Response Matrix Measurements and Analysis at DESY

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DESY – MPY –
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Motivation

- To achieve the maximum performance of an accelerator, the linear optics of the machine needs to be close to the design optics.
- The real machine has gradient errors, alignment errors, etc. which are normally unknown and distorting the optics.
- Orbit depends non-linear on the focusing of the quadrupoles. Analyze the difference orbits due to the kick of corrector magnets (Orbit-Response-Matrix) to find out the error sources.
- Correct the gradient errors and restore the linear optics.
- Analysis gives valuable information about the BPM system and the corrector magnets.
Definition of the Orbit Response Matrix

- Definition of the orbit response matrix (ORM):

\[
C_{ij}^{xx} := \frac{\Delta x_i}{\Delta \theta_{x,j}} \quad \text{for x-plane}
\]

- \( \Delta x_i \) : change of the beam position at BPM \( i \)
- \( \Delta \theta_{x,j} \) : change of the kick angle of the corrector \( j \)

- Change the kick angle of all correctors one after the other and measure the orbit change with all available BPMs

- Measurement can be written as

\[
\begin{pmatrix}
\Delta \vec{x} \\
\Delta \vec{y}
\end{pmatrix} =
\begin{pmatrix}
C_{xx} & C_{xy} \\
C_{yx} & C_{yy}
\end{pmatrix}
\cdot
\begin{pmatrix}
\Delta \vec{\theta}_x \\
\Delta \vec{\theta}_y
\end{pmatrix}
\]

- Due to coupling and/or rotated BPMs or rotated correctors the orbit changes also in the other plane. For an uncoupled machine: \( C_{xy} = C_{yx} = 0 \)
Form of the Orbit Response Matrix

Beamline

- Corrector kick is changing the trajectory downstream of corrector:

\[ C_{ij} = \begin{cases} \sqrt{\beta_i \beta_j} \sin (2\pi |\phi_i - \phi_j|) & \text{if } \phi_i > \phi_j, \\ 0 & \text{otherwise} \end{cases} \]

- Triangle above main diagonal of \( C \) is zero

Circular accelerator

- Corrector kick is changing the orbit everywhere:

\[ C_{ij} = \frac{\sqrt{\beta_i \beta_j}}{2 \sin \pi Q} \cos (2\pi |\phi_i - \phi_j| - \pi Q) - \frac{D_i D_j}{\left(\alpha_c - \frac{1}{\gamma^2}\right) C} \]

\( D \): dispersion function, \( \alpha_c \): momentum-compaction factor,

\( C \): circumference, \( \gamma \): Lorentz factor

- Second term: energy shift due to kick of the corrector
Distinguish between

- **Measured matrix** $\bar{C}$
  
  Depends on scaling factors $b_i := 1 + \Delta b_i$ and $c_j := 1 + \Delta c_j$ of BPMs and correctors. For error-less BPMs/correctors $b_i = c_j = 1$.

- **Model matrix** $C$

  The computer model of the real machine

Assume, that model matrix depends on unknown parameters $\vec{p}$ of the lattice (e.g. quadrupole gradient errors, quadrupole roll angles, ...). Design parameters are called $\vec{p}_0$.

Then $\bar{C}$ can be written as:

$$\bar{C}_{ij} = \frac{1}{b_i} \cdot C(\vec{p}_0 + \Delta \vec{p})_{ij} c_j$$

Taylor expansion ($\Delta p_k, \Delta b_i, \Delta c_j \ll 1$):

$$\bar{C}_{ij} \approx C_{ij} + \sum_k \frac{\partial C_{ij}}{\partial p_k} \bigg|_{\vec{p}_0} \Delta p_k - C_{ij} \Delta b_i + C_{ij} \Delta c_j$$
Fitting the Unknown Parameters

- Linear system of equations to solve:

\[
\begin{pmatrix}
\bar{C} - C
\end{pmatrix}
\begin{pmatrix}
\bar{y}
\end{pmatrix}
= \begin{pmatrix}
\frac{\partial C}{\partial p_k} & -C & +C
\end{pmatrix}
\begin{pmatrix}
\Delta \bar{p} \\
\Delta \bar{b} \\
\Delta \bar{c}
\end{pmatrix}
\begin{pmatrix}
\bar{A}
\end{pmatrix}
\begin{pmatrix}
\Delta \vec{p}
\end{pmatrix}
\begin{pmatrix}
\Delta \vec{b}
\end{pmatrix}
\begin{pmatrix}
\Delta \vec{c}
\end{pmatrix}
\begin{pmatrix}
\vec{x}
\end{pmatrix}
\]

with the fit-parameter vector \( \vec{x} \) and the measured response matrix in \( \bar{y} \).

