

# Results from SASE FEL Simulations

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## Outline

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1. Task definition.
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## XFEL UNDULATOR LINE FOCUSING LATTICE ERRORS; NUMERICAL SIMULATIONS

### 1. Problem definition

The impact of the undulator focusing lattice quadrupole magnets misalignments on the FEL performance has been studied numerically applying FEL simulation codes GENESIS and SIMPLEX. Since electron beam steering yields in zigzag beam orbit that goes through the centers of the Beam Position Monitors (BPM), the influence of the BPM alignment errors on the FEL parameters such as saturation power and saturation length was studied.

#### Beam parameters at the entrance of the undulator system SASE1

Electron energy [GeV]	17.5
Bunch length (RMS) [m]	$2.5 \times 10^{-05}$
Bunch charge [nC]	1
Emittance, $\varepsilon_x$ [mm-mrad]	1.4
Emittance, $\varepsilon_y$ [mm-mrad]	1.4
Energy spread [MeV]	$1.5 (8.57 \times 10^{-05})$
Peak current [kA]	5
$\gamma$	34246.6

## SASE1 FEL design parameters

K value	3.3
Period length [cm]	3.56
Segment length [m]	5
Number of segments	33
Segment interval [m]	6.1
Peak field [T]	1
Periods per segment	140
Total length [m]	201
Resonant radiation wavelength [nm]	0.1

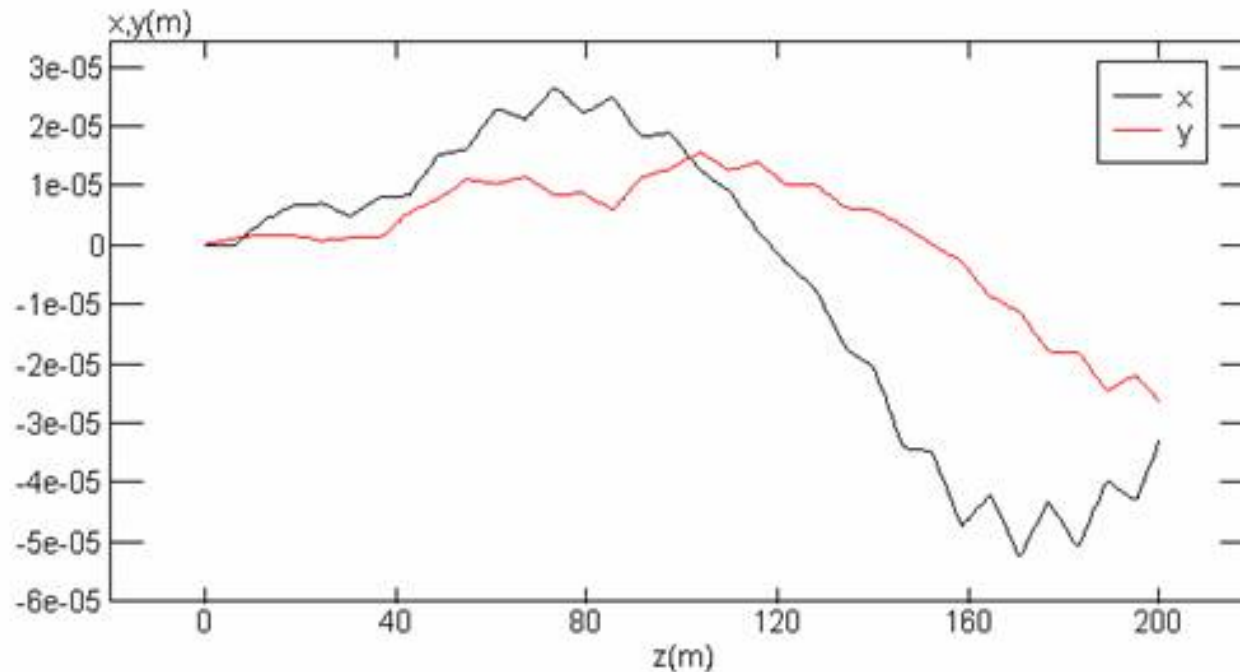
## SASE1 undulator focusing lattice

Lattice type	FODO
Focusing gradient (F) [T/m]	18
Defocusing gradient (D) [T/m]	-18
Focusing lens width [m]	0.2
Defocusing lens width [m]	0.2
F – D distance [m]	6.1
Period length [m]	12.2
Number of periods	17
Beta function [m]	32

