

# Tolerances for the XFEL undulator system

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# Outline

- Assumption
- Pierce parameter and phase shake
- Error models
- Operation modes for calculation and results  
(only SASE1 results are shown)
- Combination of different kinds of errors

# I Assumption for

- Beam remains on axis: orbit can always be corrected (**Non-steering error only**)
- No intersection between undulator segments: only complicating simulations but doesn't contain additional information

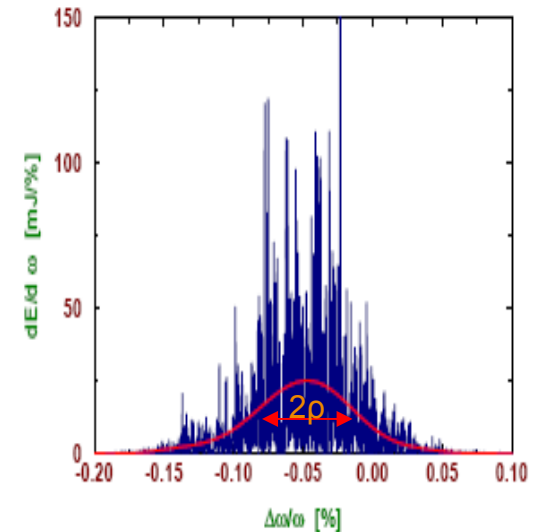
## II Pierce Parameter and Phase shake

Resonance condition:

SASE FEL bandwidth

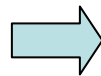
$$\lambda_s = \frac{\lambda_u}{2\gamma^2} (1 + K^2) \Rightarrow \frac{\Delta\lambda_s}{\lambda_s} = \frac{2K \cdot \Delta K}{1 + K^2} \approx \frac{2\Delta K}{K} < \rho$$

- If the error strength can be limited smaller than  $\rho$ , no matter how error distributes, electron and optical field are always synchronized, no gain degradation is expected.



$\rho \sim 10^{-4}$  For XFEL

$\Delta g < 1 \mu\text{m}$   
 $\Delta T < 0.08 \text{ degr.}$



Very strict tolerance requirement

- the error tolerance assumed by  $\rho$  is so tight, means a lot of money be expended on undulator, air controller system etc.

## Phase shake --- correlation to power degradation

Ponderomotive Phase and Phase shake:

$$\varphi(z) = (k_u + k_s)z - k_s ct = \left( k_u - \frac{k_s}{2\gamma^2} (1 + K_0^2) \right) z - \Delta\varphi(z)$$

$$\Delta\varphi(z) = \frac{k_u}{1 + K_0^2} \int_0^z \left( (K_0 + \Delta K \cdot f(z'))^2 - K_0^2 \right) dz'$$

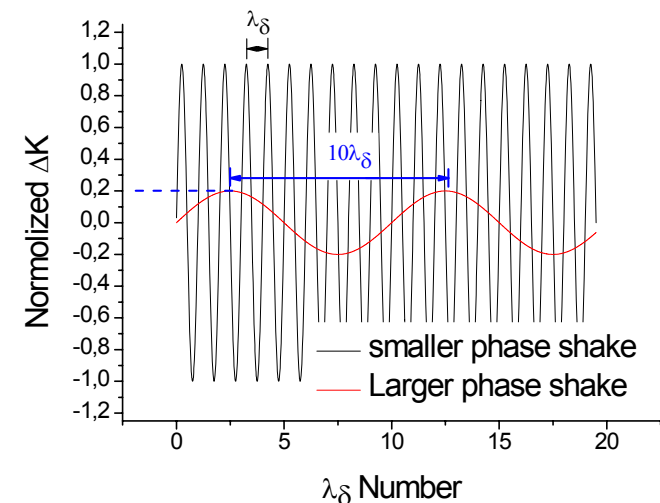
Phase shake

Field error

Rms phase shake  $\sigma_{\Delta\varphi} = \sqrt{\frac{1}{L} \int_0^L (\Delta\varphi(z') - \Delta\bar{\varphi})^2 dz'}$

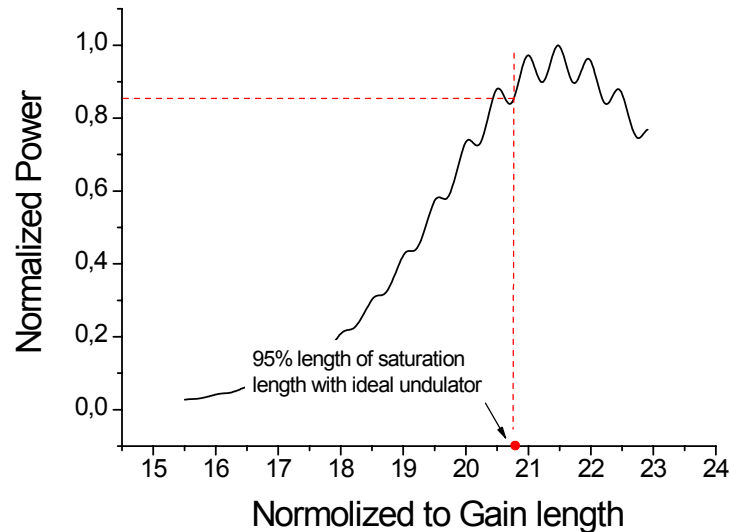
If  $f(z)$  is a periodic function,  $f(z) = f(z + \lambda_\delta)$   
by some assumption, it can be proved:

$$\sigma_{\Delta\varphi} = \alpha \cdot \frac{\Delta K}{K_0} \cdot \lambda_\delta$$



Rms phase shake is not only determined by error strength  $\Delta K$ , but also determined by the error distribution scale.

