Center for the Advancement of Natural Discoveries using Light Emission

Wake Field Model for ASTRA

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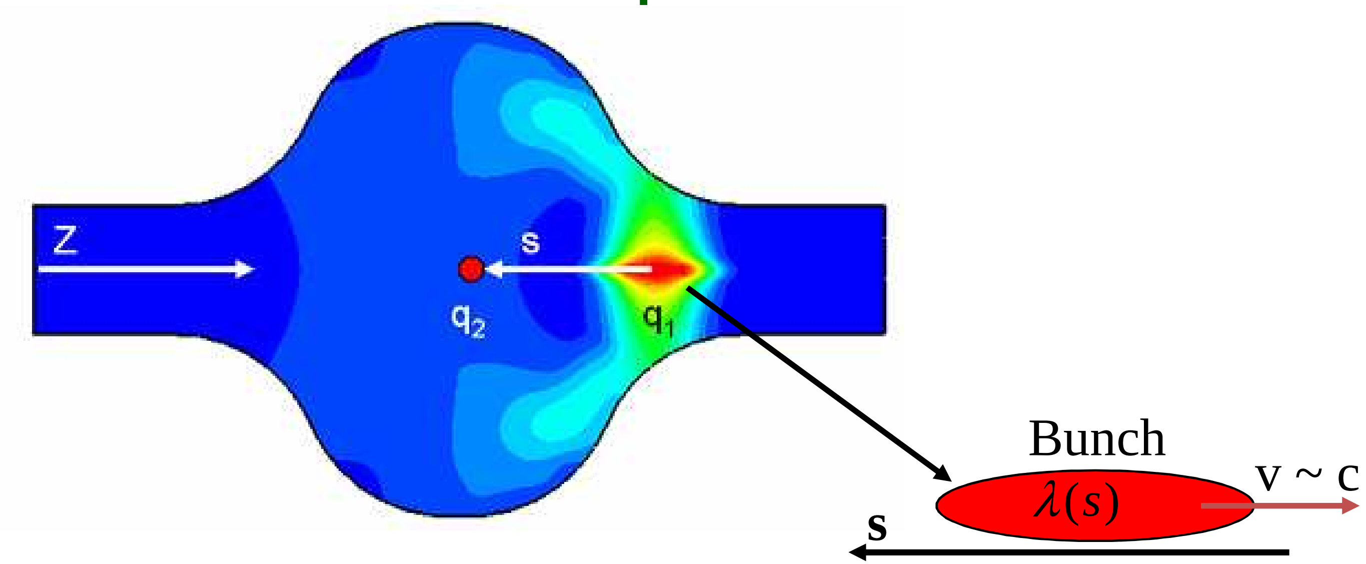
Introduction

Wakefields and wake potentials



$$\vec{w}(\vec{r}_\perp^{(q_1)},\vec{r}_\perp^{(q_2)},s)=\frac{1}{q_1}\int_{-\infty}^{+\infty}(\vec{E}+\vec{v}\times\vec{B})\Big|_{t=\frac{z+s}{c}}dz$$

$$\vec{W}^{(\lambda)}(s)=\int_{-\infty}^{s}\vec{w}(s-s')\lambda(s')\,ds'$$

$$\Delta\vec{p}^{(q_2)}=-q_1\,q_2\,\vec{w}(\vec{r}_\perp^{(q_1)},\vec{r}_\perp^{(q_2)},s)$$

Energy Loss

Transverse Kick

Panoswky-Wenzel

$$\frac{\partial}{\partial s}\vec{w}_\perp(\vec{r}_\perp^{(q_1)},\vec{r}_\perp^{(q_2)},s)=-\vec{\nabla}_\perp^{(q_2)}w_{//}(\vec{r}_\perp^{(q_1)},\vec{r}_\perp^{(q_2)},s)$$

Motivation

$$W_{//}^{(\lambda)}(s) = \int_{-\infty}^{s} w_{//}(s-s')\lambda(s')\,ds'$$

How to obtain ?