## 2. Quads misalignments impact on the saturation power

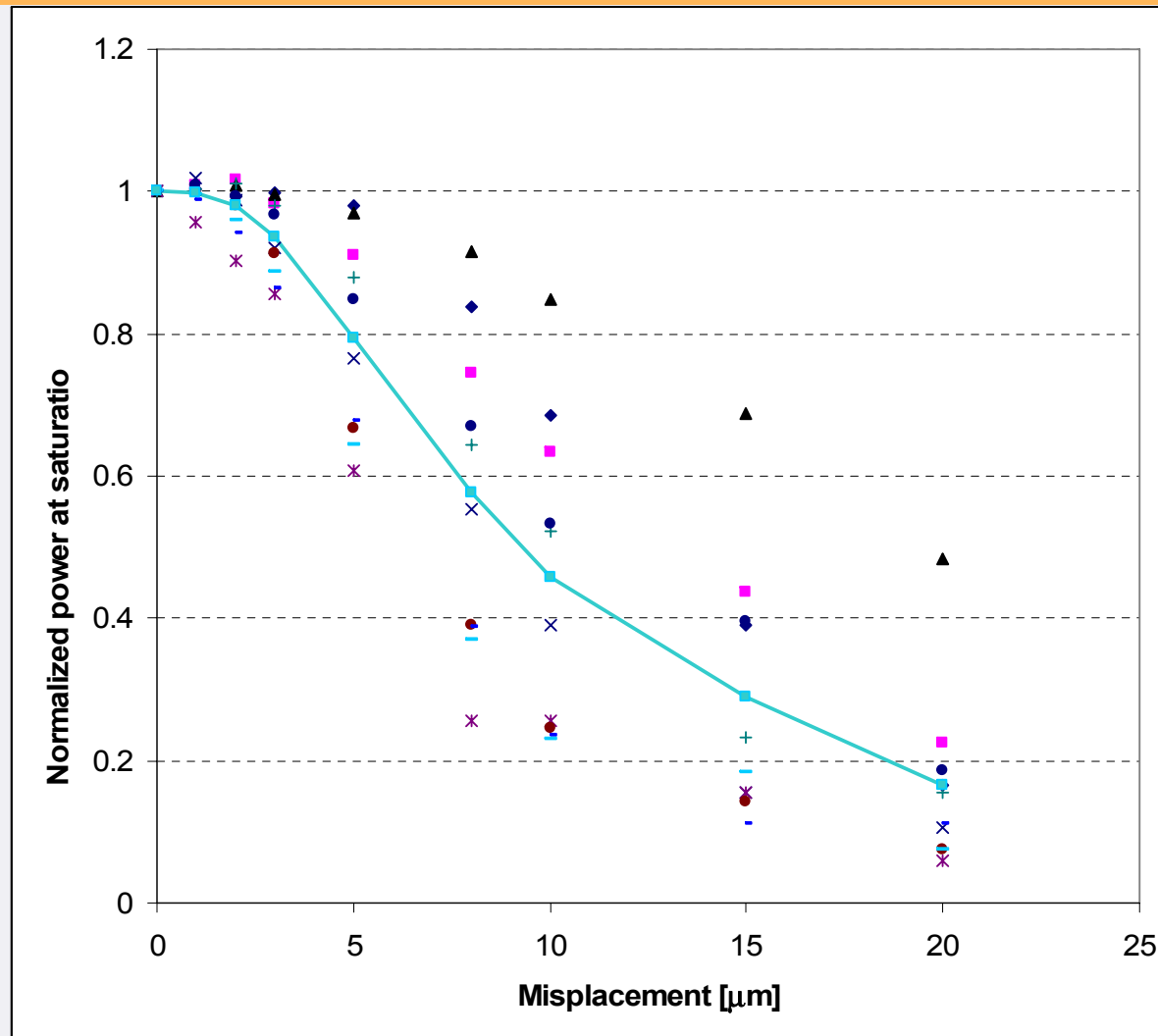
$\Delta x$  horizontal misplacement of the focusing quadrupole produces vertical dipole field  $B_y = -g\Delta x$ , where  $g$  is a gradient. That result in the horizontal bending of the beam with momentum  $p$  according to curvature radius  $\rho$  given by

