XFEL Injector matching

The present XFEL injector foresees operation of the first four cavities of the first module to operate at > 12MV/m

Can we also operate at a higher gradient?

Yes, but….

Classical Brillouin flow

Cancellation of the linear defoucusing space charge by a solenoid field requires:

$$\left(\frac{eB_z}{2m_e\gamma\beta c}\right)^2 x_{rms} = \frac{P}{4x_{rms}}$$

$$P = \frac{2I}{I_A^0\beta^3\gamma^3}$$

$$I_A^0 = \frac{4\pi\varepsilon_0 m_e c^3}{e}$$

P = generalized perveance

Classical Brillouin flow

The beam size stays constant if the contribution of the emittance can be ignored and if

$$x_{rms}^{eq} = \sqrt{P}\,\frac{m_e \gamma \beta c}{e B_z}$$

is fulfilled.

Small mismatches lead to an oscillatory solution.

Generalized Brillouin flow

Envelope equation for a beam in an acceleration section (ignoring emittance contribution):

$$x'' + \frac{\gamma'}{\gamma} x' + \frac{C}{\gamma^2} x - \frac{I}{2 I_0^A \gamma^3 x_{rms}^2} x = 0$$

$$C = \frac{\gamma'^2}{8}$$

for a cavity, on crest, linear energy gain: $\gamma = \gamma_0 + \gamma' z$

Generalized Brillouin flow

Cancellation of space charge and focusing term requires:

$$x_{rms}^2 = x_{0,\,rms}^2 \frac{\gamma_0}{\gamma}$$

$$\left(x_{rms}\right)' = -\frac{\gamma'}{2\gamma} x_{rms}$$

$$\left(x_{rms}\right)'' = \frac{3\gamma'^2}{4\gamma^2} x_{rms}$$

$$\frac{3\gamma'^2}{4\gamma^2} x_{rms} - \frac{\gamma'^2}{2\gamma^2} x_{rms} + \frac{\gamma'^2}{8\,\gamma^2} x_{rms} - \frac{I}{2 I_0^A \gamma^2 \gamma_0 x_{0,\,rms}^2} x_{rms} = 0$$

Generalized Brillouin flow

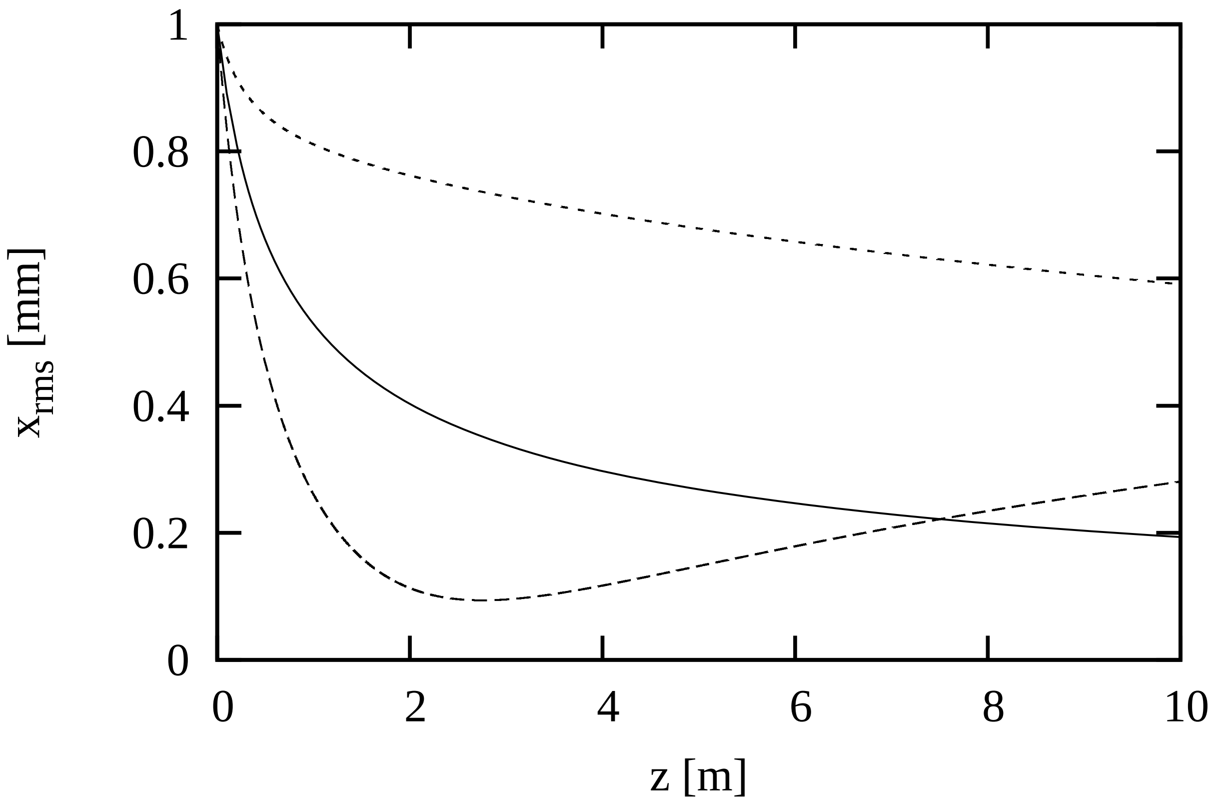
It follows:
$$\gamma' = \frac{2}{x_{0,rms}} \sqrt{\frac{I}{3 I_0^A \gamma_0}}$$

$$\left(x_{0,rms} \right)' = -\frac{\gamma'}{2\gamma_0} x_{0,rms}$$

What happens in case of small mismatches?
$$x'' + \frac{\gamma'}{\gamma} x' + \frac{C}{\gamma^2} x - \frac{I}{2 I_0^A \gamma^3 x_{rms}^2} x = 0$$

Damped oscillator?

Generalized Brillouin flow



Invariant envelope (solid) and envelope with mismatch of initial angle (± 50%).

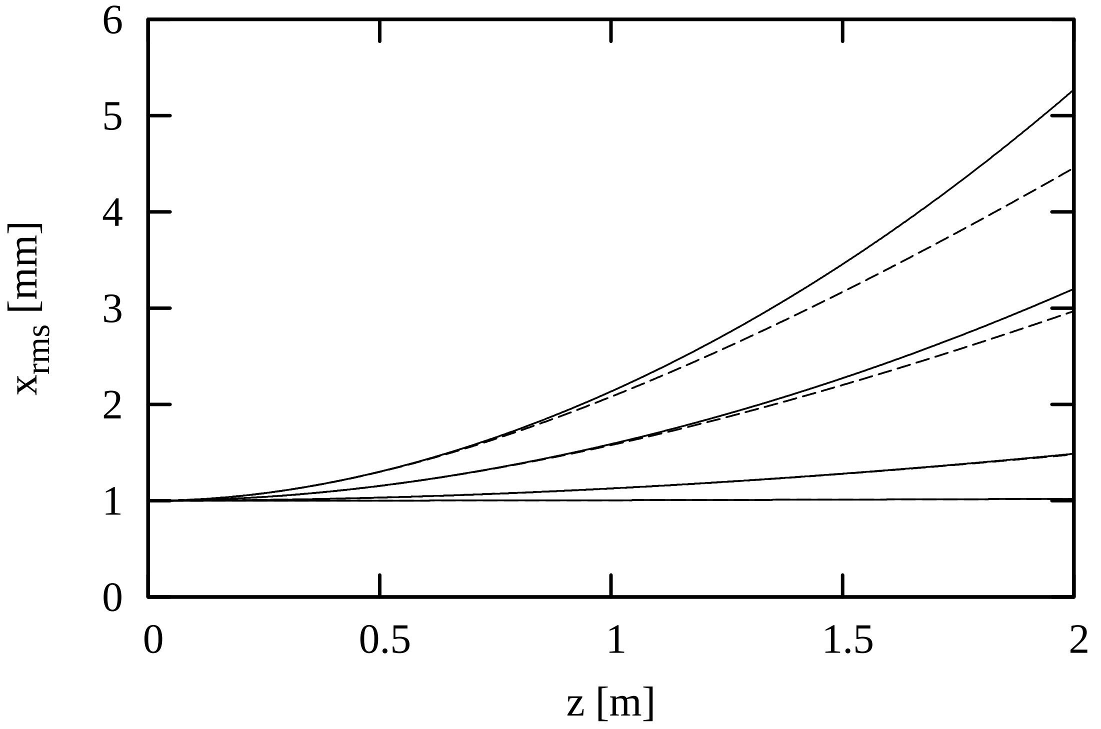
Generalized Brillouin flow

The solution describes not a static equilibrium but a dynamic equilibrium. Direct comparison of space charge and focusing force reveal that the focusing force would need to be a factor sqrt(3) larger for a static compensation.