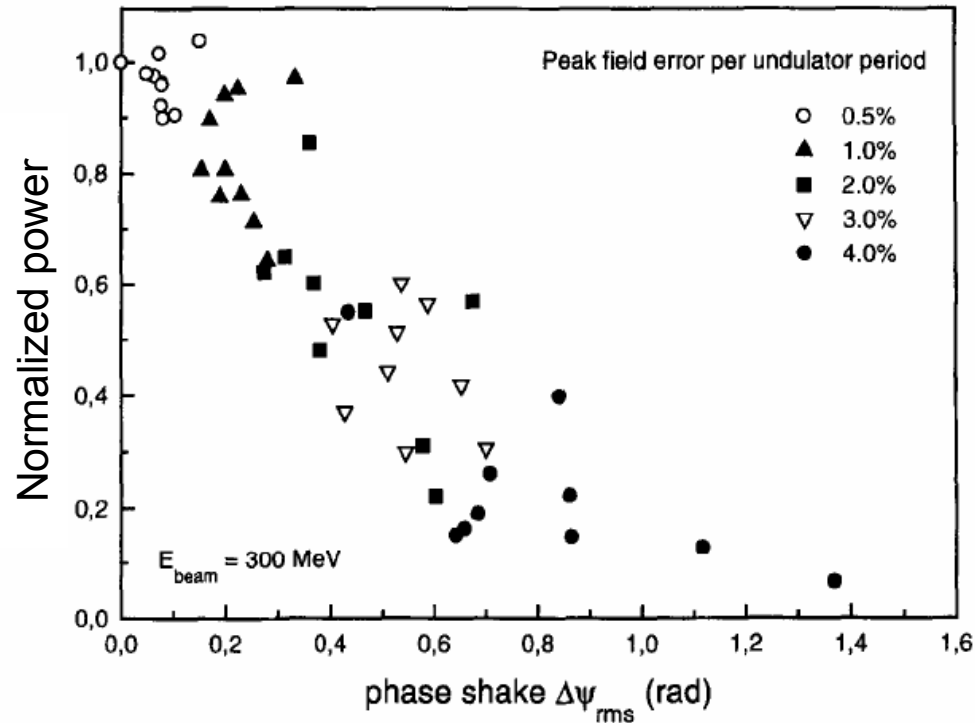
## Critical number --- power at a fixed point



- To evaluate the error impact need a critical number
- Naturally saturation length and saturation power is the number to evaluate error impact
- it is difficult to judge a suitable saturation point

Choosing the radiation power at a fixed point (95% of saturation length with ideal undulator) as the critical number

## RMS Phase shake correlates to the power degradation



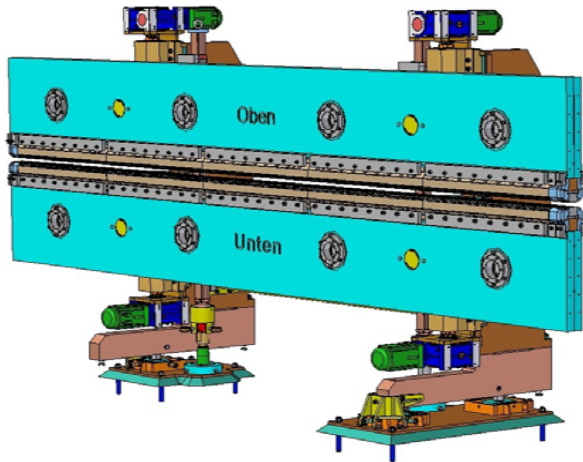
- Non-steering error
- Random distribution
- Simulation for FLASH

*B. Faatz et al. / Nucl. Instr. And Meth. In Phys. Res. A 393 (1997) 380-384*

## III Basic Error Models --- Periodical error

- Former undulator error tolerance study mainly takes into account random error on each pole
- Many error sources (girder deformation, temperature, gap tilt...) don't induce random errors on each pole but periodic variation
- Four kinds of basic periodical models --- **Sinus**, **Triangle**, **Sawtooth**, **Constant** are considered in our calculation

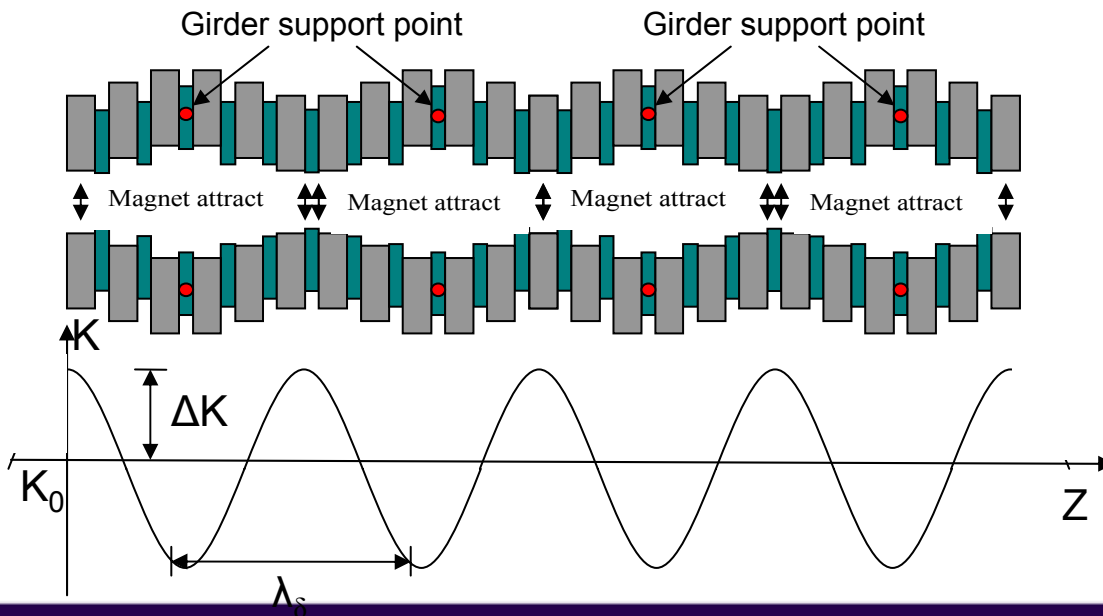
## Basic Error Models --- Sinus



By properly choose the distances of the supports, a homogeneous deformation line can be expected