Use developed analytical models for short-range wake function

Singular

Wake potential is not-singular

Use wake potential of much shorter bunch than the bunch one need to model

Not singular

Difficult to obtain

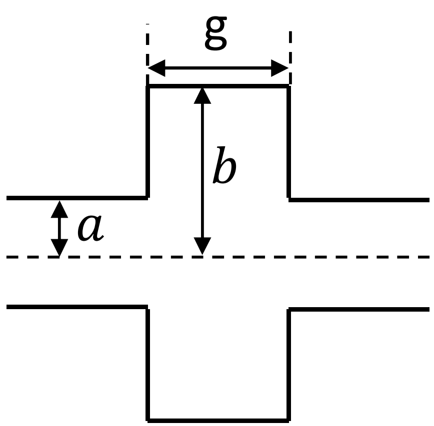
To tabulate the wake function there is need of a model that will solve singularity problem!

Such approach is used, for example, in:

- A. Novokhatsky, M. Timm, and T. Weiland, Single bunch energy spread in the TESLA cryomodule, Tech. Rep. DESY-TESLA-99-16
- T. Weiland, I. Zagorodnov, The Short-Range Transverse Wake Functionfor **TESLA** Accelerating Structure, DESY-TESLA-03-23

More Examples

Pillbox Cavity



Diffraction model $\Longrightarrow$ $\sigma_b < 2\sqrt{(b-a)^2 + (g/2)^2} - g$

$$w_{//}^{(m)}(s) = \frac{Z_0 c}{\sqrt{2}\pi^2 a}\sqrt{\frac{g}{s}}$$

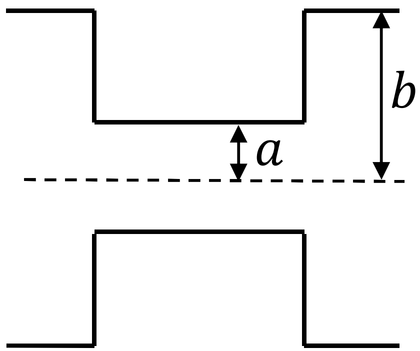
$$w_{//}^{(d)}(s) = \frac{2}{a^2} w_{//}^{(m)}(s)$$

$\Rightarrow$ $\boxed{w_{//}^{(m,d)}(s) = c\frac{\partial}{\partial s} w_{//,m,d}^{(-1)}(s)}$ $\Rightarrow$

$$w_{//,m}^{(-1)}(s) = \frac{Z_0}{\sqrt{2}\pi^2 a}\sqrt{g\, s}$$

$$w_{//,d}^{(-1)}(s) = \frac{2}{a^2} w_{//,m}^{(-1)}(s)$$

Step Collimator & Step-out transition



$$w_{//}^{(m)}(s) = \frac{Z_0 c}{\pi}\ln\left(\frac{b}{a}\right)\delta(s)$$

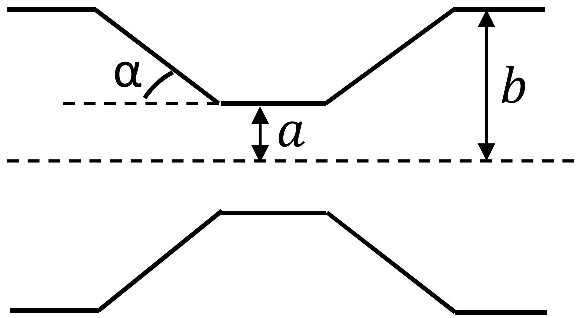
$$w_{//}^{(d)}(s) = \frac{Z_0 c}{\pi}\left(\frac{1}{a^2} - \frac{1}{b^2}\right)\delta(s)$$