$$\frac{1}{\rho} [m^{-1}] = 0.2998 \frac{B_y [T]}{p [GeV / c]}.$$



Electron beam centroid orbit in the undulator SASE1 in the presents of quadrupole random misalignments. Random Gaussian misalignments with the rms value  $10^{-5}$  m are assumed.

Dependence of normalized power at saturation on quadrupole magnets maximal misalignments calculated for XFEL SASE1 undulator (GENESIS).

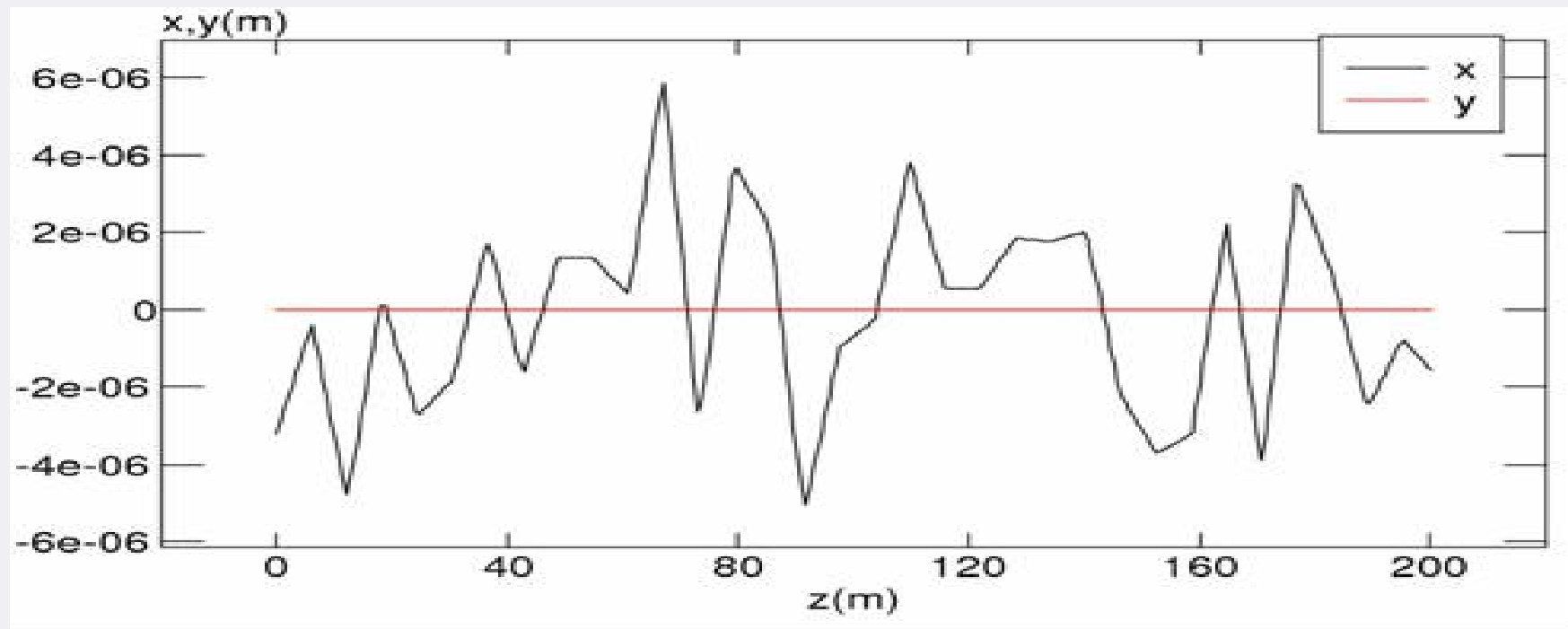


Various random sequences of the misplacements are considered. Solid line links the points that correspond to mean values of saturation power at different maximal displacements. The values of maximal misplacement considered are 1, 2, 3, 5, 8, 10, 15 and 20 microns. Bunch transverse shape is round and Gaussian. Therefore magnets misalignment in the horizontal and vertical planes will have the same effect.