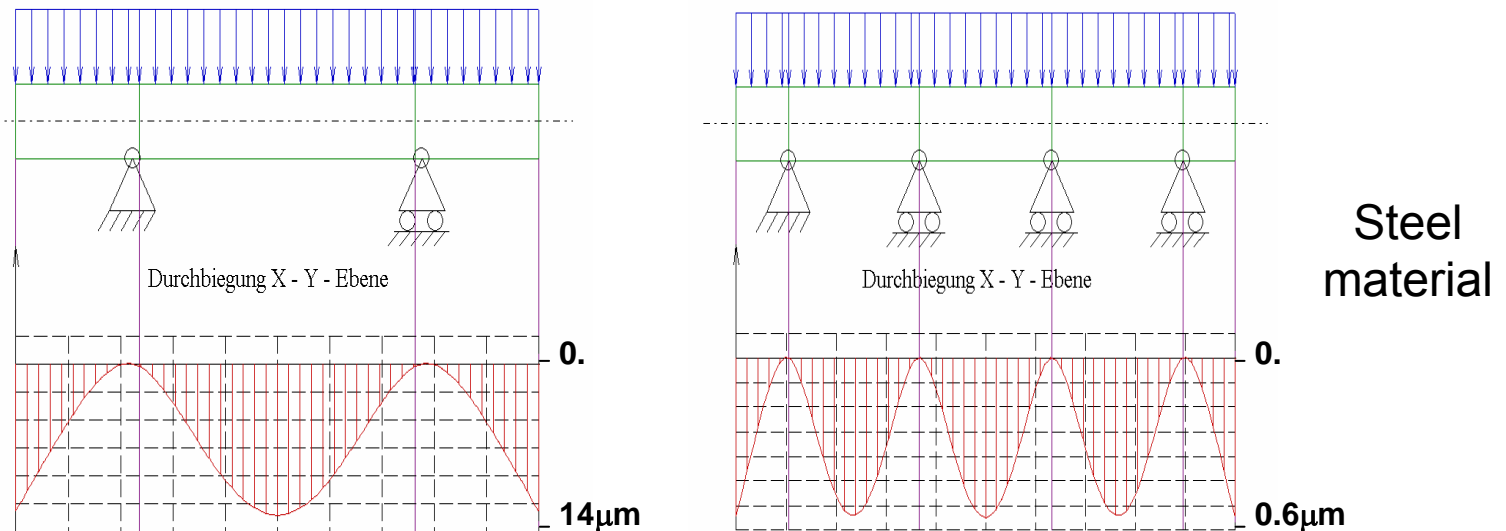


**Sinus** error for girder deformation



**Four** sinus error period  $\lambda_s$  equal to the length of undulator segment

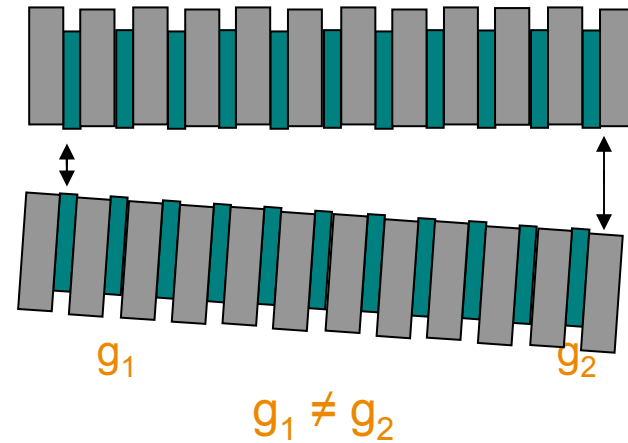
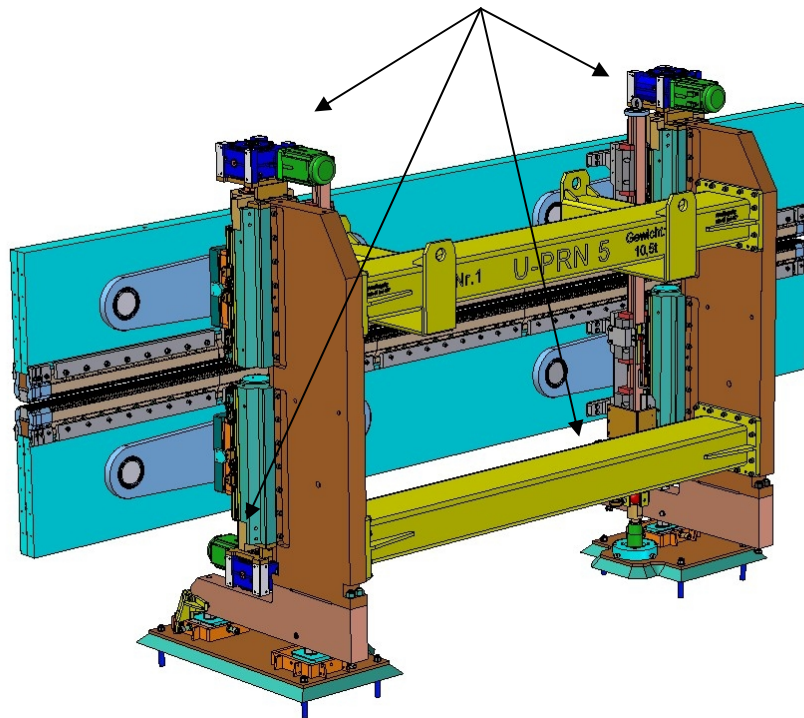
## Basic Error Models --- extent of girder deformation



For steel girder, two points support has 14 micron girder deformation, while four points support has 0.6 micron. Aluminum material has 3 times larger deformation