$\Rightarrow$ $\boxed{w_{//}^{(m,d)}(s) = c\, R_{m,d}\, \delta(s)}$ $\Rightarrow$

$$R_m = \frac{Z_0}{\pi}\ln\left(\frac{b}{a}\right)$$

$$R_d = \frac{Z_0}{\pi}\left(\frac{1}{a^2} - \frac{1}{b^2}\right)$$

Tapered Collimator



Small tapered angle $\Longrightarrow$ $\rho = tg\alpha\frac{a}{\sigma_b} < 1$

$$w_{//}^{(m)}(s) = c^2\left(\frac{Z_0}{4\pi c}\int_{Geom.} r'dr\right)\frac{\partial}{\partial s}\delta(s)$$

$$w_{//}^{(d)}(s) = c^2\left(2\frac{Z_0}{4\pi c}\int_{Geom.}\frac{r'}{r^2}dr\right)\frac{\partial}{\partial s}\delta(s)$$

$\Rightarrow$ $\boxed{w_{//}^{(m,d)}(s) = c^2\, L_{m,d}\frac{\partial}{\partial s}\delta(s)}$ $\Rightarrow$

$$L_m = \frac{Z_0}{4\pi c}\int_{Geom.} r'dr$$

$$L_d = 2\frac{Z_0}{4\pi c}\int_{Geom.}\frac{r'}{r^2}dr$$

Wake function model

Longitudinal wake function:

$$\Phi(s) = \begin{cases} 1, s > 0 \\ 0, s = 0 \end{cases}$$

$$w_{||}(s) = \underbrace{w_{||}^{(0)}(s) + \frac{\Phi(s)}{C}}_{\text{Regular part}} + \underbrace{R\, c\, \delta(s) - c\frac{\partial}{\partial s}\left[Lc\delta(s) + w_{||}^{(-1)}(s)\right]}_{\text{Singular part}}$$

Wake model:
I. Zagorodnov

$$w(s) = \int Z(\omega)e^{-j\frac{\omega}{c}s}d\omega$$

$$Z(\omega) = Z^{(0)}(\omega) - \frac{1}{j\omega C} + R + j\omega\left[L + Z^{(-1)}(\omega)\right]$$

Transverse wake function

Panowsky-Wenzel $\Rightarrow$ $\vec{w}_{\perp}(\vec{r},\vec{r}_0,s) = -\vec{\nabla}_{\perp,\vec{r}}\int_{-\infty}^{s}\vec{w}_{||}(\vec{r},\vec{r}_0,s')ds' =$

$$= -\vec{\nabla}_{\perp,\vec{r}}\left[\vec{w}_{\perp}^{(0)}(s) + \frac{\Phi(s)}{C}s + c\,R - Lc^2\delta(s) - cw_{||}^{(-1)}(s)\right]$$

Wake Potential for arbitrary bunch shape λ(s)

Longitudinal Wake Potential

$$W_{||}(s) = \int_{-\infty}^{s} w_{||}^{(0)}(s-s')\lambda(s')ds' + \frac{\Phi(s)}{C}\int_{-\infty}^{s}\lambda(s')ds' + R\,c\,\lambda(s) - c^2L\lambda'(s) - c\int_{-\infty}^{s} w_{||}^{(-1)}(s-s')\lambda'(s')ds'$$

$\Rightarrow$ No singularities!

ASTRA Format for Wake Data
(Taylor Method)

Implemented in ASTRA:
M. Dohlus

Taylor Expansion of wake function
(Test particle coordinates – $\{x_t, y_t\}$)

$$w_{//}(x,x_t,y,y_t,s) = w_0(s) + \begin{pmatrix} w_1(s) \\ w_2(s) \\ w_3(s) \\ w_4(s) \end{pmatrix}^T \begin{pmatrix} x \\ y \\ x_t \\ y_t \end{pmatrix} + \begin{pmatrix} x \\ y \\ x_t \\ y_t \end{pmatrix}^T \begin{pmatrix} w_{11}(s) & w_{12}(s) & w_{13}(s) & w_{14}(s) \\ w_{12}(s) & w_{22}(s) & w_{23}(s) & w_{24}(s) \\ w_{13}(s) & w_{23}(s) & w_{33}(s) & w_{34}(s) \\ w_{14}(s) & w_{24}(s) & w_{34}(s) & w_{44}(s) \end{pmatrix} \begin{pmatrix} x \\ y \\ x_t \\ y_t \end{pmatrix}$$