The space charge field works against the incoming beam divergence.

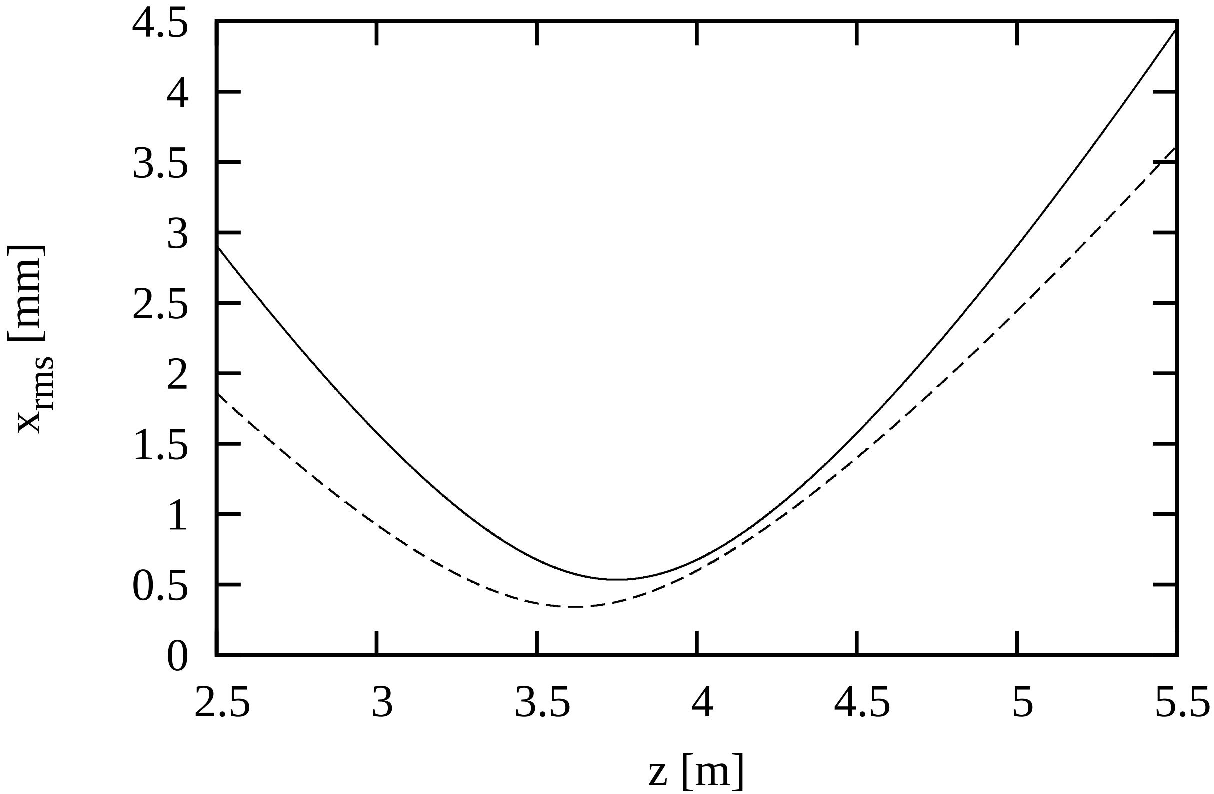
A simple Model for emittance compensation

$$\left(x_{rms}\right)'' - \frac{P}{4x_{rms}} - \frac{\varepsilon_{rms}^2}{x_{rms}^3} = 0$$



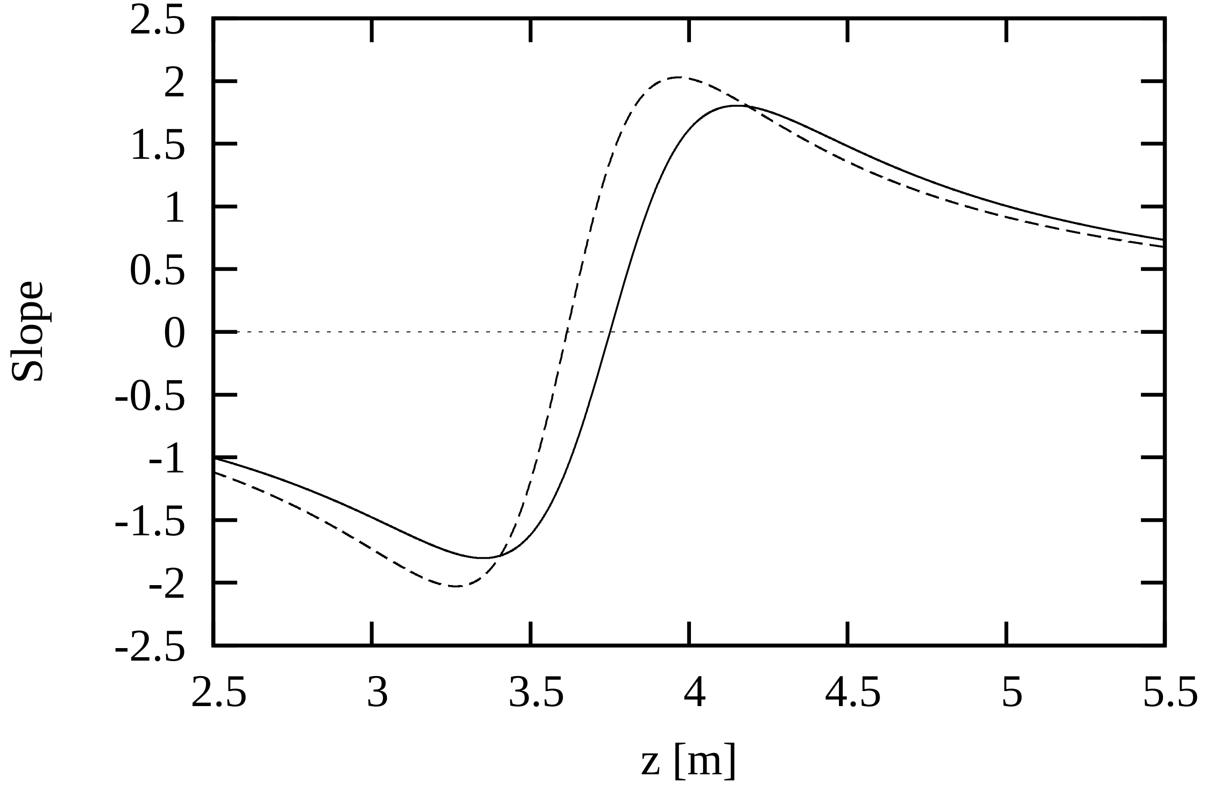
Development of the beam size in a drift with space charge. Beam parameters:
$\gamma = 10$, $\varepsilon_n / \gamma = 10^{-7}$, $P = (10, 5, 1, 0) \cdot 10^{-6}$
Dashed line = analytic approximation

