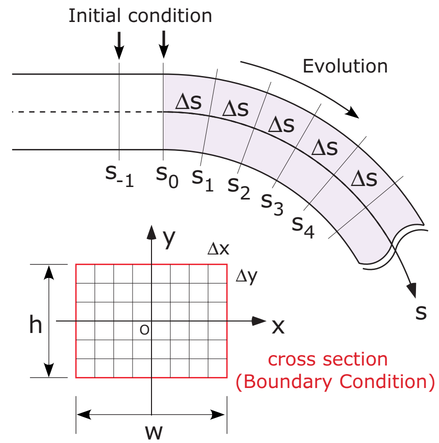
# … paraxial approximation

T. Agoh: PhD Thesis, Dec. 2004

advantages:

- (curved) rectangular beam-pipes defined by coordinate planes
- bending radius needs not to be constant
- mesh based computation (explicit, frequency by frequency)
- resistive wall effects

- generalization to arbitrary transverse cross-sections and smooth variation of longitudinal profile




special care:

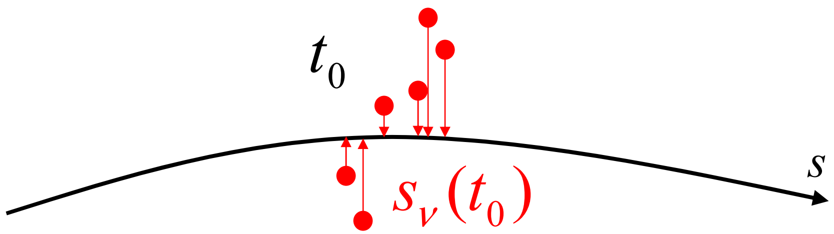
- singularity of 1d beams
- transverse beam dimensions & SC effects
- variation of bunch shape

problems:

- free space or large chamber
- non smooth variations $\rightarrow$ stimulation of backward waves
- distributions with fine structure

# “CSR” codes: 1d

$$\dot{\mathbf{p}}_\nu = q\left(\mathbf{e}_{\nu\|} E^{(\lambda)}(s_\nu, t) + \mathbf{v}_\nu \times \mathbf{B}^{(\mathrm{ext})}\right)$$




**some physics is missing**

no transverse self-forces

no transverse dimensions,
rigid 1d charge distribution:

$$\lambda^{(\delta)}(s - t_0 c) = \sum q_\nu \delta((s - t_0 c) - (s_\nu - s))$$

$$\lambda(s - t_0 c) = \lambda^{(\delta)}(s - t_0 c) \otimes \left(g\left(s/\sigma\right)/\sigma\right)$$

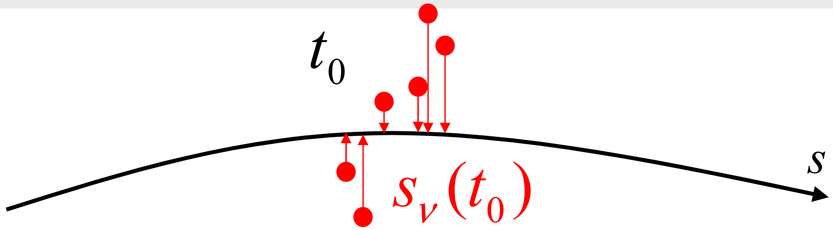
no SC effect,
1d E-field without $\gamma^{-2}$ singularity:

$$E^{(\lambda)}(s, t_0) = \int \lambda'(u + s - ct_0) K(s, u)\, du$$

no transverse dependency
of longitudinal forces

**very low numerical effort**

# … “CSR” codes: 1d

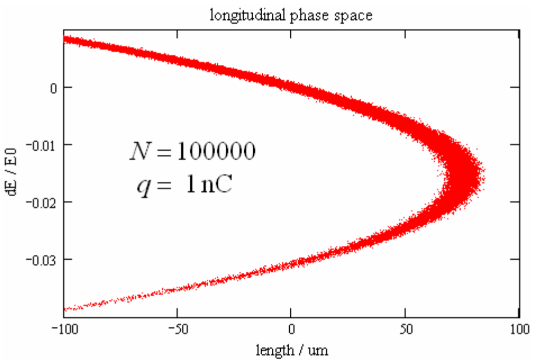


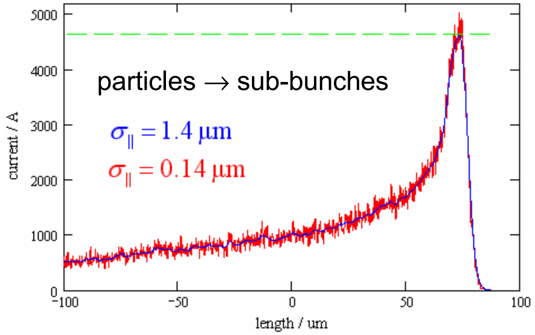

differences of implementations (ELEGANT vs. CSRtrack)

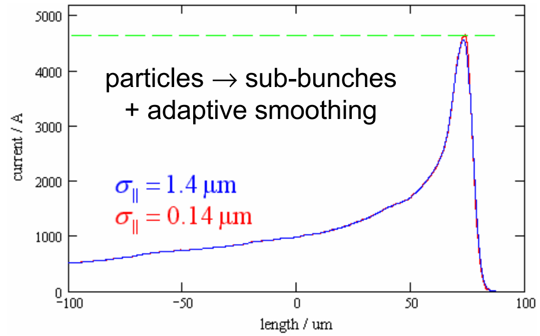
a) trajectory: arc after line or line after arc (neglects longer interactions, uses artificial damping);
general sequences of arcs and lines (→ interaction with waves from objects far beyond; that requires sometimes small track steps although the net effect is weak)

b) shielding: PEC planes

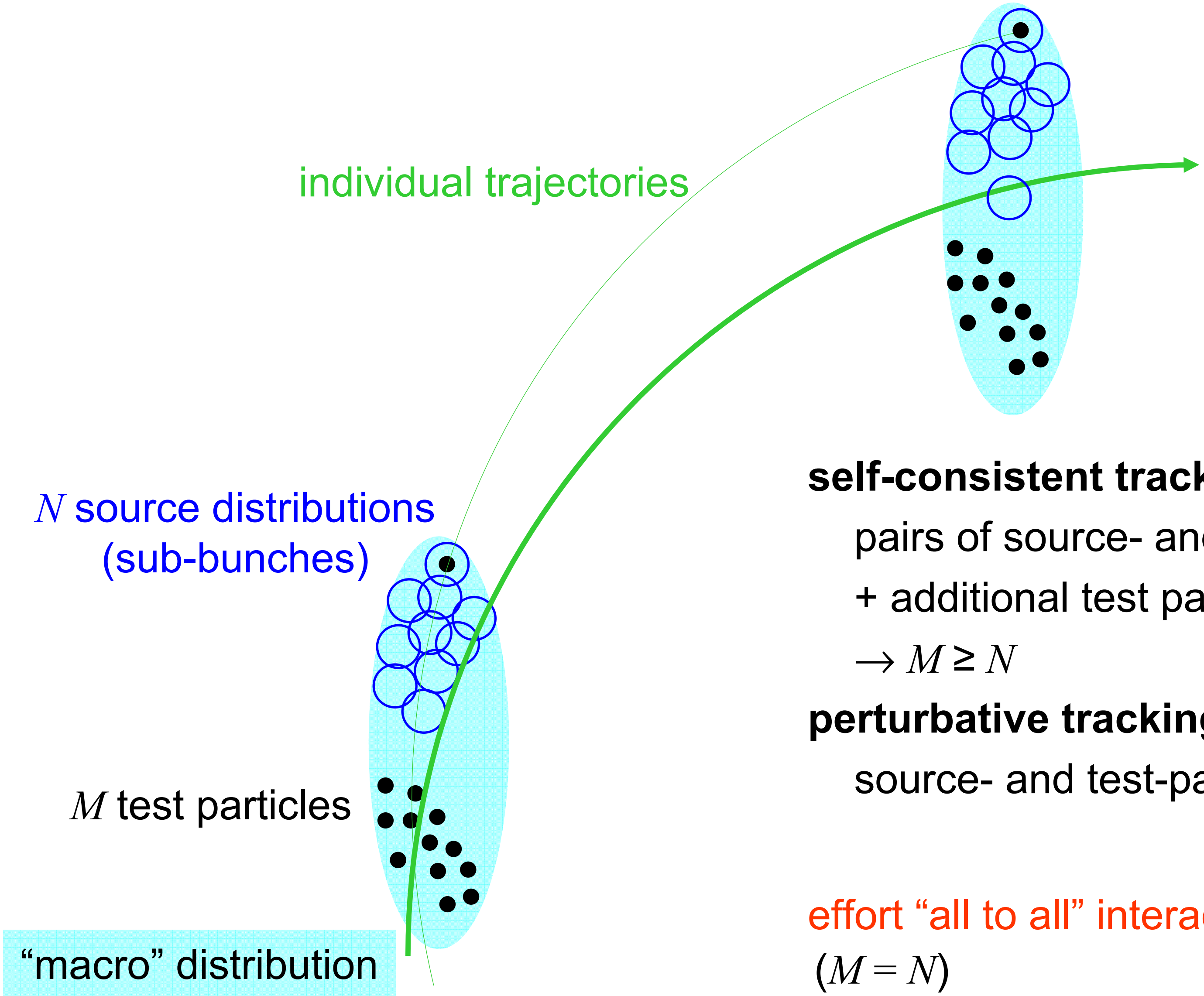
c) smoothing: crucial for suppression of artificial μ-bunch effects
binning & smoothing of histograms,
sub-bunches & density dependent adaptive filters



# “CSR” codes: sub-bunch approach




**self-consistent tracking**:

pairs of source- and test-particles ⊙

\+ additional test particles •

$\rightarrow M \geq N$

**perturbative tracking**:

source- and test-particles independent ○ •

effort “all to all” interaction = $N^2 \, E_{\mathrm{s\_to\_s}}$

$(M = N)$

# … "CSR" codes: sub-bunch approach

## calculation of sub-bunches

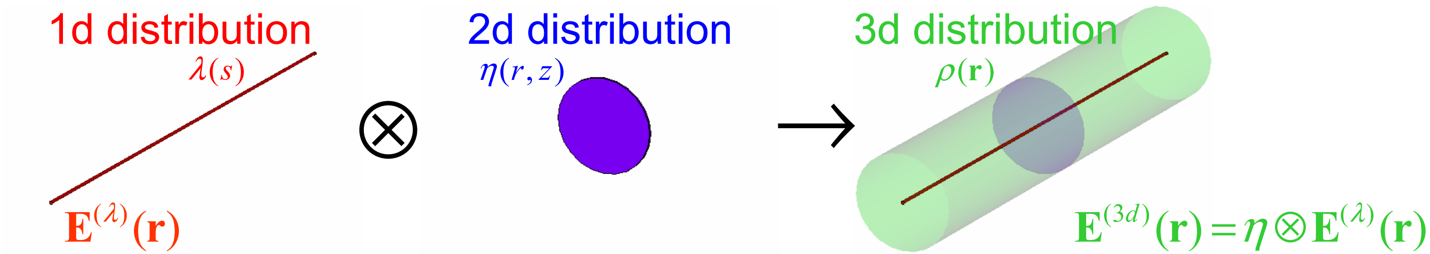
point particles & 1d sub-bunches $\rightarrow$ singular fields

3d sub-bunch $\rightarrow$ 3d integration

$$\mathbf{E}(\mathbf{r},t) = \int \frac{\mathbf{Q}(\mathbf{r}',t')}{\|\mathbf{r}-\mathbf{r}'\|} dV'$$

calculation of 3d sub-bunches by 1d integration:

a) convolution technique




(1d integral)

