about beam loading

wakes, bunch to bunch interaction

single mode beam loading

single mode beam loading on resonance

two mode beam loading on resonance (1.3GHz + 3.9GHz = main modes)

feed back (or whatever)

operation of 3rd harmonic cavity

self interaction, all modes (short range wake)

long range interaction, many modes

proposed estimation

some conclusions

multi-bunch beam loading (bunch centroids)

one bunch: $u(t) = qw(t)$

bunch $m \to n$: $u_n^{(m)} = q_m w_{n-m}$

$$w_m = \frac{\int w(t+mT)\lambda(t)dt}{\int \lambda(t)dt} \approx w(mT) \text{ for } m > 0$$

bunches $0 \ldots n \to n$: $u_n = \sum_{m=0}^{n} q_m w_{n-m}$

"centroid wakes"

bunch charge: $q_n = \bar{q} + x_n q_{rms}$

T = bunch distance

$$u_n = \bar{q} \cdot \boxed{S_n} + q_{rms} \cdot \boxed{X_n}$$

$$S_n = \sum_{m=0}^{n} w_{n-m}$$

systematic part

$$X_n = \sum_{m=0}^{n} x_m w_{n-m}$$

random part

$$R_n = rms\{X_n\} = \sqrt{\sum_{m=0}^{n} w_m^2}$$

rms jitter

single mode beam loading

$$w_m \approx \begin{cases} 0 & m<0 \\ k & m=0 \\ 2k\,\mathrm{Re}\{z^m\} & m>0 \end{cases}$$

with k, Q, ω = loss-parameter, quality, frequency

$$\tau = 2Q/\omega$$

$$z = \exp(-T/\tau + j\omega T)$$

$$u_n = \bar{q} \cdot \boxed{S_n} + q_{rms} \cdot X_n$$

$$S_n = k\,\mathrm{Re}\left\{-1 + 2\frac{1-z^{n+1}}{1-z}\right\}$$

$$R_n = rms\{X_n\}$$

$$R_n = k\sqrt{-3 + 2\,\mathrm{Re}\left\{\frac{1-z^{2(n+1)}}{1-z^2}\right\} + 2\frac{1-|z|^{2(n+1)}}{1-|z|^2}}$$

$n \to \infty$

$$S_\infty = k\,\mathrm{Re}\left\{\frac{1+z}{1-z}\right\}$$

$$R_\infty = k\sqrt{-3 + 2\,\mathrm{Re}\left\{\frac{1}{1-z^2}\right\} + 2\frac{1}{1-|z|^2}}$$

on resonance, high Q: $z = \exp(-T/\tau + j\omega T) \approx 1 - T/\tau$

$$S_\infty \to 2k\frac{\tau}{T}$$

$$R_\infty \to k\sqrt{2\frac{\tau}{T}}$$

off resonance, high Q: $\frac{1}{1-|z|^2} >> \mathrm{Re}\left\{\frac{1}{1-z^2}\right\}$

$$R_\infty \to k\sqrt{\frac{\tau}{T}}$$

single mode beam loading: on resonance, high Q

on resonance, high Q: $z = \exp(-T/\tau + j\omega T) \approx 1 - T/\tau$

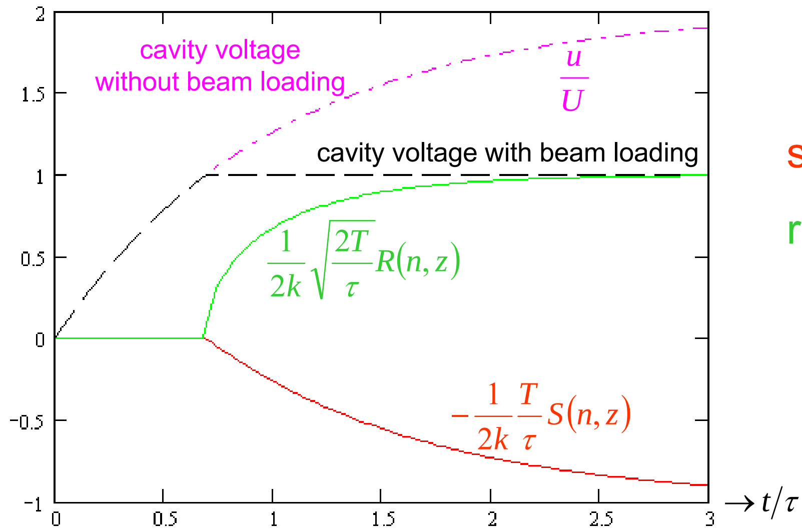
$S_\infty \to 2k\dfrac{\tau}{T}$ $\qquad$ $R_\infty \to k\sqrt{2\dfrac{\tau}{T}} = \sqrt{kS_\infty}$

simple interpretation: $u_{n\to\infty} = \sum_{m=0}^{n} q_m w_{n-m} \propto w_0 \sum_{m=n-N}^{n} q_m$