A simple Model for emittance compensation



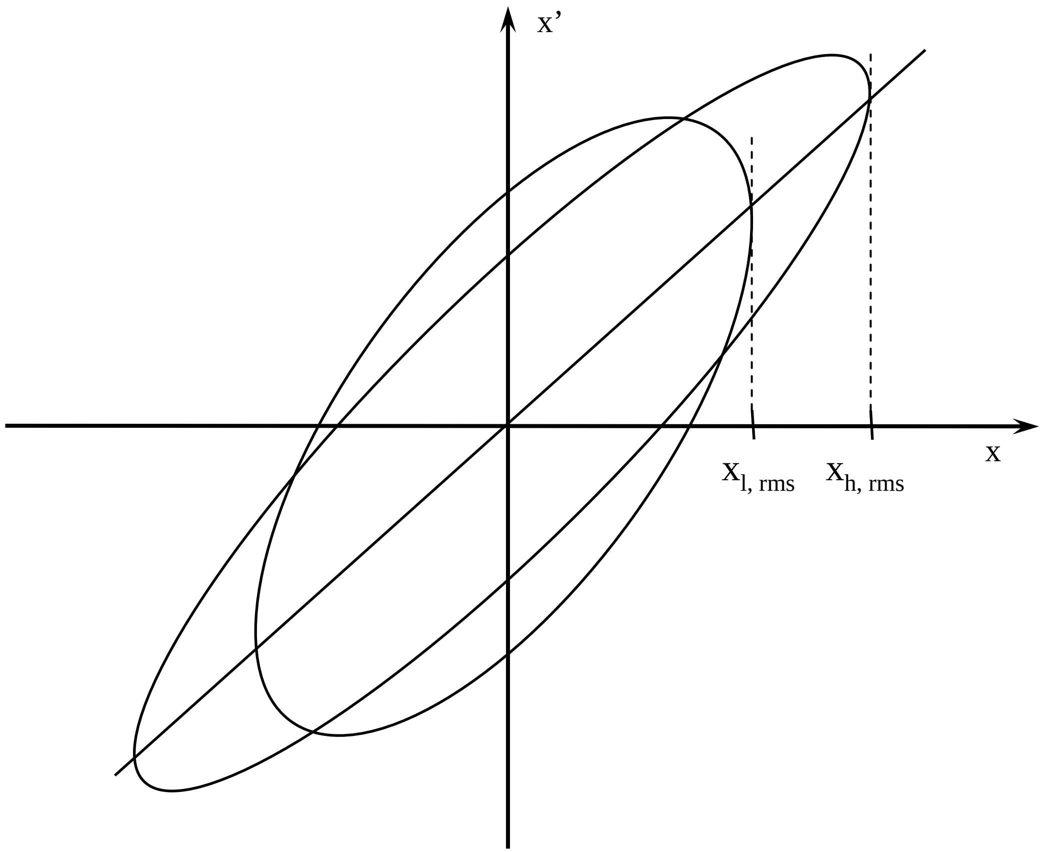
Slice size development of the high perveance slice (solid line) and the low perveance slice (broken line) after a focusing kick from a numerical integration of the envelope equation.

A simple Model for emittance compensation

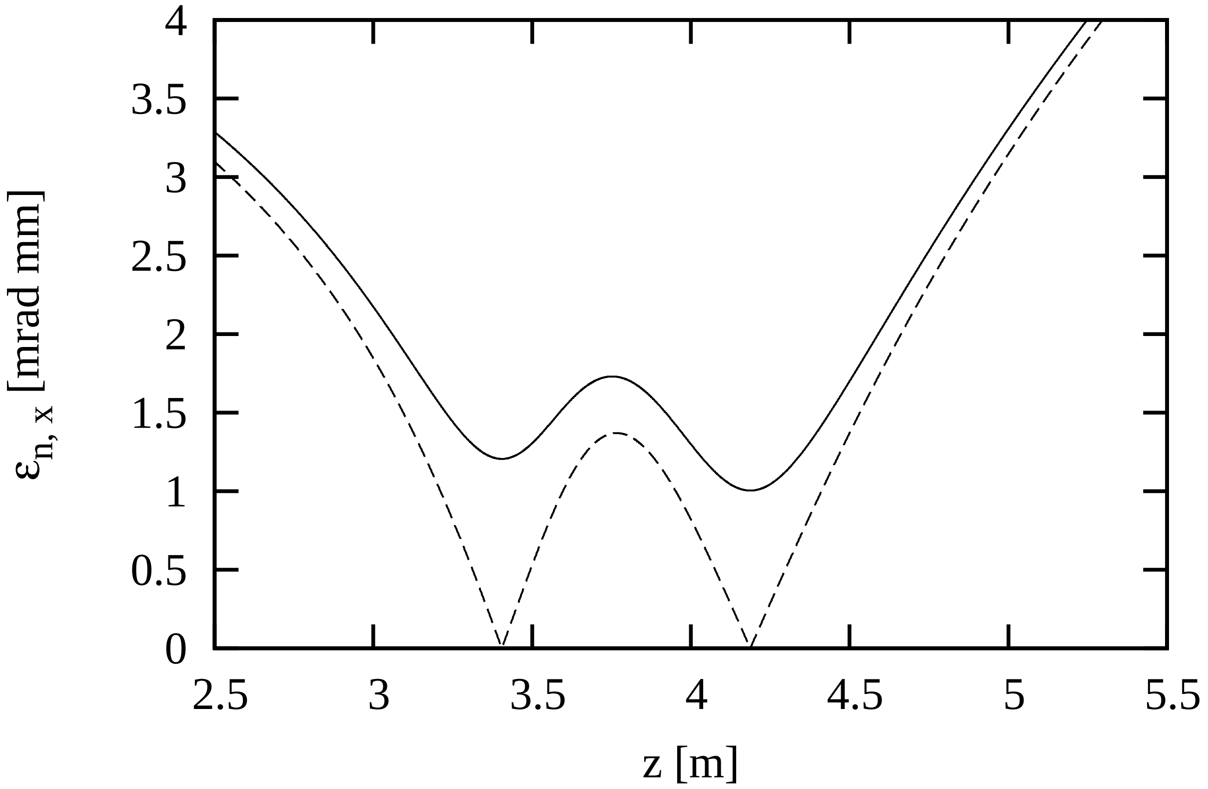


Slice size development of the high perveance slice (solid line) and the low perveance slice (broken line) after a focusing kick from a numerical integration of the envelope equation.