(approximation based on near field expansion)

b) spherical Gaussian sub-bunches

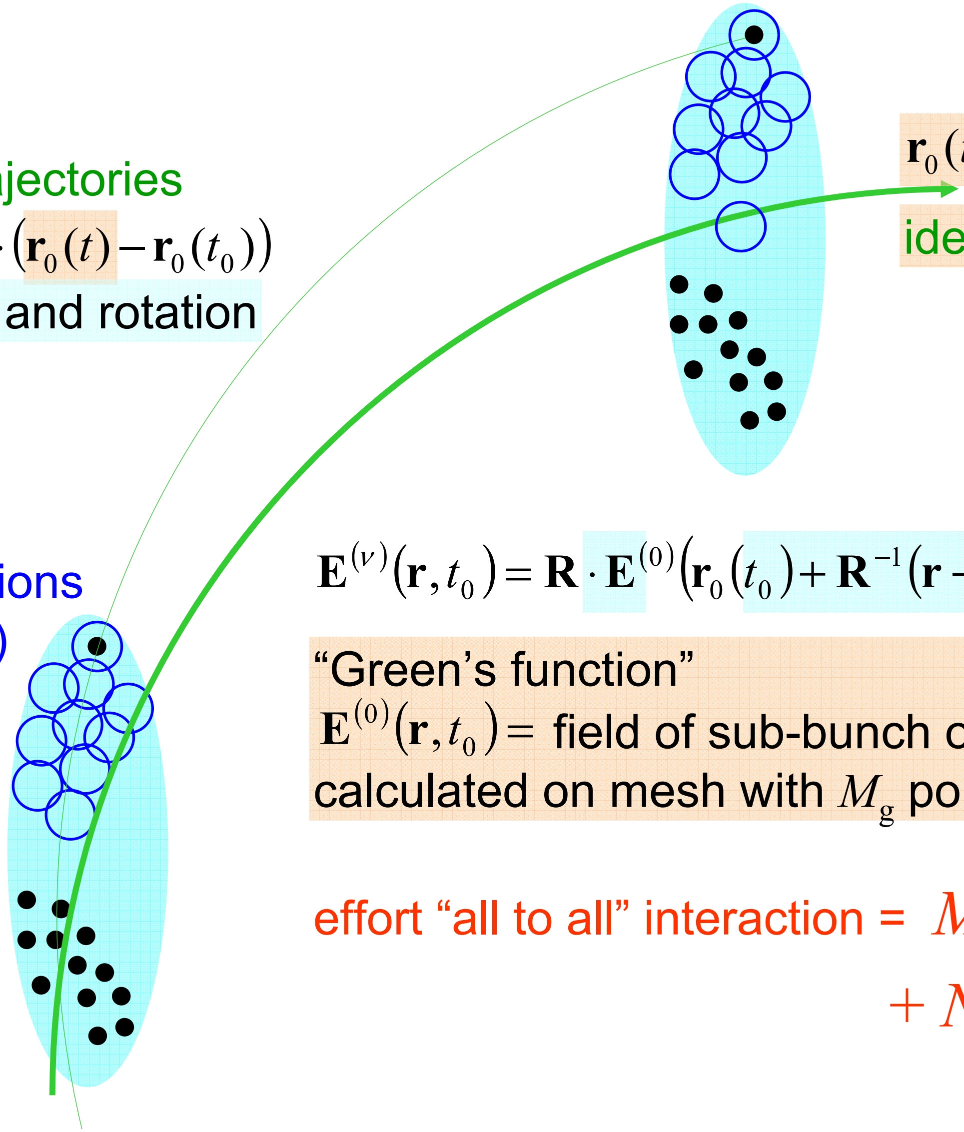
$$\rho(\mathbf{r},t) = \rho_s(\mathbf{r} - \mathbf{r}_t(t))$$

$$\Phi(\mathbf{r},t) = \frac{q}{\varepsilon(2\pi)^{3/2}\sigma^2} \int_0^\infty f\left(\frac{r'}{\sigma}, \frac{\|\mathbf{r} - \mathbf{r}_t(t - r'/c)\|}{\sigma}\right) dr'$$

with $f(a,b) = \exp\left(-\frac{a^2+b^2}{2}\right)\frac{\sinh(ab)}{b}$

# … "CSR" codes: sub-bunch approach

## reduction of effort: Green's function on mesh



$\mathbf{r}_0(t)$

ideal trajectory

individual trajectories

$$\mathbf{r}_\nu(t) \approx \mathbf{r}_\nu(t_0) + \mathbf{R}_\nu \cdot \left(\mathbf{r}_0(t) - \mathbf{r}_0(t_0)\right)$$

individual offset and rotation

$N$ source distributions (sub-bunches)

$$\mathbf{E}^{(\nu)}(\mathbf{r}, t_0) = \mathbf{R} \cdot \mathbf{E}^{(0)}\left(\mathbf{r}_0(t_0) + \mathbf{R}^{-1}\left(\mathbf{r} - \mathbf{r}_0(t_0)\right), t_0\right)$$

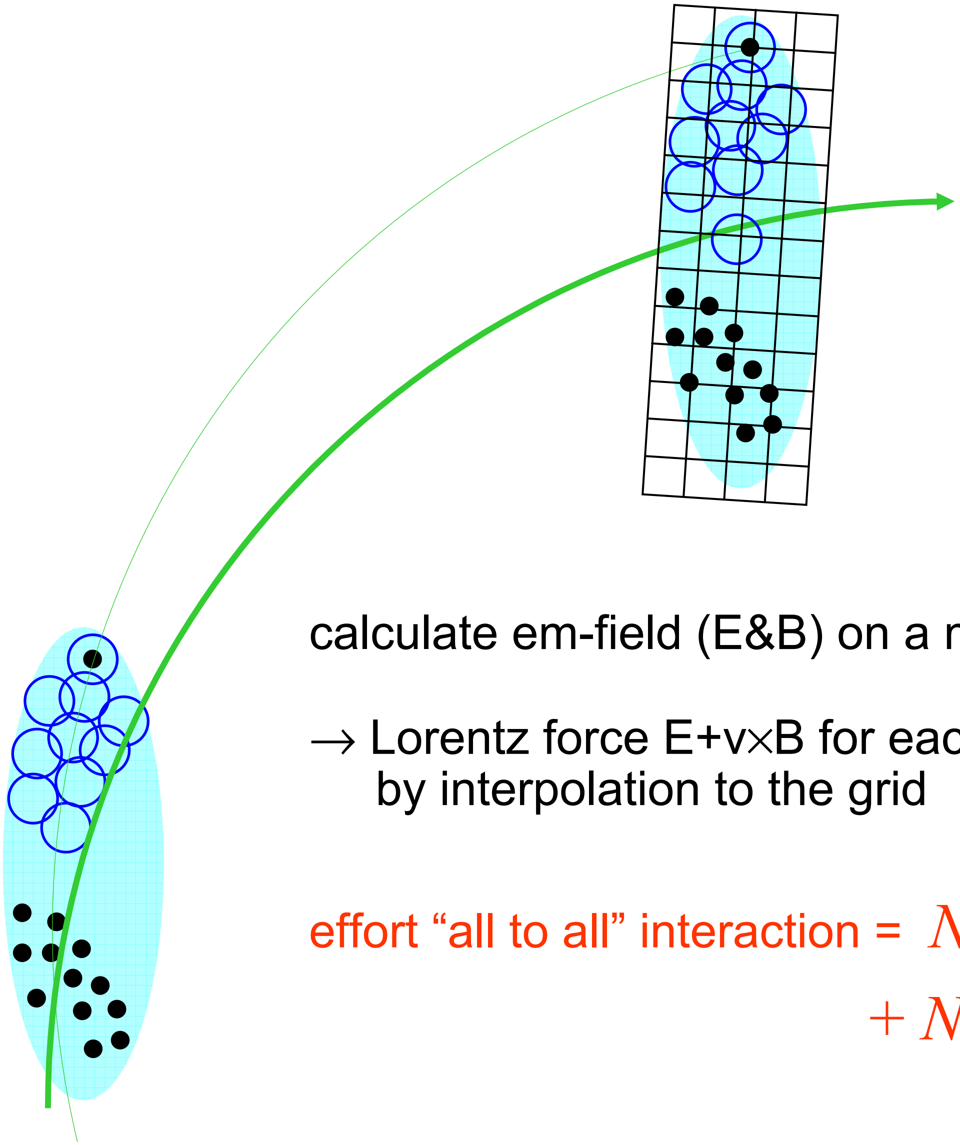
"Green's function"

$\mathbf{E}^{(0)}(\mathbf{r}, t_0) =$ field of sub-bunch on ideal trajectory calculated on mesh with $M_\mathrm{g}$ points

effort "all to all" interaction = $M_\mathrm{g}\, E_{\mathrm{s\_to\_s}} + N^2\, E_{\mathrm{interpolation}}$

# … “CSR” codes: sub-bunch approach

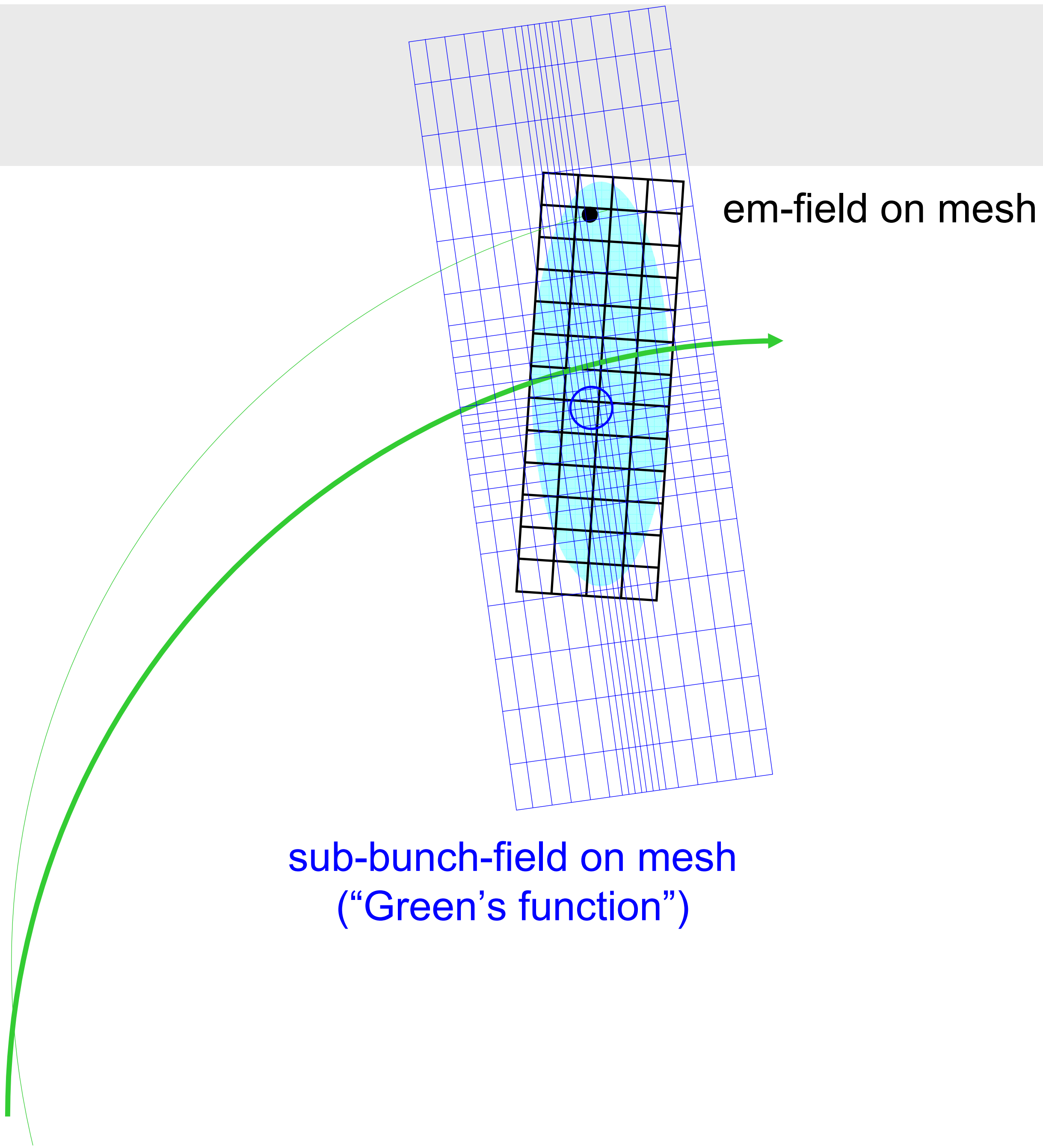
## reduction of effort: em-field on mesh



calculate em-field (E&B) on a mesh with $M_{\mathrm{em}}$ points

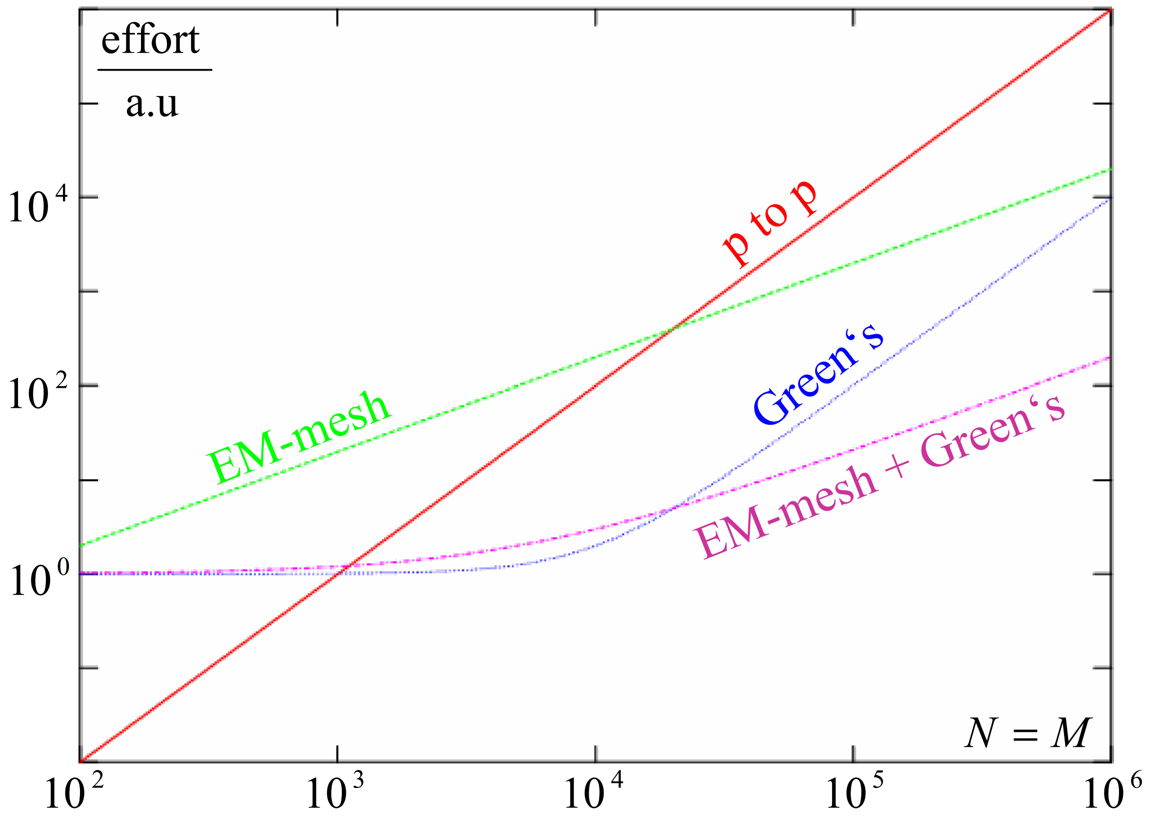
→ Lorentz force E+v×B for each point in that volume
by interpolation to the grid

effort “all to all” interaction = $N \cdot M_{\mathrm{em}} \, E_{\mathrm{s\_to\_s}} + N \cdot E_{\mathrm{em\_interpolation}}$


em-field on mesh
sub-bunch-field on mesh
("Green's function")