## Basic Error Models --- Driving motor inhomogeneous movement

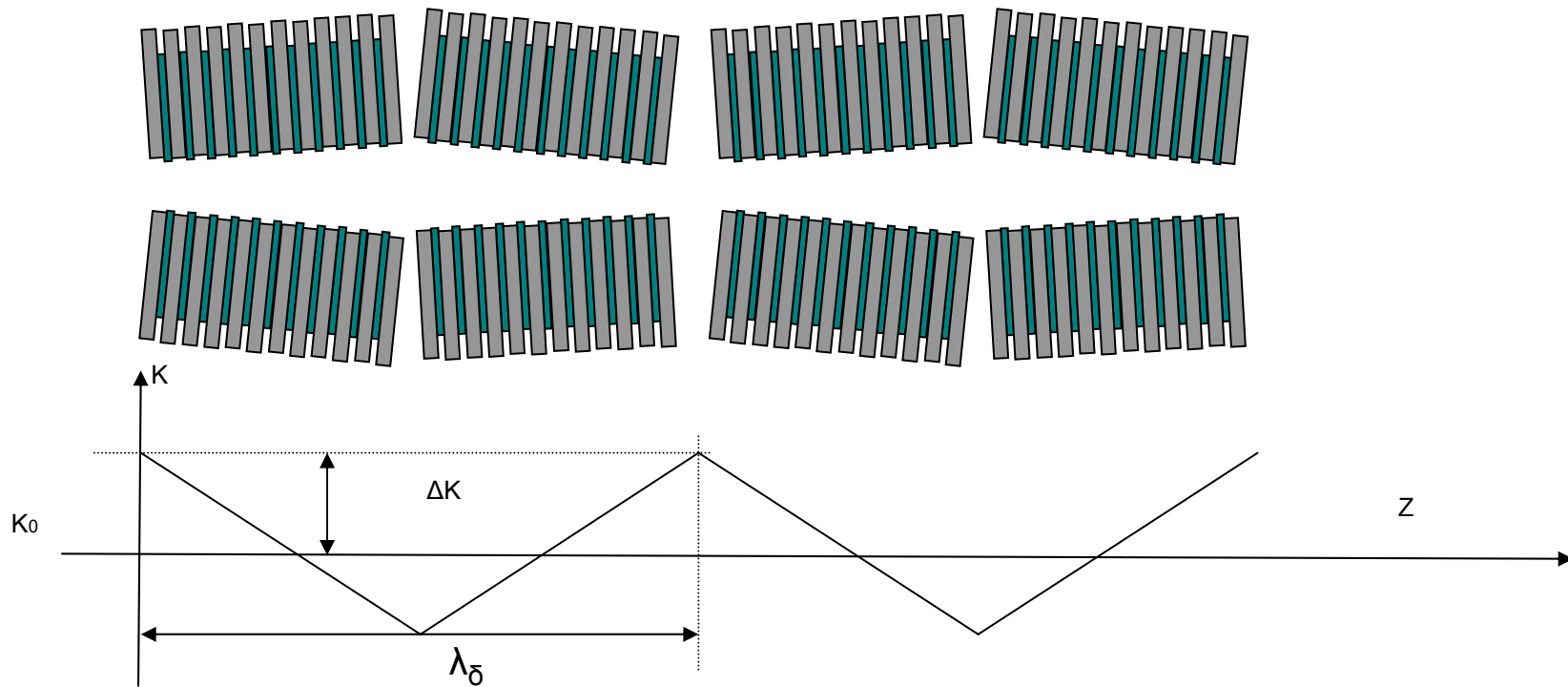
4 Driving motor



Gap taper exists due to limitation drive accuracy, **1 micron** accuracy is assumed

If temperature gradient is constant enough, it has same impact as the gap taper

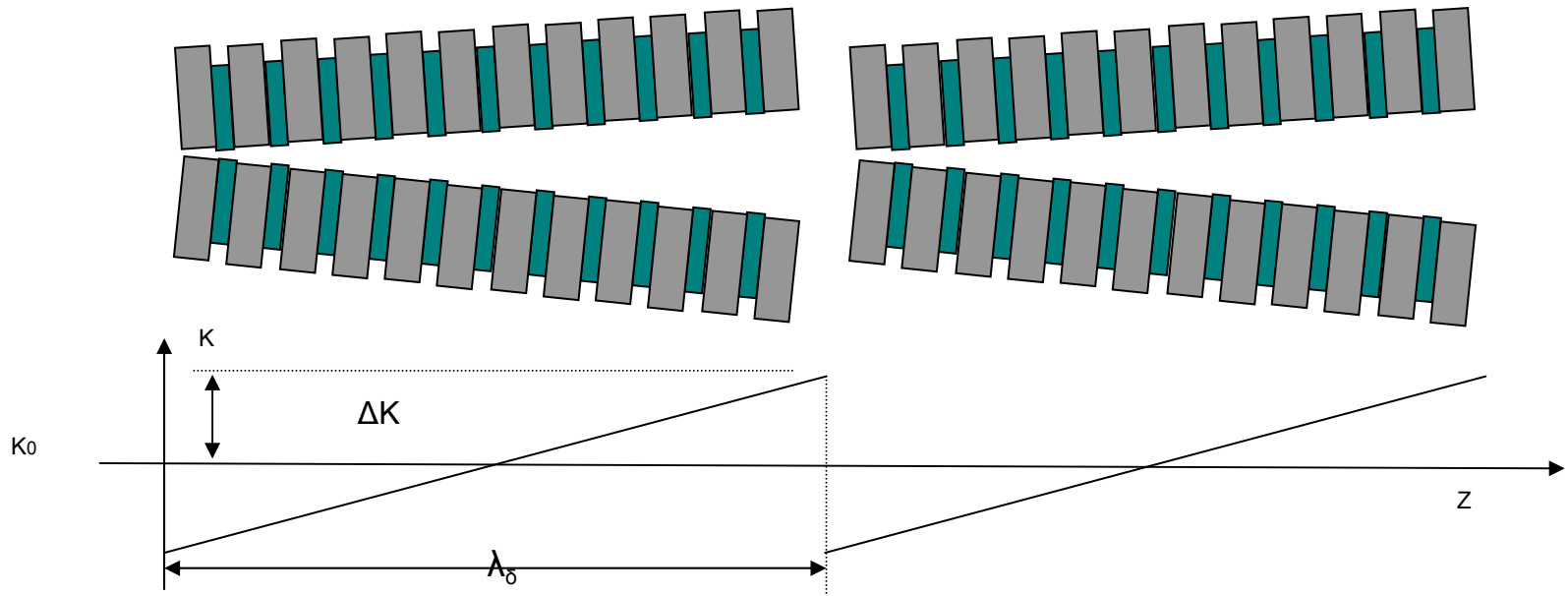
## Basic Error Models --- Triangle error



**Triangle** error for gap control error

**Half** triangle error period  $\lambda_\delta$  equal to the length of undulator segment

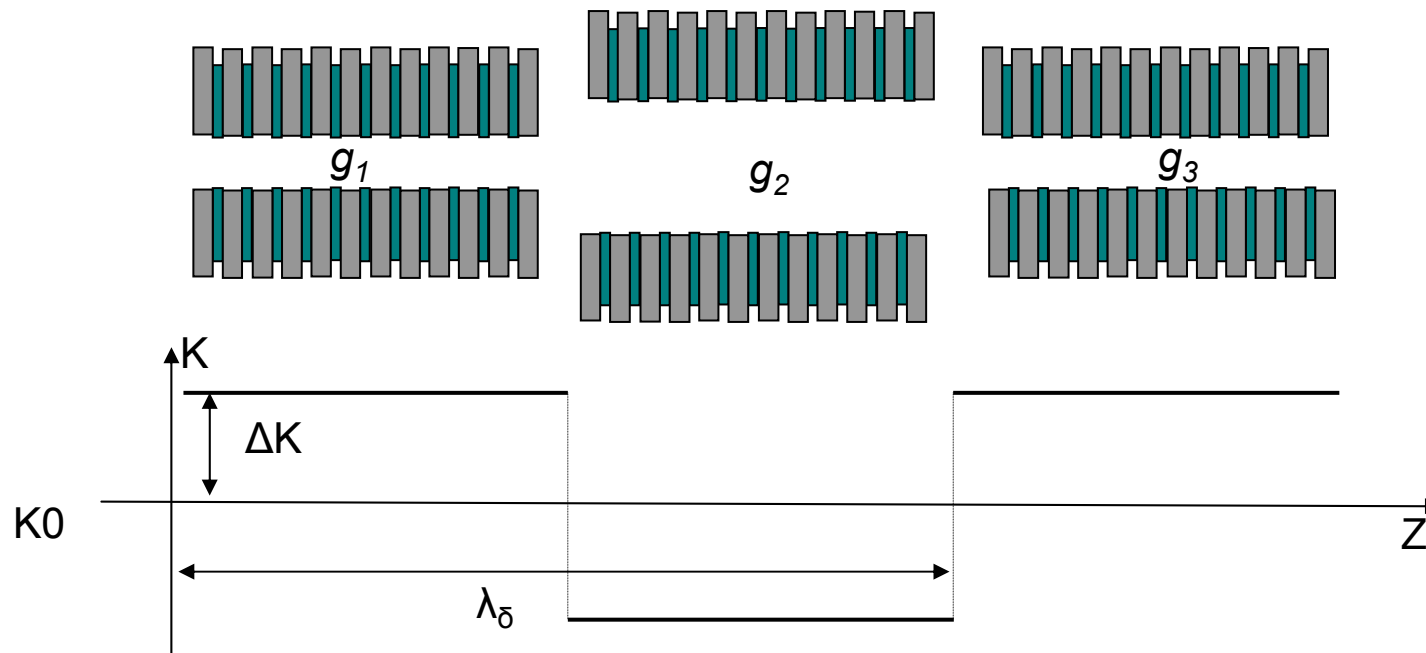
## Basic Error Models --- Sawtooth error