A simple Model for emittance compensation

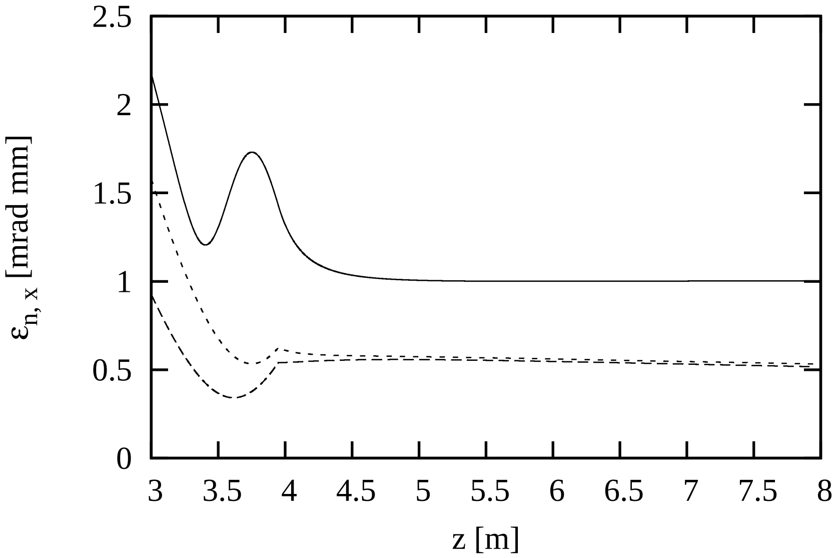


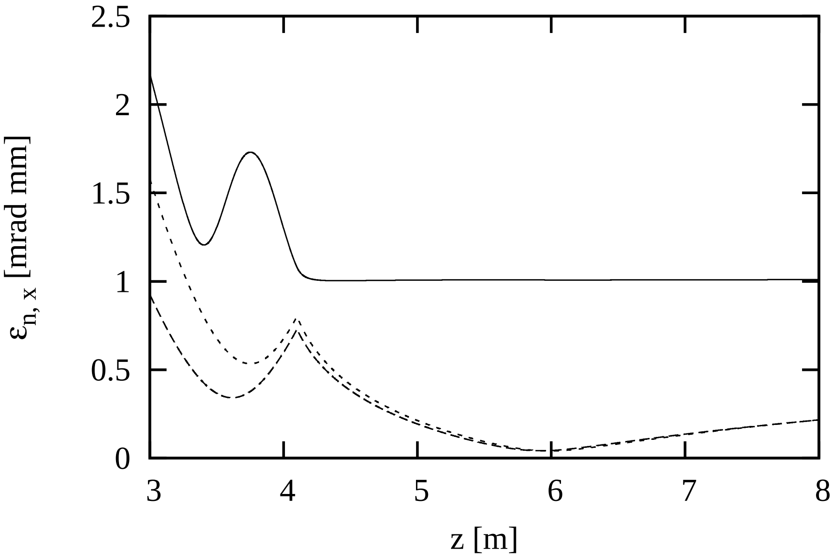
A simple Model for emittance compensation



Development of the beam emittance as calculated from the high perveance and low perveance slices. The broken line ignores the slice emittance, i.e. the slices are treated as lines.

A simple Model for emittance compensation





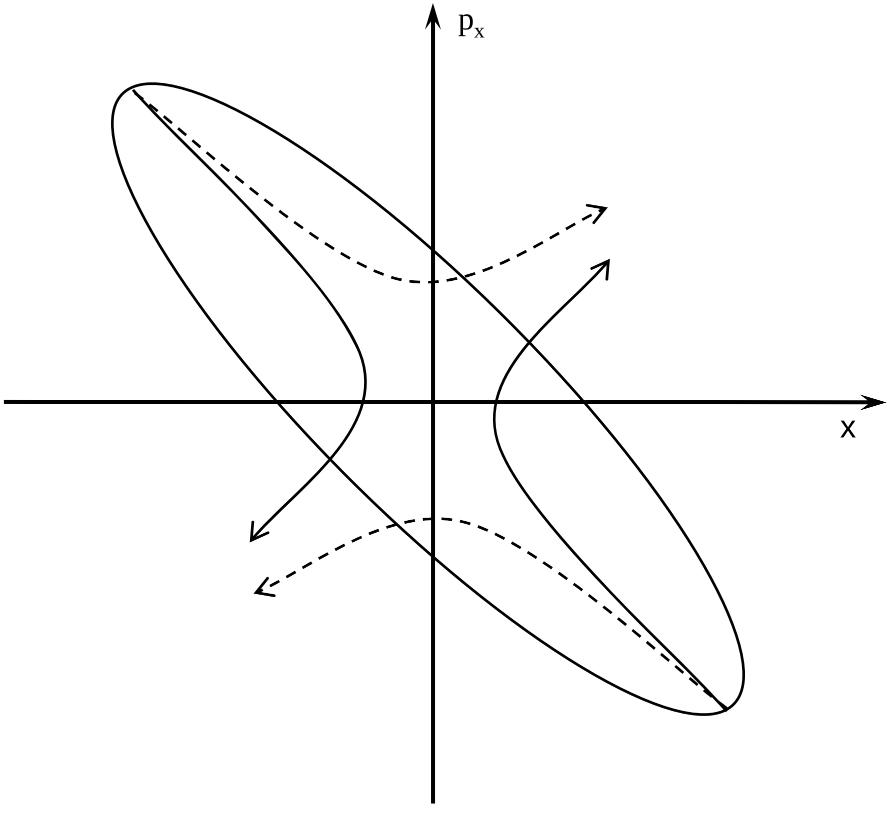
Emittance (solid line) and slice size (broken lines) in an accelerating section with

$$\gamma' = 41$$

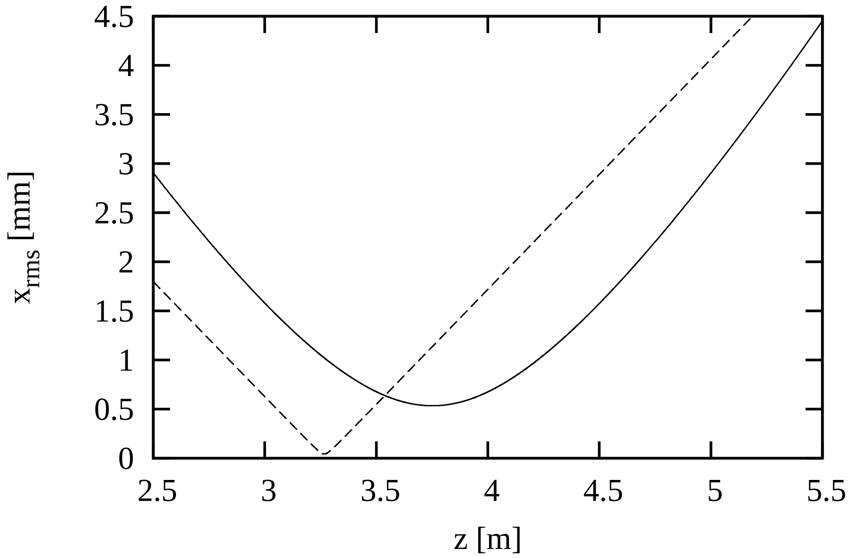
Emittance (solid line) and slice size (broken lines) in an accelerating section with:

$$\gamma' = 82$$

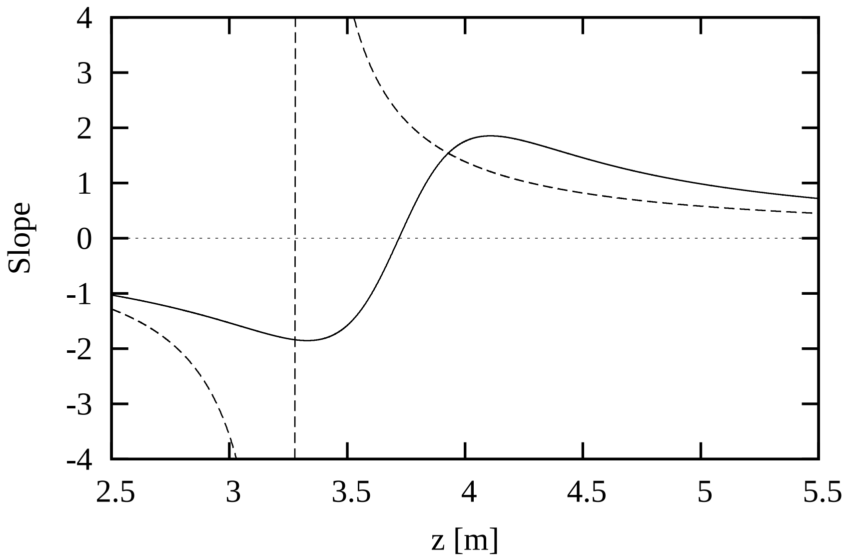
Cross over trajectories



Cross over trajectories

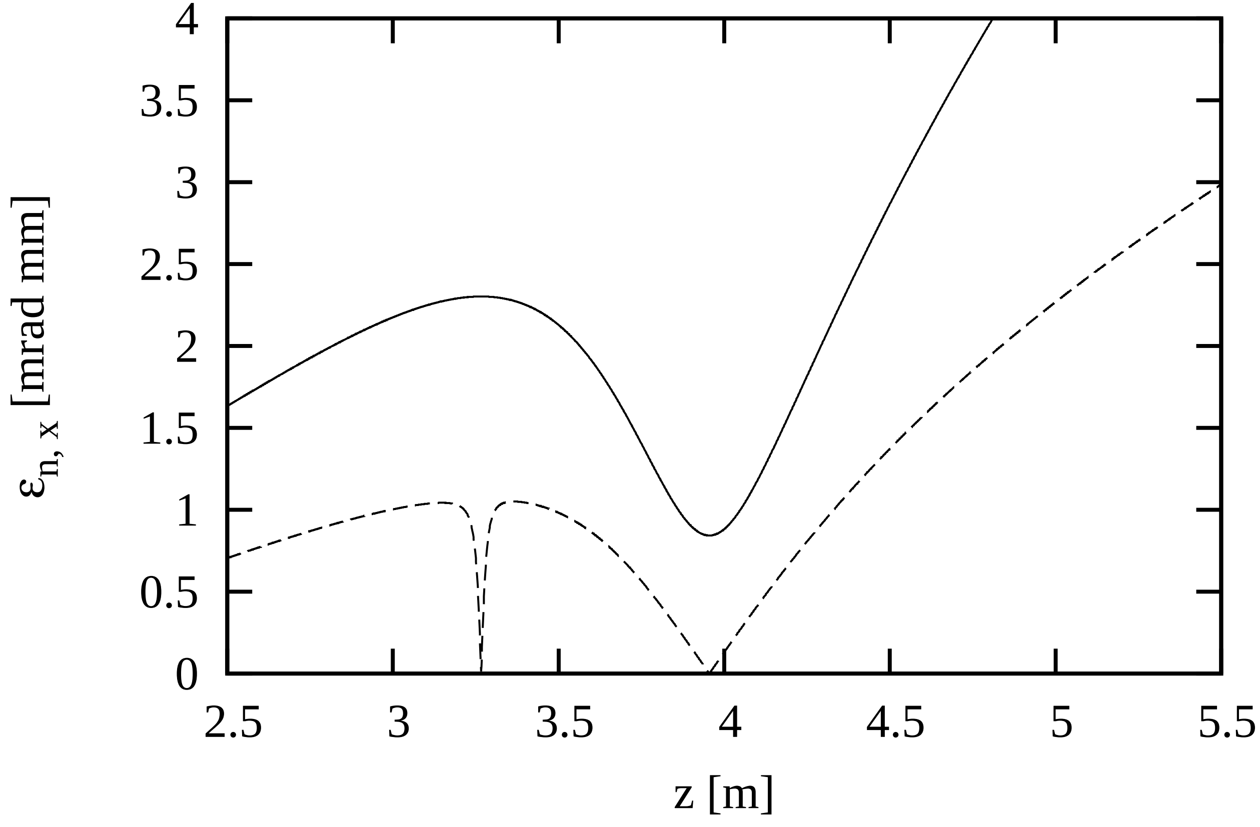


Slice size development for the high perveance slice (solid line) and a slice without space charge (broken line).



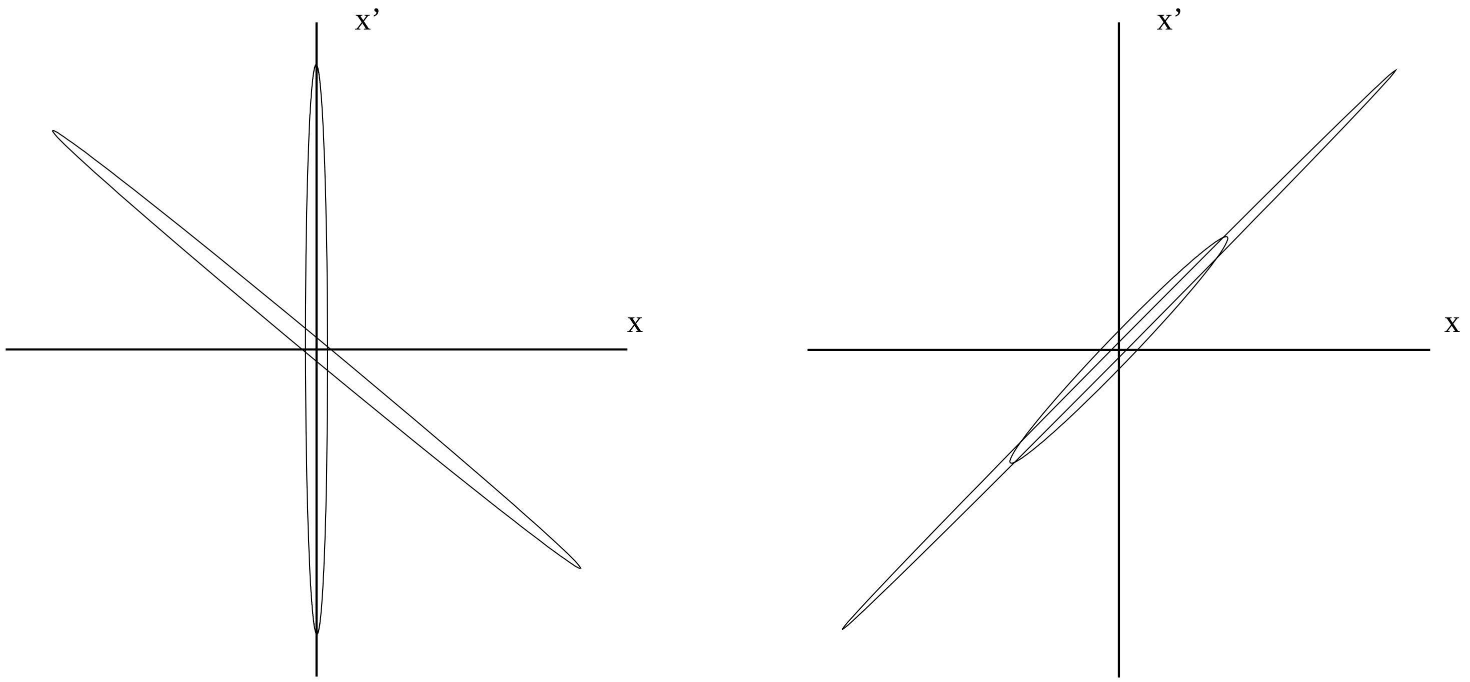
Development of the slopes of the phase space ellipses for the high perveance slice (solid line) and a slice without space charge (broken line).