# … “CSR” codes: sub-bunch approach

scaling of effort
(simplified)



# Zeuthen benchmark chicane

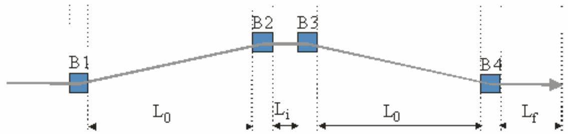
ICFA Beam Dynamics Mini-Workshop, Berlin-Zeuthen 2002, **`http://www.desy.de/csr`**

## CSR workshop 2002

Workshop | Benchmark | Elegant | P. Emma Prog. | R. Li Prog. | TraFiC4 | Tredi | Info / Contact | Links

← computed by many CSR codes
still a reference for new developments
e.g. Maxwell-Vlasov solver

The example consists of a simple four-bend chicane with parameters similar to the one required for the compression stages of the LCLS (at 5 GeV) or TESLA XFEL (at 500 MeV). It is meant to be a compromise between academic benchmarking and more practical issues. The compressor is depicted in the figure below and its parameters are gathered in the first table. Click on the graphics to download a MAD input deck.




| *Parameters* | *Symbol* | *Value* | *Unit* |
|---|---|---|---|
| Bend magnet length (projected) | $L_b$ | 0.5 | m |
| Drift length B1->B2 and B3->B4 (projected) | $L_0$ | 5.0 | m |
| Drift length B2->B3 | $L_i$ | 1.0 | m |
| Post chicane drift | $L_f$ | 2.0 | m |
| Bend radius of each dipole magnet | R | 10.35 | m |
| Bending Angle | $\phi$ | 2.77 | deg |
| Momentum compaction | $R_{56}$ | -25 | mm |
| 2nd order momentum compaction | $T_{566}$ | +37.5 | mm |
| Total projected length of chicane | $L_{tot}$ | 13.0 | m |
| Vertical half gap of bends | g | 2.5,5 | mm |

4 magnet chicane
length = 15 m
r56 = 100 mm

**The electron beam description:**

The input electron beam will test two different examples: (1) a uniform, and (2) a Gaussian distribution for the temporal profile, where the initial rms length is the same in both case ($FWHM_{uniform}$=2Sqrt[3]*rms). The transverse phasespace is assumed to be gaussian in either case. The beam should have a perfectly linear time-energy "chirp" (the bunch head has lower energy than the tail). Therefore the time and energy distribution will be identical. In addition a very small uncorrelated ("slice") energy spread should be added with, for example, a Gaussian distribution.

| *Parameter* | *Symbol* | *Value* | *Unit* |
|---|---|---|---|
| Nominal energy | $E_0$ | 0.5/5.0 | GeV |
| bunch charge | Q | 0.5, 1.0 | nC |
| incoherent rms energy spread | $(\Delta E)$ | 10 | keV |

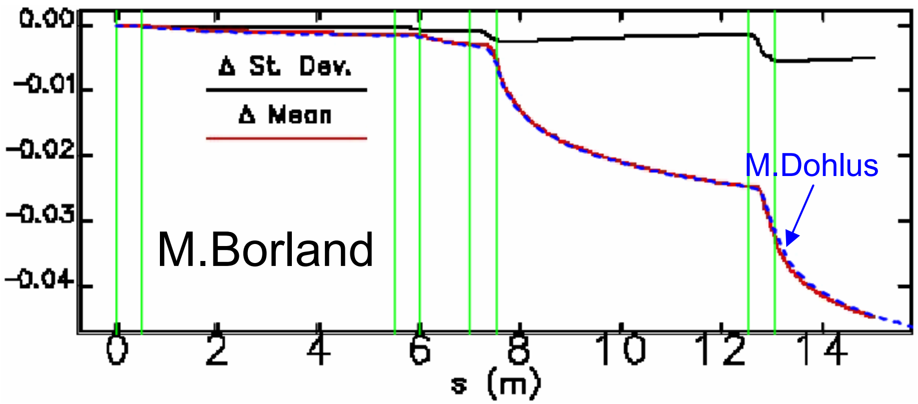
energy = 500 MeV / 5GeV
charge = 0.5 nC or 1nC
compression factor = 10
(600 A → 6 kA)
shape = Gaussian / rectangular

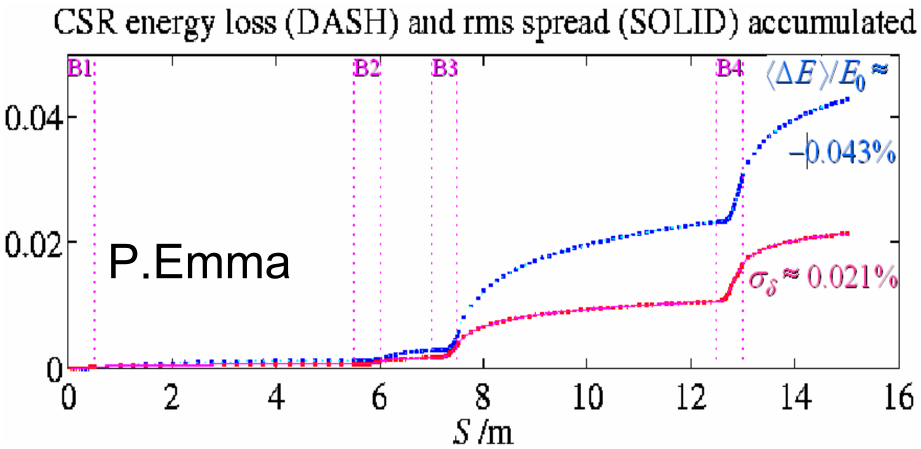
# … Zeuthen benchmark chicane

## longitudinal phase space

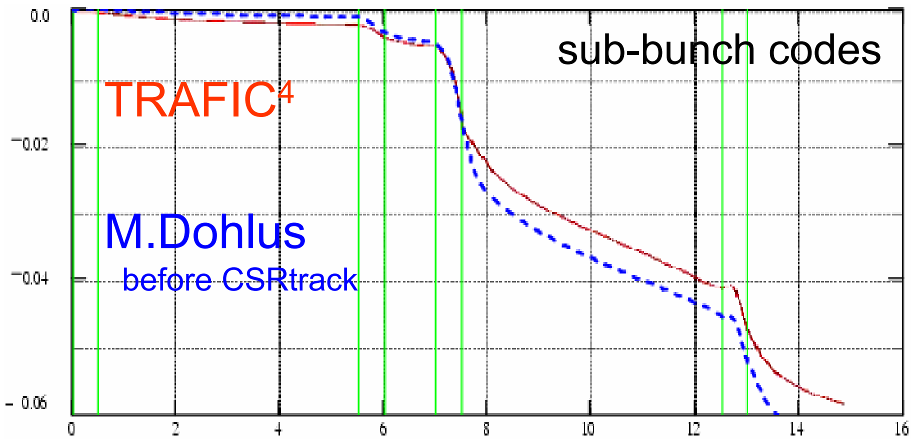
5GeV, 1nC, Gaussian

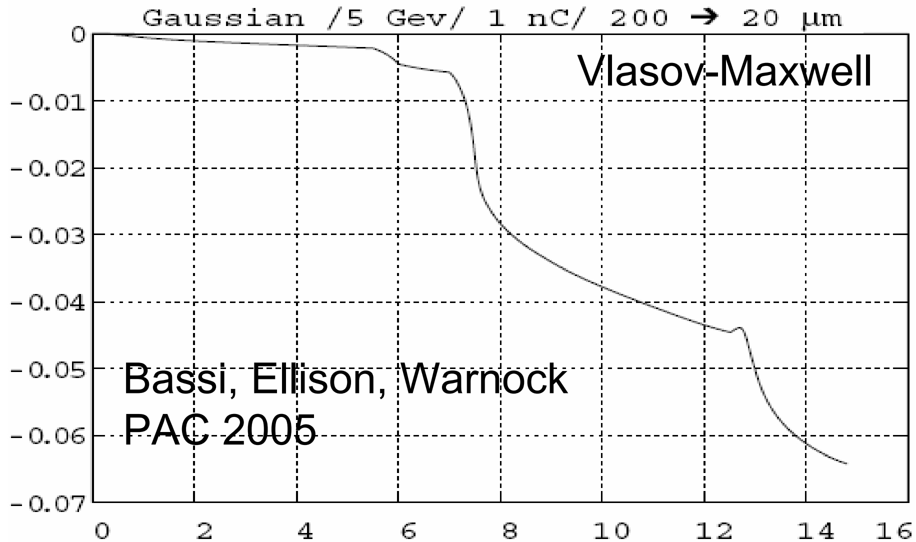
1d codes




agreement between 1d codes
e.g. relative loss @ 14m ≈ 0.04%




relative loss @ 14m ≈ 0.06%
(differences due to transverse beam dim.?!)

# … Zeuthen benchmark chicane

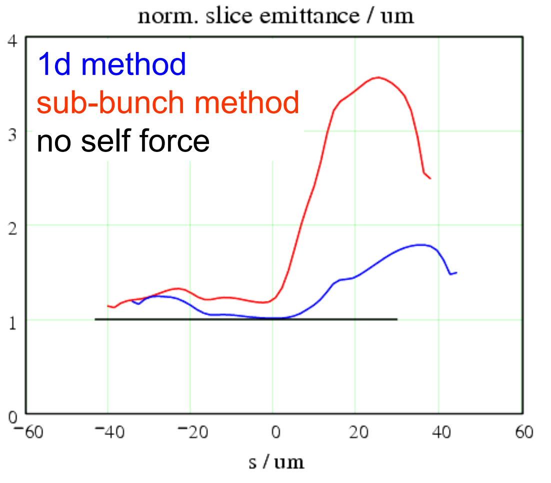
## horizontal phase space

5GeV, 1nC, Gaussian

available results are comparable:

- weak growth of slice emittance about 1% of $1 \cdot 10^{-6}$ m
- projected emittance $\approx 1.5 \cdot 10^{-6}$ m

but: 500 MeV, 1nC, trapezoid




significant differences between 1d and sub-bunch methods for lower energy; 3d & space charge effects

# part 2: simulation of BC systems

some problems

physical
numerical

- particle description (macro particles,
  ensembles, sub-bunch distributions
  phase space density)

- tracking with different methods (different particle descriptions)

- ●● μ-bunching → laser heater
  → decoupled investigation → amplification
  → noise suppression

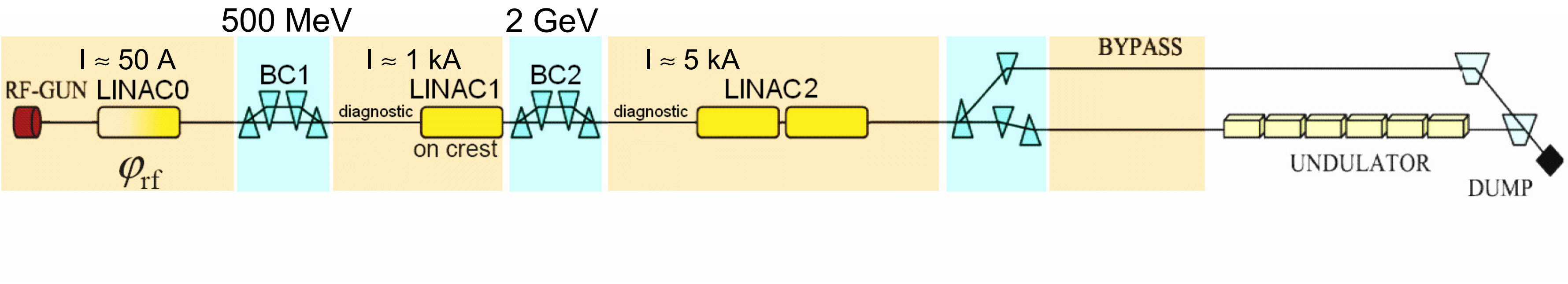
- longitudinal sensitivity → a) controlled compression
  → b) "over" compression