$u_{n\to\infty} \propto k \sum_{m=0}^{N} q_{n-m}$ $\qquad$ with $N = \tau/T$ the number of bunches that contribute to the wake

$\mathrm{av}\{u_\infty\} \propto \bar{q}kN$ $\qquad$ $\mathrm{rms}\{u_\infty\} \propto q_{rms}k\sqrt{N}$

fundamental mode:



syst. beam loading: $\bar{q}S_\infty \to U$

rms beam loading (without FB):

$$\frac{u_{rms}}{U} = \frac{q_{rms}}{\bar{q}}\underbrace{\sqrt{2\frac{\tau}{T}}}_{\approx 0.013}$$

≈0.013 (TESLA module)

two mode beam loading: on resonance, high Q

$$s_1 = 2\frac{k_1\tau_1}{T}, s_2 = 2\frac{k_2\tau_2}{T}$$

$$u_1 = 2\frac{k_1^2\tau_1}{T}, u_2 = 2\frac{k_2^2\tau_1}{T}$$

$$S_\infty = s_1 + s_2$$

$$R_\infty = \sqrt{u_1 + \frac{4u_1u_2}{u_1+u_2} + u_2}$$

acc. mode (40 TESLA cavities to 500MeV): $\frac{k}{40} = 2.08\cdot 10^{12}\,\frac{\mathrm{V}}{\mathrm{C}}$ $\quad \tau = \frac{\hat{U}\bar{I}}{2k} = 721\,\mu\mathrm{s}$ $\quad Q_e \approx 3\cdot 10^6$

$$\bar{I} \approx 5\,\mathrm{mA}$$

$$\hat{U} + U_{3rd} \approx 500\,\mathrm{MV} \rightarrow \hat{U} \approx 600\,\mathrm{MV}$$

3rd harmonic used-mode (32 cavities in 2 modules): $\frac{k}{32} = 4.8\cdot 10^{12}\,\frac{\mathrm{V}}{\mathrm{C}}$ $\quad Q_e \approx 1\cdot 10^6$

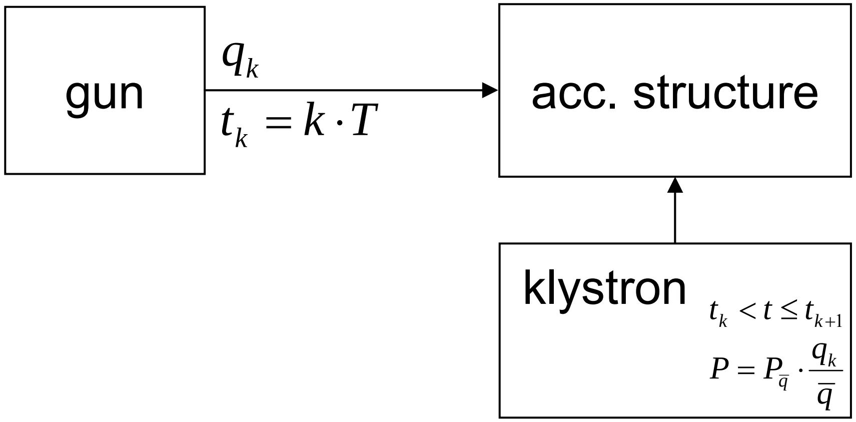
$$S_\infty \cdot 1\,\mathrm{nC} = 725\,\mathrm{MV}$$

$$R_\infty \cdot q_{rms} = \frac{q_{rms}}{\bar{q}} 11.2\,\mathrm{MV}$$

$$\frac{R_\infty \cdot q_{rms}}{500\,\mathrm{MV}} = \frac{q_{rms}}{\bar{q}} 0.0224$$

(without FB):

fundamental mode with ideal feed"back?"