Cross over trajectories



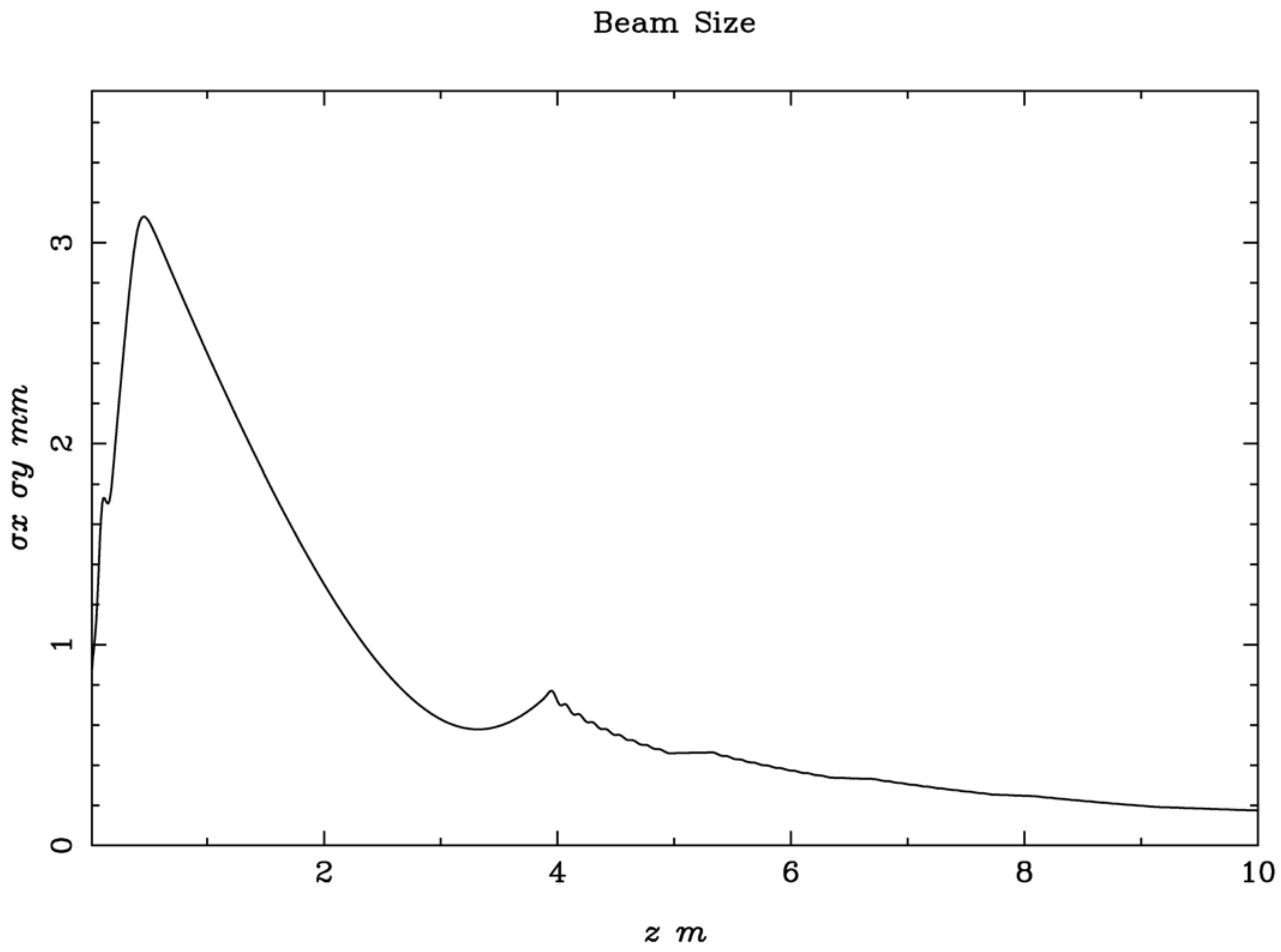
Development of the weighted emittance as calculated from the high perveance slice and the slice without space charge.

Cross over trajectories

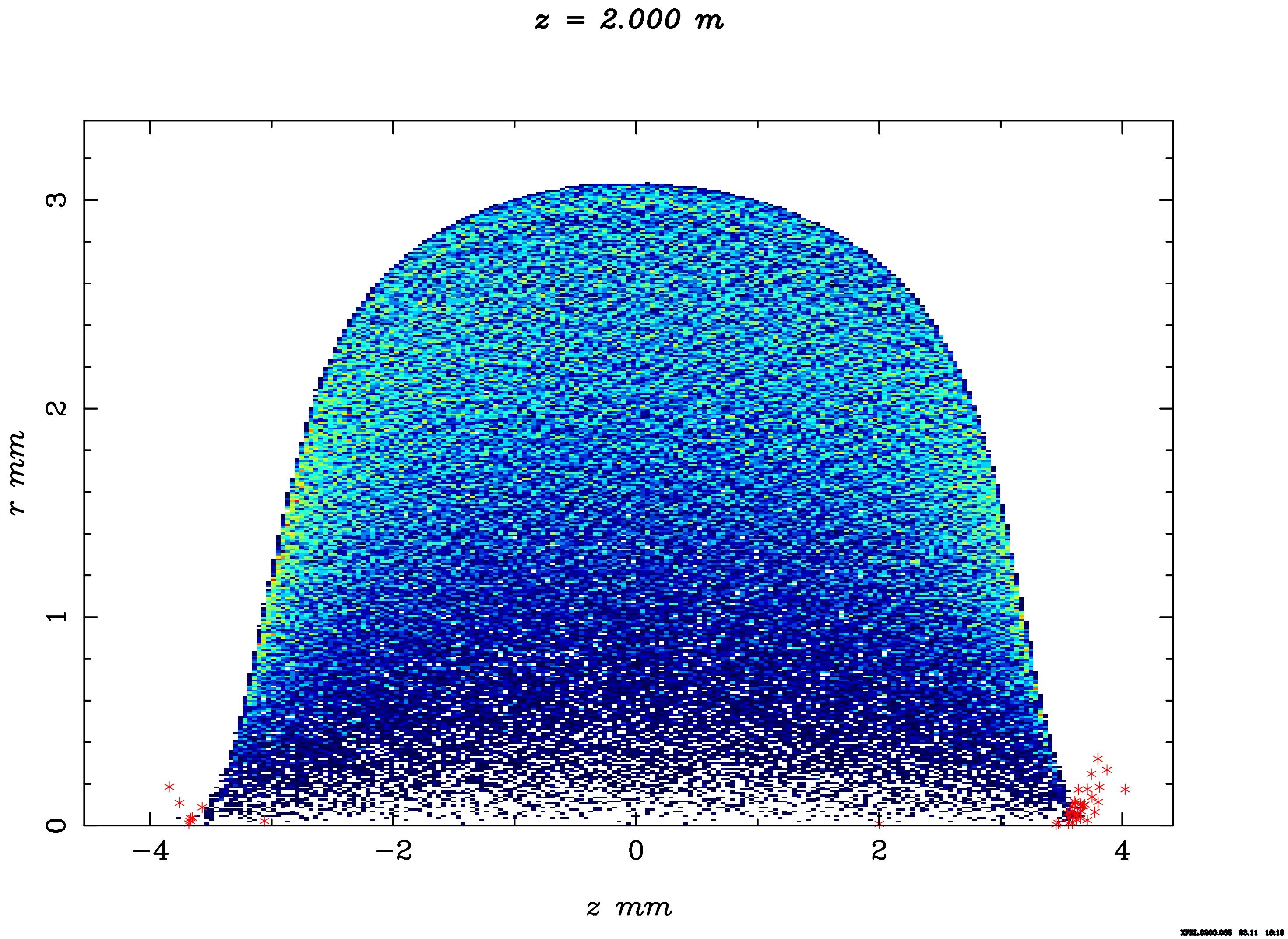


Phase space ellipses at the first (left) and second (right) coincidence of the correlation axis.