Matrix \( \bar{A} \) can be computed e.g. using MAD

- Take finite resolution of BPMs \( \sigma \) into account, by dividing each row of \( \bar{A} \) by \( \sigma/\theta_j \)

- Solve over-determined equations by least square fit using truncated SVD:

\[
\vec{x} = (\bar{A}^T \bar{A})^{-1} \bar{A}^T \bar{y}
\]

Normal equations

- Use for the next iteration optics with \( \vec{p}_0 + \Delta \vec{p} \) for the model matrix. Iterate several times, until convergence achieved.
Matrix $B := A^T A$ is singular and inverse matrix $B^{-1}$ is not existing due to an unknown global scaling factor $f$ between BPMs and correctors:

$$\frac{1}{f} \cdot \frac{1}{b_i} \cdot C_{ij} \cdot c_j f \equiv \frac{1}{b_i} \cdot C_{ij} \cdot c_j$$

→ Two small eigenvalues of $B$ ($x$ and $y$-plane)

Fix the unknown scaling factor $f$ by measuring the dispersion function $D$

Solution: Remove null-space from $B$ using singular value decomposition:

$$B = USV^T$$

with orthogonal matrices $U$ and $V$ and a diagonal matrix $S$ with singular values $s_i$.

Compute pseudo-inverse of $B$

$$B^+ = VDU^T$$

using the truncated SVD and the cutoff-parameter $\epsilon < 1$. Set $1/s_i = 0$ for small singular values. Matrix $D$ has diagonal shape with $D_{ii} = 1/s_i$ if $s_i < \epsilon s_1$, otherwise $D_{ii} = 0$. 
Matrix Sizes

Assume a ring with $N$ BPMs and $M$ correctors

- Size of response matrix: $N_{\text{MAT}} = (N_x + N_y) \cdot (M_x + M_y)$
- Minimum number of fit parameters: $N_{\text{FIT}} = (N_x + N_y) + (M_x + M_y)$

Examples:

<table>
<thead>
<tr>
<th>Machine</th>
<th>$N_x$</th>
<th>$N_y$</th>
<th>$M_x$</th>
<th>$M_y$</th>
<th>$N_{\text{MAT}}$</th>
<th>$N_{\text{FIT}}$</th>
<th>Memory(A)</th>
</tr>
</thead>
<tbody>
<tr>
<td>El-Weg</td>
<td>6</td>
<td>6</td>
<td>11</td>
<td>12</td>
<td>276</td>
<td>35</td>
<td>77 kB</td>
</tr>
<tr>
<td>PETRA-e</td>
<td>113</td>
<td>113</td>
<td>118</td>
<td>111</td>
<td>51754</td>
<td>455</td>
<td>188 MB</td>
</tr>
<tr>
<td>HERA-p</td>
<td>141</td>
<td>141</td>
<td>128</td>
<td>126</td>
<td>71628</td>
<td>536</td>
<td>307 MB</td>
</tr>
<tr>
<td>HERA-e</td>
<td>287</td>
<td>287</td>
<td>281</td>
<td>277</td>
<td>320292</td>
<td>1132</td>
<td>2.9 GB</td>
</tr>
</tbody>
</table>

Problems for HERA-e with memory and computation time

→ Working on a subset of all corrector magnets/BPMs
→ Or: Use a different approach for optics correction!
Assumption: FODO lattice with $N$ BPMs and $M$ corrector magnets; same kick $\theta$ of all correctors; same $\beta$ function at all BPMs and correctors; all BPMs have the same resolution $\sigma$ (V. Ziemann, EPAC 2002).

- Fitting $N$ BPM scaling factors:
  \[ \sigma(b) \approx \frac{\sigma}{\beta \theta \sqrt{M}} \]

- Fitting $M$ corrector scaling factors:
  \[ \sigma(c) \approx \frac{\sigma}{\beta \theta \sqrt{N}} \]

- Fitting $Q$ gradient errors:
  \[ \sigma(\Delta k l) \approx \frac{\sigma}{\beta^2 \theta \sqrt{N \cdot M}} \frac{48\pi}{\sqrt{N \cdot M}} \]

Small BPM resolution $\sigma$ is crucial for the sensitivity to fit errors. Use big corrector kicks $\theta$ and as much BPMs $N$ and correctors $M$ as possible.
Beta/Phase function fit

Matrix element of response matrix:

\[ \Delta x_{ij} = \sqrt{\beta_i} \frac{\sqrt{\beta_j} \Delta \theta_j}{2 \sin \pi Q} \cos(\pm \phi_j \mp \phi_i + \pi Q) \]

for \( \phi_i > \phi_j \)