bunches $0 \ldots n \to n$: $$u_n = 2k\left[\frac{q_n}{2} + \sum_{m=0}^{n-1} q_m \operatorname{Re}\left\{z^{n-m}\right\}\right]$$

$$u_n = \bar{q} \cdot k + q_{rms} \cdot k$$

systematic part

random part

relative rms energy spread → $$\frac{kq_{rms}}{2k\bar{q}\frac{\tau}{T}} = \frac{q_{rms}}{\bar{q}}\frac{T}{2\tau}$$

e.g.: $\approx 0.00017\frac{q_{rms}}{\bar{q}}$

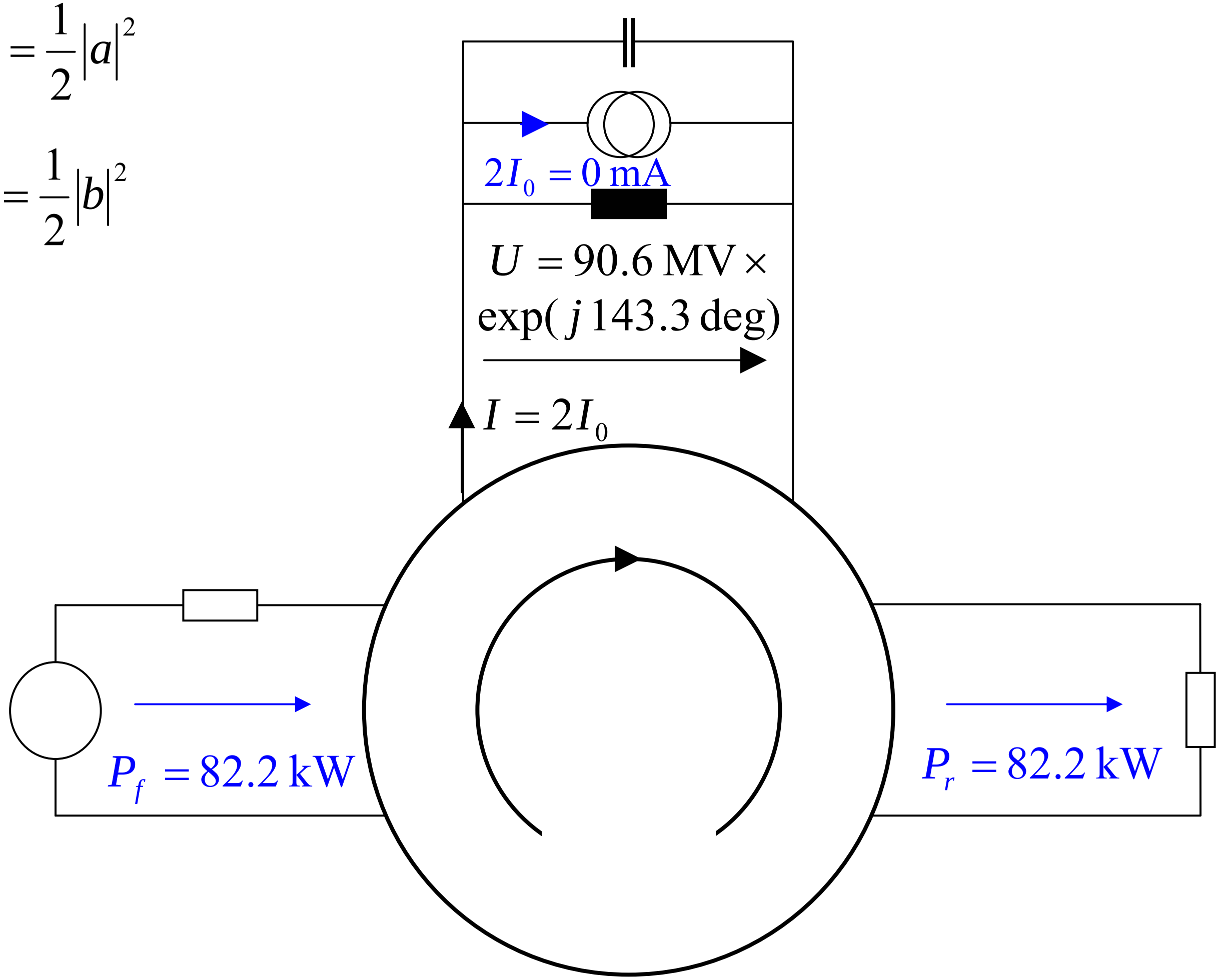
operation of 3rd harm. cavity: $I_{beam} = 0$

(steady state)

$$Q = 10^6 \qquad \frac{k}{32} = 4.8 \cdot 10^{12}\,\frac{\mathrm{V}}{\mathrm{C}} \qquad R = \frac{2kQ}{\omega}$$

$$a = \frac{1}{2}\left(\frac{U}{\sqrt{R}} + I\sqrt{R}\right) \qquad P_f = \frac{1}{2}|a|^2$$

$$b = \frac{1}{2}\left(\frac{U}{\sqrt{R}} - I\sqrt{R}\right) \qquad P_r = \frac{1}{2}|b|^2$$