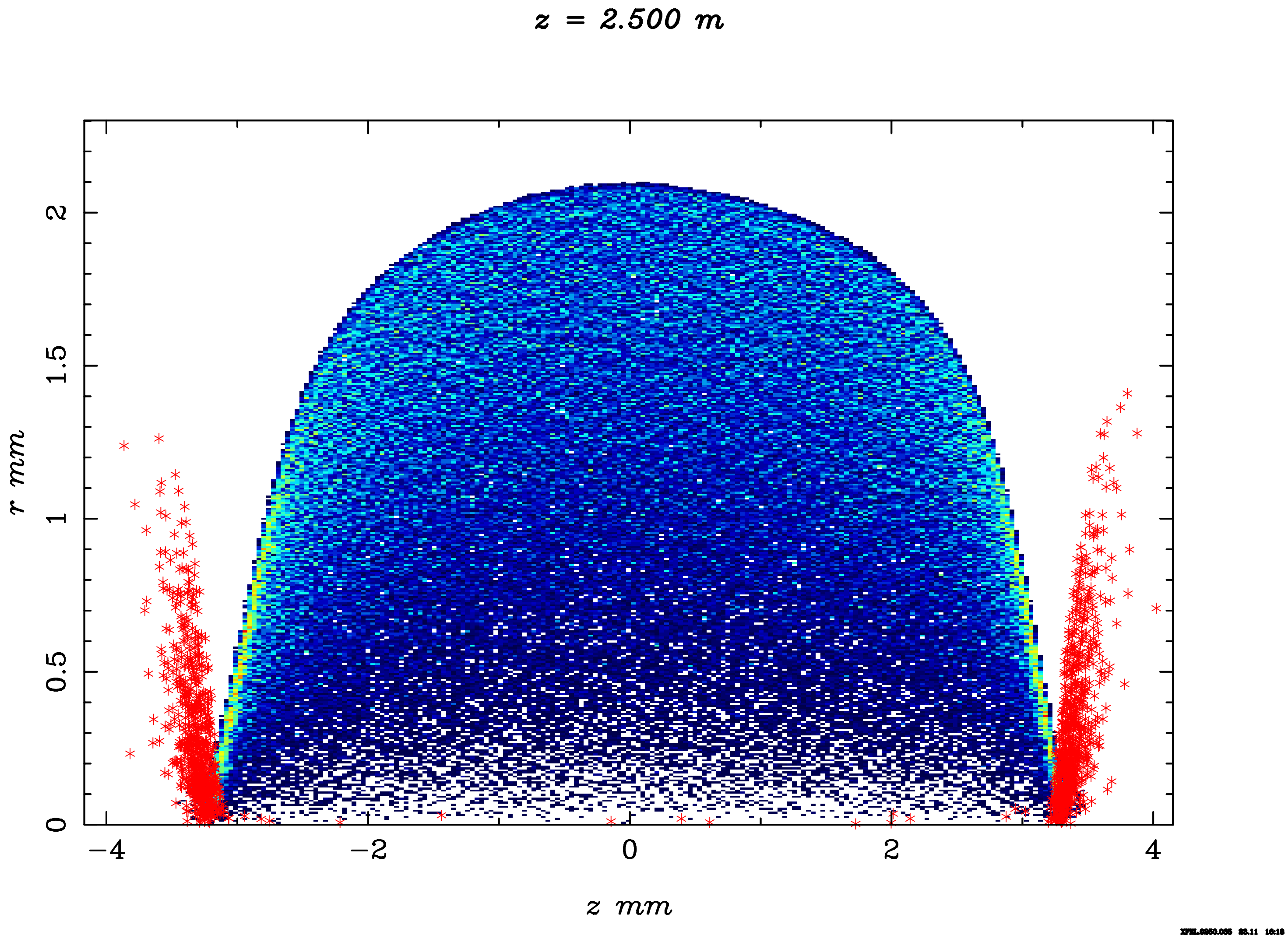
XFEL Injector with 20MV/m in the Module



XFEL Injector with 20MV/m in the Module



XFEL Injector with 20MV/m in the Module



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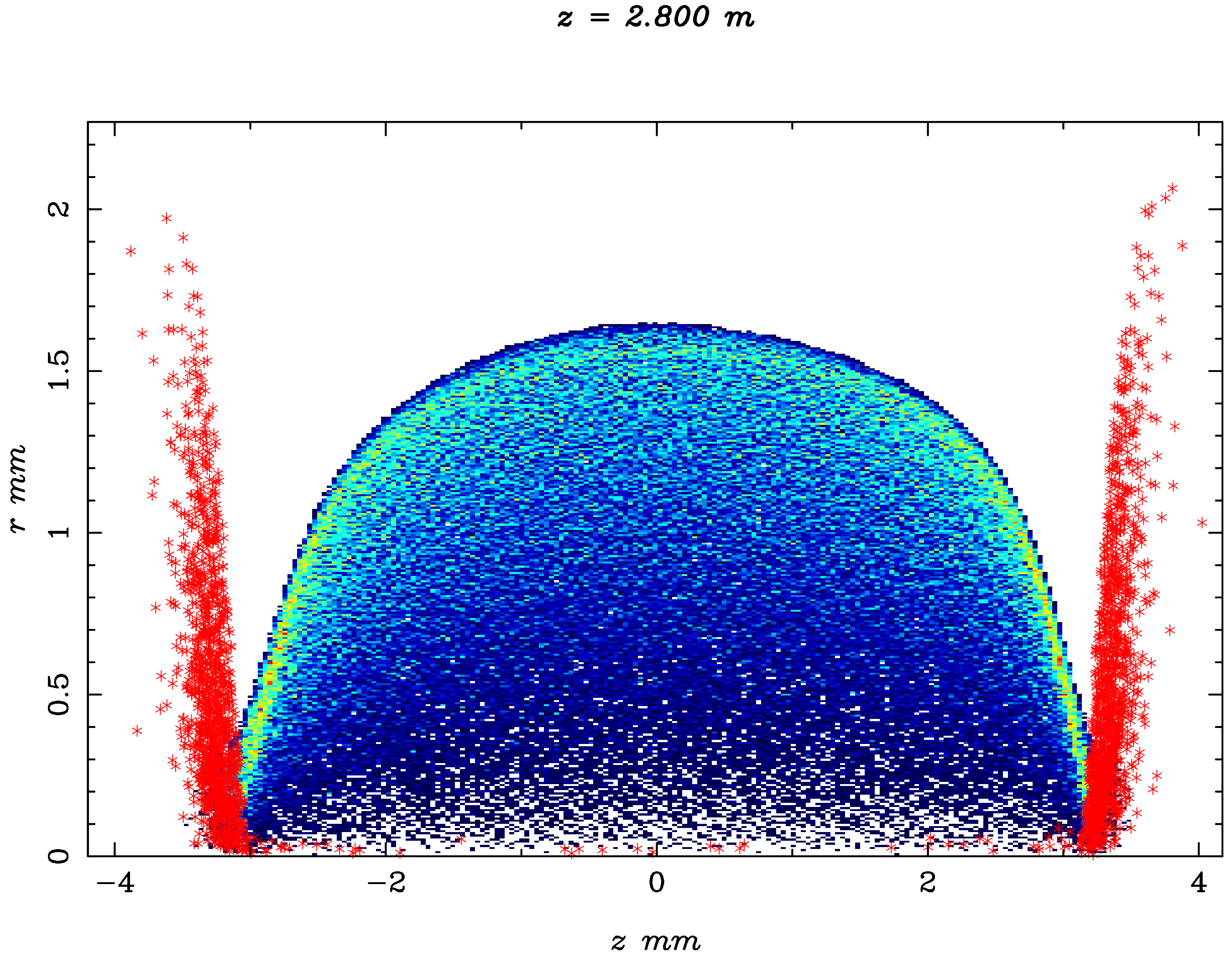

