• Self acceleration: Results from CSRtrack vs. GPT vs. HOMDYN
• Self acceleration: Theoretically
• GPT: The applied calculation method
• Longitudinal phase space
• Spatial distribution
• Considering slices
Simulation

“Extreme” case:

- PIC-Simulations -> $\sigma_x = \sigma_y = \sigma_z = 1 \ \mu m$
- Charge: 1.25 nC -> $I = 150 \ \text{kA}$
- Initial distribution: Gaussian

Comparison between CSRtrack and GPT
Self Acceleration

Gain of kinetic energy of approx. 20%

Initial distribution

After a free drift of approx. 20mm

CSRtrack

GPT

Longitudinal Phase Space

Longitudinal Phase Space
**Self Acceleration**

**HOMDYN** results do not comply to CSRtrack and GPT, predicting strong “debunching”.

Locking for the origin of these contradictions is work in progress.
Self Acceleration

Electron's rest frame:
Potential energy leads to a mass defect:
\[ E_{\text{pot}} = m_d c^2 \]

Total energy in the electrons' mean rest frame:
\[ E' = E_0 + E_{\text{pot}} = m_0 c^2 + E_{\text{pot}} = (m_0 + m_d) c^2 \]
\[ E' = m' c^2 = \gamma' m_0 c^2 \text{ after potential energy "released"} \]

Potential energy in average is estimated to be of the order of 100 keV / electron

Mean Lorentz factor due to Coulomb explosion:
\[ \gamma' = 1 + \frac{m_d}{m_0} \]

Laboratory frame: Electron bunch is leaves the bubble with \( \gamma_0 \)

Total energy:
\[ E = \gamma_0 m' c^2 = \gamma_0 m_0 c^2 \cdot (1 + \frac{m_d}{m_0}) \]
\[ E = \gamma_0 \gamma' m_0 c^2 = \gamma m_0 c^2 \text{ after potential energy "released"} \]
GPT

- GPT adapts time steps dynamically
- Calculation precision is defined by relative momentum changes stepwise
- Space charge is considered point to point using Lorentz transformation. Calculation time \( t \sim N^2 \)

For equation of motion:

\[
F_i = q(E_i + v_i \times B_i)
\]

Interaction in rest frame:

\[
E'_{j\rightarrow i} = \frac{Qr'_{ji}}{4\pi\varepsilon_0 |r'_{ji}|^3}
\]

From Lorentz transformations:

\[
r_{ji} = r_i - r_j
\]

\[
r'_{ji} = r_{ji} + \frac{\gamma_j^2}{\gamma_j + 1}(r_{ji} \cdot \beta_j)\beta_j
\]

\[
E_i = \sum_{j \neq i} \gamma_j \left[ E'_{j\rightarrow i} - \frac{\gamma_j}{\gamma_j + 1}(\beta_j \cdot E'_{j\rightarrow i})\beta_j \right]
\]

\[
B_i = \sum_{j \neq i} \frac{\gamma_j \beta_j \times E'_{j\rightarrow i}}{c}
\]
GPT- Energy Conservation

Our personal “Lackmustest” for the evaluation of GPT results
Experimental Overview

- **Electron Beam**
  - Peak energy: 130 MeV
- **Quadrupole Triplet**
  - Field gradient: 500 T/m
- **Undulator**
  - Period: 5 mm
  - $K \sim 0.5$
- **Grating**
- **Spectrometer**
- **Aperture**
- **FEL Radiation at VUV**
- **TW Laser**
- **Gas Jet**

Acceleration fields in the order of TV/m
Evolution from “hat” to an almost linear energy chirp
Longitudinal Phase Space

- Length of free (divergent) drift: $L$
- Difference of propagation distance of axial electrons compared to electrons of angle $\theta$: $\Delta s$

Criteria for the phase space to become a linear energy shift:

- Axial electron propagates with $\gamma_0$
- Electron at angle $\theta$ propagates with $(\alpha + 1) \cdot \gamma_0$

$$\Delta s \approx L \left( \frac{\theta^2}{2} - \frac{\alpha}{\gamma_0^2} \right) < 0$$

Phase space becomes “linear” chirp for $\theta < \frac{\sqrt{2} \alpha}{\gamma_0}$
Coulomb explosion of resting electrons of a respective bunch yields:

\[ \beta' = 0.7 \Rightarrow \alpha = \frac{\beta'}{\gamma_0} \approx 0.25 \]

\[ \Rightarrow \frac{\sqrt{2\alpha}}{\gamma_0} \approx 3 \text{ mrad} > \theta \]

This \( \theta \) is smaller than expected, especially after collimating the beam.
After refocusing, the spacial shape of the bunch reminds of... a fish...
The presented calculation is far from being optimized. The “fish” in this case is not “slim” but rather looks like a “hammerhead shark”.

Optimization using linear beam optics is not possible, as the beam keeps gaining kinetic energy while propagating though the lens.

Optimization hence is work in progress as being very time consuming.

Longitudinal beam prolongation
Slices

Just qualitatively, as beam is not optimized !!!
Slices

Just qualitatively, as beam is not optimized !!!
Summary

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- Considering slices - qualitatively