**Sawtooth** error for gap control error

One **Sawtooth** error period  $\lambda_\delta$  equal to the length of undulator segment

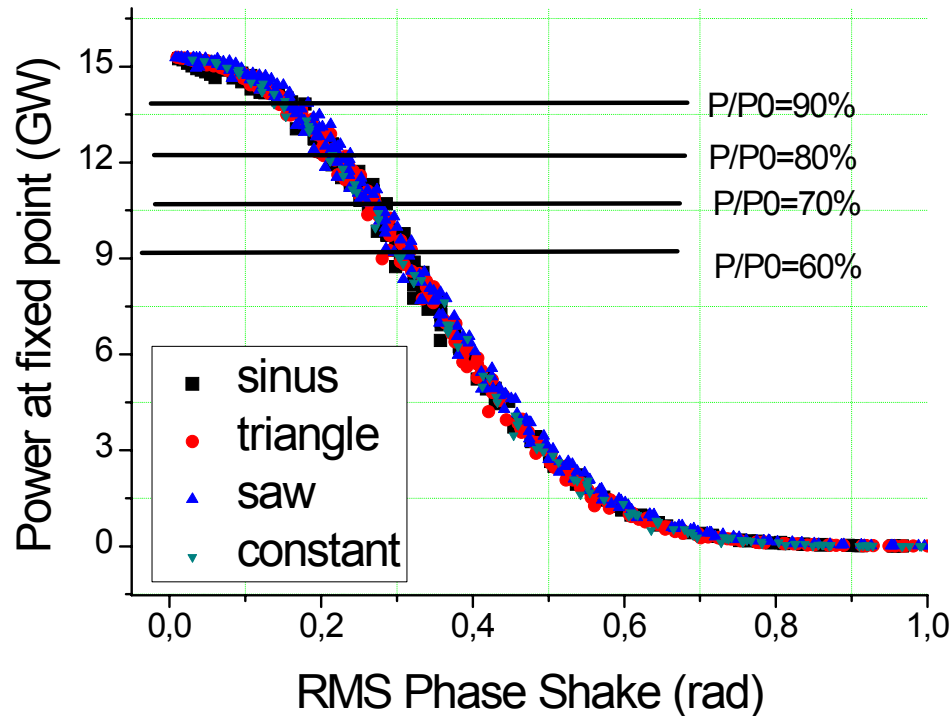
## Basic Error Models --- Constant error



**Constant** error for gap control error or temperature variation

**One Constant** error period  $\lambda_\delta$  equal to the length of undulator segment

## Different error distributions give same results



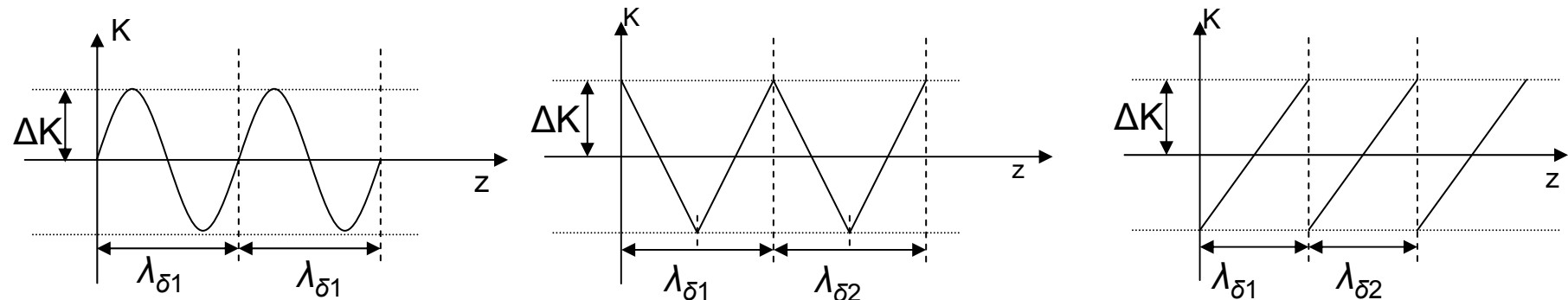
- Periodical calculation
- Good correlation between rms Phase shake and power
- SASE1 parameters are used in the example

- No matter what kind of error it is, the **same phase shake** gives **same power reduction**
- For any other error, the power degradation can be estimated by it's phase shake without performing FEL simulation

## Rms Phase shake for the basic error models

For all of the four errors analyzed ,

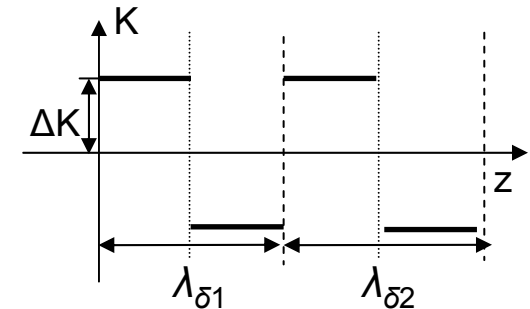
$$\sigma_{\Delta\varphi} = \alpha \cdot \frac{\Delta K}{K_0} \cdot \lambda_\delta$$



$$\alpha_{\text{sinus}} = A \frac{1}{2\sqrt{2}\pi}; \quad \alpha_{\text{triangle}} = A \frac{1}{\sqrt{120}};$$

$$\alpha_{\text{sawtooth}} = A \frac{1}{\sqrt{180}}; \quad \alpha_{\text{constant}} = A \frac{1}{2\sqrt{12}}; \quad A = \frac{4\pi K_0^2}{\lambda_u (1 + K_0^2)} \approx 2k_u$$

$$\alpha_{\text{sinus}} : \alpha_{\text{triangle}} : \alpha_{\text{sawtooth}} : \alpha_{\text{constant}} = 1.51 : 1.23 : 1 : 1.95$$