\( \phi_i < \phi_j \)

Factorization of monitor and corrector parameters:

\[ \Delta x_{ij} = f_j \cos(\pi Q \pm \phi_j) \cdot \sqrt{\beta_i} \cos(\phi_i) \pm f_j \sin(\pi Q \pm \phi_j) \cdot \sqrt{\beta_i} \sin(\phi_i) \]

\[ = \sqrt{\beta_i} \cos(\pi Q \mp \phi_i) \cdot f_j \cos(\phi_j) \mp \sqrt{\beta_i} \sin(\pi Q \mp \phi_i) \cdot f_j \sin(\phi_j) \]

Alternating fit of \( (\beta_i, \phi_i) \) or \( (f_j, \phi_j) \):

\[ \chi^2 = \sum_{i,j} \left( \frac{\Delta x_{ij}^{\text{meas}} - \Delta x_{ij}^{\text{model}} (\beta_i, \phi_i, f_j, \beta_j)}{\sigma(\Delta x_{ij}^{\text{meas}})} \right)^2 \to \text{min.} \]

For optics correction phases \( \phi_i \) and \( \phi_j \) are used (not sensitive to scaling errors of BPMs and correctors!)

BPMs and correctors have unknown scaling factors.

Scaling factors will lead to an error in the beta function but not in the phase function. 

⇒ Use phase function \((\varphi_i, \varphi_j)\) for correction!

Phase beating due to gradient error of a quadrupole \(\Delta k_q\):

\[
\Delta \varphi = \frac{\beta_q \Delta k_q l}{4 \sin \pi Q} \{ \sin(2\pi Q) + \sin(2\varphi_q - 2\pi Q) \\
+ \text{sign}(\varphi - \varphi_q)[\sin(2\pi Q) + \sin(2|\varphi - \varphi_q| - 2\pi Q)] \}
\]

Global correction of beta beating:

Solve for quadrupole corrections \(\Delta k_q\) using SVD or MICADO:

\[
\|\varphi_{i,j} - \sum_q \frac{\partial \varphi_{i,j}}{\partial k_q} \Delta k_q\|^2 \to \min.
\]
Before correction; ZEUS calorimeter closed; luminosity optics

**Beta beating**

\[ \Delta \beta_x / \beta_x \]

\( \phi_x / (2\pi) \) / rad

**Phase beating**

\[ \Delta \phi_x / (2\pi) \] / rad

\( \phi_x / (2\pi) \) / rad
Before correction; ZEUS calorimeter closed; luminosity optics

Beta beating

Phase beating
After correction with 10 quadrupoles ($\Delta k/k$ up to 4%)
Example: HERA-e $y$-plane, corrected

After correction with 10 quadrupoles ($\Delta k/k$ up to 4%)
**Response-Matrix Analysis: Accuracy**

**Top:** Difference orbits (*Measurement, unfitted and fitted model*) for corrector OR17 CI

**Bottom:** Difference between measurement and model before and after fit
Response-Matrix Analysis: Accuracy

Orbit difference at all BPMs before (blue) and after (red) fit (BPM resolution $\sigma \approx 7/4 \mu m$):

\[ \sigma_x \approx 8.3 \mu m \]

\[ \sigma_y \approx 6.9 \mu m \]
HERA: Bugs found with ORM

HERA-e:

- Wrong longitudinal position of corrector magnets (VO, VG) in lattice file
- 20% magnetic field reduction for CV 27 corrector magnets
- Wrong longitudinal position of 8 BPMs in rotator section N & S
- Global scaling factor of BPM system (software bug)
- Many BPMs with wrong cabling or bad buttons signals
- Longitudinal permutation of three BPMs in HERA-e

HERA-p:

- Interchanged cables of s.c. quadrupoles QP33/35 NL
- Wrong length entry in magnet database for QP33/35 NL & QP33/35 SL
- Wrong calibration curve of IR quadrupole family GA/GB
- Wrong calibration curve of corrector magnet CZ 27
- Many bad BPMs
Example: PETRA-e

- **BPMs:**
  - **x & y-plane:** 113 BPMs
  - In control system (2003): 3 different BPM types
    - (octagonal shape, round chamber $\varnothing = 100$ mm and $\varnothing = 120$ mm)
  - But: 8 different BPM types installed in PETRA!
  - POISSON: $K_x$ of BPMs with octagonal shape 20% too small (50% of all BPMs)!