- transverse: space charge Q shift

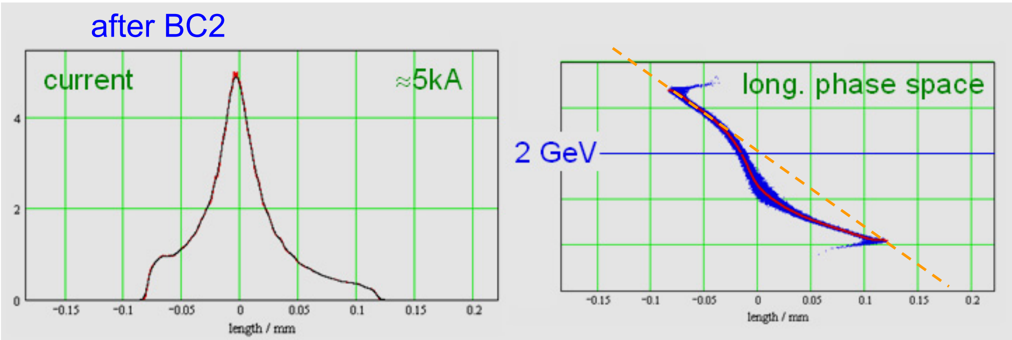
# SC contribution to longitudinal phase space

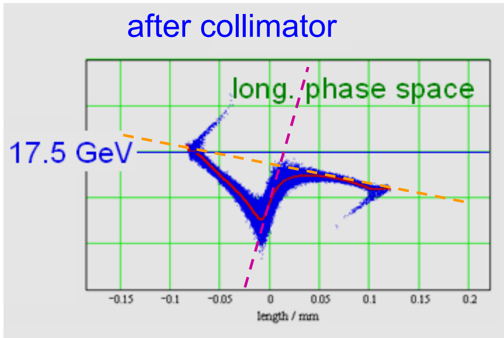
longitudinal SC effects per length $\propto I/\gamma^2$

longitudinal SC effect in accelerator ($\gamma_1 \to \gamma_2 >> \gamma_1$) $\propto \dfrac{I}{\gamma_1} \cdot \dfrac{1+\ln(\gamma_1)}{\gamma'}$




e.g. European XFEL:



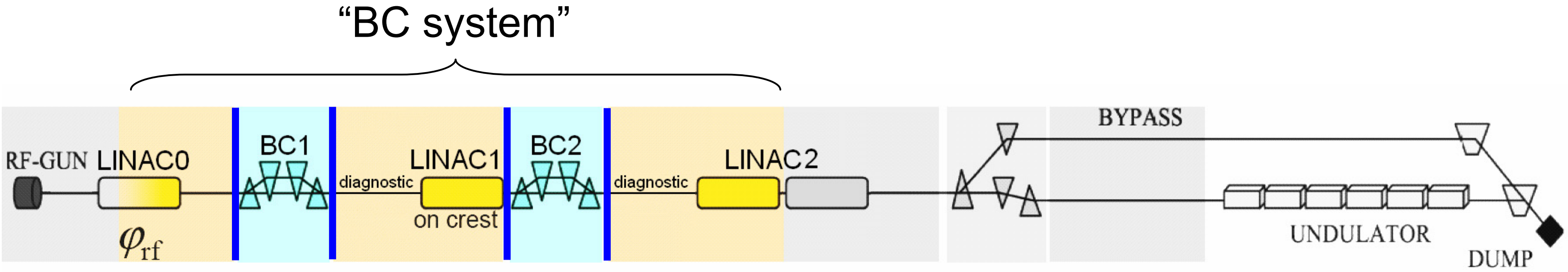




negative chirp compensated by LINAC wakes

positive chirp induced by space charge !

# simulation of BC systems

## codes & tools




CSR codes

utility programs:

format and/or phase space conversion
some simple manipulations of phase space:
add cavity wakes
add space charge wakes (semi analytic model)
transverse matching, …

linear trajectory codes (=LT codes):

1st principles method: particle in cell codes as MAFIA T3

working horse: Runge-Kutta tracker + Poisson solver
PARMELA, ASTRA, GPT, …
or ELEGANT + external SC calculation

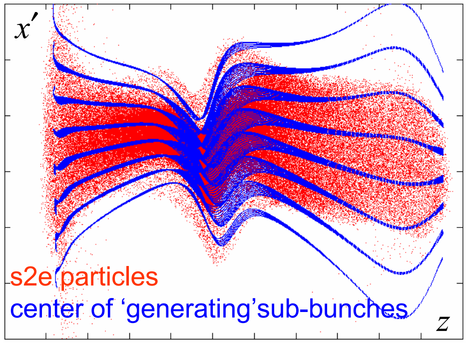
# simulation of BC systems

## particle distribution

try to simulate the complete BC system (or even s2e) with one set of particles

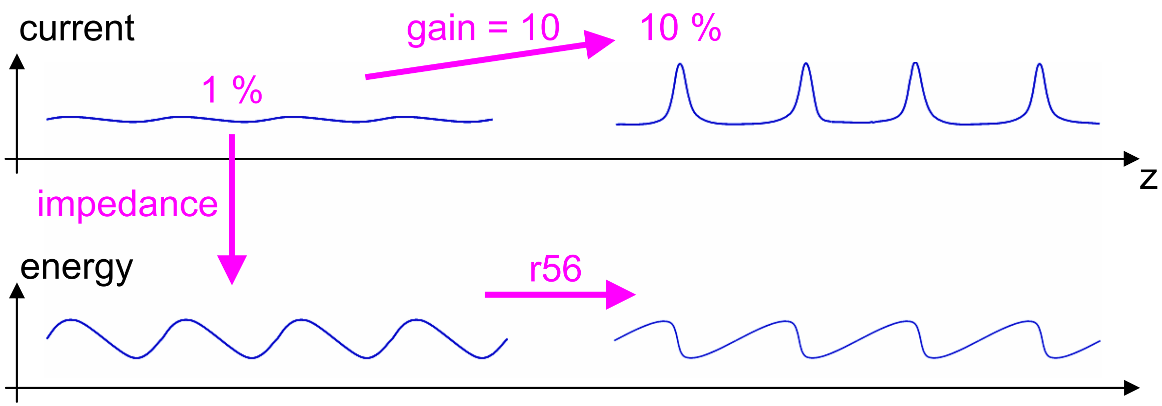
injector simulation and linear trajectory codes:
- typical number of particles ~ $10^5$ … $10^6$
- equal charged
- random or semi-random distribution in 6d phase space

noise of particle distribution is a problem in general $\rightarrow$ amplification of $\mu$-bunching
- 1d: binning, filtering (e.g. sub-bunches), adaptive to density
- mesh: too few particles per cell (e.g. of Poison solver) are a problem $\rightarrow$
  - increase number of particles or
  - decrease resolution of mesh or
  - reduce dimension of mesh (e.g. rz in ASTRA or xy in CSR codes)
- use smooth source distribution:
  - track original (s2e) particles in the field that is created by the smooth source ($\rightarrow$ and track smooth source with self interaction)




number of particles is a problem for some CSR methods
- 1d method: similar as LT codes
- sub-bunch methods with point to point interaction*: N ~ $10^3$ ... $10^4$
- sub-bunch methods + mesh techniques*: N ~ $10^5$ ... $10^6$
- (*) 10 .. 20 CPUs

# μ-bunching - amplification




picture based on: Z. Huang FLS2006

impedances (steady state):

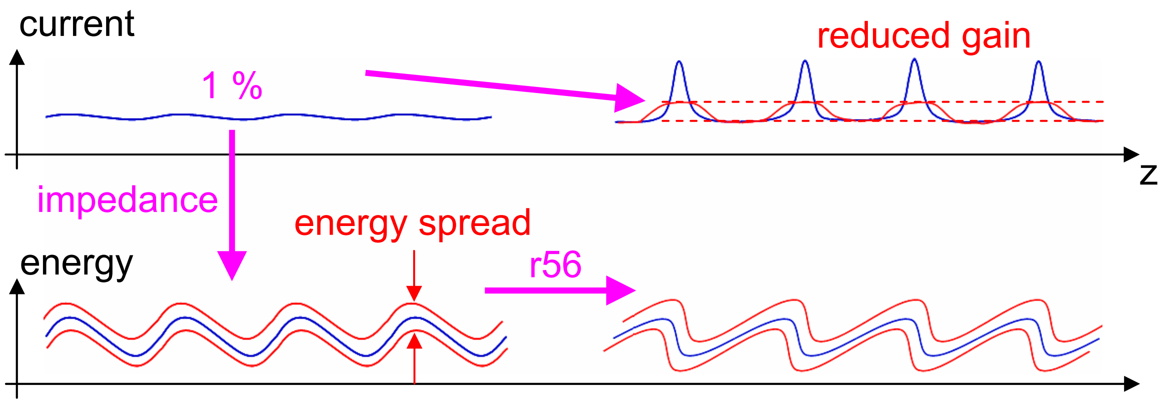
$$Z'_{sc}\left(k,\sigma_r,R_{pipe},\gamma\right) \approx \frac{iZ_0 k}{2\pi\gamma^2}\ln\left(\frac{\gamma}{k\sigma_r}\right)$$ (free space, $k\sigma_r/\gamma \ll 1$) "SC-instability"

$$Z'_{CSR}\left(k,R_{curv}\right) \approx Z_0\frac{\Gamma(2/3)}{2\pi}\sqrt[3]{\frac{k}{3iR_{curv}^2}}$$ "CSR-instability"

# … μ-bunching - amplification

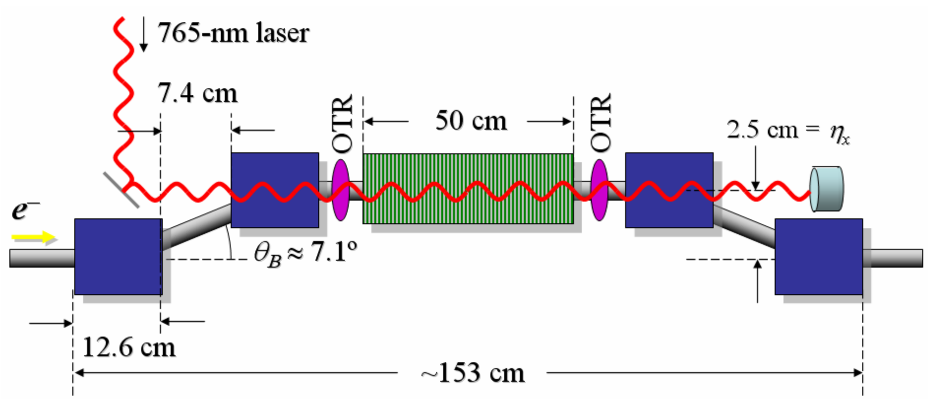
## laser heater

proposed by E. Schneidmiller 2002




picture based on: Z. Huang FLS2006

‘laser heater’ System (LCLS layout)



# … µ-bunching - amplification

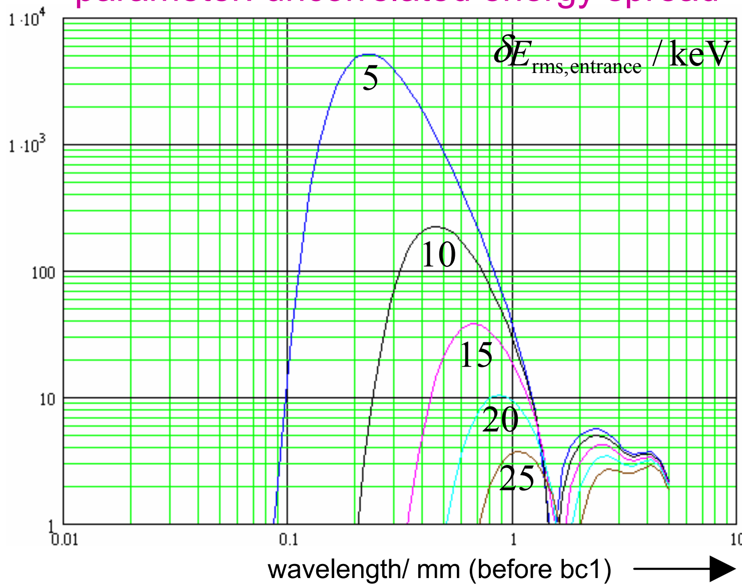
## numerical aspects

1) it is difficult to simulate macroscopic & microscopic effects together
(very high resolution, very many particles required)