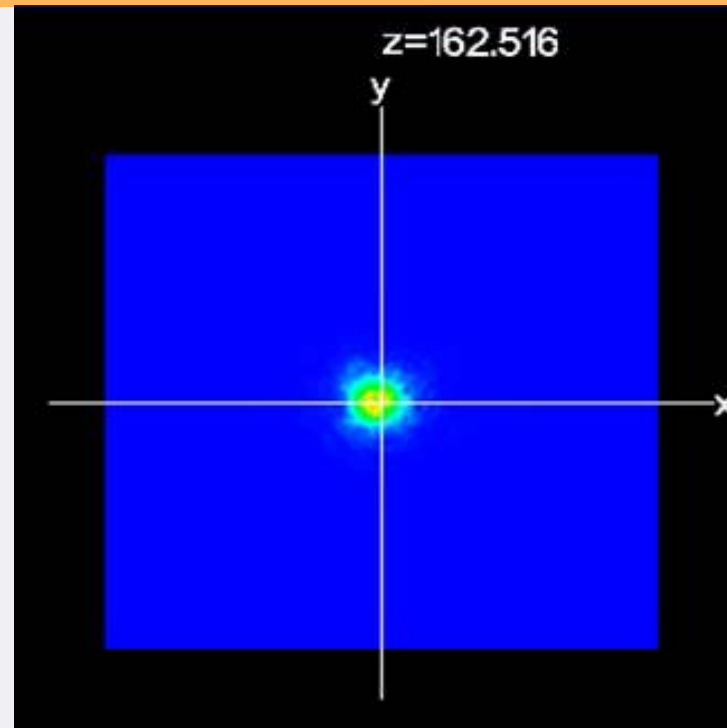


### 3. BPM misalignment influence on the undulator performance

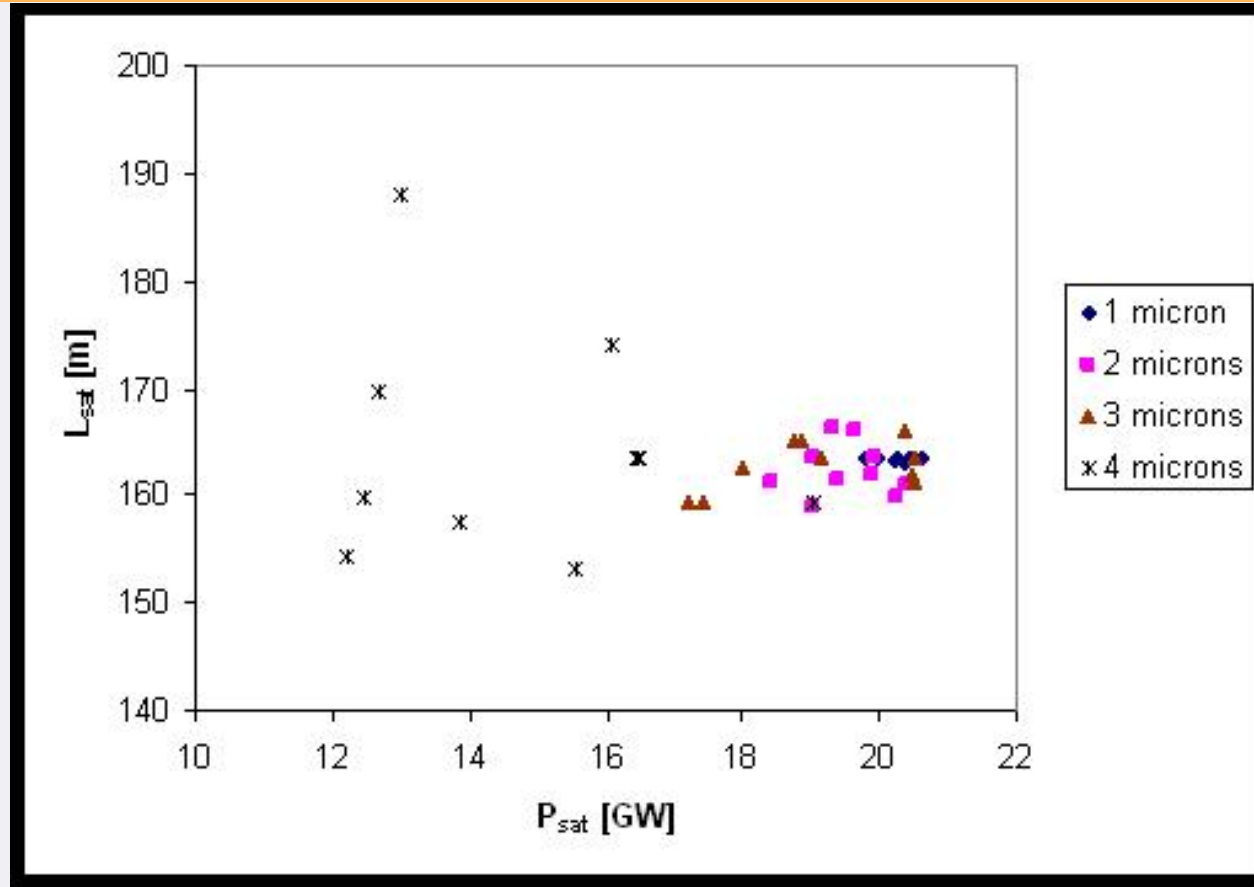
Beam centroid follows error trajectory going through the centers of the BPMs in the result of the steering by the correctors.



Error orbit due to BPM Gaussian horizontal misalignments ( $\sigma_x = 3 \mu\text{m}$ ).



Radiation power density at 162.516 m from undulator entrance simulated by SIMPLEX. BPM Gaussian horizontal misalignments is  $\sigma_x = 5$  mm.



BPM horizontal misalignment influence on the saturation power and saturation length calculated by SIMPLEX. 10 different Gaussian random misalignments samples are considered for each RMS value  $\sigma_x$ .

Dependence of the normalized saturation power ( $\langle P_{\text{sat}}/P_0 \rangle$ ) and normalized saturation length ( $\langle L_{\text{sat}}/L_0 \rangle$ ) on the RMS value of the BPM horizontal misalignments ( $\sigma_x$ ).

$\sigma_x$ [ $\mu\text{m}$ ]	$\langle P_{\text{sat}}/P_0 \rangle$	$\langle L_{\text{sat}}/L_0 \rangle$
1	0.984239	1.001654
2	0.947139	0.995098
3	0.927061	0.997733
4	0.717022	1.006863

## 3. Conclusions

- ✓ Uncorrected errors due to quadrupole magnet random misplacements in the range of 3 micron can reduce radiation power at saturation by ~15%.
- ✓ If one corrector is associated with each BPM, then trajectory errors due to correction cause reduction of saturation power by about 10% in the presence of BPM random Gaussian misalignment with the  $\sigma = 3$  micron.