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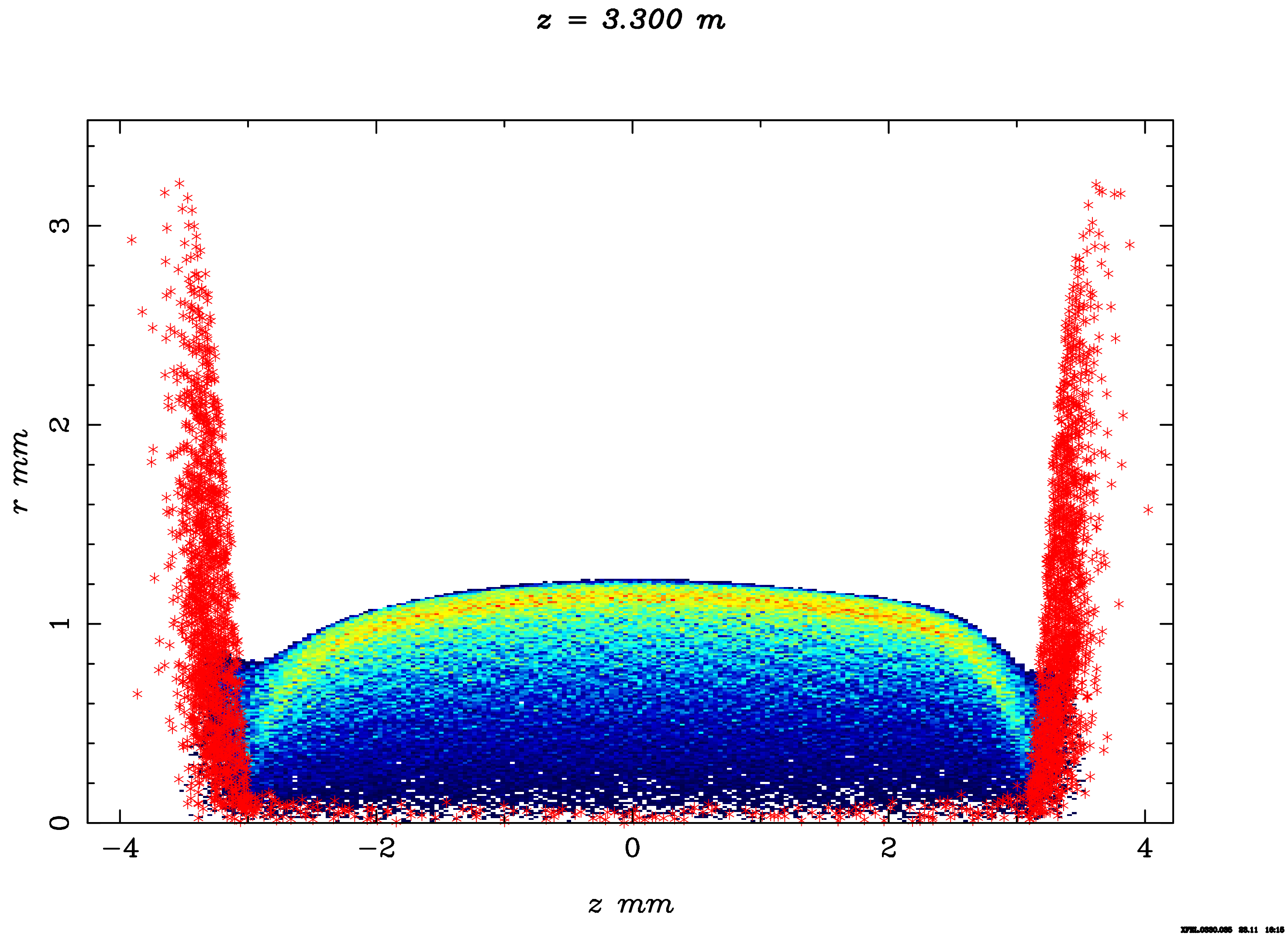
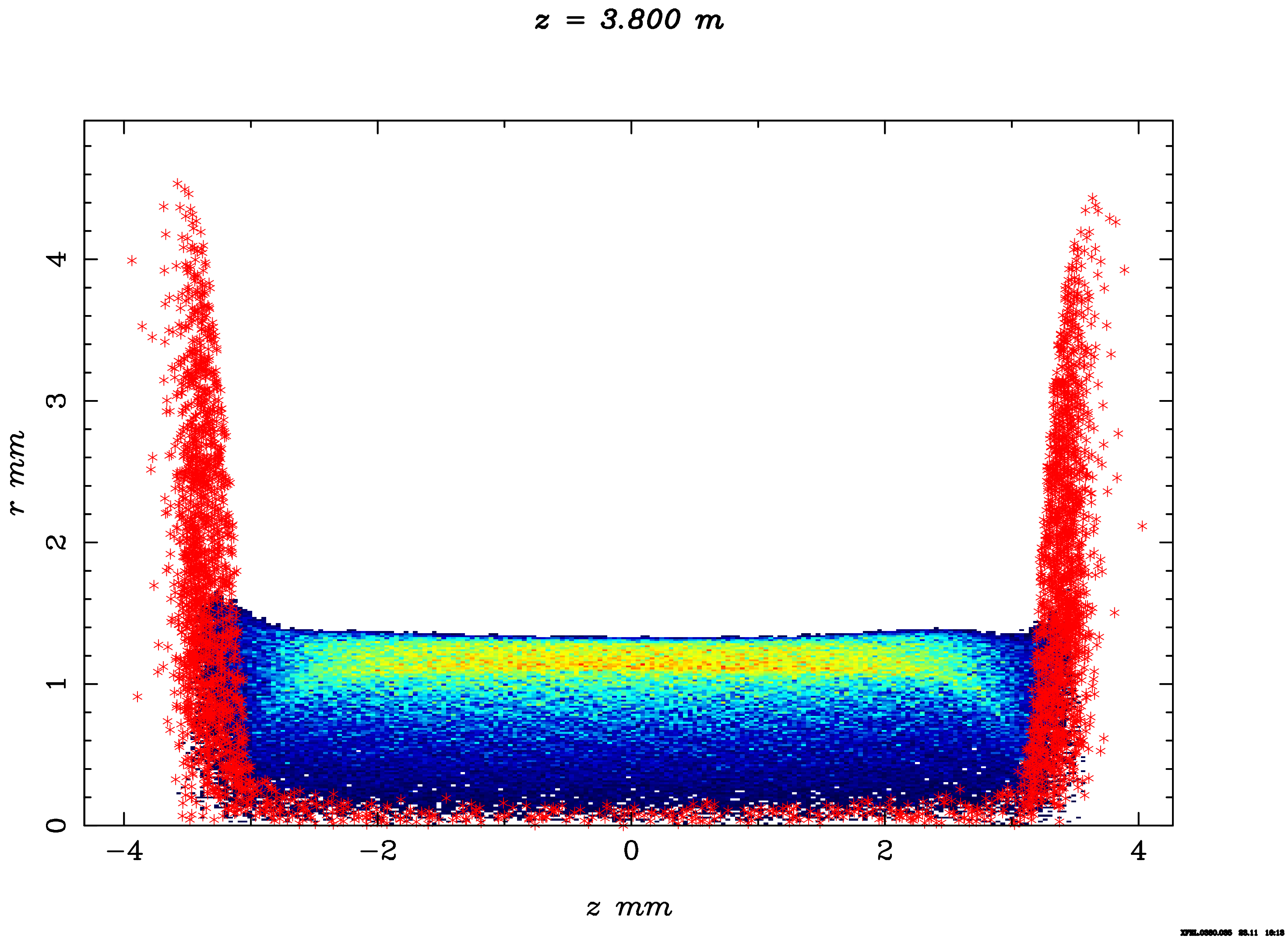


K. Floettmann 30th Nov. 2009

XFEL Injector with 20MV/m in the Module



XFEL Injector with 20MV/m in the Module