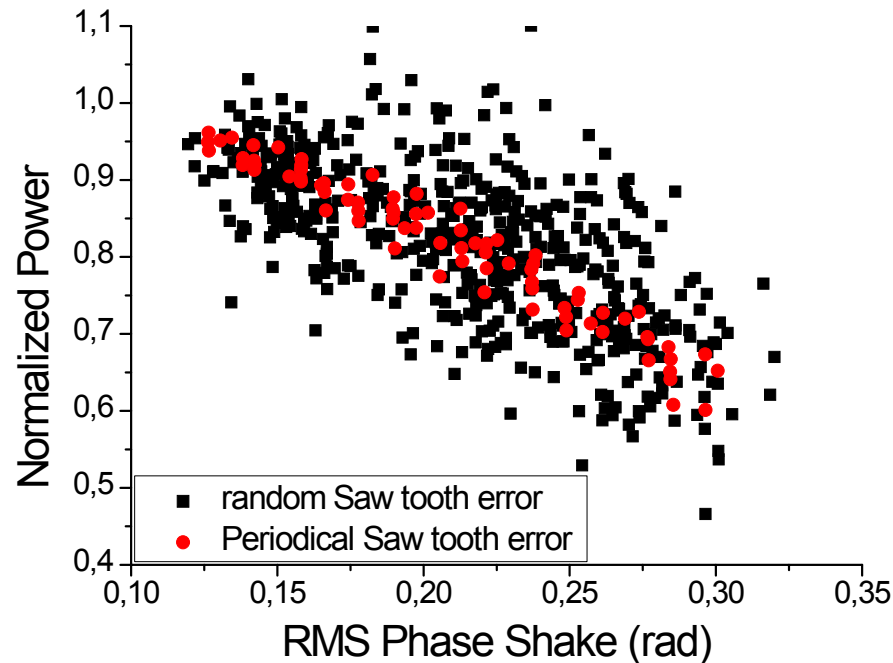
## IV Tolerance Table --- Power degradation is 10% (SASE1 17.5&8.75 GeV)

Type	wavelength	Error period	RMS Phase shake	Error strength	Gap error ( $\mu$ m)
SASE1, sinus	0.1nm	1.2m	0.146	0.366%	30
SASE1, triangle	0.1nm	10m	0.156	0.058%	5
SASE1, sawtooth	0.1nm	5m	0.165	0.148%	13
SASE1, constant	0.1nm	10m	0.183	0.043%	4
SASE1, sinus	0.4nm	1.2m	0.200	0.501%	42
SASE1, triangle	0.4nm	10m	0.199	0.074%	6
SASE1, sawtooth	0.4nm	5m	0.199	0.179%	15
SASE1, constant	0.4nm	10m	0.203	0.048%	4

## Periodical error and random error

- For a certain girder material and magnet force, the deformation is constant --- Periodical **sinus** error can be expected
- Errors generated by driving motor movement is however random --- Random **Sawtooth, Triangle, Constant** error are expected

## Random error strength simulation --- random sawtooth error



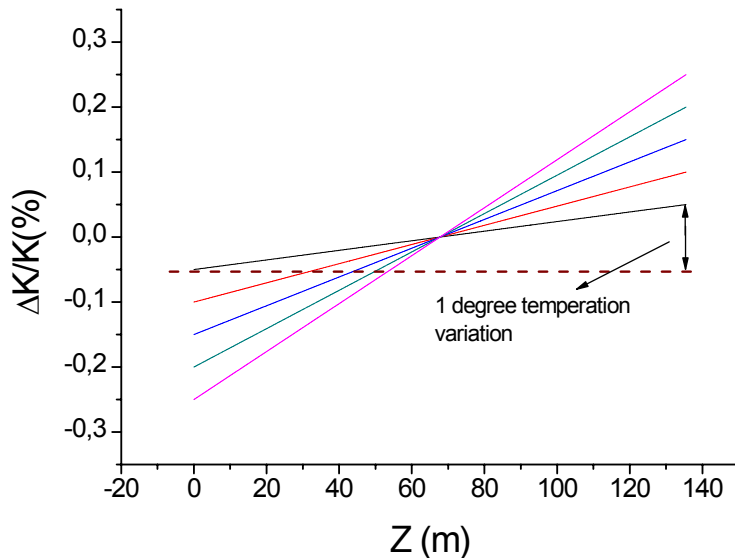
- Power correlate with phase shake too with random error considered
- For a certain phase shake, the mean power is same for periodic and random error
- For a certain phase, power deviation by random error is larger than that generated by periodic error

Sawtooth error is used  
as the example

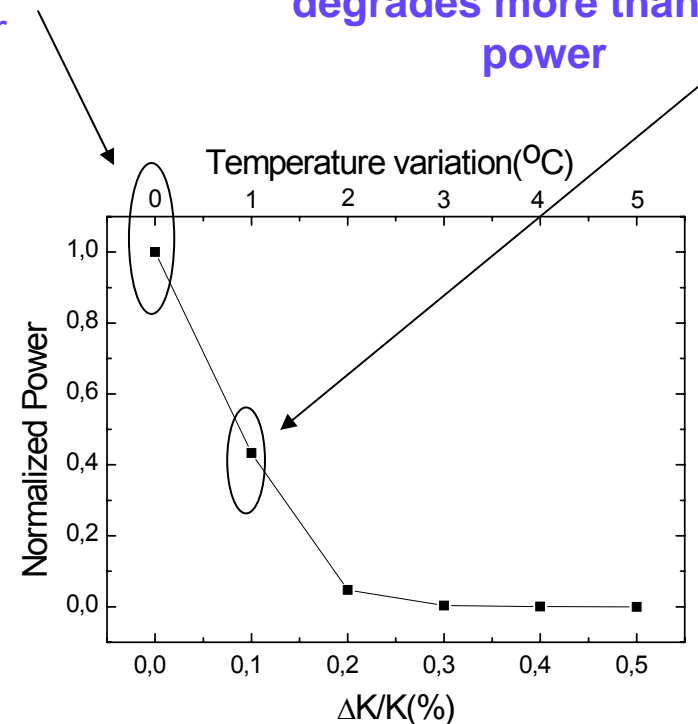
## V Influence of global temperature variation and compensation --- K linearly increases

- 1 degree temperature variation corresponds to 0.1%  $\Delta K/K$
- If the temperature gradient is homogeneous enough, the K value changes linearly