2) → separate investigation of µ-bunching
CSR: integral equation method (limited applicability)
projected method: modulated beam, 1- and 2-stage compression
SC: impedance + r56

example: European XFEL

gain vs. modulation wavelength
parameter: uncorrelated energy spread

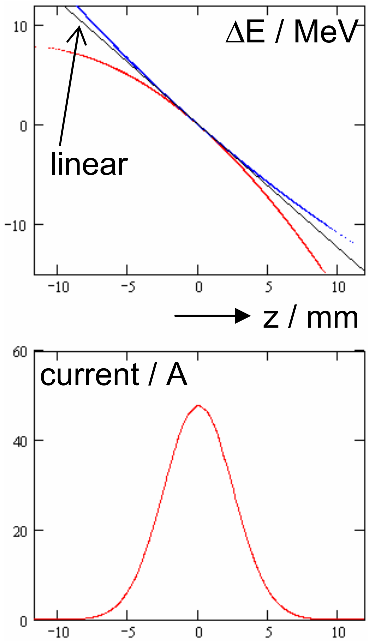



3) s2e simulations without µ structure:
avoid artificial instability
e.g. due to shot noise of few macro-particles → noise reduction

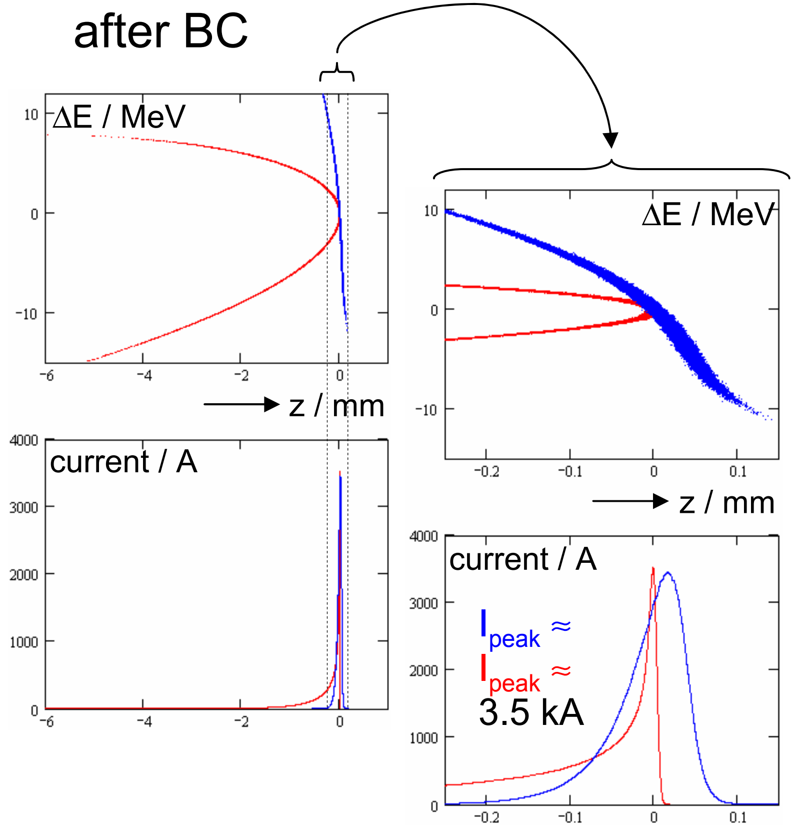
# non linear effects in long. phase space

## controlled compression vs. rollover compression

### before BC

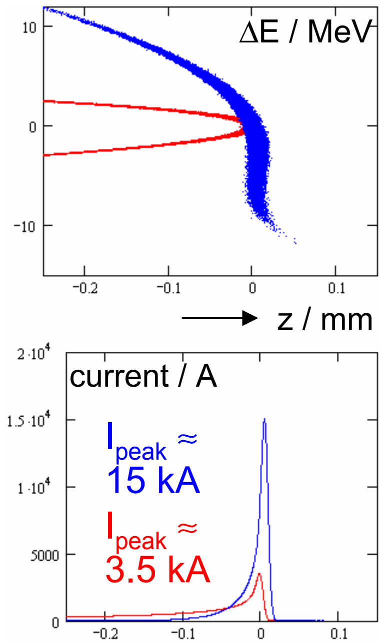



### after BC




### lost control:

magnet strength changed by 0.5%




**controlled compression:**

ΔE = non lin. function(z)

$z_2$-$z_1$ = non lin. function(ΔE)