## PART 2.OPTIMAL BETA FUNCTION AND NUMBER OF QUADRUPOLE MAGNETS; NUMERICAL SIMULATIONS

### 1.Task definition

From the condition of the best overlapping in the phase space of the electron beam and diffraction limited radiation being produced uniformly along the whole undulator length L one can obtain the following expression for the optimal beta function in the horizontal or vertical plane

$$\beta_{x,y} = \frac{L}{2\pi}$$

(See: Helmut Wiedemann, Electromagnetic Radiation from Relativistic Electron Beams, School on Synchrotron Radiation November 13-16, 2000, ICTP, Trieste, Italy.)

Parameters of the undulator systems. The electron beam energy is 17.5 GeV (TDR)

	$\lambda$ [nm]	$\lambda_u$ [mm]	$g$ [mm]	$B_{\max}$ [T]	$K$	$\beta$ [m]	$L_{\text{sat}}^+$ [m]	$N_{\text{tot}}^{++}$ [m]	$L_{\text{tot}}^{+++}$ [m]
SASE 1*	0.1	35.6	10	1.0	3.3	32	133	33	201.3
SASE 2*	0.1	48	19	0.63	2.8	46	174	42	256.2
	0.4		10	1.37	6.1	15	72		
SASE 3**	0.4	80.0	23	0.44	3.3	15	81	21	128.1
	1.6		10	0.91	6.8	15	50		
U 1/U 2***	0.06	26	6	1.24	3.0	15	-	10	61
	0.014		16	0.28	0.7	15	-		
<b>Total:</b>								<b>116</b>	<b>707.6</b>

According to SSY optimal beta function corresponding to absolute minimum of the gain length is approximated by the formulae

(see: E.L. Saldin, E. A. Schneidmiller, and M.V. Yurkov, Design formulas for short wavelength FELs, Opt. Commun. 235 (2004) 415.)

$$L_g = 1.67 \left( \frac{I_A}{I} \right)^{1/2} \frac{(\epsilon_n \lambda_u)^{5/6} (1 + K_{rms}^2)^{1/3}}{\lambda^{2/3} K_{rms} A_{JJ}} (1 + \delta),$$

$$\beta_{opt} = 11.2 \left( \frac{I_A}{I} \right)^{1/2} \frac{\epsilon_n^{3/2} \lambda_u^{1/2}}{\lambda K_{rms} A_{JJ}} (1 + 8\delta)^{-1/3},$$