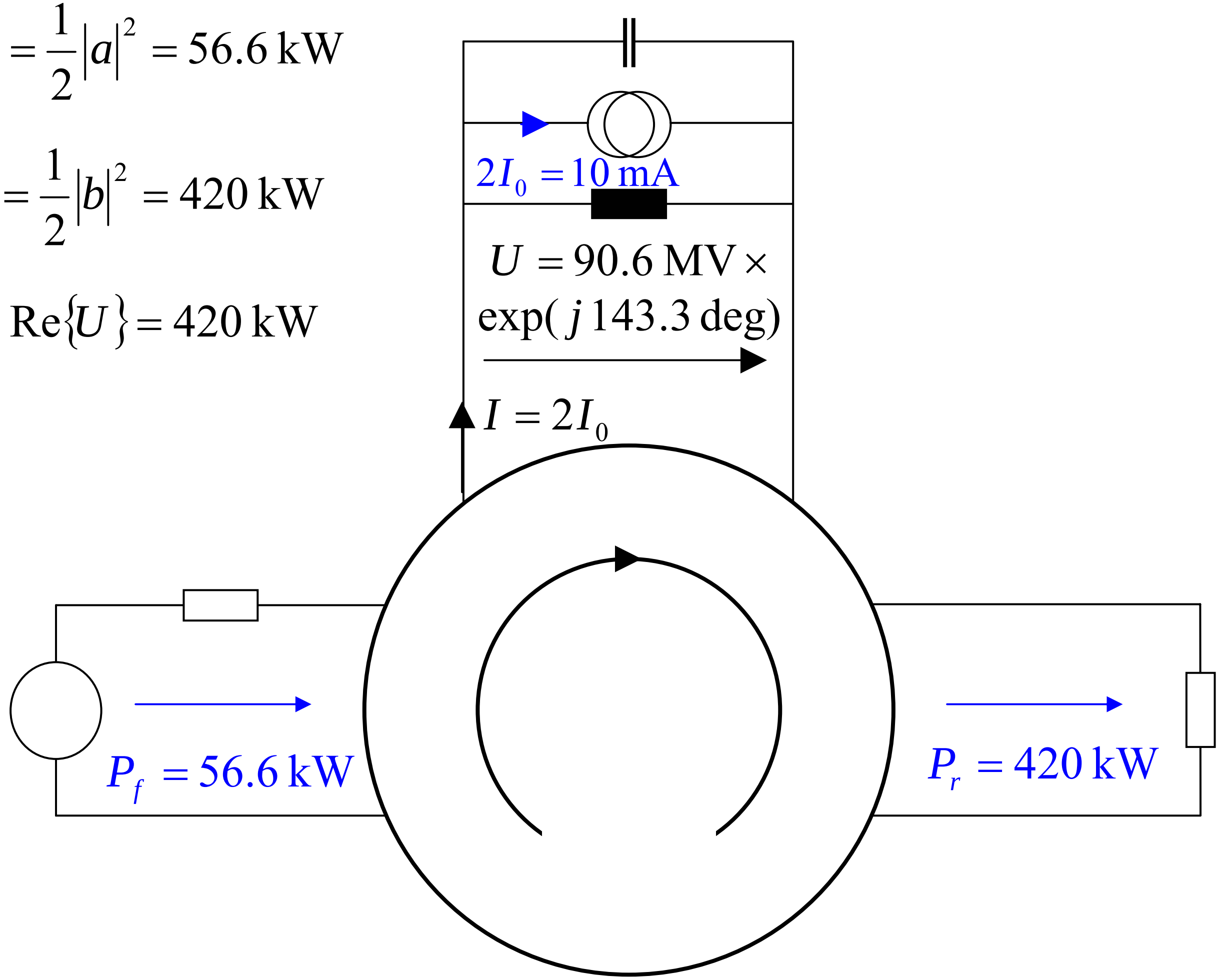
operation of 3rd harm. cavity: I_{beam} = 5 mA
(steady state)

$$Q = 10^6 \qquad \frac{k}{32} = 4.8 \cdot 10^{12}\,\frac{\mathrm{V}}{\mathrm{C}} \qquad R = \frac{2kQ}{\omega}$$

$$a = \frac{1}{2}\left(\frac{U}{\sqrt{R}} + I\sqrt{R}\right) \qquad P_f = \frac{1}{2}|a|^2 = 56.6\,\mathrm{kW}$$

$$b = \frac{1}{2}\left(\frac{U}{\sqrt{R}} - I\sqrt{R}\right) \qquad P_r = \frac{1}{2}|b|^2 = 420\,\mathrm{kW}$$