In the special case (monopole+dipole wake)
non-vanishing coefficients are:

$$w_0(s) = w_{//}^{(monopole)}(s)$$
$$w_{13}(s) = w_{24}(s) = 0.5 \cdot w_{//}^{(dipole)}(s)$$

Wake file is in ASCII format and is a "multi-table" describing up to 14 coefficient functions.
Each function is described by following model:

$$w_{ij}(s) = w^{(0)}(s) + \frac{\Phi(s)}{C} + R\, c\, \delta(s) - c\frac{\partial}{\partial s}\left[Lc\delta(s) + w_{||}{}^{(-1)}(s)\right]$$

The format is:

K	0
Table 1	
Table 2	
…	
Table K	

Each Table format

$$\begin{array}{ll} N_0 & N_1 \\ R & L \\ \tilde{C} & i\ or\ i+10j \\ s_1^{(0)} & w^{(0)}\left(s_1^{(0)}\right) \\ \vdots & \vdots \\ s_{N_0}^{(0)} & w^{(0)}\left(s_{N_0}^{(0)}\right) \\ s_1^{(-1)} & w^{(-1)}\left(s_1^{(-1)}\right) \\ \vdots & \vdots \\ s_{N_1}^{(-1)} & w^{(-1)}\left(s_{N_1}^{(-1)}\right) \end{array}$$

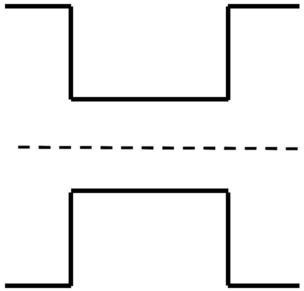
Here $\{i\ or\ i+10\,j\}$ is a number that indicates the wake coefficient indexes $[\boldsymbol{w_i}$ or $\boldsymbol{w_{i+10\,j}}]$

Special cases	
Mon.+Dip.	K=3
Mon.	K=1
Dip.	K=2

ASTRA check - Monopole Wakes

Particle dist. Gauss (x,y,z) – 25 µm
Kin. Energy – 5MeV
Num. particles - 2000
No Divergence

Step Collimator



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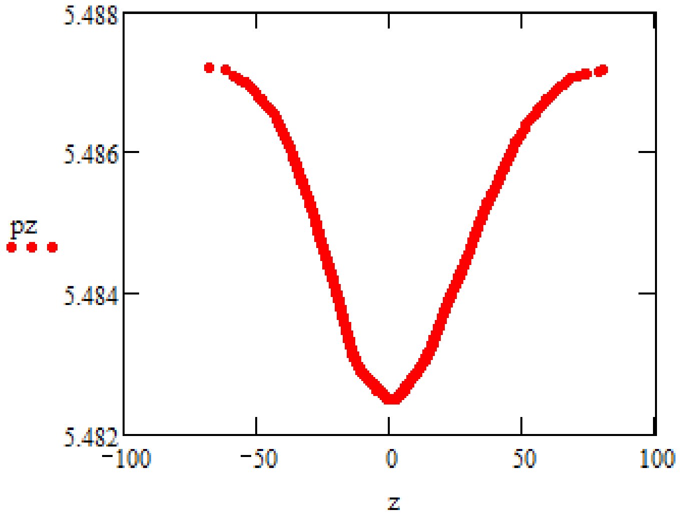
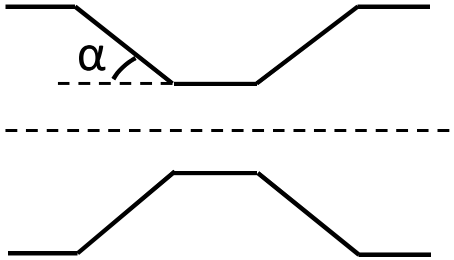