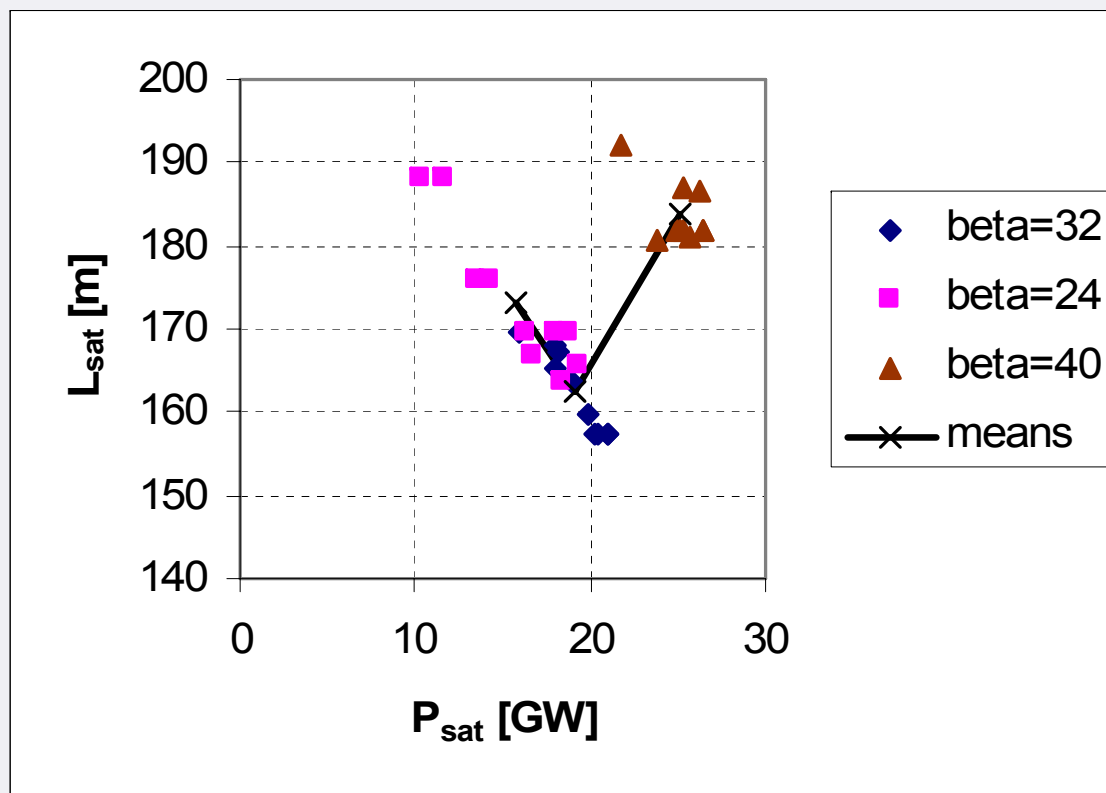
$$\delta = 131 \frac{I_A}{I} \frac{\epsilon_n^{5/4}}{\lambda^{1/8} \lambda_u^{9/8}} \frac{\sigma_\gamma^2}{(K_{rms} A_{JJ})^2 (1 + K_{rms}^2)^{1/8}},$$

For the SASE1 parameters one gets

$$\delta = 1.286 \times 10^{-11}, \quad L_g = 8.06m, \quad \beta_{opt} = 23.8m$$



## 2. SASE1 undulator performance dependence on the different average beta function values and different numbers of lattice quadrupoles



Distribution of events with different random seeds over saturation power and saturation length. Different series correspond to mean beta values of 32, 24 and 40. Solid line links mean points of each series.

SASE1 FEL performance parameters dependence on the lattice parameters ( $N_q$  – is the number of the quadrupoles,  $\beta$ - is average beta function).

	$\langle P_{\text{sat}} \rangle$ [GW]	$\langle L_{\text{sat}} \rangle$ [m]	Predicted brilliance $\times 10^{33}$	Integrated gradient [T]
$N_q=34, \beta=24\text{m}$	15.72	173.3	3.99	5
$N_q=34, \beta=29\text{m}$	19.06	173	4.77	4
$N_q=34, \beta=32\text{m}$	19.15	162.4	5.15	3.6
$N_q=34, \beta=40\text{m}$	25.11	183.7	6	2.8
$N_q=18, \beta=32\text{m}$	18.5	165.1	5.16	3.6
$N_q=18, \beta=40\text{m}$	23.44	173.4	6	2.8

Brilliance (in the units photons/s/mrad<sup>2</sup>/mm<sup>2</sup>/0.1%B.W.) is calculated by SIMPLEX using analytical and empirical formulae.

PPMQ type quadrupoles to be used can provide at least 8 T integrated gradient.

## 3. Conclusions

Preliminary results of the simulations suggest that with the present arrangement of the SASE1 undulator lattice 32m value is the best choice for the average beta function, if one uses the minimization of the saturation length and thus gain length as a performance sole criterion.

Higher values of beta function result in higher saturation power.

There are some arguments supporting the idea that if other criteria of the FEL performance are being used (e. g. brilliance) reduction of the number of lattice quadrupoles can be possible without degradation of the performance.