$$P_f - I_0\,\mathrm{Re}\{U\} = 420\,\mathrm{kW}$$

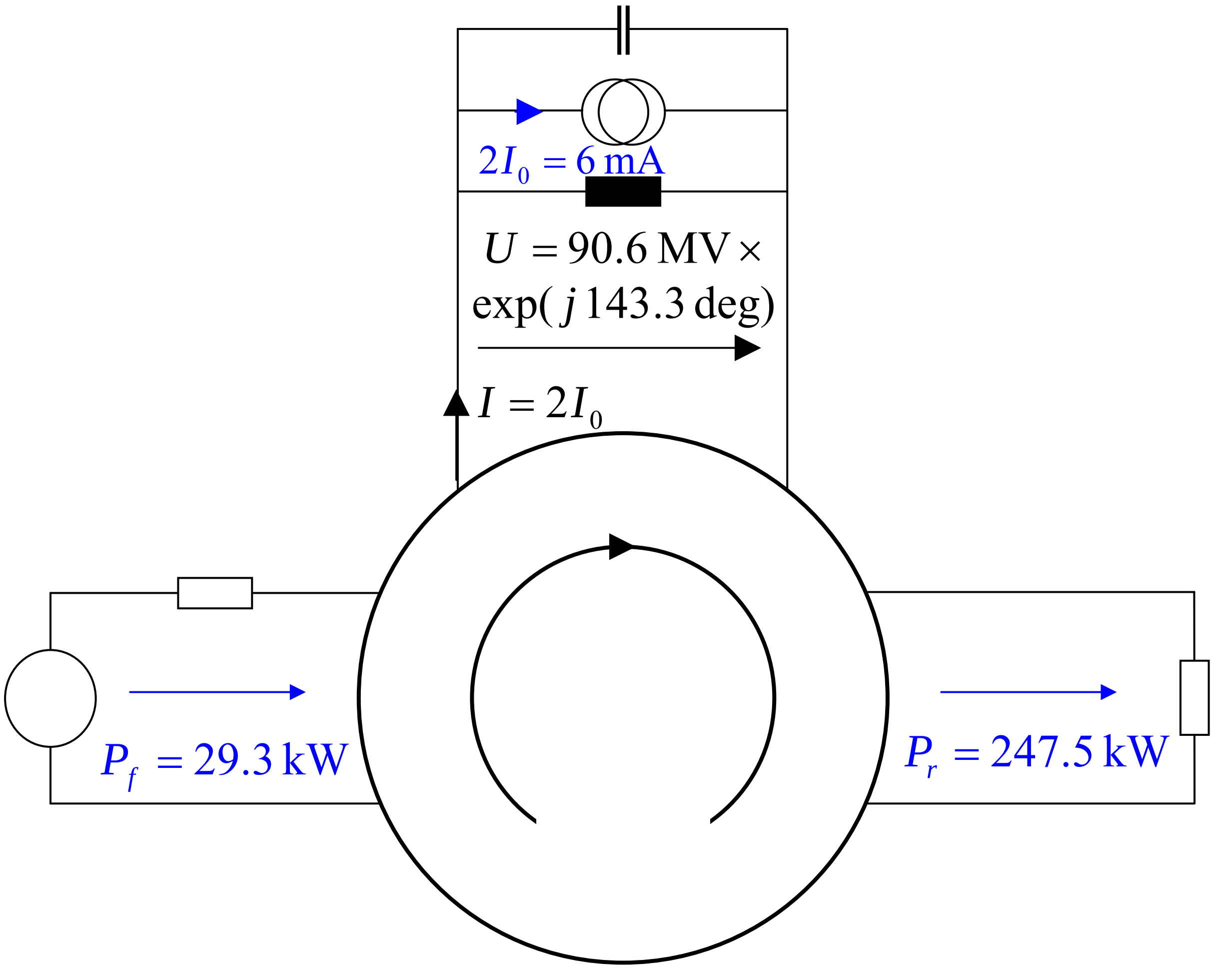


operation of 3rd harm. cavity: I_{beam} = 3 mA
(steady state)

$$Q = 10^6 \qquad \frac{k}{32} = 4.8\cdot 10^{12}\,\frac{\text{V}}{\text{C}} \qquad R = \frac{2kQ}{\omega}$$

$$a = \frac{1}{2}\left(\frac{U}{\sqrt{R}} + I\sqrt{R}\right)$$

$$b = \frac{1}{2}\left(\frac{U}{\sqrt{R}} - I\sqrt{R}\right)$$



n=0; self interaction
(all modes / short range)