compensation of both effects with higher harmonics rf

**rollover compression:**

use rollover

**controlled compression:**

~ uniform compression of complete distribution

very sensitive to parameter fluctiations

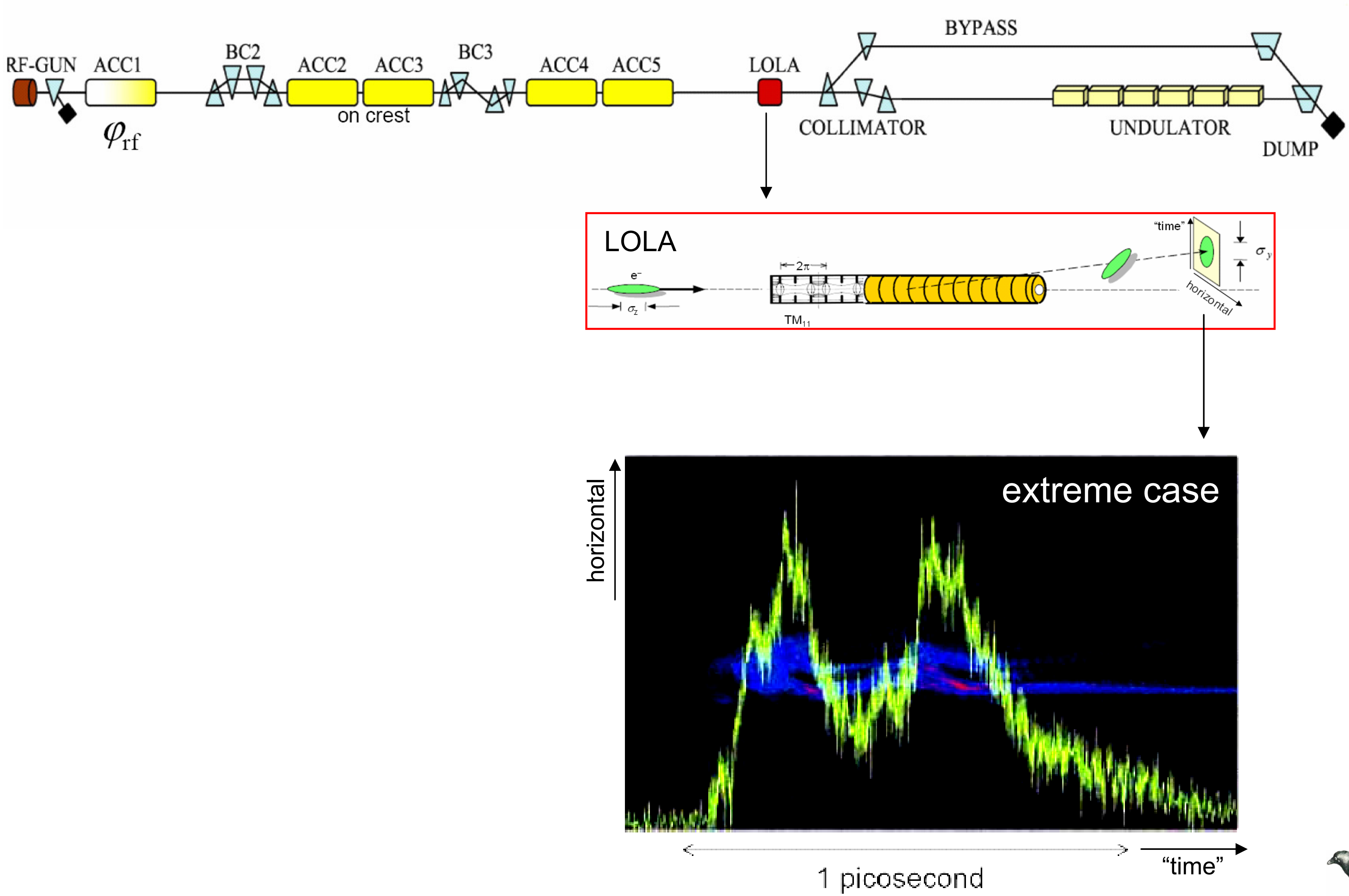
**rollover compression:**

sharp spike with less charge

insensitive to parameter fluctuations, few knobs

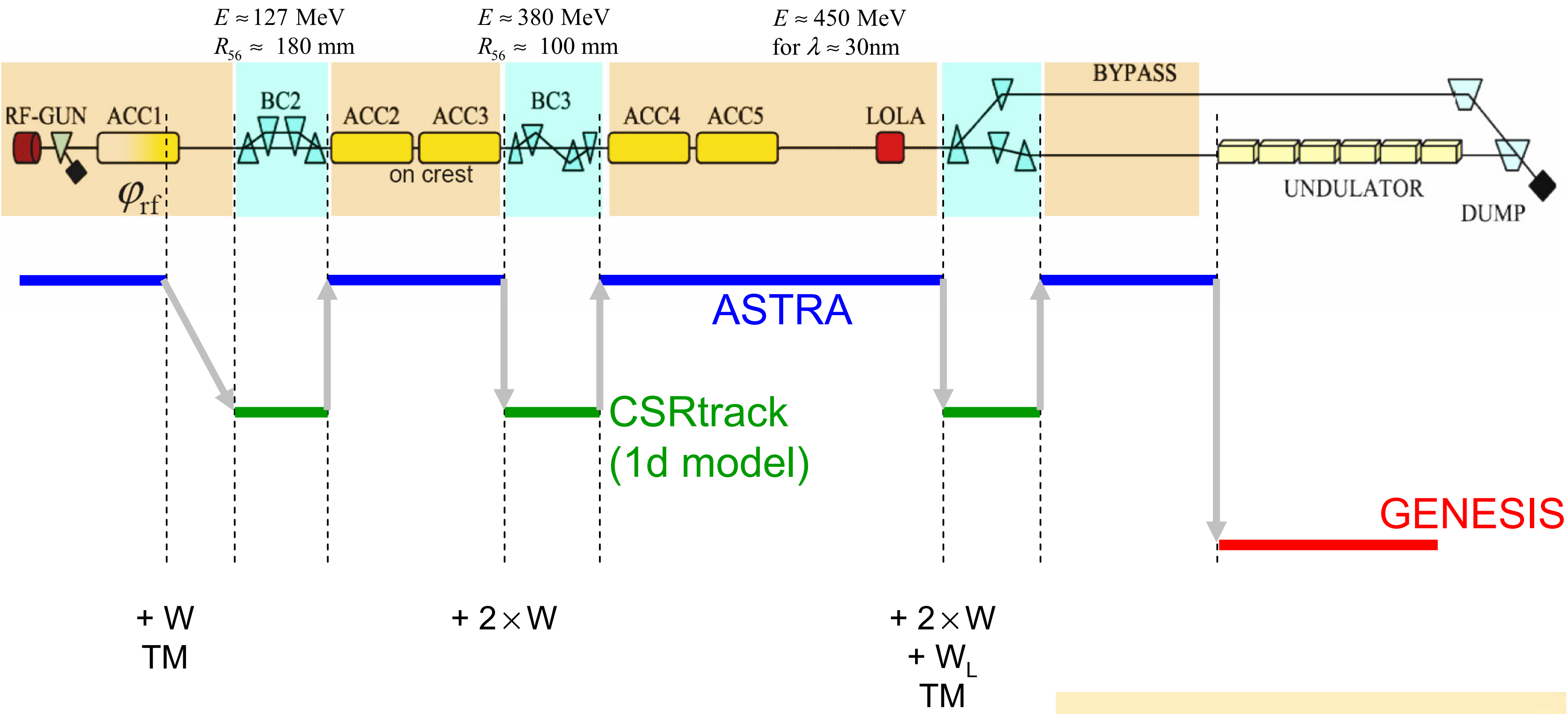
# rollover compression

## example: FLASH (=VUF-FEL=TTF2)



# … rollover compression

## example: FLASH s2e simulation




W = wake of one TTF module

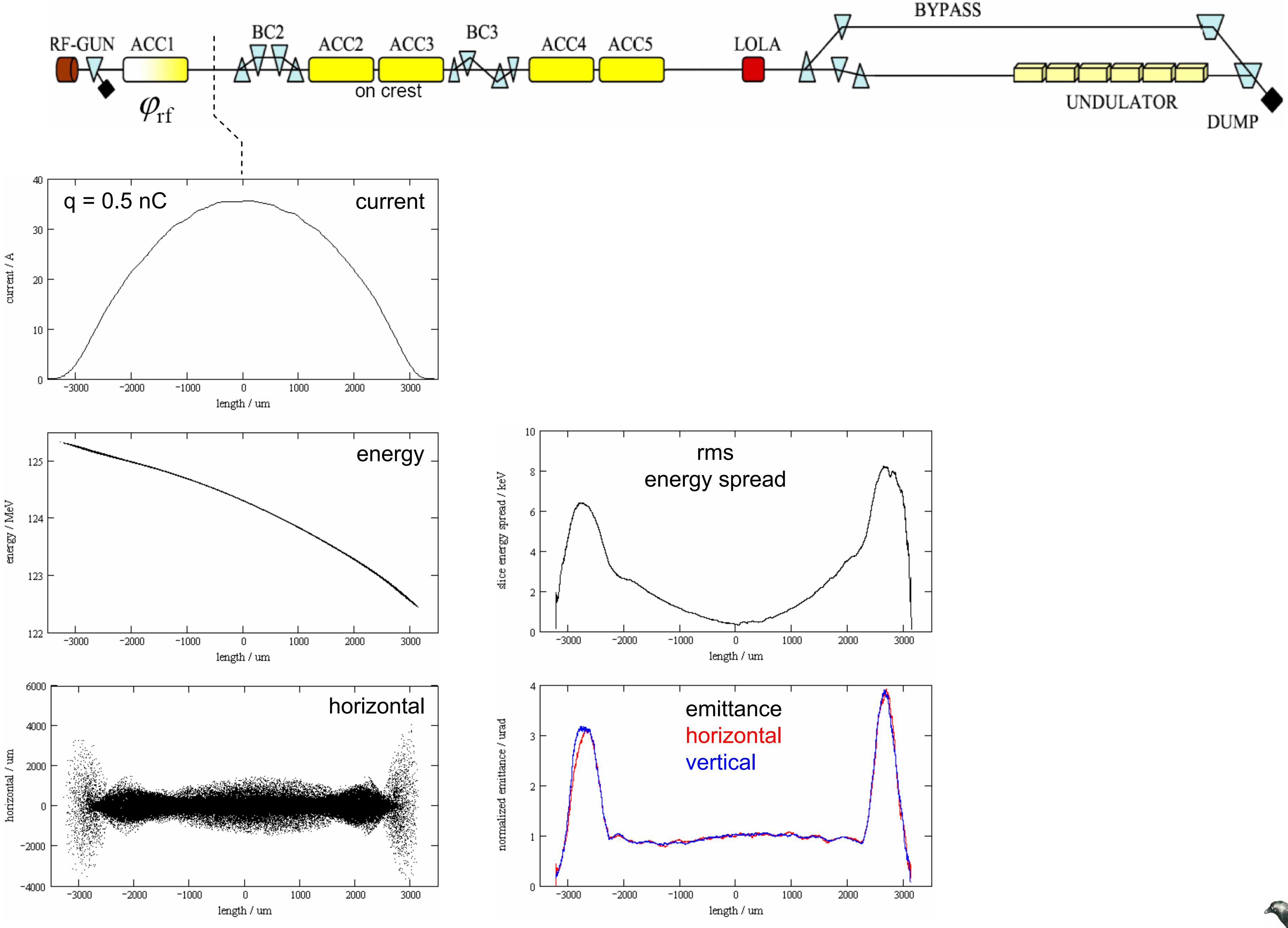
$W_L$ = wake of LOLA structure

TM = transverse matching to design optic

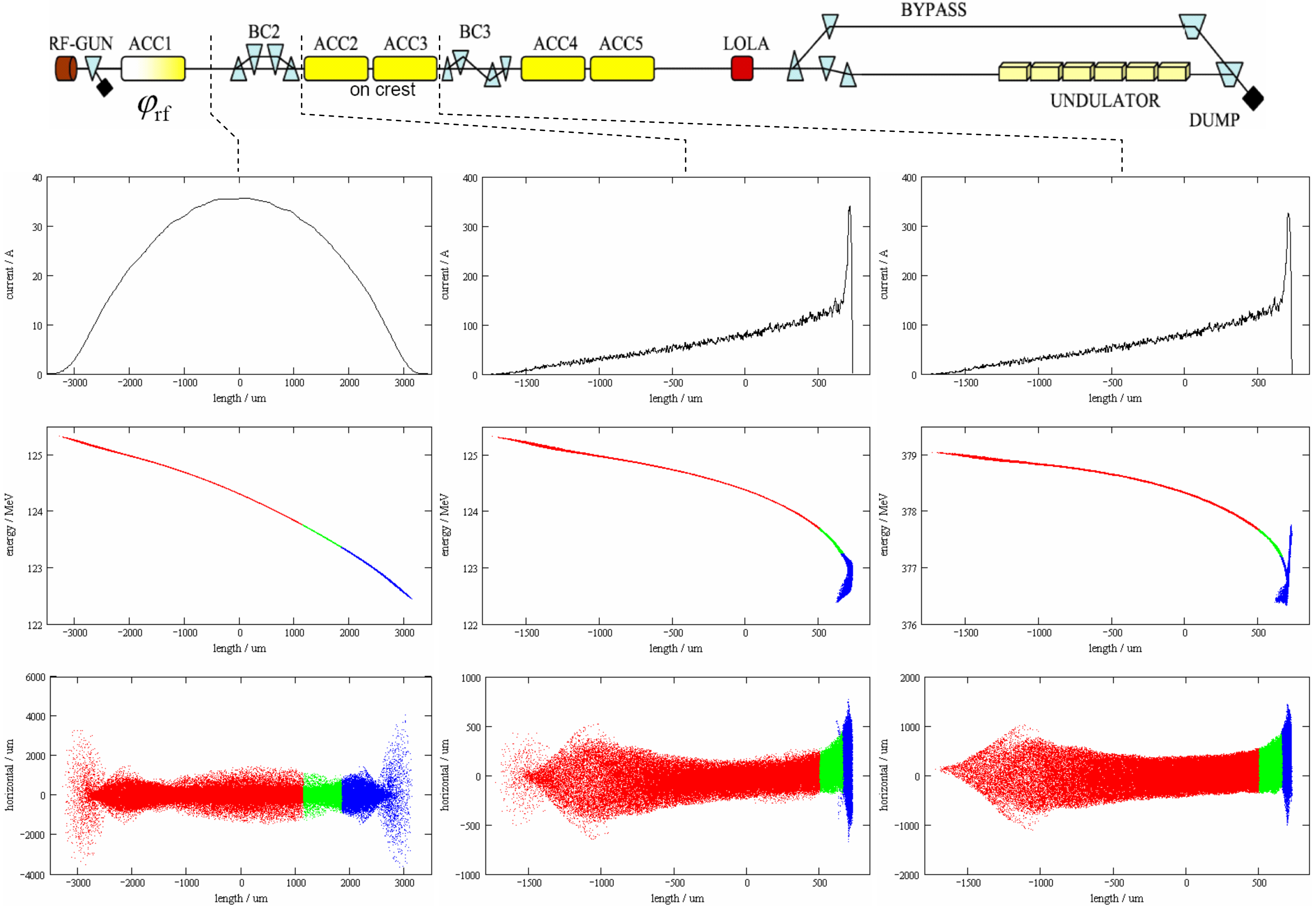
CSR & SC, guiding fields
dispersive

SC, guiding fields & rf
linear

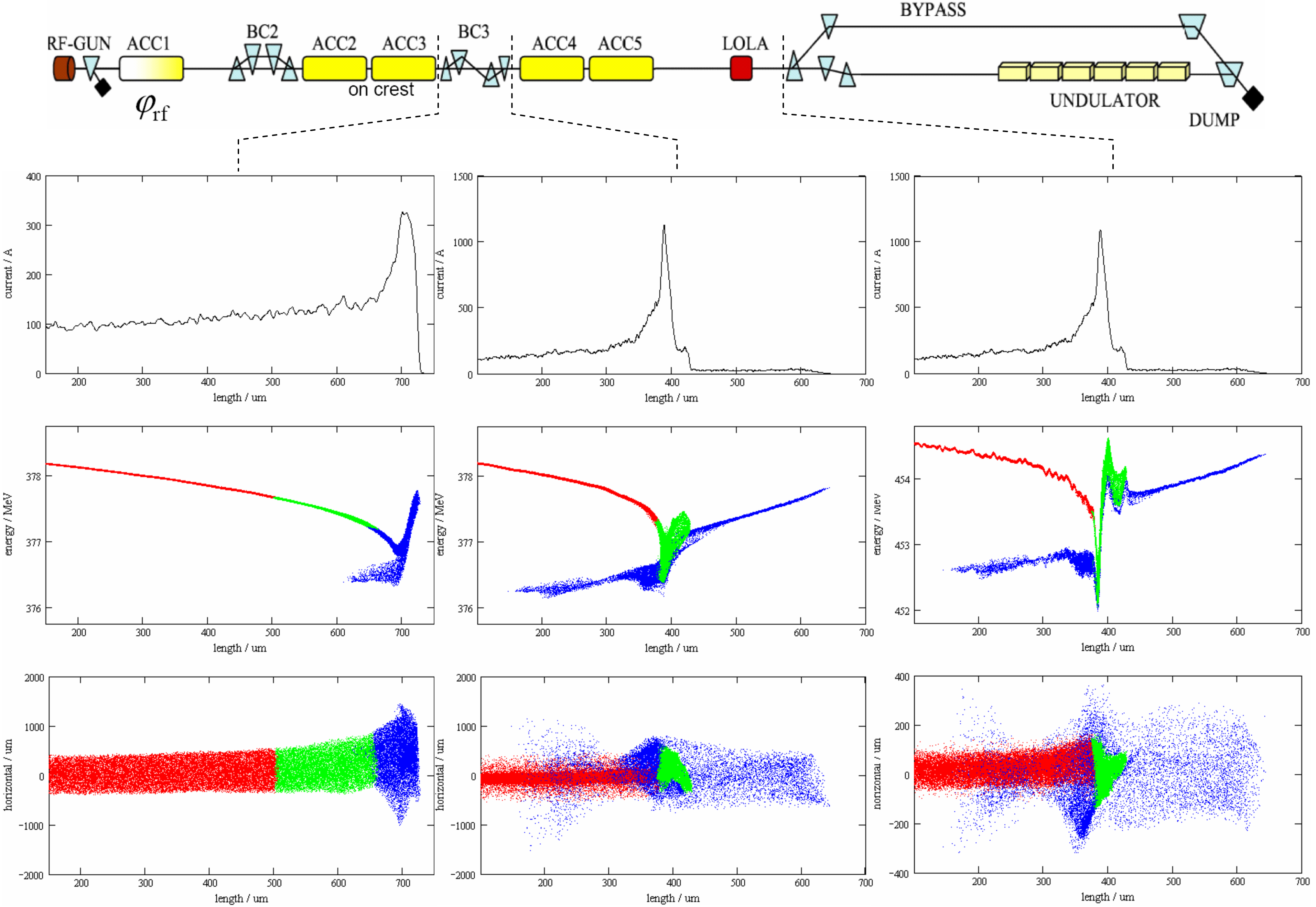