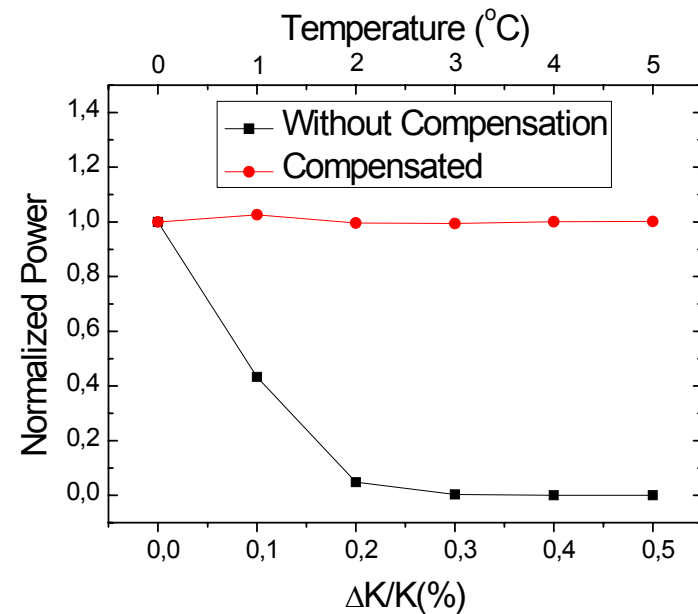
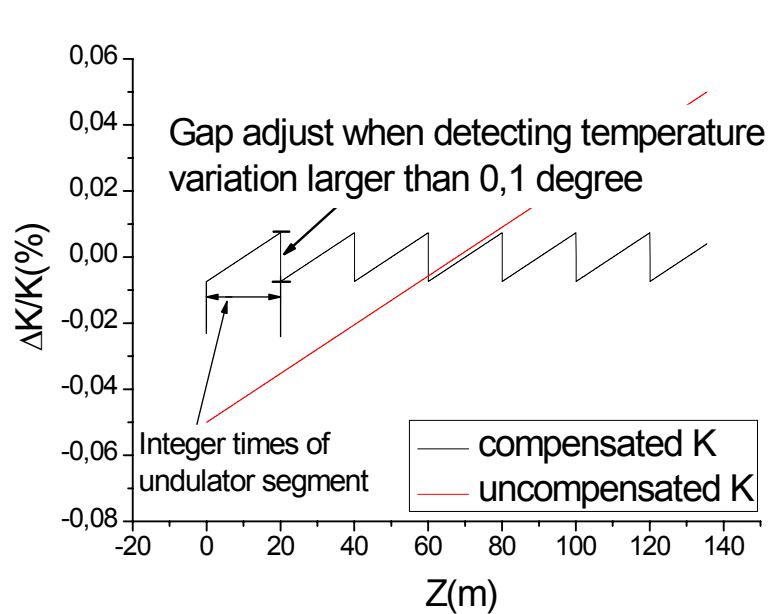
If the temperature variation is limited in 0.1 degree (by  $\rho$ ), the power degradation is small.



only 1 degree temperature variation over 140 meters degrades more than 50% power



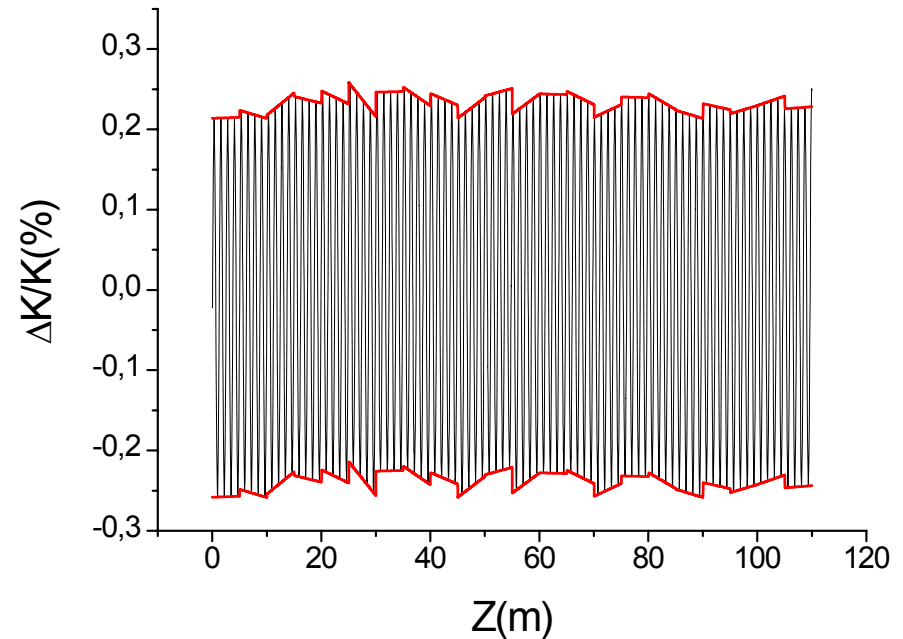
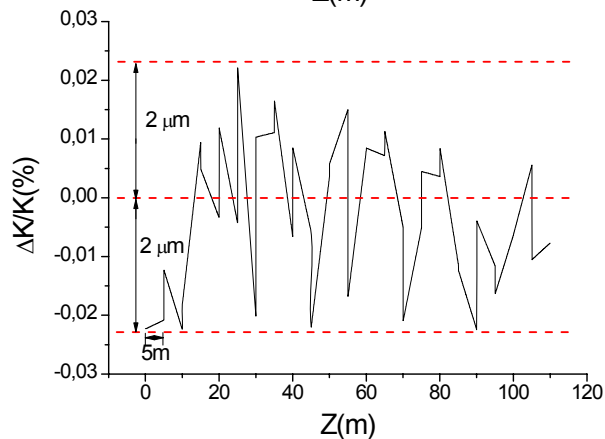
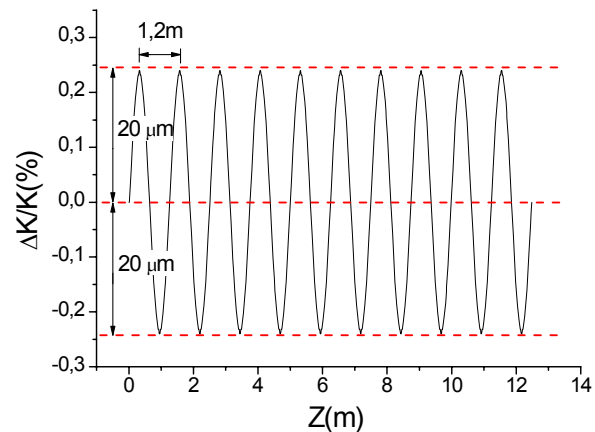
# Influence of global temperature variation and compensation --- K linearly increases