$$u_n = \sum_{m=0}^{n} q_m w_{n-m}$$

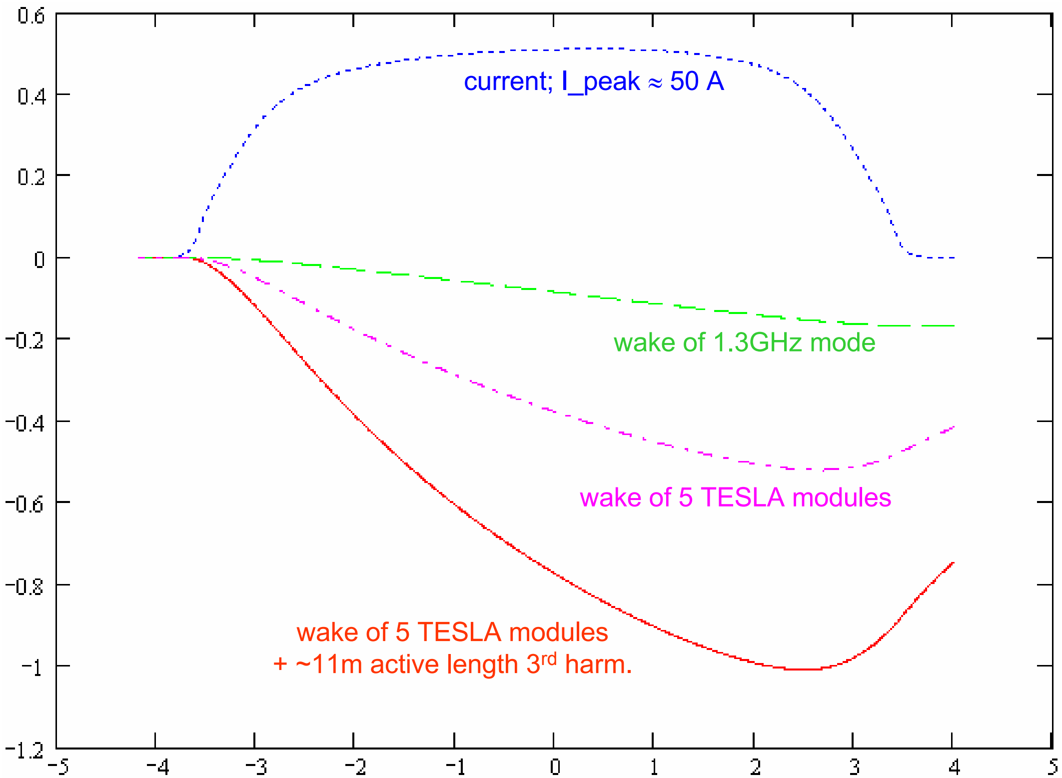
$$w_0 = \frac{\int w(t)\lambda(t)dt}{\int \lambda(t)dt}$$

$$u_0 = \bar{q} \cdot \boxed{w_0} + q_{rms} \cdot \boxed{w_0}$$

$R_0 = w_0$ rms jitter

systematic part   random part

before BC1:



n=0; self interaction
(all modes / short range)

$$u_n = \sum_{m=0}^{n} q_m w_{n-m}$$

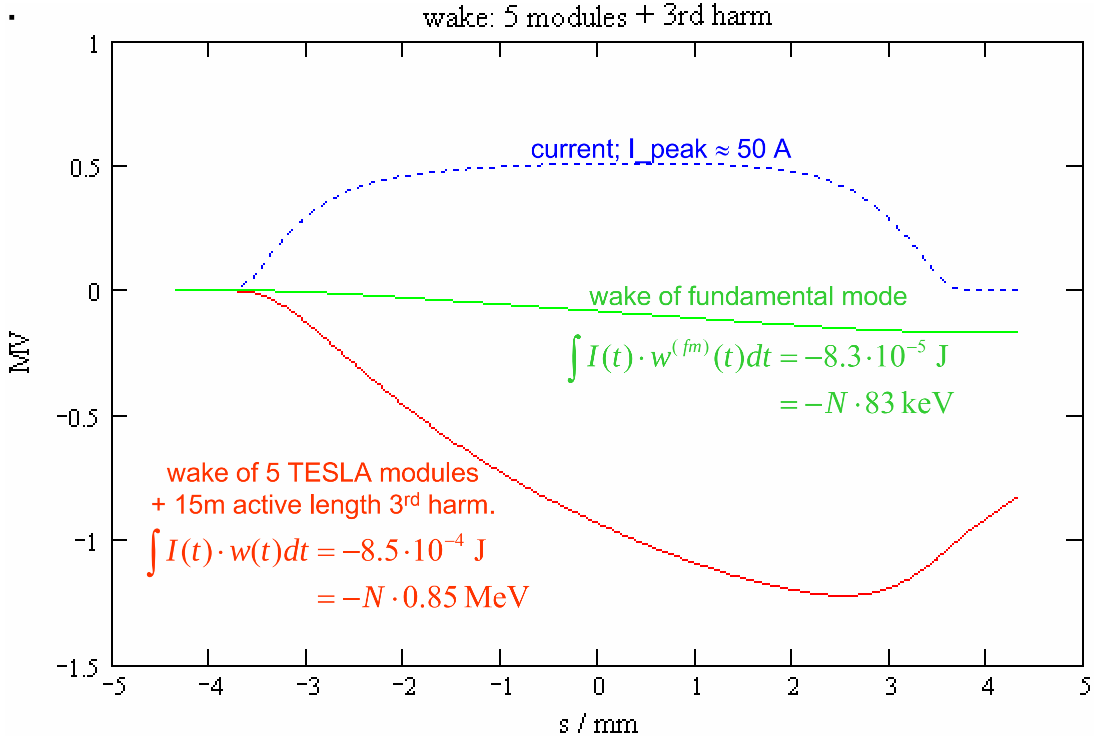
$$w_0 = \frac{\int w(t)\lambda(t)dt}{\int \lambda(t)dt}$$