Tapered Collimator



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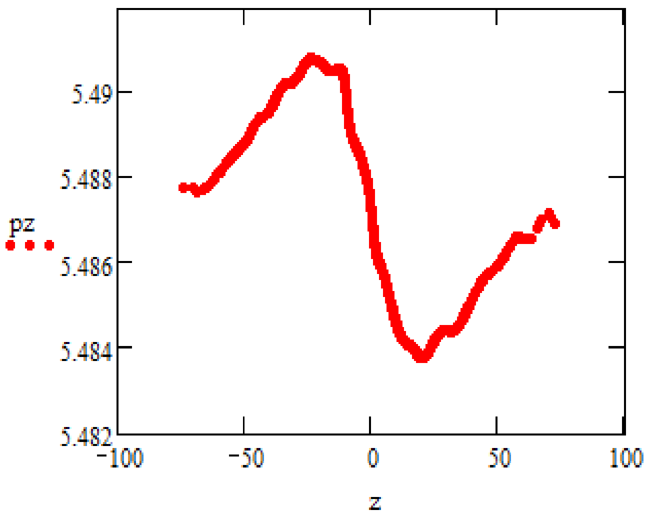
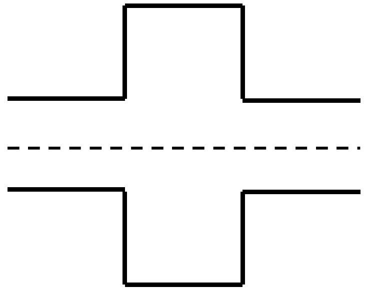
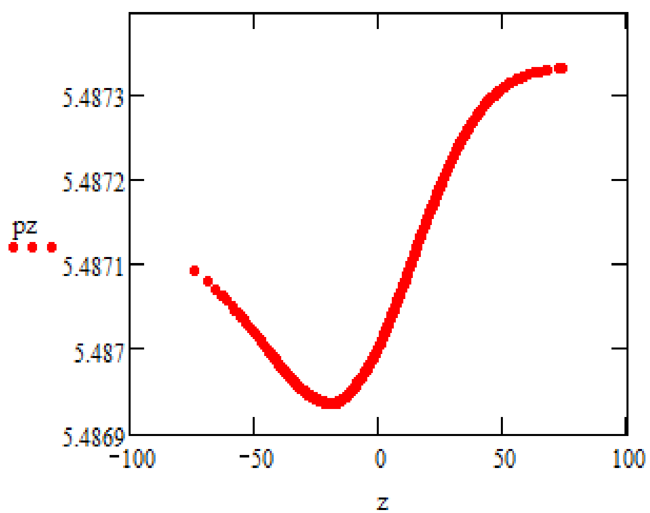


Pillbox Cavity



Scaling= -100



Wake input files used in ASTRA

0	0
10	0
0	0

0	0
0	1E-9
0	0

FLANGE data (positive)
$w^{(-1)}(s)$

Summary

- Wake model to resolve singularities of Green function
- ASTRA with wake field option

Next step

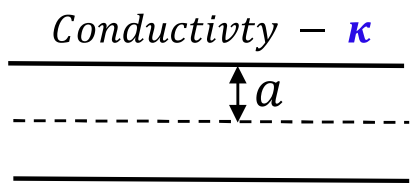
- Include transverse wakes in XFEL & FLASH Impedance Databases.
- Develop a model for fast estimation of emittance growth.

Thank You for Attention

Resistive wakes (per unit length)

Analytical solution ($\kappa = const$)

$$s_0 = \left(\frac{2a^2}{Z_0 \kappa}\right)^{1/3}$$



$$w_{//}^{(m)}(s) = 8\frac{Z_0 c}{\pi a^2}\left[\frac{1}{3}e^{-\frac{s}{s_0}}\cos\left(\sqrt{3}\frac{s}{s_0}\right) - \frac{\sqrt{2}}{\pi}\int_0^{\infty}\frac{x^2}{x^6+8}e^{-x^2\frac{s}{s_0}}dx\right]$$

$$\boxed{w_{//}^{(d)}(s) = \frac{2}{a^2}w_{//}^{(m)}(s)}$$

For dipole wake the relation holds when we add surface roughness, frequency dependent conductivity & metal oxidation.