- **Corrector magnets:**
  - **x-plane:** 118 correctors
    - 23 CH (separate)
    - 83 CB, 6 C4, 6 C5 (backleg winding)
  - **y-plane:** 111 CV (separate)

- **Quadrupoles:**
  - 23 independent quadrupole families
Before correction of monitor constants of BPMs with octagonal shape

ORM analysis PETRA-e, 7 GeV, 25.6.2003, Sextupoles off

NOL 47 MO
\[ \Delta s \approx -10 \text{ m!} \]
After correction of monitor constants of BPMs with octagonal shape

ORM analysis PETRA-e, 7 GeV, 25.6.2003, Sextupoles off
Four correctors *longitudinally permutated*: NR 8 CH ↔ NR 12 CH, NR 9 CH ↔ NR 11 CH
Found four groups of corrector scaling factors. In addition two correctors with increased field near DESY II and DESY III beam line (SOL/SOR34 CV) were found.
Vertical correctors (CV) near quadrupoles, sextupoles and dipoles
→ Magnetic short-circuit between CV and adjacent magnet?

Classification by distance between CV and nearby magnets

- B = bending magnet
- MQA, MQA1 = quadrupoles
- MS = sextupole

**Case #1:**
\[ \Delta s_1 = 10.3 \text{ cm} \]
\[ \Delta s_2 = 4.7 \text{ cm} \]

**Case #2:**
\[ \Delta s_1 = 10.3 \text{ cm} \]
\[ \Delta s_2 = 9.7 \text{ cm} \]

**Case #3:**
\[ \Delta s_1 = 10.3 \text{ cm} \]
\[ \Delta s_2 = 43.6 \text{ cm} \]

**Case #4:**
\[ |\Delta s| > 30 \text{ cm} \]
Result of fitting the gradients of quadrupole families with CALIF (PEM04 optics, 7 GeV)

Relative deviations of the $k$-values from theory ($Q1 =$ doublet, $Q4A =$ triplet A, $Q4B =$ triplet B)

Currents of quadrupole families had to be changed to achieve nominal tunes

Explanation: calibration curves of quadrupoles are based on a different magnet cycling procedure; empirical corrections did correct this effect
PETRA-e: Beta & phase function

Beta function and phase function fit, PEM04 optic, 7 GeV
Example: El-Weg

- El-Weg is e\(\pm\)-transport line between PETRA and HERA-e
- PETRA and HERA are located on different levels and have different slopes
  - Coupled beamline
  - \(x\)- and \(y\)-bending
- Transfer efficiency in HERA II was sometimes \(\ll 100\%\) and non-reproducible
- Summer 2004: six BPMs were installed
- Dec. 2004: Optic was checked by measuring a response matrix
Hardware components of the El-Weg:

- **BPMs**: $N_x = 6, N_y = 6$
- **Correctors**: $M_x = 11, M_y = 11$
- **Quadrupoles**: $Q = 19$ in 8 families
  - Matrix elements: $N_{x,\text{MAT}} = 36, N_{y,\text{MAT}} = 33$
  - BPM/corrector fit parameters: $N_{\text{FIT}} = 6 + 11 = 17$

Bad: Number of fit parameters $\approx$ number of matrix elements

Strategy: Use a precise BPM-model for position reconstruction, rely on correct calibration of correctors, fit only quadrupole families

Result of ORM analysis: Quadrupole families are 2-4% too strong
El-Weg-BPM Calibration

Isolines of BPM EL 003 ($r = 17$ mm)

Isolines of BPM EL 037 ($r = 30$ mm)

$q = \Delta / \Sigma$ of the four BPM buttons were calculated as function of $(x, y)$ on grid

Positions are the solution of the non-linear equations using interpolated $q$:

$$
\begin{align*}
q_{x}^{\text{theo}}(x, y) &= q_{x}^{\text{meas}} \\
q_{y}^{\text{theo}}(x, y) &= q_{y}^{\text{meas}}
\end{align*}
$$
Example: Trajectory change due to the kick angle of vertical corrector VEL 37 and theoretical prediction with old and new optics model

Old optics model

New optics model
Measured QEL-quadrupole field was different from calibration curve assumed!

From optics calculations: $\Delta \beta / \beta \approx 100\%$ in second part of beamline
Response matrix analysis is a valuable tool to understand and debug the accelerator.

Fit of the response matrix allows to find out gradient errors, calibration errors of BPMs and calibration errors of corrector magnets.

But: not everything can be fitted; it depends on the number of BPMs/correctors, the kick amplitude and the resolution of the BPMs.

Comprehensive analysis of data can give information about faulty hardware.

Many errors found in HERA-e, HERA-p, PETRA and El-Weg.

Method also useful for VUV-FEL and XFEL?