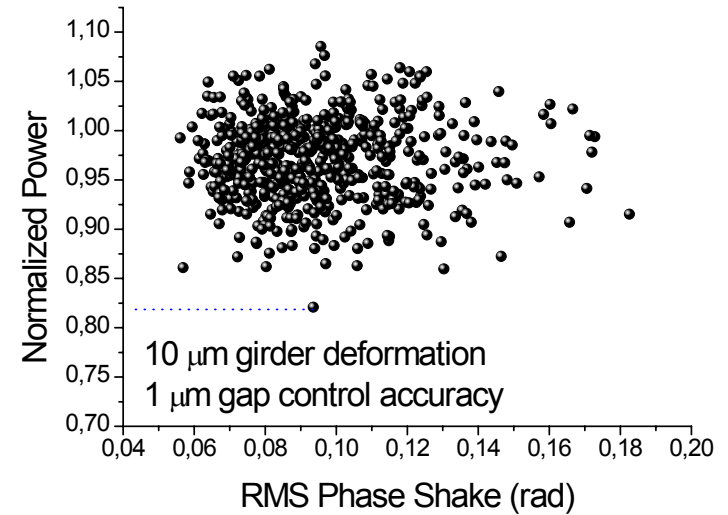
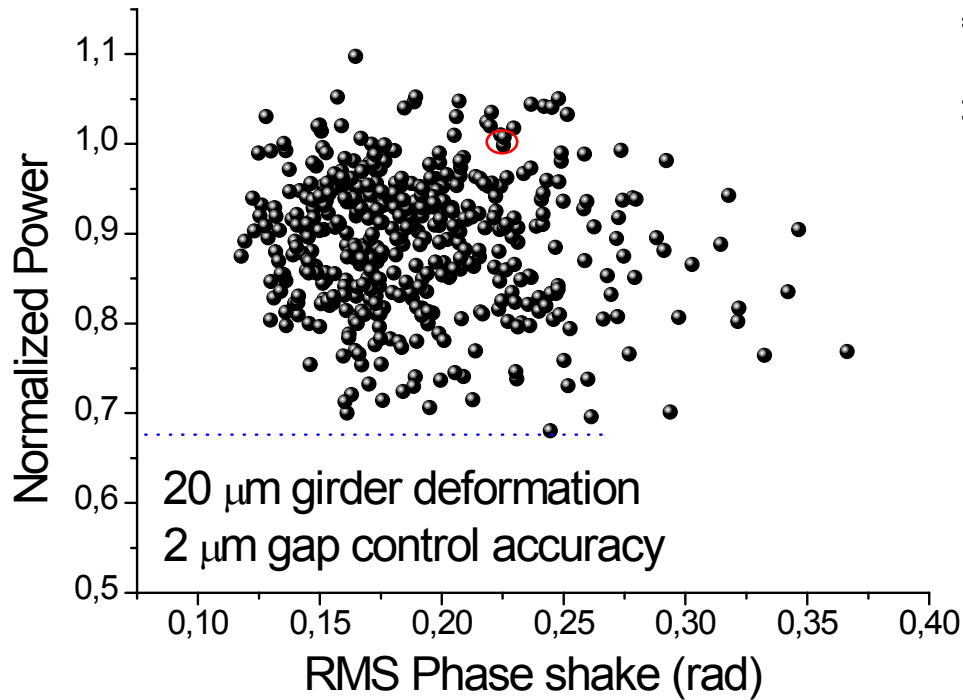
By adjusting undulator gap, the global temperature impact can be compensated

## VI Combining errors --- sinus, constant, sawtooth

- periodic sinus error is expected for girder deformation --- 20 micron
- random sawtooth and constant errors are expected for gap control motor error --- 2 undulator

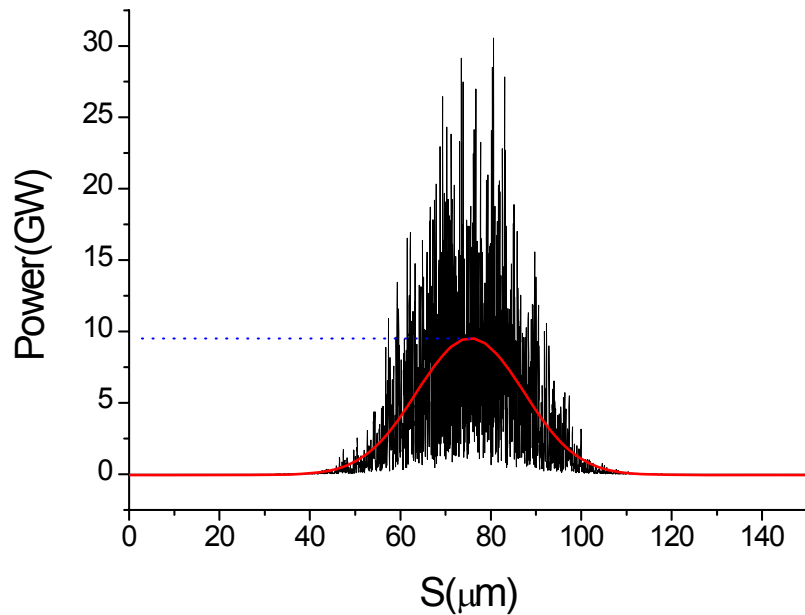


# Combining error together --- sinus, constant, sawtooth

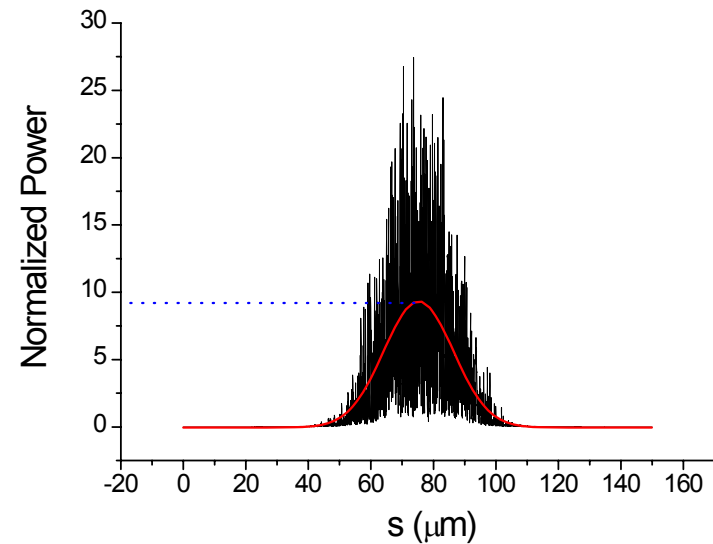


# Time dependent simulation for real structure --- Power at 130m (20 micron & 2 micron)

No error



Error included



## Conclusion & Summary

- For the error types we considered, radiation power correlates to the phase shake. Phase shake is proportional to the product of error period and error strength
- For the operation modes we simulated, when the rms phase shake is smaller than 0.15rad and 0.2rad for SASE1 0.1nm mode and 0.4nm mode respectively, the power degradation is smaller than 10%
- For 20 microns girder deformation and 2 micron motor movement accuracy, the output power is larger than 70% of the power without error
- Global temperature variation reduces the power, but its impact can be compensated by adjusting the undulator gap

The end