Transverse space charge forces in XFEL optics after final compression

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Beam lines:

A. Main linac (max 30 cells):
   - 90° cell: XFEL.19NOV04.mad
   - 60° cell.

B. Main linac with upstream diagnostic section.

Beam parameters:

- Energy: (0.5 - 3.0) GeV.
- Current: 5 kA.
- Normalized emittance: 1 mm · mrad.
- Matched Gaussian beam truncated at 3 σ.

Acceleration in linac:

- $E_{acc} = 20.27 \text{ MV/m}$, on-crest.
2 Definitions

- Statistical emittance:
  \[ \epsilon_x = \sqrt{\langle x^2 \rangle \langle p_x^2 \rangle - \langle xp_x \rangle^2} \]

- Statistical \( \beta \)-function:
  \[ \beta_x = \frac{\langle x^2 \rangle}{\epsilon_x} \]

- Moment invariant of coupled 2D linear motion (includes linear space charge when treated in the Vlasov approximation) (first discovered by W.Lysenko):
  \[ I_{xy}^2 = \frac{\epsilon_x^2 + \epsilon_y^2 + 2 \cdot (\langle xy \rangle \langle pxpy \rangle - \langle xp_y \rangle \langle yp_x \rangle)}{2} \]

- Statistical normalized emittance:
  \[ \epsilon_{x,n} = \beta_0 \gamma_0 \epsilon_x \]

where \( p_x, p_y \) are particle momentum divided by design momentum.
3 Some tests
Figure 1: Main linac: 90° cells (10 cells). $I = 0$. Blue: periodic solution without RF cavities. Green: statistical beta functions without RF cavities. Red: statistical beta functions with RF cavities. In two last cases the initial Twiss parameters are periodic solution without RF. Bottom right figure is the energy.
4 Beam parameters: $E = 500 \text{ MeV}$.
$\varepsilon_n = 1 \text{ mm} \cdot \text{ mrad}$. $I = 5 \text{kA}$. 
Figure 2: Main linac without (middle) and with (top, bottom) upstream diagnostic section. Beam parameters: \( E = 500 \text{ MeV} \), \( \varepsilon_n = 1 \text{ mm} \cdot \text{mrad} \). \( I = 0 \) (top), \( 5 \text{kA} \) (middle and bottom).
Figure 3: Exit of diagnostic section (linac entrance). Test 1 $\sigma$ and 3 $\sigma$ particles (which feel the space charge forces of main particles but not contribute in these forces). $\varepsilon_n = 1 \text{ mm} \cdot \text{mrad}$. $I = 0$ (red), 5 (blue) kA. $E = 500$ MeV (top), 1 GeV (bottom).
5 Main Linac (90° cell):
initial energy (0.5 - 3) GeV
Figure 4: Main linac (90° cell, 30 cells). $\varepsilon_n = 1 \text{mm} \cdot \text{mrad}$. $I = 5 \text{kA}$.
Initial energy: $E = 0.5, 1.0, 1.5 \text{GeV}$ (from top to bottom).
Figure 5: Main linac (90° cell, 30 cells). $\varepsilon_n = 1mm \cdot mrad$. $I = 5kA$. Initial energy: $E = 2.0, 2.5, 3.0$ GeV (from top to bottom).
6 Linac with upstream diagnostic section (90° cell): initial energy (0.5 - 3) GeV
Figure 6: Main linac with upstream diagnostic section. $\varepsilon_n = 1 \text{mm} \cdot \text{mrad}$. $I = 5 \text{kA}$. Initial energy: $E = 0.5, 1.0, 1.5 \text{GeV}$ (from top to bottom).
Figure 7: Main linac with upstream diagnostic section. $\varepsilon_n = 1 \text{mm} \cdot \text{mrad}$. $I = 5\,\text{kA}$. Initial energy: $E = 2.0, 2.5, 3.0$ GeV (from top to bottom).
Figure 8: Moment invariants (divided by their initial values and multiplied by $\gamma(z)$). $\varepsilon_n = 1 \text{ mm} \cdot \text{mrad}$. $I = 5 \text{kA}$. $E = (0.5 - 3.0) \text{GeV}$ (from top to bottom with 0.5 GeV step). Right: linac ($90^\circ$, 30 cells). Left: linac with upstream diagnostic section.
6.1 Main Linac: 60° cell
Figure 9: Main linac: 90° (left) and 60° (right) cells (10 cells). $\varepsilon_n = 1 \text{mm} \cdot \text{mrad}$. $I = 5 \text{kA}$. Initial energy: $E = 0.5$ (magenta), 1 (green), 1.5 (blue) GeV. Red: $I = 0$. 
Figure 10: Main linac: 60° cell: FODO (left) and DOFO (right) (10 cells). \( \varepsilon_n = 1 \text{mm \cdot mrad} \). Initial energy: \( E = 0.5 \) (magenta), 1 (green), 1.5 (blue) GeV. Red: \( I = 0 \).