# example: FLASH s2e simulation



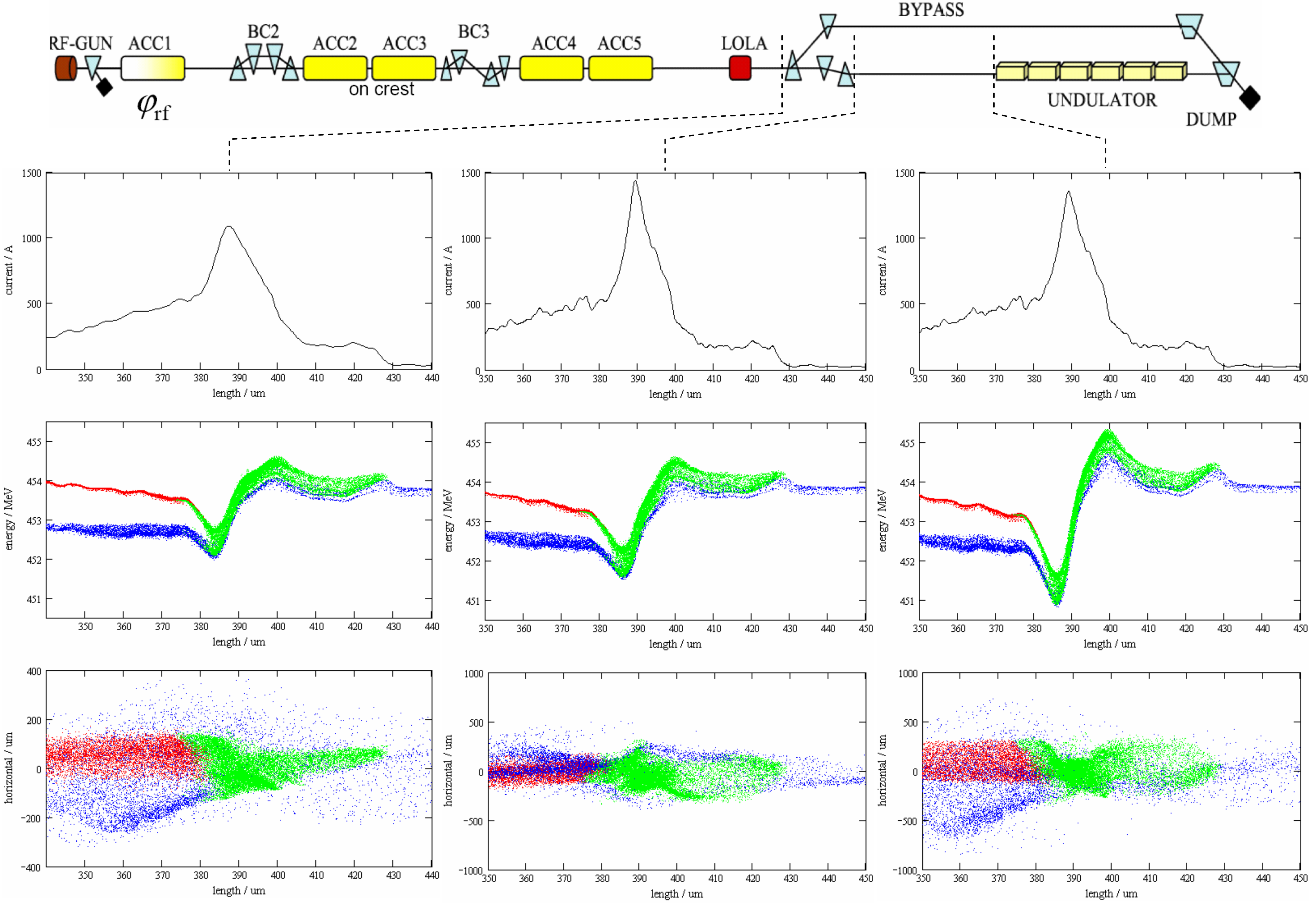
# example: FLASH s2e simulation



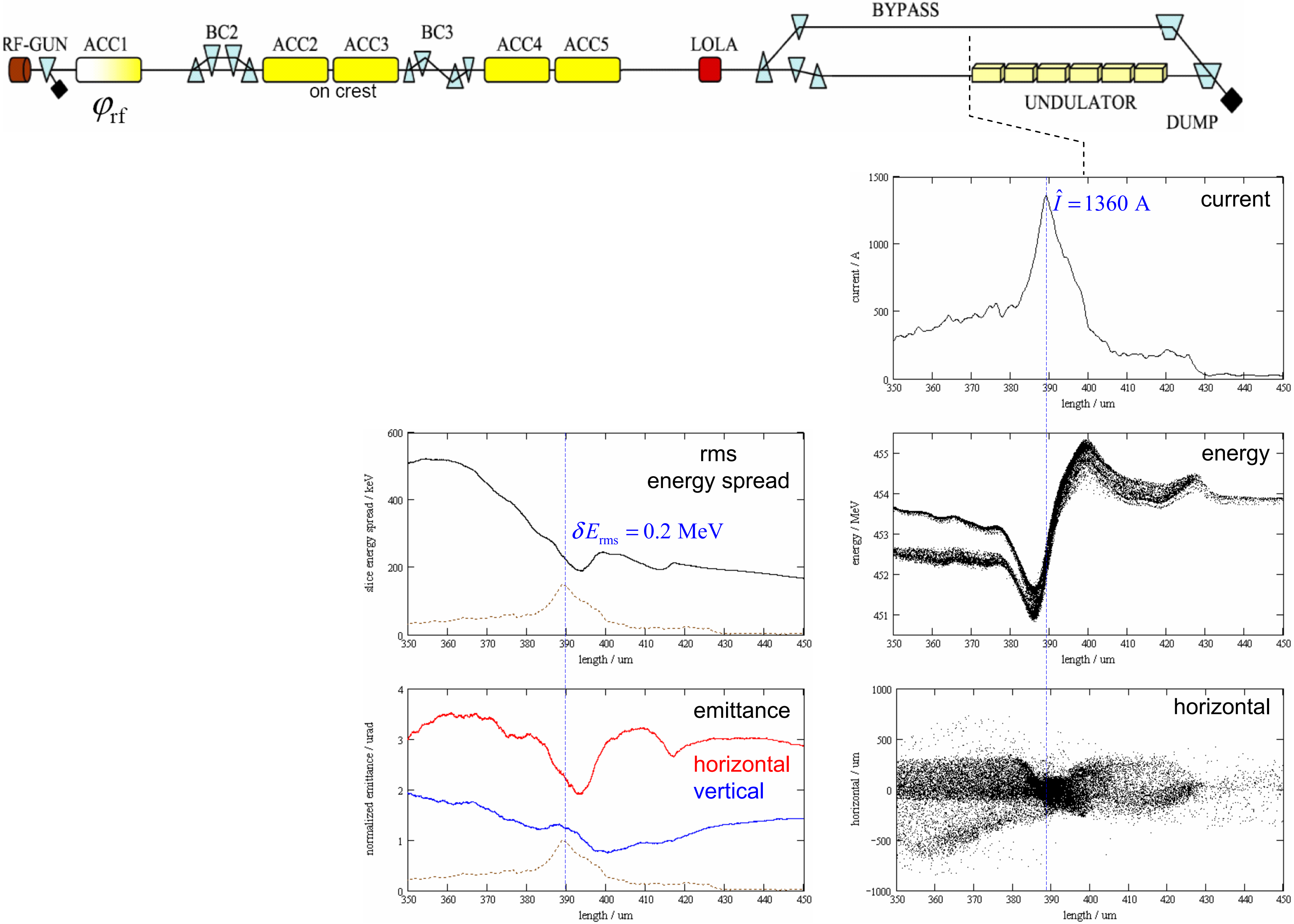
# example: FLASH s2e simulation



# example: FLASH s2e simulation



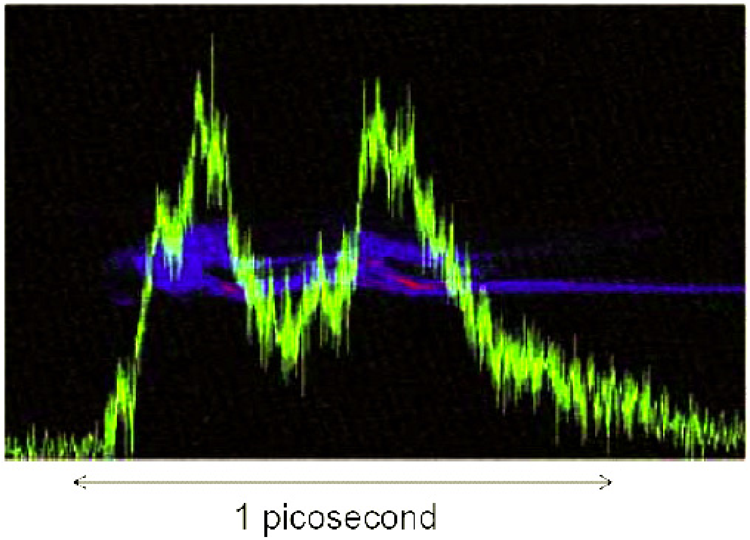
# example: FLASH s2e simulation

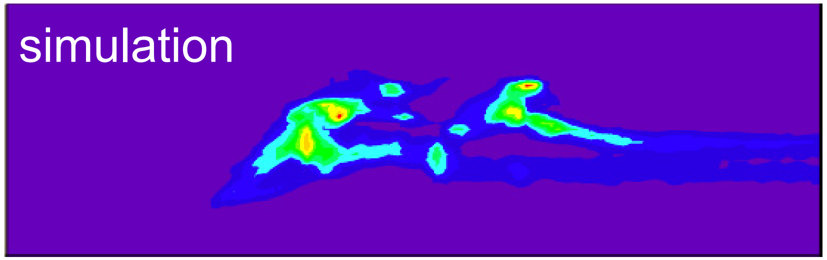


# … rollover compression
## example: FLASH, extreme case

strong ‘over compression’

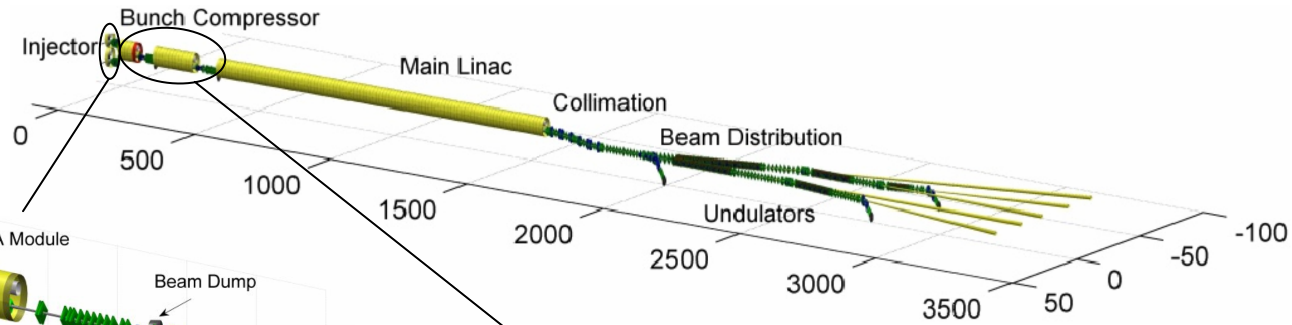
FLASH, 1nC, more chirp

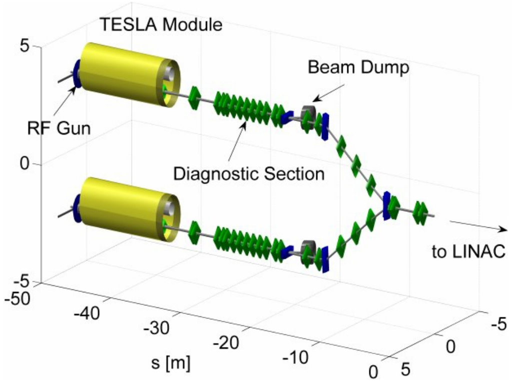


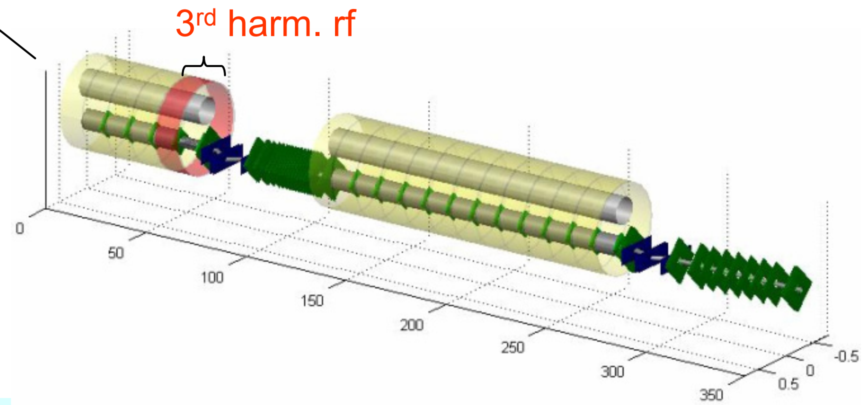



# controlled compression

## example: European XFEL




gun → 1nC ~50A ~7MeV
1 x module → 130MeV
dogleg
4 x module + 2 x module-3rd → 500MeV
bc1 → ~1kA
12 x module → 2GeV
bc2 → ~5kA
main linac → 17.5GeV
collimator
beam distribution … undulators …