$$u_0 = \bar{q} \cdot \boxed{w_0} + q_{rms} \cdot \boxed{w_0}$$

$R_0 = w_0$ rms jitter

systematic part random part

before BC1:



$$\frac{rms\{E\}}{av\{E\}} \approx \frac{q_{rms}}{\bar{q}} \frac{0.85\ \text{MeV}}{500\ \text{MeV}} \approx \frac{q_{rms}}{\bar{q}} \cdot 1.7 \cdot 10^{-3}$$

n→∞; “steady state”
(all modes / long range)

$$R_\infty = rms\{X_\infty\} = \sqrt{\sum_{m=0}^{\infty} w_m^2}$$

with

$$w_m \approx \begin{cases} 0 & m < 0 \\ \sum k_\nu & m = 0 \\ 2\,\mathrm{Re}\left\{\sum k_\nu z_\nu^m\right\} & m > 0 \end{cases}$$

$$z_\nu = \exp\left(T \left(\frac{-\omega_\nu}{2Q_\nu} + j\omega_\nu \right) \right)$$

Rainer’s table

mode	f /GHz	$k^{(0)}$/ V/(pC)	G_1 /Ω	$(R/Q)^{(0)}$ / Ω	Q_0/Q_{0FM}	φ /°
Band 1						
MM- 1	1.2756	$0.848\ 10^{-06}$	252.7	0.0002	1.027	20.0
MM- 2	1.2776	$0.239\ 10^{-06}$	252.9	0.0001	1.025	39.9
MM- 3	1.2807	$0.523\ 10^{-05}$	253.2	0.0013	1.021	59.9
MM- 4	1.2845	$0.187\ 10^{-05}$	253.5	0.0005	1.017	79.8
MM- 5	1.2885	$0.217\ 10^{-05}$	253.9	0.0005	1.012	99.8
MM- 6	1.2924	$0.776\ 10^{-05}$	254.2	0.0019	1.007	119.7
MM- 7	1.2955	$0.138\ 10^{-03}$	254.5	0.0339	1.003	139.6
MM- 8	1.2976	$0.662\ 10^{-04}$	254.7	0.0163	1.001	159.2
MM- 9	1.2983	2.08	254.8	511.0652	1.000	176.1
Band 2						
MM-10	2.3800	$0.746\ 10^{-05}$	370.6	0.0010	0.433	159.9
MM-11	2.3856	$0.147\ 10^{-03}$	370.7	0.0196	0.431	139.9
MM-12	2.3943	$0.248\ 10^{-03}$	370.9	0.0329	0.428	119.9
MM-13	2.4055	$0.414\ 10^{-03}$	371.2	0.0547	0.424	100.1
MM-14	2.4181	$0.376\ 10^{-02}$	371.3	0.4943	0.420	80.6
MM-15	2.4308	$0.573\ 10^{-04}$	371.2	0.0075	0.416	61.4
MM-16	2.4419	0.08	370.6	10.2352	0.411	43.0
MM-17	2.4499	0.60	369.0	77.6533	0.407	25.9
MM-18	2.4539	0.57	365.9	73.8717	0.402	11.5
Band 3						
MM-19	2.6695	$0.363\ 10^{-03}$	546.8	0.0433	0.508	14.9
MM-20	2.6756	$0.291\ 10^{-02}$	548.7	0.3465	0.507	30.6
MM-21	2.6858	$0.118\ 10^{-02}$	550.9	0.1395	0.505	47.2
MM-22	2.6993	$0.141\ 10^{-02}$	554.2	0.1659	0.503	64.8
MM-23	2.7148	$0.166\ 10^{-02}$	559.7	0.1948	0.502	83.2
MM-24	2.7307	$0.198\ 10^{-03}$	567.6	0.0231	0.504	102.1
MM-25	2.7453	$0.825\ 10^{-03}$	577.1	0.0957	0.507	121.4
MM-26	2.7571	$0.236\ 10^{-05}$	586.4	0.0003	0.510	140.8
MM-27	2.7648	$0.965\ 10^{-04}$	593.3	0.0111	0.513	160.4
MM-28	3.0971	0.02	464.6	1.8325	0.320	—
MM-29	3.0971	$0.377\ 10^{-04}$	464.6	0.0039	0.320	—
Band 4						
MM-30	3.3898	$0.981\ 10^{-02}$	542.9	0.9210	0.313	180.0
MM-31	3.3921	$0.388\ 10^{-02}$	541.8	0.3638	0.311	180.0
MM-32	3.4055	$0.592\ 10^{-02}$	553.4	0.5537	0.316	147.9
MM-33	3.4261	$0.977\ 10^{-02}$	565.7	0.9075	0.319	127.8
MM-34	3.4541	$0.144\ 10^{-02}$	580.9	0.1327	0.322	108.4
MM-35	3.4885	$0.689\ 10^{-02}$	597.6	0.6283	0.325	89.1
MM-36	3.5283	$0.538\ 10^{-02}$	614.1	0.4851	0.326	69.7
MM-37	3.5719	$0.163\ 10^{-05}$	627.8	0.0001	0.326	50.2
MM-38	3.6174	$0.954\ 10^{-02}$	633.8	0.8397	0.320	30.4
MM-39	3.6650	0.03	623.6	2.8606	0.307	0.0
MM-40	3.6783	$0.455\ 10^{-03}$	596.0	0.0394	0.291	0.0
MM-41	3.7727	$0.152\ 10^{-02}$	816.4	0.1286	0.379	19.4
MM-42	3.8005	0.01	730.7	1.0079	0.335	37.8
MM-43	3.8206	$0.180\ 10^{-02}$	661.5	0.1501	0.300	57.4
MM-44	3.8319	$0.858\ 10^{-02}$	615.5	0.7127	0.277	79.5