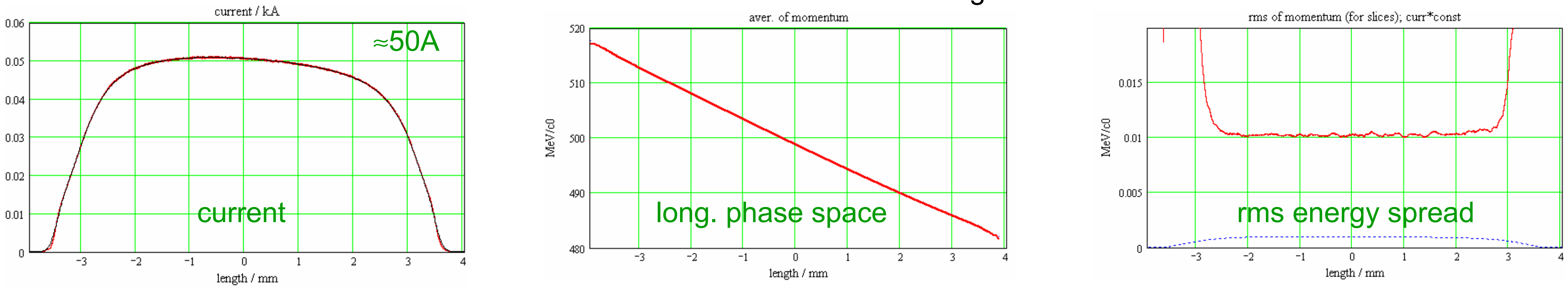
CSR & SC, guiding fields
dispersive

SC, guiding fields & rf
linear

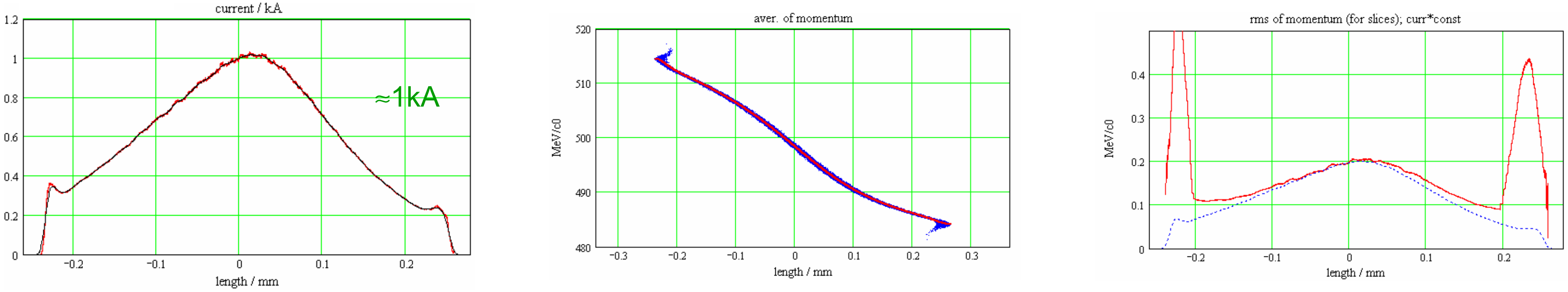
# example: European XFEL

## before BC1

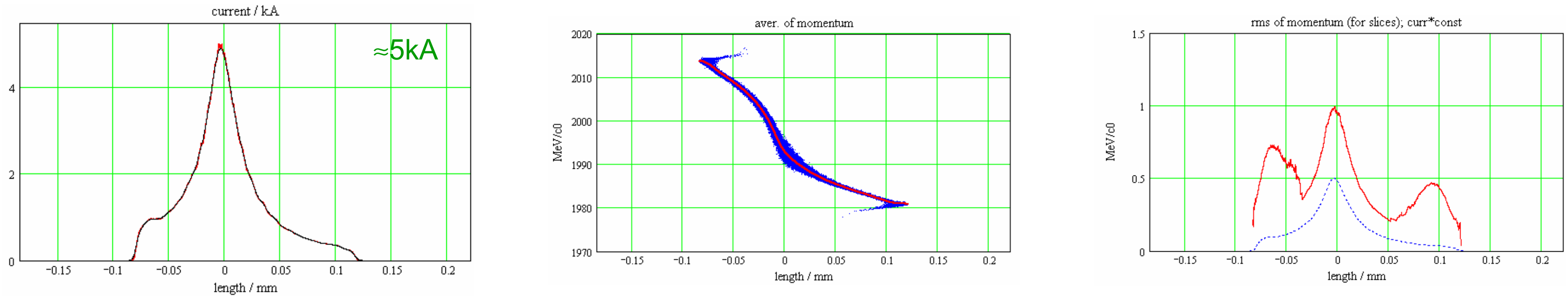
1.3GHz: 442.85 MV 1.42 deg
3.9GHz: 90.63 MV 143.35 deg




## after BC1

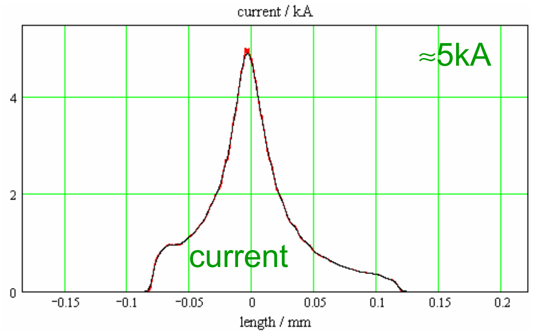


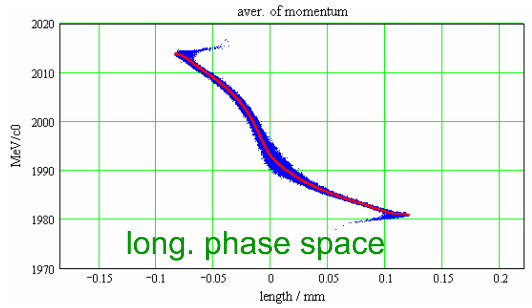

## after BC2

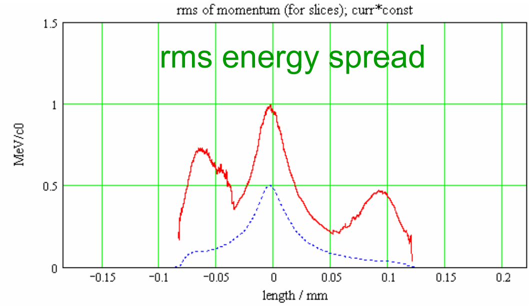


# example: European XFEL

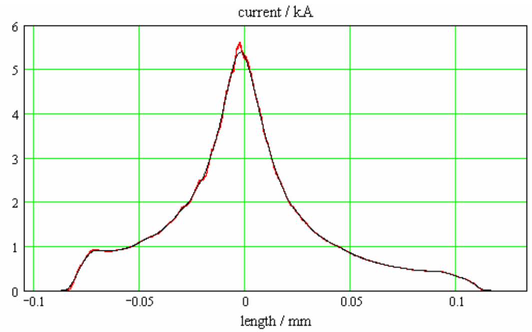
## after BC2

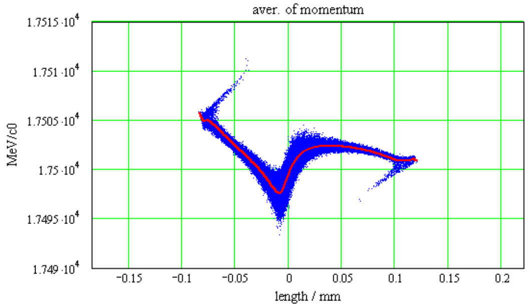


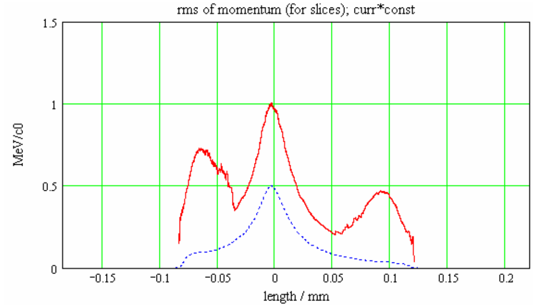



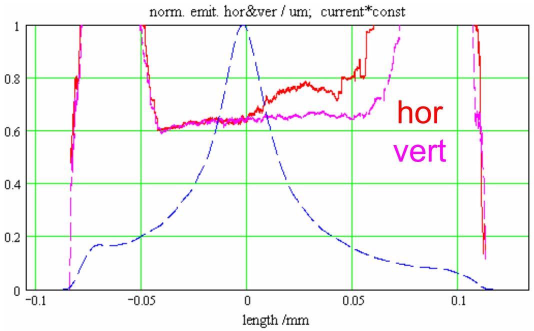

## after collimator



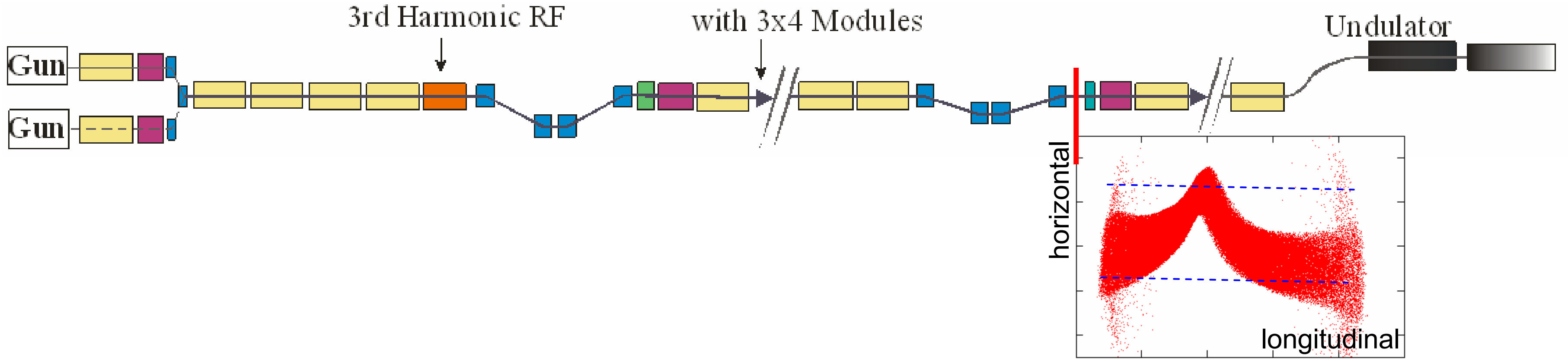



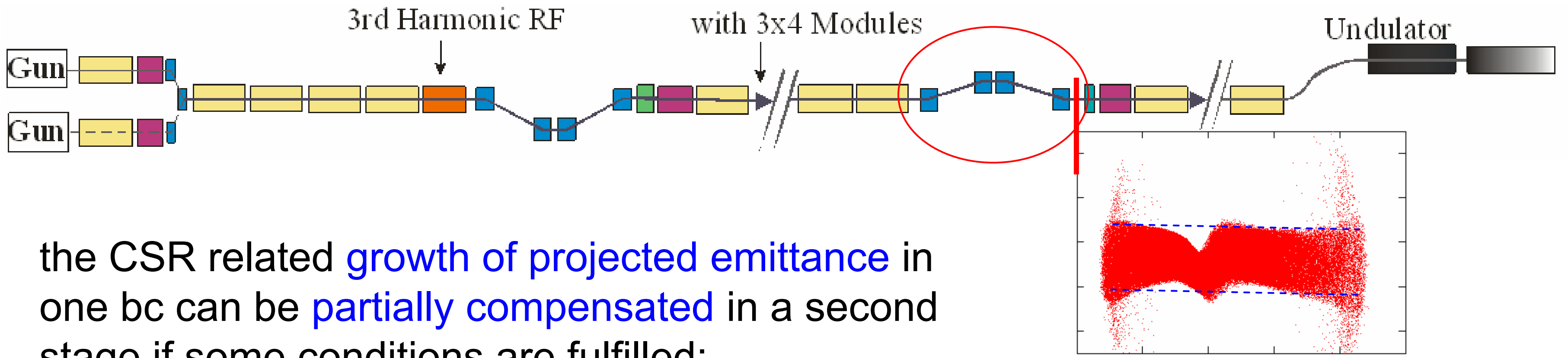

norm. emittance




# compensation in 2-bc systems

## shielding & resistive walls



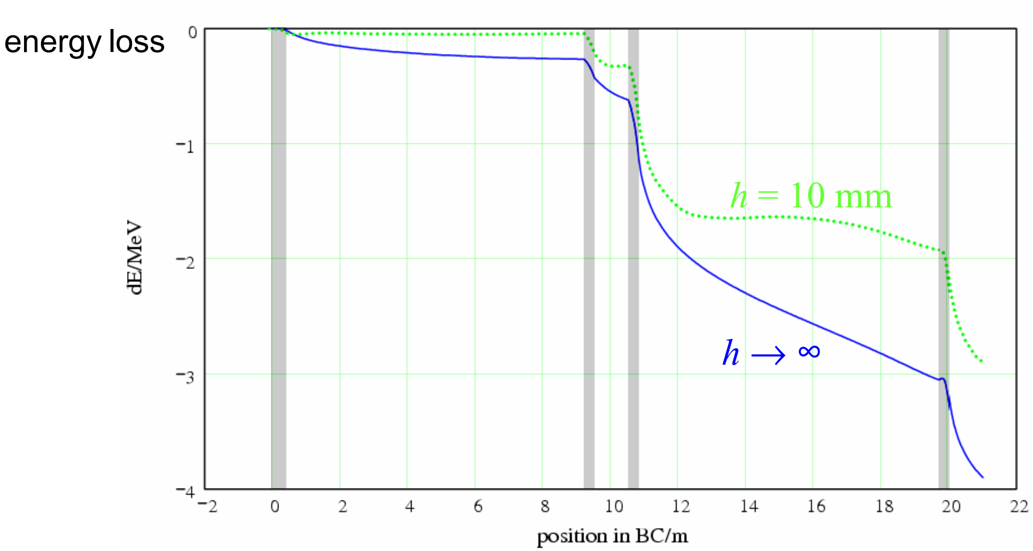

the CSR related growth of projected emittance in one bc can be partially compensated in a second stage if some conditions are fulfilled:

- right phase advance,
- right compression ratio (chirp as well as r56),
- no interference with other effects as shielding or resistive walls

# … compensation in 2-bc systems

## shielding & resistive walls

example: compression from 100 μm → 20 μm with gap $h$




free space condition for CSR in circular motion: $\frac{h^3}{\sigma^2} >> R_0$ curvature radius $\frac{h^3}{\sigma^2} \propto 10^3$

free space condition for wave propagation after bend: $\frac{2}{\pi^2}\frac{h^2}{\sigma} >> L_d$ length of drift $\frac{2}{\pi^2}\frac{h^2}{\sigma} \propto 1$

inside of a chicane:

if is difficult to avoid shielding

shielding in a long drift: resistive wall effects

# Conclusion

part II

- effects in BC **systems** are challenging (many physical effects are involved)
- μ-bunching effects beyond the resolution of non-1d-codes
- several types of codes needed (LT- and CSR-codes)

part I

- 1d- and sub-bunch codes are available
  Vlasov-Maxwell approach and paraxial approximation under development
- resolution of sub-bunch method increased
- ‘CSR’ methods cover all important physical effects
  (SC, CSR, shape variation, shielding, resistive walls)

**in reach: code that covers all effects**