from:
Monopole, Dipole and Quadrupole Passbands of the TESLA 9-cell Cavity
R. Wanzenberg, TESLA 2001-33

estimation: $$R_\infty \approx \sqrt{\sum_{\nu=0}^{N} \left(R_\infty^{(\nu)}\right)^2} \approx \sqrt{\sum_{m=0}^{N} k_\nu^2 \frac{\tau_\nu}{T}}$$

limited to the known modes!

proposed estimation:
(for perfect main-modes feedback)

$$R_\infty = rms\{X_\infty\} = \sqrt{w_0^2 + \sum_{m=1}^{\infty} w_m^2} \approx \sqrt{\boxed{w_0^2} + \boxed{\sum_{\text{without m.m.}} k_\nu^2 \frac{\tau_\nu}{T}}}$$

short range; time domain   long range, modal

e.g. (40 TESLA cavities + ~45 3rd-harm. cavities):

$$\sqrt{w_0^2} \approx 850\,\text{kV/nC}$$

$$\sqrt{\sum_{\text{without m.m.}} k_\nu^2 \frac{\tau_\nu}{T}} \approx 500\,\text{kV/nC}$$

only TESLA cavity modes (Rainer's list)
pessimistic estimation of HOM absorption
($Q_{ext} = 10^6$, pessimistic enough ???)

$$\sqrt{w_0^2 + \sum_{\text{without m.m.}} k_\nu^2 \frac{\tau_\nu}{T}} \approx 1\,\text{MV/nC}$$

→ strong contribution from self interaction !

$$\boxed{\frac{rms\{E\}}{av\{E\}} \approx \frac{q_{rms}}{\bar{q}} \frac{1\,\text{MeV}}{500\,\text{MeV}} \approx \frac{q_{rms}}{\bar{q}} \cdot 2 \cdot 10^{-3}}$$

some conclusions

systematic beam is loading important

- comes always to steady state
- transients not investigated now; (some unknown parameters)

random beam loading (by charge fluctuation)

- main modes: feed back required
- significant contribution from 3rd harmonic rf
- transverse deflecting cavities are not considered now
- self interaction $E_{rms}/E \sim q_{rms}/q_{av} * 0.0017$
- self & multi-bunch interaction $E_{rms}/E \sim q_{rms}/q_{av} * 0.002$
  (unknown parameters estimated)

interference with compression process

- has been investigated earlier (self interaction)
- $\rightarrow$ constraints for charge fluctuation